船體振動の減衰力

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Abstract.

Damping Forces in Vibrations of a Ship.

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Since in the case of a ship the surrounding water is seemingly so deformable that the dissipation of vibrational energy of that ship in the form of radiating waves is hardly possible, the reason why ship vibrations shall actually be damped not slightly, is as a matter of fact difficult.

There are at least four possible causes of damping in ship vibrations, namely, (1) the water friction, (2) generation of pressure waves, (3) generation of surface waves, and (4) the structural damping force. Air damping is obviously out of consideration.

The solution of the problem was obtained mathematically for the case of flexural vibrations under an unbalanced force, its result being compared with reliable experimental data, so as to determine what kind of force is the most plausible one for the damping of the ship vibrations.

In the case of a ship of shallow draught the generation of surface waves is the primary cause, whereas in the case of a ship with full load and deep draught the effective structural damping resistance becomes the cause under consideration.

The details of the problem as well as its theoretical treatment will be described in Appendix towards the end of this paper.

陸上にある構造物や機關の振動が自由振動に於ても强制振動に於ても或る減衰された形となつて現れるのは、多くの場合に振動勢力が波動の形となつて其の周圍の部分へ流れることが多いからである。自分自身の中や又は一度外部へ波動してから振動勢力が熱の形に變ることも勿論相當の位地を占めて居るけれども、上述の如く外部へ傳播する事柄は見逃し得ないものである。

此の外部へ傳播する勢力量は外部の物體の效果的剛度と自身の效果的剛度が近い場合に最も甚しいのである。しかるに鋼の臺の上に載せられたゴムの振動や、又は同じ鋼にしても平面の鋼の臺の上に立つてゐる鋼の線では波動勢力の傳播は極く僅かなものである。

傳播の少い物は振動體の效果的剛度が外部の夫れよりも著しく高い場合にも矢張りあり得るのであつて、船體の振動は其の著しい例である。飛行機は尙更この仲間に入りそうなものであるけれども事實はそう簡單ではない。其の場合には前進速度が高い爲に流體力學的の減衰力が强く働いて、恰も周圍の流體の剛度が相當高いのと同じ結果になるからである。

偖て船體の場合に前記の如く周圍の剛度に當るものが無いかと云へば必ずしもそうでない事は當然である。卽ち船體が振動する時に水の壓力波や表面波として傳播する部分がある。其の外、水の粘性からの抵抗や水の中に渦流を起して所謂渦粘性の形の抵抗があり、尙、船體の構造中の粘性、特に材料同志の接觸面や積荷等の中へ振動勢力として流れる所謂廣い意味の固體粘性があるのである。空氣抵抗は始めから大して問題にならぬものと考へられる。

此の前に報告した機關室の位置と船體の振動との關係の場合には、不衡力のある機關の組織が船體の剛度に比して比較的に軟い彈性的性質があると云ふ考を以て問題を解いた爲に、昔から普通云はれて居る事柄と異なる結果まで出たのであるが、然し考へ方によつては不衡力が直接船體に働くと見た方が寧ろ實際に近い樣に思はれたので、玆には矢張り舊來の考へ方の樣に問題を取扱ふことにしたのである。但し强い減衰力を入れて考へると、舊來の行き方でも結果として前回の論文と同じやうなことが出るものである。此の論文では減衰力が夫程大きくない場合しか出て來ないのである。

減衰力に就いては其の大さを假定するのでなく、前から判つて居る力學的性質から何の減衰力が効くかが自然に出るものである。又、此の論文では簡單の爲に不衡力が働き且つ船の屈曲振動のみ起る場合の數理を作つた。數値的計算には船尾に不衡力が働いゝ其の點の變位のみ出したのである。之は推進機に不衡力がある時の船尾の振幅に當る譯である。種々異なる物理的減衰力を入れて此樣な計算を試みた上、其の何れの場合が今迄に判つて居る實船の振動驗測結果に能く當嵌まるかを比較して見て、其の一番近い場合が最も可能な振動減衰力であることが知られるのであり、著しく喰ひ違ひのある場合は問題外とすることが出來るのである。

最初に水の粘性を取つて見る。而かも普通の粘性だけでは問題にならぬから、普通の粘性の 1000 倍位もある渦粘性を取つて計算したけれども、强制共振に於ける振幅が共振でない振動中の最大振幅と比較して 1000 倍にも大きくなつたのである。渦粘性を 100,000 倍にしても矢張り振幅比は 100 倍位にもなるのである。之は實際の結果と餘りにも差異がある。

次に壓力波を取つて考へると、船體振動の振動數位では勢力の波動逸散が極めて少なく、共振の振幅と然らざる場合の夫れとの比が矢張り 1000 倍近くになる。然し船の振動數が著しく增加出來れば此の比は多少小さくなるけれども、普通の場合にはあり得ぬことである。

水の表面波を取つて見ると、吃水が比較的に少い場合には逸散波の勢力が著しく大きくなり、共振に於ける振幅と然らざる場合の夫れとの比が普通の場合でも 50 倍位であり、特別に淺い吃水では 10 倍位にもなり得るのである。

船體の構造中で固體粘性の爲の減衰は考へ方によつて差異がある。物理實驗室で出したやうな粘性を用ひること、換言すれば船體に繼目がなく、且つ內部の載荷の爲に勢力が熱として喰はれることが無いと假定すると、共振振幅と然らざる場合の最大振幅との比は 200 近くにもなるものである。之に反して材料に繼目があつて其の運動摩擦力が働いたり、載荷の爲に勢力が熱に變つて生ずるらしく思

はれる減衰性の數値を他の複合振動體の實驗から推定して計算に當嵌めると、上記の比は 20 位になつてしまふのである。靜力學的に大きな摩擦力も動力學的には小さくなり、只振動減衰力として働く様になるものである。吃水の深い場合にはこの條件を滿足して居る様に思はれる。

以上の如く船體の振動が適當に減衰される機構は、或場合即ち吃水の淺い場合には、表面波の成生に依つて爲され、他の場合即ち吃水の深い場合には廣い意味での構造的減衰力に依る事が判る。

此の理由から船體の振動を出來るだけ少くするには、構造的減衰力には限りがある。從つて殘るものは振動による表面波が出來るだけ成生するやうにすれば良い譯である。勿論不衡力を減じたり、船體の剛度を高くすることは當然であるが、然しながら剛度を高くした爲に其の共振週期に出る振幅は剛度が低い時の夫れよりも寧ろ大きくなつて惡い結果を與へる傾向があるものである。表面波が出來るだけ强くなるにはビルヂ•キールや元良式安定板の如きものを水面近くに置くのも一つの考へ方である。Inglis 式の考は附録にもある様に望みが少ない。

此の研究は單に數理的に船體振動の一つの問題を取扱つただけであるが、我々の目的は平賀教授の御勸めもあり、寧ろ實船の振動を測定して其の性質を直接知るにあるのであつて、只今其の準備中である。夫れの豫備研究としても減衰力の場合には模型實驗は寧ろ不適當かと思はれ、特に數理的に研究したのである。實は我々は絶えず船體の問題に興味を持ちながら最近 11 年間他方面の研究を爲すべき境遇にあり、此の様な研究を滿足に爲す餘裕が無かつたのである。然るに今回、近藤記念海事財團の御援助によつて我々の希望に向つて相當滿足が出來るまで研究を進める機會を得たのである。玆に同財團及び其の役員諸氏に厚く御禮を申述べる次第である。只目下重工業が餘り盛んな爲に此様な研究には故障が多く、仕事の進捗がはかばかしくないのは甚だ申譯のない次第である。

Appendix. Theory of and Conclusion to the Problem of the Damping Forces in Vibrations of a Ship.

