The Society of Naval Architects of Japan

(昭和 31 年 11 月造船協会秋季講演会に於て講演)

The Forces on a Body moving under the Surface of Water (second report)

By Hajime Maruo, *Kogakushi*, *Member*

Abstract

In this paper, a general formula for the forces acting on a body moving under the surface of water is obtained as an application of the extended Lagally's formula. The problem of the wave resistance experienced by a body moving under the regular sea waves is also discussed.

1. Introduction

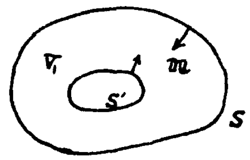

Fig. 1

The formula given in the first report is quite incomplete, because, first, we can apply this to the pure translatory motion but it is inadequate for the motion including rotation. The other reason is that it is applicable only when the body is represented by a special distribution of singularities since the expression involves integrals over the surface of the body. Recently W. Cummins extended Lagally's formula to the general unsteady motion. [1] † Applying this result, we can find more general expression for the forces acting on a body moving under the surface of water with a quite arbitrary manner.

2. Extended Lagally's formula

Consider an arbitrary potential flow outside a moving body whose surface S is realized by some distribution of singularities imagined within S. Assume a control surface S' within S which encloses the singularities, and consider the region V_1 between the surfaces S and S'. (Fig. 1)

When the unit normal to the surface S or S' drawn inward to V_1 is denoted by $\boldsymbol{n}$, the force acting on the surface S is expressd by

$$\int_S p\,\boldsymbol{n}\,dS=\rho\int_S\left[\frac{\partial\phi}{\partial t}-\frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{q})-gz\right]\boldsymbol{n}\,dS, \tag{1}$$

where $\boldsymbol{q}=-\nabla\phi$, the velocity of the fluid. From the last term in the brackets we obtain the statical buoyancy in the direction of z taken vertically upwards.

$$F_b=\rho gV. \tag{2}$$

The surface integral of the first term can be transformed by means of Gauss' theorem. If we assume the surface S or S′ moves or deforms with the normal velocity v_n, we can write.

$$\begin{aligned}\int_S\frac{\partial\phi}{\partial t}\boldsymbol{n}\,dS&=-\int_{V_1}\nabla\left(\frac{\partial\phi}{\partial t}\right)d\tau-\int_{S'}\frac{\partial\phi}{\partial t}\boldsymbol{n}\,dS\\&=\frac{d}{dt}\int_{V_1}\boldsymbol{q}\,d\tau+\int_{S+S'}v_n\boldsymbol{q}\,dS-\int_{S'}\frac{\partial\phi}{\partial t}\boldsymbol{n}\,dS.\end{aligned} \tag{3}$$

Again making use of Gauss′ theorem

$$\frac{d}{dt}\int_{V_1}\boldsymbol{q}\,d\tau=\frac{d}{dt}\int_{S+S'}\phi\,\boldsymbol{n}\,dS, \tag{4}$$

原稿受付. 7 月 11 日

* Assistant Professor of The National University of Yokohama.

† Number in the square brackets means the number of the reference.

NII-Electronic Library Service

The Society of Naval Architects of Japan

If the position vector of a point on the surface is denoted by $\boldsymbol{r}_1$, we can express

$$\boldsymbol{n}=\partial \boldsymbol{r}_1/\partial n, \tag{5}$$

and making use of Green's reciprocal theorem, we have

$$\frac{d}{dt}\int_{S+S'}\phi\,\boldsymbol{n}\,dS=\frac{d}{dt}\int_{S+S'}\phi\frac{\partial \boldsymbol{r}_1}{\partial n}\,dS$$

$$=\frac{d}{dt}\int_{S+S'}\frac{\partial\phi}{\partial n}\boldsymbol{r}_1 dS=-\frac{d}{dt}\int_{S+S'}(\boldsymbol{nq})\boldsymbol{r}_1\,dS. \tag{6}$$

Since on the surface of the body

$$v_n=\boldsymbol{nq}, \tag{7}$$

we can write

$$\int_S v_n\,\boldsymbol{q}\,ds=\int_S(\boldsymbol{nq})\boldsymbol{q}\,ds. \tag{8}$$