1. *Introduction.*

One of the interesting problems of ship vibrations is that by what forces those vibrations shall be damped. In the case of a structure or a machine on land the damping is chiefly due to the radiation of vibrational energy from its boundary, so that the vibration amplitude would not assume infinitely large value even should the inner damping be very small. In the case of a ship, nevertheless, the surrounding medium, namely the water, is enormously deformable compared with the ship's body, the outward radiation of vibrational energy appears to be hardly possible. On the other hand, the result of observation of ship's vibrations shows that the vibration amplitudes under resonance condition are not specially large in the ship's case only. From the result of experiments made by Schlick[1] on board Deutschland or any other one since then made, the vibration amplitudes under resonance condition are almost twenty or thirty times those of non-resonance condition in the majority of cases. It appears furthermore that under a special kind of vibration excitement the value of the resonance amplitude is not likely to exceed a certain limit.[2] The apparent difference thus revealed between actual resonance amplitudes and theoretically probable ones is the reason of why the investigation of the damping forces is, at all events, of special interest.

There are at least four possible damping forces in ship's vibrations, namely (1) the water friction, (2) generation of pressure waves, (3) generation of surface waves, and (4) the structural damping force. Damping due to the air resistance is obviously out of consideration. A close examination of every case shows that the generation of surface waves as well as the structural damping force are the main sources of damping. In a ship of shallow draught the damping due to the generation of surface waves is most pronounced, whereas for the damping of a ship with full load and deep draught the structural resistance becomes the principal force. Neither the water friction nor generation of pressure waves shall be a sensible source of damping.

To ascertain the nature of damping consisting of different kinds of forces it is rather advisable to use mathematical methods than to conduct model experiments. The result of experiments made on an actual ship, on the other hand, is available to confirm the conclusion obtained theoretically or to obtain an insight into such vibration phenomena that cannot be examined mathematically. With this idea in mind, the present paper has been written as a preliminary report of our new investigation regarding the vibrations of a ship. For simplicity, the case which we are here discussing is the flexural vibrations of a ship under an unbalanced force.

II. *General Solution of Flexural Vibrations of a Ship under Unbalanced Force.*

The simplest way of mathematical calculation is to assume the ship as a free-free uniform bar and it still gives a general idea of the problem. Let the position of an unbalanced force be l_1 from the one end of a ship, its full length being $l=$

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(1) O. Schlick, "On Some Experiments made on Board the Atlantic Liner 'Deutschland' during her Trial Trip in June, 1900", *T. I. N. A.*, **43** (1901), 48-66.

(2) S. Yokota, "On Vibration of Steamers", *Journ. Coll. Eng., Tokyo Imp. Univ.*, **5** (1910), 1-24.

l_1+l_2. Taking the origin of x at the position of unbalanced force, the equation for the deflection of the vibratory motion $y=Ye^{i\sigma t}$ generally satisfies the form such that

$$\frac{d^4Y}{dx^4}=m^4Y, \qquad (1)$$

where m^4 is complex, the solution thus being

$$y=\{A\cosh mx+B\cos mx+C\sinh mx+D\sin mx\}e^{i\sigma t}. \qquad (2)$$

The boundary conditions are as follows:

$$\left.\begin{array}{ll} x=l_1; & y''=0, \quad y'''=0, \\ x=-l_2; & y''=0, \quad y'''=0, \\ x=0; & y_{x>0}=y_{x<0}, \quad y'_{x>0}=y'_{x<0}, \\ & y''_{x>0}=y''_{x<0}, \quad y'''_{x>0}=y'''_{x<0}+\dfrac{Fe^{i\sigma t}}{EI}, \end{array}\right\} \qquad (3)$$

where $Fe^{i\sigma t}$, E, I are unbalanced force, Yung's modulus, and moment of inertia of section respectively. In the previous paper[(3)] we assumed that the unbalanced force acts on the ship somewhat indirectly, namely through elastic deformation of engine parts including their beds, resulting quite uncommon facts—a condition which is possible to exist in the case of a large ship with an elastically connected large unbalance. More strictly speaking, the spring force between the unbalance and the ship was assumed to be very weak so that there was a certain phase difference between the movement of unbalance and that of the ship at the position of the same unbalance. But, seeing that such a condition is rather improbable in the actual rigid condition of the engine or propeller fixing, we shall now suppose that the unbalance immediately imparts its force on the ship's hull as will be shown by the final condition of (3).

Substituting (1) in (3) we obtain

$$y=\frac{Fe^{i\sigma t}}{4EIm^3}\frac{P\cosh mx+Q\cos mx+R\sinh mx+S\sin mx}{\Phi}, \qquad (4)$$

where

$$\Phi=\cosh m(l_1+l_2)\cos m(l_1+l_2)-1, \qquad (5)$$

$$\begin{aligned}P=&-\sinh m(l_1+l_2)\cos m(l_1+l_2)+\cosh m(l_1-l_2)\sin m(l_1+l_2)\\&+\cosh ml_1\sin ml_1-\sinh ml_1\cos ml_1+\cosh ml_2\sin ml_2-\sinh ml_2\cos ml_2,\end{aligned} \qquad (6)$$

$$\begin{aligned}Q=&\cosh m(l_1+l_2)\sin m(l_1+l_2)-\sinh m(l_1+l_2)\cos m(l_1-l_2)\\&+\cosh ml_1\sin ml_1-\sinh ml_1\cos ml_1+\cosh ml_2\sin ml_2-\sinh ml_2\cos ml_2,\end{aligned} \qquad (7)$$

(3) K. Sezawa, "The Effect of Difference in the Position of Engine Room on Ship Vibrations", *Journ. Soc. Nav. Arch.*, **57** (1935), 103-113.

$$\left.\begin{aligned} R_{x>0}&=\cosh m(l_1+l_2)\cos m(l_1+l_2)-\sinh m(l_1-l_2)\sin m(l_1+l_2)\\ &\quad+\cosh ml_1\cos ml_1-\sinh ml_1\sin ml_1-\cosh ml_2\cos ml_2+\sinh ml_2\sin ml_2-1,\\ R_{x<0}&=-\cosh m(l_1+l_2)\cos m(l_1+l_2)-\sinh m(l_1-l_2)\sin m(l_1+l_2)\\ &\quad+\cosh ml_1\cos ml_1-\sinh ml_1\sin ml_1-\cosh ml_2\cos ml_2+\sinh ml_2\sin ml_2+1, \end{aligned}\right\}\quad(8)$$

$$\left.\begin{aligned} S_{x>0}&=-\cosh m(l_1+l_2)\cos m(l_1+l_2)-\sinh m(l_1+l_2)\sin m(l_1-l_2)\\ &\quad-\cosh ml_1\cos ml_1-\sinh ml_1\sin ml_1+\cosh ml_2\cos ml_2+\sinh ml_2\sin ml_2+1,\\ S_{x<0}&=\cosh m(l_1+l_2)\cos m(l_1+l_2)-\sinh m(l_1+l_2)\sin m(l_1-l_2)\\ &\quad-\cosh ml_1\cos ml_1-\sinh ml_1\sin ml_1+\cosh ml_2\cos ml_2+\sinh ml_2\sin ml_2-1. \end{aligned}\right\}\quad(9)$$

These expressions are the same as those shown by equations (10), (12), (13), (14), (15) in our previous paper.[4]

Under a somewhat different criterion we obtained previously

$$y=\frac{Fe^{i\omega t}}{4EIm^3}\frac{P\cosh mx+Q\cos mx+R\sinh mx+S\sin mx}{\sqrt{\Phi^2+(M_1E_1\theta p^2/4E^2I^2m^6)\Psi^2}},\quad\ldots\ldots\ldots(4')$$

where

$$\Psi=-\sinh m(l_1+l_2)\cos ml_1\cos ml_2+\sin m(l_1+l_2)\cosh ml_1\cosh ml_2$$
$$+\cosh ml_1\sin ml_1-\sinh ml_1\cos ml_1+\cosh ml_2\sin ml_2-\sinh ml_2\cos ml_2.$$

If we were to put $M_1E_1\theta p^2/4E^2I\,m^6=0$ in the equation (4′) we obtain the solution shown in (4).

Since in the case of damped forced vibrations m^4 is a complex quantity, Φ, P, Q, R, S become also complex. Let

$$m^4=\mu-i\nu\equiv(\gamma-i\delta)^4,\quad\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

then Φ, P, Q, R, S assume the forms

$$\left.\begin{aligned}\Phi=\xi-i\eta,\quad P=P_1-iP_2,\quad Q=Q_1-iQ_2,\\ R=R_1-iR_2,\quad S=S_1-iS_2,\end{aligned}\right\}\quad\ldots\ldots\ldots(11)$$

where ξ, η, P_1, P_2, ... are as follows:

$$\xi=\cosh\gamma l\cosh\delta l\cos\gamma l\cos\delta l+\sinh\gamma l\sinh\delta l\sin\gamma l\sin\delta l-1,\quad\ldots\ldots(12)$$

$$\eta=\sinh\gamma l\cosh\delta l\cos\gamma l\sin\delta l-\cosh\gamma l\sinh\delta l\sin\gamma l\cos\delta l,\quad\ldots\ldots(13)$$

$$\begin{aligned}P_1&=-\sinh\gamma l\cosh\delta l\cos\gamma l\cos\delta l-\cosh\gamma l\sinh\delta l\sin\gamma l\sin\delta l\\ &\quad+\cosh\gamma(l_1-l_2)\cosh\delta l\sin\gamma l\cos\delta(l_1-l_2)\\ &\quad-\sinh\gamma(l_1-l_2)\sinh\delta l\cos\gamma l\sin\delta(l_1-l_2)\\ &\quad+G_1,\quad\ldots\ldots\ldots\ldots\ldots\ldots(14)\end{aligned}$$

$$\begin{aligned}P_2&=-\cosh\gamma l\cosh\delta l\cos\gamma l\sin\delta l+\sinh\gamma l\sinh\delta l\sin\gamma l\cos\delta l\\ &\quad+\sinh\gamma(l_1-l_2)\cosh\delta l\sin\gamma l\sin\delta(l_1-l_2)\end{aligned}$$

(4) K. Sezawa, *loc. cit.*[3]

$$+\cosh\gamma(l_1-l_2)\sinh\delta l\cos\gamma l\cos\delta(l_1-l_2)$$
$$+G_2, \qquad (15)$$

$$Q_1=\cosh\gamma l\cosh\delta l\sin\gamma l\cos\delta l-\sinh\gamma l\sinh\delta l\cos\gamma l\sin\delta l$$
$$-\sinh\gamma l\cosh\delta(l_1-l_2)\cos\gamma(l_1-l_2)\cos\delta l$$
$$-\cosh\gamma l\sinh\delta(l_1-l_2)\sin\gamma(l_1-l_2)\sin\delta l$$
$$+G_1, \qquad (16)$$

$$Q_2=\sinh\gamma l\cosh\delta l\sin\gamma l\sin\delta l+\cosh\gamma l\sinh\delta l\cos\gamma l\cos\delta l$$
$$-\cosh\gamma l\cosh\delta(l_1-l_2)\cos\gamma(l_1-l_2)\sin\delta l$$
$$+\sinh\gamma l\sinh\delta(l_1-l_2)\sin\gamma(l_1-l_2)\cos\delta l$$
$$+G_2, \qquad (17)$$

$$\left.\begin{matrix}R_1\,(x>0)\\R_1\,(x<0)\end{matrix}\right\}=-\sinh\gamma(l_1-l_2)\cosh\delta l\sin\gamma l\cos\delta(l_1-l_2)$$
$$+\cosh\gamma(l_1-l_2)\sinh\delta l\cos\gamma l\sin\delta(l_1-l_2)$$
$$+\cosh\gamma l_1\cosh\delta l_1\cos\gamma l_1\cos\delta l_1+\sinh\gamma l_1\sinh\delta l_1\sin\gamma l_1\sin\delta l_1$$
$$-\sinh\gamma l_1\cosh\delta l_1\sin\gamma l_1\cos\delta l_1+\cosh\gamma l_1\sinh\delta l_1\cos\gamma l_1\sin\delta l_1$$
$$-\cosh\gamma l_2\cosh\delta l_2\cos\gamma l_2\cos\delta l_2-\sinh\gamma l_2\sinh\delta l_2\sin\gamma l_2\sin\delta l_2$$
$$+\sinh\gamma l_2\cosh\delta l_2\sin\gamma l_2\cos\delta l_2-\cosh\gamma l_2\sinh\delta l_2\cos\gamma l_2\sin\delta l_2$$
$$\pm\xi, \qquad (18)$$

$$\left.\begin{matrix}R_2\,(x>0)\\R_2\,(x<0)\end{matrix}\right\}=-\cosh\gamma(l_1-l_2)\cosh\delta l\sin\gamma l\sin\delta(l_1-l_2)$$
$$-\sinh\gamma(l_1-l_2)\sinh\delta l\cos\gamma l\cos\delta(l_1-l_2)$$
$$+\sinh\gamma l_1\cosh\delta l_1\cos\gamma l\sin\delta l_1-\cosh\gamma l_1\sinh\delta l_1\sin\gamma l_1\cos\delta l_1$$
$$-\cosh\gamma l_1\cosh\delta l_1\sin\gamma l_1\sin\delta l_1-\sinh\gamma l_1\sinh\delta l_1\cos\gamma l_1\cos\delta l_1$$
$$-\sinh\gamma l_2\cosh\delta l_2\cos\gamma l_2\sin\delta l_2+\cosh\gamma l_2\sinh\delta l_2\sin\gamma l_2\cos\delta l_2$$
$$+\cosh\gamma l_2\cosh\delta l_2\sin\gamma l_2\sin\delta l_2+\sinh\gamma l_2\sinh\delta l_2\cos\gamma l_2\cos\delta l_2$$
$$\pm\eta, \qquad (19)$$