If the control surface moves with the velocity $\boldsymbol{v}$, we have

$$v_n=\boldsymbol{nv} \qquad \text{on } S'$$

and the last term of (3) is transformed into

$$\int_{S'}\frac{\partial\phi}{\partial t}\boldsymbol{n}\,ds=\frac{d}{dt}\int_{S'}\phi\,\boldsymbol{n}\,ds-\int_{S'}(\boldsymbol{v}\nabla)\phi.\,\boldsymbol{n}\,ds-\int_{S'}\phi\frac{\partial\boldsymbol{n}}{\partial t}ds$$

$$=\frac{d}{dt}\int_{S'}\phi\,\boldsymbol{n}\,ds+\int_{S'}(\boldsymbol{vq})\,\boldsymbol{n}\,ds-\int_{S'}\phi\frac{\partial\boldsymbol{n}}{\partial t}ds. \tag{9}$$

If the control surface has a velocity of translation $\boldsymbol{V}_1$ and the angular velocity of rotation $\boldsymbol{\omega}_1$ about a point $\boldsymbol{r}_0=\boldsymbol{r}-\boldsymbol{r}_1$, we have

$$\boldsymbol{v}=\boldsymbol{V}_1+\boldsymbol{\omega}_1\times\boldsymbol{r}_1 \text{ and } \partial\boldsymbol{n}/\partial t=\boldsymbol{\omega}_1\times\boldsymbol{n}, \tag{10}$$

and we can readily prove

$$\int_{S'}\left[(\boldsymbol{vn})\boldsymbol{q}-(\boldsymbol{vq})\boldsymbol{n}+\phi\frac{\partial\boldsymbol{n}}{\partial t}\right]ds=0 \tag{11}$$

Collecting the above results and making use of the relation [2]

$$\int_{S+S'}\left[\frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{q})\boldsymbol{n}-(\boldsymbol{nq})\boldsymbol{q}\right]ds=0 \tag{12}$$

we obtain finally

$$\boldsymbol{F}=-\rho\int_{S'}\left[(\boldsymbol{nq})\,\boldsymbol{q}-\frac{1}{2}(\boldsymbol{q}\cdot\boldsymbol{q})\,\boldsymbol{n}\right]ds-\rho\frac{d}{dt}\int_{S'}\left[(\boldsymbol{nq})\boldsymbol{r}_1+\phi\boldsymbol{n}\right]ds-\rho\frac{d}{dt}\int_S v_n\boldsymbol{r}_1 ds \tag{13}$$

When the boundary form of the body is given, the last term of (13) is a known quantity and the force is expressed completely by the integrals over the control surface.

If the singularities generating the boundary surface S are discrete, the control surface is taken as infinitely small spheres having their centres at the position of each singularity and the formula becomes a quite simple form. Consider a body moving with the velocity of translation $\boldsymbol{V}_0$ and the angular velocity $\boldsymbol{\omega}$ about a point $\boldsymbol{r}_0$, and the boundary surface being generated by a distribution of sources of strength σ and doublets of the vector moment $\boldsymbol{\mu}$, which are situated at points denoted by the vector $\boldsymbol{r}_0+\boldsymbol{r}_i$ fixed to the body. Now we write the velocity potential in the form

$$\phi=\phi_i+\phi_e \tag{14}$$

in which ϕ_i is the velocity potential due to singularities within the body. Then the force is expressed by the formula

NII-Electronic Library Service

The Society of Naval Architects of Japan

$$\boldsymbol{F}=-4\pi\rho\int\Big[\{\sigma+(\boldsymbol{\mu}\nabla)\}\boldsymbol{q}_e-\sigma(\boldsymbol{V}_0+\boldsymbol{\omega}\times\boldsymbol{r}_i)+\boldsymbol{r}_i\frac{d\sigma}{dt}+\frac{d\boldsymbol{\mu}}{dt}\Big]d\tau-\Big\{\boldsymbol{r}_g\times\frac{d\boldsymbol{\omega}}{dt}$$

$$-\boldsymbol{\omega}(\boldsymbol{r}_g\boldsymbol{\omega})+\boldsymbol{r}_g(\boldsymbol{\omega}\cdot\boldsymbol{\omega})-\frac{d\boldsymbol{V}_0}{dt}\Big\}\rho V, \tag{15}$$

where

$$\boldsymbol{q}_e=-\nabla\phi_e \ at\ \boldsymbol{r}_i, \tag{16}$$

$\boldsymbol{r}_g$ is the position vector of the centre of gravity of the body relative to the centre of rotation, and V is the volume of the body.