$$\left.\begin{matrix}S_1\,(x>0)\\S_1\,(x<0)\end{matrix}\right\}=-\sinh\gamma l\cosh\delta(l_1-l_2)\sin\gamma(l_1-l_2)\cos\delta l$$
$$+\cosh\gamma l\sinh\delta(l_1-l_2)\cos\gamma(l_1-l_2)\sin\delta l$$
$$-\cosh\gamma l_1\cosh\delta l_1\cos\gamma l_1\cos\delta l_1-\sinh\gamma l_1\sinh\delta l_1\sin\gamma l_1\sin\delta l_1$$
$$-\sinh\gamma l_1\cosh\delta l_1\sin\gamma l_1\cos\delta l_1+\cosh\gamma l_1\sinh\delta l_1\cos\gamma l_1\sin\delta l_1$$
$$+\cosh\gamma l_2\cosh\delta l_2\cos\gamma l_2\cos\delta l_2+\sinh\gamma l_2\sinh\delta l_2\sin\gamma l_2\sin\delta l_2$$
$$+\sinh\gamma l_2\cosh\delta l_2\sin\gamma l_2\cos\delta l_2-\cosh\gamma l_2\sinh\delta l_2\cos\gamma l_2\sin\delta l_2$$
$$\mp\xi, \qquad (20)$$

$$\left.\begin{matrix} S_2(x>0) \\ S_2(x<0) \end{matrix}\right\} = -\cosh\gamma l\cosh\delta(l_1-l_2)\sin\gamma(l_1-l_2)\sin\delta l$$

$$-\sinh\gamma l\sinh\delta(l_1-l_2)\cos\gamma(l_1-l_2)\cos\delta l$$

$$-\sinh\gamma l_1\cosh\delta l_1\cos\gamma l_1\sin\delta l_1+\cosh\gamma l_1\sinh\delta l_1\sin\gamma l_1\cos\delta l_1$$

$$-\cosh\gamma l_1\cosh\delta l_1\sin\gamma l_1\sin\delta l_1-\sinh\gamma l_1\sinh\delta l_1\cos\gamma l_1\cos\delta l_1$$

$$+\sinh\gamma l_2\cosh\delta l_2\cos\gamma l_2\sin\delta l_2-\cosh\gamma l_2\sinh\delta l_2\sin\gamma l_2\cos\delta l_2$$

$$+\cosh\gamma l_2\cosh\delta l_2\sin\gamma l_2\sin\delta l_2+\sinh\gamma l_2\sinh\delta l_2\cos\gamma l_2\cos\delta l_2$$

$$\mp\eta, \qquad\qquad (21)$$

where ξ, η are given in (12), (13), and

$$\left.\begin{aligned} G_1 &= \cosh\gamma l_1\cosh\delta l_1\sin\gamma l_1\cos\delta l_1-\sinh\gamma l_1\sinh\delta l_1\cos\gamma l_1\sin\delta l_1 \\ &\quad -\sinh\gamma l_1\cosh\delta l_1\cos\gamma l_1\cos\delta l_1-\cosh\gamma l_1\sinh\delta l_1\sin\gamma l_1\sin\delta l_1 \\ &\quad +\cosh\gamma l_2\cosh\delta l_2\sin\gamma l_2\cos\delta l_2-\sinh\gamma l_2\sinh\delta l_2\cos\gamma l_2\sin\delta l_2 \\ &\quad -\sinh\gamma l_2\cosh\delta l_2\cos\gamma l_2\cos\delta l_2-\cosh\gamma l_2\sinh\delta l_2\sin\gamma l_2\sin\delta l_2, \\ G_2 &= \sinh\gamma l_1\cosh\delta l_1\sin\gamma l_1\sin\delta l_1+\cosh\gamma l_1\sinh\delta l_1\cos\gamma l_1\cos\delta l_1 \\ &\quad -\cosh\gamma l_1\cosh\delta l_1\cos\gamma l_1\sin\delta l_1+\sinh\gamma l_1\sinh\delta l_1\sin\gamma l_1\cos\delta l_1 \\ &\quad +\sinh\gamma l_2\cosh\delta l_2\sin\gamma l_2\sin\delta l_2+\cosh\gamma l_2\sinh\delta l_2\cos\gamma l_2\cos\delta l_2 \\ &\quad -\cosh\gamma l_2\cosh\delta l_2\cos\gamma l_2\sin\delta l_2+\sinh\gamma l_2\sinh\delta l_2\sin\gamma l_2\cos\delta l_2. \end{aligned}\right\} \qquad (22)$$

The solution for displacement thus becomes

$$y=\frac{F}{4EI}\frac{(X^2+Y^2)^{1/2}}{(\mu^2+\nu^2)^{3/8}(\xi^2+\eta^2)^{1/2}}e^{i\left(\sigma t+\frac{3}{4}\tan^{-1}\frac{\nu}{\mu}+\tan^{-1}\frac{\eta}{\xi}-\tan^{-1}\frac{Y}{X}\right)}, \qquad (23)$$

where

$$\left.\begin{aligned} X &= (P_1\cos\delta x-R_2\sin\delta x)\cosh\gamma x+(R_1\cos\delta x-P_2\sin\delta x)\sinh\gamma x \\ &\quad +(Q_1\cosh\delta x-S_2\sinh\delta x)\cos\gamma x+(S_1\cosh\delta x+Q_2\sinh\delta x)\sin\gamma x, \\ Y &= (R_1\sin\delta x+P_2\cos\delta x)\cosh\gamma x+(P_1\sin\delta x+R_2\cos\delta x)\sinh\gamma x \\ &\quad +(S_1\sinh\delta x+Q_2\cosh\delta x)\cos\gamma x+(-Q_1\sinh\delta x+S_2\cosh\delta x)\sin\gamma x. \end{aligned}\right\} \qquad (24)$$

When $\delta\to 0$, (24) reduces to the equation (4).

If we put $x=0$ in the special case $l_1=0$, $l_2=l$, we have

$$\left.\begin{aligned} X &= P_1+Q_1 \\ &= 4\{-\sinh\gamma l\cosh\delta l\cos\gamma l\cos\delta l-\cosh\gamma l\sinh\delta l\sin\gamma l\sin\delta l \\ &\qquad +\cosh\gamma l\cosh\delta l\sin\gamma l\cos\delta l-\sinh\gamma l\sinh\delta l\cos\gamma l\sin\delta l\}, \\ Y &= P_2+Q_2 \\ &= 4\{-\cosh\gamma l\cosh\delta l\cos\gamma l\sin\delta l+\sinh\gamma l\sinh\delta l\sin\gamma l\cos\delta l \\ &\qquad +\sinh\gamma l\cosh\delta l\sin\gamma l\sin\delta l+\cosh\gamma l\sinh\delta l\cos\gamma l\cos\delta l\}. \end{aligned}\right\} \qquad (25)$$

In every numerical example of damped vibrations we shall assume that the vibrating force is the propeller unbalance at the ship's end and calculate the vibration amplitudes at that end, the equations (25) thus being availed of.

The resonance of the vibration is conditioned by the minimum of $(\xi^2+\eta^2)^{1/2}$ in (28). In the case wherein no damping force exists, η tends to zero, so that y assumes an infinitely large value under resonance. It should however be borne in mind that, when the unbalanced force is present at any of the nodal points, of the free vibrations, the vibration amplitudes are not necessarily too large, the reason being as follows:

In the special case, $\delta=0$, namely, the case of no damping force,

$$(X^2+Y^2)^{1/2}=(P_1+Q_1)_{\delta=0}=-2(\Gamma_1\chi_2+\Gamma_2\chi_1), \quad\ldots\ldots\ldots\ldots(26)$$

where

$$\left.\begin{aligned}&\Gamma_1=1+\cosh ml_1\cos ml_1, \quad \Gamma_2=1+\cosh ml_2\cos ml_2,\\&\chi_1=\sinh ml_1\cos ml_1-\cosh ml_1\sin ml_1, \quad \chi_2=\sinh ml_2\cos ml_2-\cosh ml_2\sin ml_2.\end{aligned}\right\}\quad(27)$$

$\Gamma_1=0$, $\Gamma_2=0$ are the frequency equations of clamped free bars of lengths l_1, l_2 respectively, whereas $\chi_1=0, \chi_2=0$ those of hinged-free bars of lengths l_1, l_2 respectively. Thus, if the position of the unbalanced force be at any of the nodal points of the ship, we have

$$(X^2+Y^2)^{1/2}=(P_1+Q_1)_{\delta=0}\rightarrow 0, \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(28)$$

the numerator as well the denominator thus tending to zero, which after some mathematical examination shows that the vibration amplitudes in the special case in question are not infinitely large. It will therefore be seen that the postulation of a slightly different condition of the engine part results an evidence that is opposite to our previous conclusion. In the actual ship, however, the condition of the engine part is possible to be somewhat elastic, so that the nature of the vibration, particularly with respect to the position of engine room, would be somewhat intermediate between the present and the previous answers even should the effect of the damping which we are discussing be neglected. If further the damping action were taken into consideration, the nature of the problem would be much more changed, the result being that the phenomena similar to those of our previous paper tend to manifest themselves. This will be seen from (4), (12)—(17) in the case $x=0$. With a view to confirming the problem let us specially assume that the damping is fairly large, then the resonance frequency of the forced vibration approximately satisfies the equation

$$\sinh\gamma l\cos\gamma l-\cosh\gamma l\sin\gamma l=0, \quad\ldots\ldots\ldots\ldots\ldots\ldots(29)$$

which corresponds to the frequency equation of a hinged-free bar of length l. The principal form of numerator of the expression (4), on the other hand, does not much change even in the condition of high damping, so that the usual conception concerning the node and the loop for a free-free bar is quite reversed. The resonance vibration rather results relatively large amplitudes under the force near the node of a free-free bar, and relatively small amplitudes under the force near the loop of the same bar.

To ascertain the damping nature of ship's vibrations it is obviously preferable to use numerical constants mostly agreeing with those for an actual ship. The prediction of the nature from a model experiment is not satisfactory unless its result can be extended to the case of a full-scale ship by means of any reasonable theory. With the idea under consideration in mind, the numerical value used are as follows:

Ship's length $=l=202$ m (length of Deutschland),

Ship's breadth $=B=24$ m,

Ship's draught $=H=8{\cdot}65$ m (using the block coefficient 0·6 this virtually corresponds to a ship of 25,000 tons displacement).

The stiffness of the ship is such that the natural frequency per min. is $60\sigma_0/2\pi = 60{\cdot}6/2\pi = 57{\cdot}3$/min., with the exception for the calculation of the case of the generation of pressure waves, wherein $60\sigma_0/2\pi = 60.7{\cdot}194/2\pi = 68{\cdot}8$/min.

III. *Damping due to Water Friction.*

Let the side walls of a ship be resisted by the viscous force of the surrounding water; then the vibratory motion of the ship is approximately represented by

$$M\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} - 2H\mu_1\left(\frac{\partial u}{\partial z}\right)_{z=0} = 0, \quad \cdots\cdots\cdots\cdots (29)$$

where M is the mass of the ship per its unit length, μ_1 the viscosity of the water, z the coordinate directed transversely from the ship's side, and u the oscillatory velocity of the water particle.

The determination of $\partial u/\partial z$ may be made as follows:

Since

$$\frac{\partial u}{\partial t} = \frac{\mu_1}{\rho}\frac{\partial^2 u}{\partial z^2}, \quad \cdots\cdots\cdots\cdots (30)$$

where ρ is the water density, the proper solution for u is

$$u = Ae^{-(1+i)\beta z + i\sigma t},$$ [5]

where $\beta = (\sigma\rho/2\mu_1)^{1/2}$. If $y = y_1 e^{i\sigma t}$, then

(5) H. Lamb, *Hydrodynamics*, § 345.

$$u = i\sigma y_1 e^{i\sigma t}$$

at $z=0$, so that $A = i\sigma y_1$. Thus we get

$$-\mu_1\left(\frac{\partial u}{\partial z}\right)_{z=0} = (1+i)\, i\sigma y_1 \sqrt{\frac{\sigma\mu_1\rho}{2}}\, e^{i\sigma t}. \quad \text{(31)}$$

Substituting (31) in (29) it follows

$$M\frac{\partial^2 y}{\partial t^2} + EI\frac{\partial^4 y}{\partial x^4} + (i-1)\sigma\sqrt{\frac{\sigma\mu_1\rho}{2}}\, y = 0. \quad \text{(32)}$$

Writing again $y = y_1 e^{i\sigma t}$, we obtain

$$EI\frac{d^4 y_1}{dx^4} = \left\{\left(M\sigma^2 + 2H\sigma\sqrt{\frac{\sigma\rho\mu_1}{2}}\right) - i2H\sigma\sqrt{\frac{\sigma\mu_1\rho}{2}}\right\} y_1. \quad \text{(33)}$$

Comparing this with (1), (10) we get

$$\left.\begin{aligned} m^4 &= \frac{M}{EI}\left\{\left(M\sigma^2 + 2H\sigma\sqrt{\frac{\sigma\mu_1\rho}{2}}\right) - i2H\sigma\sqrt{\frac{\sigma\mu_1\rho}{2}}\right\}, \\ \mu &= \frac{M}{EI}\left(\sigma^2 + \frac{2H\sigma}{M}\sqrt{\frac{\sigma\mu\,\rho}{2}}\right), \\ \nu &= \frac{M}{EI}\frac{2H\sigma}{M}\sqrt{\frac{\sigma\mu_1\rho}{2}}. \end{aligned}\right\} \quad \text{(34)}$$

If we assume that the natural frequency of the ship without damping $=57{\cdot}3$/min., namely $\sigma = 6\,\mathrm{s}^{-1}$, then we have $(M/EI)^{1/4} = 0{\cdot}00957\,\mathrm{m}^{-1}\,\mathrm{s}^{1/2}$. Again, in the case of a wall-sided vessel of a uniform distribution either in weight or in section along the ship, M may be written as $M = \rho BH$, so that

$$\mu = \frac{M}{EI}\left(\sigma^2 + \frac{\sigma}{B}\sqrt{\frac{2\sigma\mu_1}{\rho}}\right), \quad \nu = \frac{M}{EI}\frac{\sigma}{B}\sqrt{\frac{2\sigma\mu_1}{\rho}}, \quad \text{(34')}$$

where H is $8{\cdot}65$ m in the present example. While ρ is taken to be 1000 kg mass/m^3 in the case of pure water, the assumption as to the value of μ_1 is somewhat arbitrary. In the case of laminar shearing motion

$$\mu_1 = 1{\cdot}7.10^{-3}\ \text{kg mass/m/s}$$

for the water. But, since the existence of such a laminar condition is hardly possible and furthermore the damping force due to such a viscosity is extremely small, the idea of eddy viscosity is rather availed of, assuming

$$\mu_1 = 1{\cdot}7\ \text{kg mass/m/s}$$

for the value of eddy viscosity, that is the value a 1000 times the usual μ_1. Such an enormously large value of μ_1 is rather improbable even in the case of rapid flow in a pipe. From Stanton's experiment it appears that the value of eddy viscosity is not higher than a 100 times that of the actual viscosity.