3. A body moving under the surface of water

Now we assume a body moving under the surface of water, and the boundary condition of the rigid surface is satisfied by a surface distribution of sources of strength $\sigma(t)$ and doublets of the vector moment $\boldsymbol{\mu}(t)$ whose position is denoted by $\boldsymbol{r}'=\boldsymbol{r}_0+\boldsymbol{r}_1$. Then the velocity potential is

$$\phi=\int_S\{\sigma-(\boldsymbol{\mu}\nabla)\}\frac{1}{r_1}ds-\frac{1}{2\pi}\int_S ds\int_{-\pi}^{\pi}d\theta\int_0^{\infty}(\sigma+\kappa\boldsymbol{E}\boldsymbol{\mu})\exp[\kappa\tilde{\omega}(t)]d\kappa$$

$$+\frac{1}{\pi}\int_S ds\int_{-\infty}^{t}d\tau\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\{\sigma(\tau)+\kappa\boldsymbol{E}\boldsymbol{\mu}(\tau)\}\exp[\kappa\tilde{\omega}(\tau)]\sin\{\sqrt{g\kappa}\,(t-\tau)\}\sqrt{g\kappa}\,d\kappa \tag{17}$$

where $r_1=|\boldsymbol{r}-\boldsymbol{r}'|$, $\boldsymbol{E}$ is a vector ($i\cos\theta$, $i\sin\theta$, 1), and

$$\tilde{\omega}(t)=\boldsymbol{r}\bar{\boldsymbol{E}}+\boldsymbol{r}'_{(t)}\boldsymbol{E}, \tag{18}$$

When we define the integral

$$H(\kappa,\theta,t)=\int_S\{\sigma(t)+\kappa\boldsymbol{E}\mu(t)\}\exp[\kappa\boldsymbol{r}'(t)\boldsymbol{E}]\,ds, \tag{19}$$

the velocity of the external motion becomes

$$\boldsymbol{q}_e=\frac{1}{2\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}H(\kappa,\theta,t)\exp(\kappa\boldsymbol{r}\bar{\boldsymbol{E}})\kappa\bar{\boldsymbol{E}}\,d\kappa$$

$$-\frac{1}{\pi}\int_{-\infty}^{t}d\tau\int_{-\pi}^{\pi}d\theta\int_0^{\infty}H(\kappa,\theta,\tau)\exp(\kappa\boldsymbol{r}\bar{\boldsymbol{E}})\sin\{\sqrt{g\kappa}\,(t-\tau)\}\sqrt{g\kappa}\,\kappa\bar{\boldsymbol{E}}d\kappa. \tag{20}$$

Then the first part of the force due to the external motion, viz. the effect of the free surface, becomes

$$\boldsymbol{F}_1=-4\pi\rho\int\{\sigma+(\boldsymbol{\mu}\nabla)\}\boldsymbol{q}_e ds=-2\rho\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\int H(\kappa,\theta,t)\,\bar{H}(\kappa,\theta,t)\kappa\bar{\boldsymbol{E}}\,d\kappa$$

$$+4\rho\int_{-\infty}^{t}d\tau\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\bar{H}(\kappa,\theta,t)\,H(\kappa,\theta,\tau)\sin\{\sqrt{g\kappa}\,(t-\tau)\}\sqrt{g\kappa}\,\kappa\bar{\boldsymbol{E}}\,d\kappa. \tag{21}$$

Now we consider a pure translation. According to Green's formula, the density of the surface distribution of sources and doublets on the surface S is given by

$\sigma=(1/4\pi)(\partial\phi/\partial n)$ (22) $\qquad$ $\boldsymbol{\mu}=-(1/4\pi)\phi\boldsymbol{n}$ (23)