Substituting (34′) in (23) for the special case, $l_1 = 0$, $l_2 = l$, $x = 0$, we obtain the vibration amplitudes for different values of $(M\sigma^2/EI)^{1/4}l$, as shown in Fig. 1, the dimensions of y, F/EI are m, m^{-2} respectively, In the actual case F generally increases in accordance with the increase of σ^2, namely in the form $F=f\sigma^2$, where

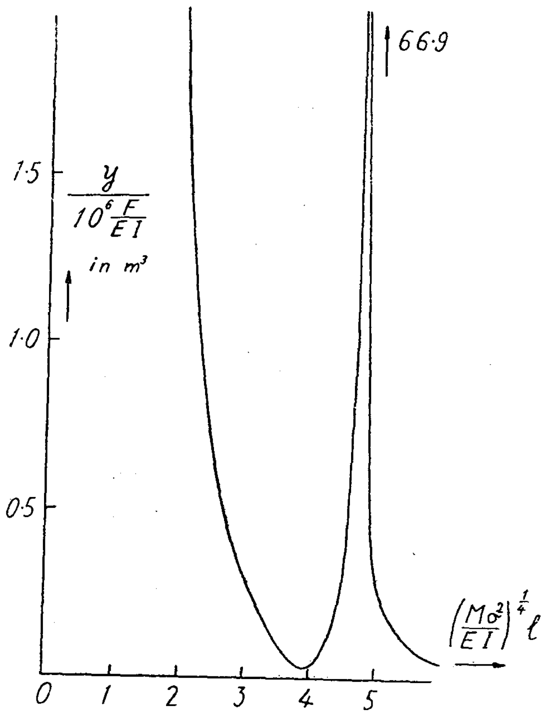

Fig. 1. **Resonance Curve in the Case of** Viscosity Damping; F=Constant.

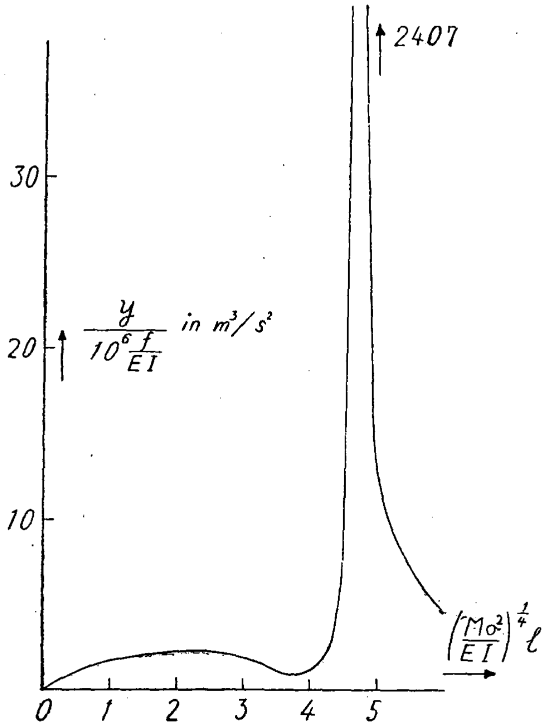

Fig. 2. Resonance Curve in the Case of Viscosity Damping; f=Constant.

f is not necessarily constant. For example, a rotating unbalance is expressed by $F=m_1r\sigma^2$, where m_1, r are the unbalanced mass and the distance of that mass from the rotary centre respectively. From this consideration the result in Fig. 1 is reproduced in Fig. 2.

It will be seen from these figures that the vibration amplitudes under a resonance condition, namely $(M\sigma^2/EI)^{1/4}l \fallingdotseq 4{\cdot}73$, are too large, which is obviously contradictory with the experimental result[6] of the forced vibrations of a ship at different frequencies. In the present case we assumed such large value of μ_1 as a 1000 times its actual value. If we were to assume the actual μ_1 value, the ordinate corresponding to the resonance condition would be about 32×2407. It is therefore improbable that the water viscosity under any criterion would take the large part of the damping action in ship's vibrations.

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[6] For example, see Schlick's paper, *loc. cit.*[1]

IV. *Damping due to Generation of Pressure Waves.*

Let us assume that the form of the ship's section particularly for the present case is of a circular cylinder of diameter B, the immersed part of the ship being therefore of a semi-circular form. The equation of the vibratory motion is expressed by

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\rho\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{\partial\phi}{\partial t}\right)_{r=\frac{B}{2}}\frac{B}{2}d\theta=0, \quad\cdots\cdots(35)$$

where ϕ is the potential of the generated pressure waves. Since

$$\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\theta^2}+k^2\phi=0, \quad\cdots\cdots(36)$$

in which $h=\sigma/c$, c being the velocity of sound waves in the water. In the present calculation c is taken as $c=1430$ m/s. The appropriate solution of (36) is

$$\phi=AH_1^{(2)}(hr)\cos\theta e^{i\sigma t}. \quad\cdots\cdots(37)$$

It is possible to write $y=y_1e^{i\sigma t}$. Since

$$\frac{\partial y}{\partial t}\cos\theta=\frac{\partial\phi}{\partial r} \quad\cdots\cdots(38)$$

at $r=B/2$, we have

$$i\sigma y_1=A\frac{dH_1^{(2)}\left(\frac{hB}{2}\right)}{d\left(\frac{B}{2}\right)}, \quad\cdots\cdots(39)$$

so that

$$A=i\sigma y_1\left|\frac{dH_1^{(2)}\left(\frac{hB}{2}\right)}{d\left(\frac{B}{2}\right)}\right. \quad\cdots\cdots(40)$$

Substituting this in (37),

$$\phi=\frac{i\sigma yH_1^{(2)}(hr)\cos\theta}{\frac{dH_1^{(2)}\left(\frac{hB}{2}\right)}{d\left(\frac{B}{2}\right)}}. \quad\cdots\cdots(41)$$

From (35), (41) we obtain

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\frac{i\rho\sigma BH_1^{(2)}\left(\frac{hB}{2}\right)}{\frac{dH_1^{(2)}\left(\frac{hB}{2}\right)}{d\left(\frac{B}{2}\right)}}\frac{\partial y}{\partial t}=0. \quad\cdots\cdots(42)$$

This is the equation of the damped vibration of the present case. If we again write $y=y_1e^{i\sigma t}$, we get

$$EI\frac{d^4y_1}{dx^4}=\left\{\left(M\sigma^2-\frac{\rho\sigma cB(J_1J_1'+Y_1Y_1')}{J_1'^2+Y_1'^2}\right)-i\frac{\rho\sigma cB(Y_1J_1'-J_1Y_1')}{J_1'^2+Y_1'^2}\right\}y_1.\ldots(43)$$