If the body is regarded as a very slender symmetric body with respect to the direction of motion, the contribution of the doublet is comparatively small. Hence the usual approximation in such a case is to take only the sources into account. Thus we have

$$\sigma=-(1/4\pi)\boldsymbol{V}_0\boldsymbol{n}, \tag{24}$$

where $\boldsymbol{V}_0$ is the velocity of the body, on accunt of the boundary condition, or we have a uniform space distribution of doublets of the vector moment $(1/4\pi)$ $\boldsymbol{V}_0$ within the body. In this case we can write

NII-Electronic Library Service

The Society of Naval Architects of Japan

$$H(\kappa,\theta,t)=(1/4\pi)(\kappa\boldsymbol{V}_0(t)\boldsymbol{E})\exp[\kappa\boldsymbol{r}_0(t)\boldsymbol{E}]G(\kappa,\theta) \tag{25}$$

where

$$G=\int_V \exp(\kappa\boldsymbol{r}_1\boldsymbol{E})d\tau \tag{26}$$

Then we find

$$\boldsymbol{F}_1=-\frac{\rho}{8\pi^2}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}|G(\kappa,\theta)|^2e^{2\kappa z_c}|\boldsymbol{V}_0\boldsymbol{E}|^2\bar{\boldsymbol{E}}\kappa^3d\kappa$$

$$+\frac{\rho}{4\pi^3}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}|G(\kappa,\theta)|^2\Big[(\boldsymbol{V}_0(t)\bar{\boldsymbol{E}})\Big]\bar{\boldsymbol{E}}\sqrt{g\kappa}\,\kappa^3d\kappa$$

$$\times\int_{-\infty}^{t}\Big[\boldsymbol{V}_0(\tau)\boldsymbol{E}\Big]\exp\Big[\kappa\bar{\boldsymbol{E}}\boldsymbol{r}_0(t)+\kappa\boldsymbol{E}\boldsymbol{r}_0(\tau)\Big]\sin\{\sqrt{g\kappa}\,(t-\tau)\}d\tau, \tag{27}$$

where $\boldsymbol{r}_0$ (t) is the instantaneous position of a point fixed to the body whose z componenent is z_0 The remainder of the force is given by the formula

$$\boldsymbol{F}_2=-4\pi\rho\int_S(\sigma\boldsymbol{V}_0+\boldsymbol{r}_1d\sigma/dt)ds+\rho Vd\boldsymbol{V}_0/dt, \tag{28}$$

and substituting (24) we find

$$\boldsymbol{F}_2=0. \tag{29}$$

Thus the inertia force does not exist and the force is solely due to the effect of the free surface. This result is paradoxical. The reason lies in the fact that the inertia force is induced by the doublet distribution of (23) which has been omitted. In fact, substitution of (23) into the formula (15) shows

$$\boldsymbol{F}_2=\rho\int_S(\partial\phi/\partial n)\boldsymbol{n}ds, \tag{30}$$

that is the inertia force. If we wish to express the fluid motion by means of the source distribution only, we have to adopt a source strength-$(1/4\pi)$ (1+k) $(\boldsymbol{V}_0\boldsymbol{n})$ where k is the inertia coefficient of the body in the direction of motion. Then the resulting wave resistance is augmented by the rate of $(1+k)^2$. This result has been obtained more explicitly by M. Bessho.[3] On the other hand, if we compare the effect of the free surface given by (27) with the result in the previous paper, some difference still exists.

This is also due to the omission of the doublet distribution. Now consider the doublet

$$\boldsymbol{\mu}_e=-(1/4\pi)\phi_e\boldsymbol{n}, \tag{31}$$

which is attributed to the external motion. Since ϕ_e is harmonic within the surface S, Green's theorem gives

$$\int_S\phi_e\frac{\partial}{\partial n}\left(\frac{1}{r_1}\right)ds=\int_S\frac{\partial\phi_e}{\partial n}\,\frac{1}{r_1}ds,$$

and the doublet distribution given by (31) is equivalent to the source distribution of the intensity-$(1/4\pi)$ $\partial\phi_e/\partial\boldsymbol{n}$ on the surface S, or by virtue of Gauss' theorem, it is equivalent to the volume distribution of doublets of the vector moment-$(1/4\pi)$ $\boldsymbol{q}_e$ within the body. Again taking an approximation of (25) we find the force due to the effect of the free surface being

$$\boldsymbol{F}_e=\frac{\rho}{8\pi^2}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\left\{\frac{d\boldsymbol{V}_0}{dt}\boldsymbol{E}+2\kappa V_z(\boldsymbol{V}_0\boldsymbol{E})\right\}|G(\kappa,\theta)|^2e^{2\kappa z_0}\kappa^2\bar{\boldsymbol{E}}d\kappa$$

$$-\frac{\rho}{8\pi^2}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}|G(\kappa,\theta)|^2e^{2\kappa z_0}|\boldsymbol{V}_0\boldsymbol{E}|^2\bar{\boldsymbol{E}}\kappa^3d\kappa$$

NII-Electronic Library Service

The Society of Naval Architects of Japan

$$-\frac{\rho g}{4\pi^2}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}|G(\kappa,\theta)|^2\bar{\boldsymbol{E}}\kappa^3 d\kappa\int_{-\infty}^{t}(\boldsymbol{V}_0(\tau)\boldsymbol{E})\exp[\kappa\bar{\boldsymbol{E}}\boldsymbol{r}_0(t)+\kappa\boldsymbol{E}\boldsymbol{r}_0(\tau)]$$

$$\times\cos\{\sqrt{g\kappa}\,(t-\tau)\}d\tau, \tag{32}$$

where V_z is the z component of $\boldsymbol{V}_0$. This result coincides with the previous one.* If the motion includes rotation, G is also a function of time and the expression for the force becomes some-what complicated.

4. A body moving under waves

The wave resistance of a ship in a seaway has been considered first by T.Hanaoka[4]. He treated the effect of the on-coming waves in a rather approximate way. Now we can apply the formula developed in the previous paragraph to this problem.

When a body is moving in the direction of x with a uniform speed of advance V_0 under waves whose direction of propagation makes an angle α with the axis of x, it makes small oscillations. Then the velocity potential is written in the form,

$$\phi=\phi_w+\phi_i+\phi_e, \tag{33}$$

where ϕ_w is a velocity potential of the incident waves, ϕ_i is that due to singularities generating the surface of the body and ϕ_e is the effect of the free surface. Then we have

$$\phi_w=(gh/\omega)\exp(kz-ikx\cos\alpha-iky\sin\alpha+i\omega t), \tag{34}$$

$$\phi_i=\int[\sigma-(\mu\nabla)]r_1^{-1}ds \tag{35}$$

where

$$k=2\pi/\lambda=\omega^2/g. \tag{36}$$

If we pnt

$$H(\kappa,\theta,t)=\int[\sigma+\kappa(\boldsymbol{\mu}\boldsymbol{E})]\exp[\kappa(\boldsymbol{r}(t)\boldsymbol{E})]ds$$

$$=\exp(i\kappa V_0 t\cos\theta)\{H_0(\kappa,\theta)+e^{i\omega_1 t}H_1(\kappa,\theta)\} \tag{37}$$

substitute in (17) and perform the integration with respect to τ, we find

$$\phi_e=-\frac{1}{2\pi}\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\{H_0(\kappa,\theta+e^{i\omega_1 t}H_1(\kappa,\theta)\}\exp[\kappa(\boldsymbol{r}_1\bar{\boldsymbol{E}})]d\kappa$$

$$-\frac{\kappa_0}{\pi}\int_{-\pi}^{\pi}d\theta\ \mathrm{P.\,V.}\int_0^{\infty}(\kappa\cos^2\theta-\kappa_0)^{-1}H_0(\kappa,\theta)\exp[\kappa(\boldsymbol{r}_1\bar{\boldsymbol{E}})]d\kappa$$

$$+i\kappa_0\left\{\int_{-\pi}^{-\pi/2}-\int_{-\pi/2}^{\pi/2}+\int_{\pi/2}^{\pi}\right\}H_0(\kappa_0\sec^2\theta,\theta)\exp[\kappa_0\sec^2\theta(\boldsymbol{r}_1\bar{\boldsymbol{E}})]\sec^2\theta d\theta$$

$$-\frac{\kappa_0}{\pi}e^{i\omega_1 t}\int_{-\pi}^{\pi}d\theta\ \mathrm{P.\,V.}\int_0^{\infty}\frac{\kappa}{(\kappa\cos\theta+\omega_1/V_0)^2-\kappa_0\kappa}H_1(\kappa,\theta)\exp[\kappa(\boldsymbol{r}_1\bar{\boldsymbol{E}})]d\kappa$$