The argument of J_1, Y_1 being $\sigma B/2c$, where $c=1430$ m/s. Comparing this with (1), (10) we have

$$\left.\begin{aligned}m^4&=\frac{1}{EI}\left\{\left(M\sigma^2-\frac{\rho\sigma cB(J_1J_1'+Y_1Y_1')}{J_1'^2+Y_1'^2}\right)-i\frac{\rho\sigma cB(Y_1J_1'-J_1Y_1')}{J_1'^2+Y_1'^2}\right\},\\ \mu&=\frac{M\sigma^2}{EI}\left\{1-\frac{\rho cB}{M\sigma}\frac{(J_1J_1'+Y_1Y_1')}{(J_1'^2+Y_1'^2)}\right\},\\ \nu&=\frac{M\sigma^2}{EI}\frac{\rho cB}{M\sigma}\frac{(Y_1J_1'-J_1Y_1')}{(J_1'^2+Y_1'^2)}.\end{aligned}\right\}\ldots(44)$$

If we assume that the natural frequency without damping be 68·8/min. (this frequency used merely in the present example), namely $\sigma_0=7{\cdot}194\ \mathrm{s}^{-1}$, we obtain $(M/EI)^{1/4}=0{\cdot}00874\ \mathrm{m}^{-1}\ \mathrm{s}^{1/2}$. Were the immersed part of the ship be of a semi-circular section, it would be possible to write $M=\pi\rho B^2/8=226.10^3$ kg mass/m. Since the dif-

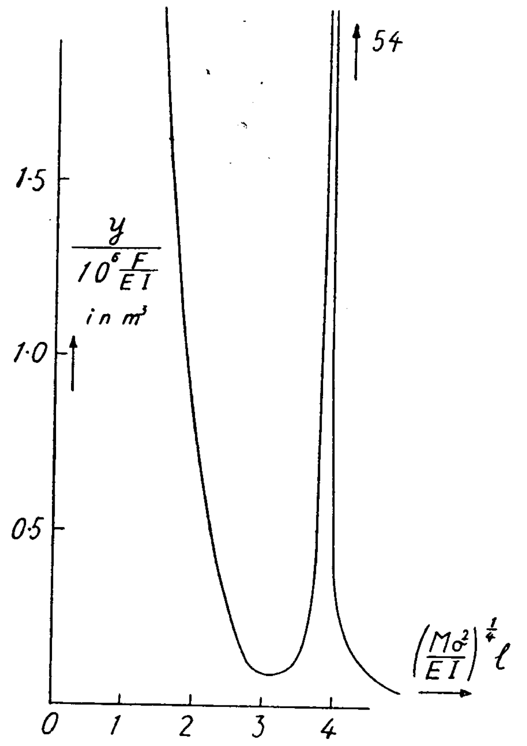

Fig. 3. Resonance Curve in the Case of Pressure Wave Generation; F=Constant.

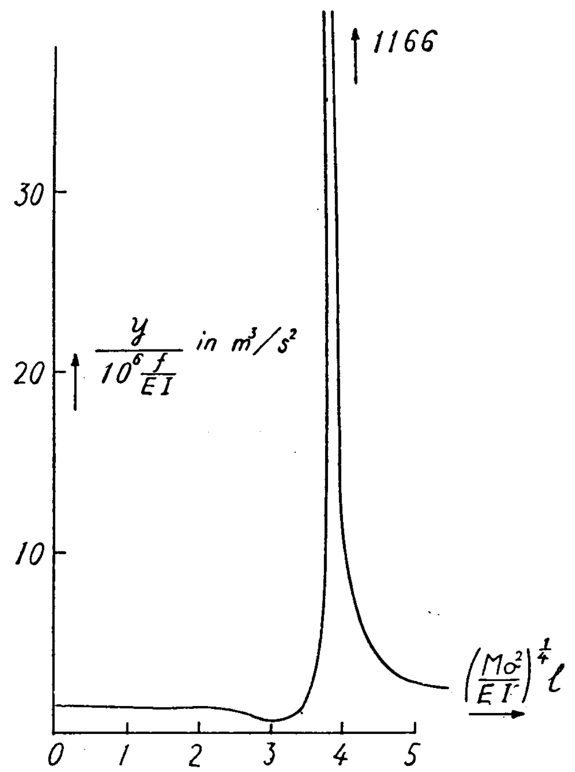

Fig. 4. Resonance Curve in the Case of Pressure Wave Generation; f=Constant.

ference of this value from the one $M=\rho BH=208.10^3$ kg mass/m is merely 8 percent, the effect of such difference on the damping is not marked. Now

$$\left.\begin{aligned}\mu&=\frac{M\sigma^2}{EI}\left(1-\frac{8c}{\pi B\sigma}\frac{J_1J_1'+Y_1Y_1'}{J_1'^2+Y_1'^2}\right),\\ \nu&=\frac{M\sigma^2}{EI}\frac{8c}{\pi B\sigma}\frac{Y_1J_1'-J_1Y_1'}{J_1'^2+Y_1'^2}.\end{aligned}\right\}\quad\ldots\ldots\ldots\ldots\ldots(44')$$

Substituting (44′) in (23) for the special case, $l_1=0$, $l_2=l$, $x=0$, we calculated the vibration amplitudes for different values of $(M\sigma^2/EI)^{1/4}l$, the result being plotted in Fig. 3. F is again assumed to be a constant for any σ. If the unbalanced force, instead of being constant, were assumed to be proportional to σ^2 namely $F=f\sigma^2$, the resonance curve would then be modified to the type shown in Fig. 4.

In the present case too the amplitude under resonance condition is so large as in the case of the viscous damping. If, on the other hand, the ship's natural frequency σ_0 were increased, the amplitude under resonance would be decreased owing to the fact that the factor involving Bessel's functions within the expression of ν is much increased. But, for a much larger value of σ_0 the same amplitude tends again to be larger, for the reason that at such high σ_0, $8c/\pi B\sigma$ participates in the change of the value of ν.

V. *Damping due to Generation of Surface Waves.*

In this case we assume that the ship's section is of a rectangular form, of which the breadth and the draught are B, H respectively. For simplicity, let R be the effective breadth of the ship's bottom to cause surface waves of length $2\pi/k$, of which $k=\sigma^2/g$. Then, taking the pressure reaction as shall generate surface waves, the approximate vibration equation of the ship becomes

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\rho gBy+B\rho\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{\partial\phi_1}{\partial t}\right)_{w=0}d\left(\frac{z}{R}\right)=0,\ldots\ldots\ldots\ldots(45)$$

ϕ_1 being the potential of the generated waves. We shall assume $R/(2\pi/k)=1/2$. This corresponds to the condition that the ship effectively rests on half length of the generated waves. The expressions of ϕ_1 may be put under some approximation as follows:

$$\phi_1=Ae^{i\sigma t-kw-ikz},\qquad\left(\frac{z}{R}>0\right)\quad\ldots\ldots\ldots\ldots\ldots\ldots(46\text{ a})$$

$$\phi_1=Ae^{i\sigma t-kw+ikz},\qquad\left(\frac{z}{R}<0\right)\quad\ldots\ldots\ldots\ldots\ldots\ldots(46\text{ b})$$

the axes of z, w being directed horizontally and vertically from the intersection of the water surface with the ship's middle plane. ϕ_1 obviously satisfies

$$\frac{\partial^2\phi_1}{\partial z^2}+\frac{\partial^2\phi_1}{\partial w^2}=0, \quad \cdots\cdots (47)$$

and the uniformity of pressure on the wave surface, namely

$$\frac{\partial^2\phi_1}{\partial t^2}+g\frac{\partial\phi_1}{\partial w}=0 \quad \cdots\cdots (48)$$

gives the condition $k=\sigma^2/g$.