$$+i\kappa_0 e^{i\omega_1 t}\left\{\int_{-\pi}^{-\pi/2}-\int_{-\pi/2}^{-\theta_1}-\int_{\theta_1}^{\pi/2}+\int_{\pi/2}^{\pi}\right\}\frac{a_1}{a_1-a_2}H_1(a_1,\theta)\exp[a_1(\boldsymbol{r}_1\bar{\boldsymbol{E}})]\sec^2\theta d\theta$$

$$-i\kappa_0 e^{i\omega_1 t}\left\{\int_{-\pi}^{-\theta_1}+\int_{\theta_1}^{\pi}\right\}\frac{a_2}{a_1-a_2}H_1(a_2,\theta)\exp[a_2(\boldsymbol{r}_1\bar{\boldsymbol{E}})]\sec^2\theta d\theta, \tag{38}$$

where

* There are errors of signs in the expressions of the previous report.

NII-Electronic Library Service

The Society of Naval Architects of Japan

$$\left.\begin{array}{l}\omega_1=\omega-kV_0\cos\alpha,\quad \boldsymbol{r}_1=\boldsymbol{r}-\boldsymbol{V}_0t,\quad \kappa_0=g/V_0^2\\ \left.\begin{array}{l}a_1\\ a_2\end{array}\right\}=\dfrac{\kappa_0-2\cos\theta(\omega_1/V_0)\pm\sqrt{\kappa_0^2-4\kappa_0\cos\theta(\omega_1/V_0)}}{2\cos^2\theta}\\ \theta_1=\cos^{-1}(V_0\kappa_0/4\omega_1)\end{array}\right\} \tag{39}$$

Then the mean value of the force during one period is given by

$$\bar{\boldsymbol{F}}=\bar{\boldsymbol{F}}_e+\bar{\boldsymbol{F}}_w, \tag{40}$$

where

$$\bar{\boldsymbol{F}}_e=\text{the mean value of } 4\pi\rho\int[\sigma+(\mu\nabla)]\nabla\phi_e ds$$

$$=-2\rho\int_{-\pi}^{\pi}d\theta\int_0^{\infty}\left\{|H_0(\kappa,\theta)|^2+\frac{1}{2}|H_1(\kappa,\theta)|^2\right\}\kappa\bar{\boldsymbol{E}}d\kappa$$

$$-4\rho\kappa_0\int_{-\pi}^{\pi}d\theta\ P.V.\int_0^{\infty}(\kappa\cos^2\theta-\kappa_0)^{-1}|H_0(\kappa,\theta)|^2\kappa\bar{\boldsymbol{E}}d\kappa$$

$$+4\pi i\rho\kappa_0^2\left\{\int_{-\pi}^{-\pi/2}-\int_{-\pi/2}^{\pi/2}+\int_{\pi/2}^{\pi}\right\}|H_0(\kappa_0\sec^2\theta,\theta)|^2\bar{\boldsymbol{E}}\sec^4\theta d\theta$$

$$-2\rho\kappa_0\int_{-\pi}^{\pi}d\theta\ P.V.\int_0^{\infty}\{(\kappa\cos\theta+\omega_1/V_0)^2-\kappa_0\kappa\}^{-1}|H_1(\kappa,\theta)|^2\kappa^2\bar{\boldsymbol{E}}d\kappa$$

$$+2\pi i\rho\kappa_0\left\{\int_{-\pi}^{-\pi/2}-\int_{-\pi/2}^{-\theta_1}-\int_{\theta_1}^{\pi/2}+\int_{\pi/2}^{\pi}\right\}\frac{a_1^2}{a_1-a_2}|H_1(a_1,\theta)|^2\bar{\boldsymbol{E}}\sec^2\theta d\theta$$

$$-2\pi i\rho\kappa_0\left\{\int_{-\pi}^{-\theta_1}+\int_{\theta_1}^{\pi}\right\}\frac{a_2^2}{a_1-a_2}|H_1(a_2,\theta)|^2\bar{\boldsymbol{E}}\sec^2\theta d\theta \tag{41}$$

$$\bar{\boldsymbol{F}}_w=\text{the mean value of } 4\pi\rho\int[\sigma+(\mu\nabla)]\nabla\phi_w ds$$

$$=2\pi\rho(gh/\omega)H_1(k,\alpha)k\boldsymbol{E}(\alpha). \tag{42}$$

$\boldsymbol{E}(\alpha)$ is the value of $\boldsymbol{E}$ when $\theta=\alpha$, and only the real part is to be taken. The x component of $\bar{\boldsymbol{F}}_e$ gives the usual wave resistance which is the same thing as Hanaoka's findings. On the other hand, the x component of $\bar{\boldsymbol{F}}_w$ gives the resistance increase due to waves indicated by Havelock. [5] When the translatory oscillation is expressed by the vector $\boldsymbol{h}\exp i\omega_1 t$ and the rotational oscillation by the vector $\boldsymbol{\theta}\exp i\omega_1 t$, the velocity on the surface S becomes

$$\boldsymbol{v}=\boldsymbol{V}_0+i\omega_1 e^{i\omega_1 t}(\boldsymbol{h}+\boldsymbol{\theta}\times\boldsymbol{r}_1), \tag{43}$$

where $\boldsymbol{V}_0$ means the uniform velocity in x direction. If we take the approximate source distribution due to the motion of the body

$$\sigma=-(1/4\pi)\boldsymbol{vn}, \tag{44}$$

on the surface S (the slender body assumption), we get

$$H_1(k,\alpha)=-(1/4\pi)\{kV_0\boldsymbol{E}(\alpha)+i\omega_1\}\{\boldsymbol{h}\int_S\exp[k\boldsymbol{r}_1\boldsymbol{E}(\alpha)]\boldsymbol{n}\,ds$$

$$+\boldsymbol{\theta}\int_S\exp[k\boldsymbol{r}_1\boldsymbol{E}(\alpha)](\boldsymbol{r}_1\times\boldsymbol{n})ds\} \tag{45}$$

On the other hand, the force evaluated by the undisturbed wave pressure (Froude Kriloff hypothesis) may be written as

$$i\rho gh e^{i\omega_1 t}\int_S\exp[k\boldsymbol{r}_1\bar{\boldsymbol{E}}(\alpha)]\boldsymbol{n}\,ds=e^{i\omega_1 t}\boldsymbol{f}_w, \tag{46}$$

NII-Electronic Library Service

The Society of Naval Architects of Japan

and the moment is expressed by

$$i\rho g h e^{i\omega_1 t}\int_S \exp[k r_1 \bar{E}(\alpha)](r_1 \times n)ds = e^{i\omega_1 t} m_w. \qquad (47)$$

Then the mean force becomes

$$\bar{F}_w = \text{real part of } \frac{1}{2} k E(\alpha)(h\bar{f_w} + \theta\overline{m_w}), \qquad (48)$$

since

$$kV_0 E(\alpha) + i\omega_1 = i\omega.$$

This is equivalent to Havelock's result.

5. Conclusion

The author has obtained a general formula for the force acting on the body moving under the surface of water, taking the surface disturbance of water into consideration,

Vast applications of this formula are permitted if a suitable distribution of singularities is found so as to satisfy the boundary condition on the surface of the body.

The problem of the wave resistance of a ship in a sea way is an application of practical importance and will be discussed in future.

This work is a part of the researches about the propulsive performance of a ship in a sea way subsidized by the Ministry of Education.

References

[1] W. Cummins, The forces and moments acting on a body moving in an arbitrary potential stream. T. M. B. Rep. No. 780 (1953)

[2] L. M. Milne-Thomson, Theoretical hydrodynamics, 3 rd ed. (1955) p.84

[3] M. Bessho, On the wave resistance theory of a submerged body. J. Z. K. vol.99 (1956)

[4] T. Hanaoka, The motion of a ship among waves and the theory of wave resistance. J. Z. K. vol.98 (1956)

[5] T. H. Havelock, Notes on the theory oy heaving and pitching. T. I. N. A. vol. 87 (1945)

[6] H. Maruo, The forces on a body moving under the surface of water· J. Z. K. vol. 97 (1955)

NII-Electronic Library Service