Since, on the other hand, it is possible to write

$$\frac{\partial y}{\partial t}=-\frac{\partial\phi_1}{\partial w} \quad \cdots\cdots (49)$$

at $z=0$, $w=0$, where $y=y_1e^{i\sigma t}$, we obtain

$$A=\frac{i\sigma y_1}{k}. \quad \cdots\cdots (50)$$

By means of the condition already given, namely $R/(2\pi/k)=1/2$, and (50),

$$B\rho\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{\partial\phi_1}{\partial t}\right)_{w=0}d\left(\frac{z}{R}\right)=i\frac{2B}{R}\frac{\rho\sigma^2}{k^2}\left(1-e^{-\frac{ikR}{2}}\right)y_1e^{i\sigma t}$$

$$=\frac{2\rho Bg}{\pi\sigma}\frac{\partial y}{\partial t}. \quad \cdots\cdots (51)$$

The equation (45) is thus reduced to

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\rho gBy+\frac{2\rho Bg}{\pi\sigma}\frac{\partial y}{\partial t}=0. \quad \cdots\cdots (52)$$

The condition (46) between $1/2>z/R>-1/2$ should rather be the addition of (46 a) and (46 b) together, namely

$$\phi_1=2Ae^{i\sigma t-kw}\cos kz, \quad \cdots\cdots (46')$$

but, owing to the determinateness of the integrated value in (51) we have used the respective original expressions in (46).

Now, writing again $y=y_1e^{i\sigma t}$, we obtain

$$EI\frac{d^4y_1}{dx^4}=\left\{(M\sigma^2-\rho gB)-i\frac{2\rho gB}{\pi}\right\}y_1. \quad \cdots\cdots (53)$$

Comparing this with (1), (10) we have

$$\left.\begin{aligned} m^4&=\frac{M\sigma^2}{EI}\left\{\left(1-\frac{\rho gB}{M\sigma^2}\right)-i\frac{2\rho gB}{\pi M^2\sigma^2}\right\},\\ \mu&=\frac{M\sigma^2}{EI}\left(1-\frac{\rho gB}{M\sigma^2}\right),\\ \nu&=\frac{M\sigma^2}{EI}\frac{2\rho gB}{\pi M\sigma^2}.\end{aligned}\right\} \quad \cdots\cdots (54)$$

In the calculation for the case under consideration we shall use $\sigma_0=6\ \mathrm{s}^{-1}$, so that

$(M/EI)^{1/4}=0{\cdot}00957\ \mathrm{m}^{-1}\,\mathrm{s}^{-1/2}$, whereas $M=\rho BH=208.10^3$ kg mass/m, H being $8{\cdot}65$ m. The simplified expression of (54) is

$$\left.\begin{aligned} m^4&=\frac{M\sigma^2}{EI}\left\{\left(1-\frac{g}{H\sigma^2}\right)-i\frac{2g}{\pi H\sigma^2}\right\},\\ \mu&=\frac{M\sigma^2}{EI}\left(1-\frac{g}{H\sigma^2}\right),\\ \nu&=\frac{M\sigma^2}{EI}\frac{2g}{\pi H\sigma^2}. \end{aligned}\right\}\qquad\ldots\ldots\ \ldots\ldots(54')$$

Under the first resonance condition we have $2g/(\pi H\sigma_0^2)=1/50$, so that $\nu/\mu\approx1/50$ at that condition.

The result of calculation for the case, $l_1=0$, $l_2=l$, $x=0$, is plotted in Fig. 5, F being assumed to be a constant. Writing $F=f\sigma^2$, the modified result under the assumption that f is a constant is shown in Fig. 6.

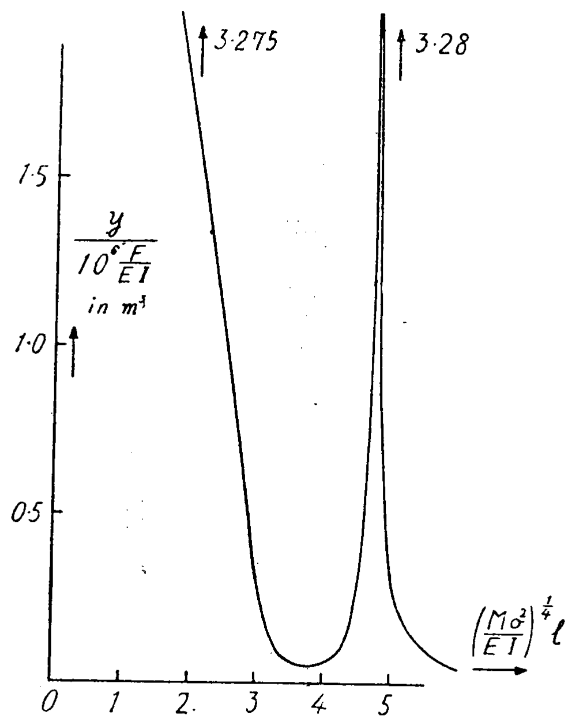

Fig. 5. Resonance Curve in the Case of Surface Wave Generation; F=Constant.

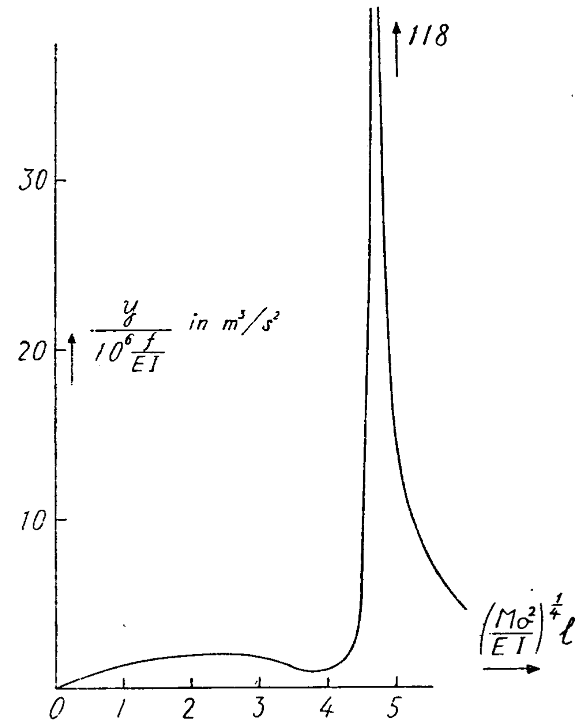

Fig. 6. Resonance Curve in the Case of Surface Wave Generation; f=Constant.

In this case the vibration amplitudes under resonance condition is not excessively large, though they are still somewhat too large. Assuming that f is invariably constant, the amplitude under the first resonance condition is about sixty times the greatest one of the amplitudes at frequencies below that resonance. Thus, there is a great probability of a fairly large damping due to the generation of the surface waves.

Although the formation of capillary waves is also possible, the waves do not much participate in the damping at such frequencies as not exceeding 100 or 200 per minute.

It should be borne in mind that, the smaller the value of H or higher the ship's natural frequencies, the more the vibration amplitude under resonance condition is diminished in virtue of the expressions μ, ν in (54′). The rough ratio of the amplitude, A_r, under resonance to the greatest amplitude, A_g, outside and below the first resonance is shown below.

H in m	8·65	4	1	0·5	0·147
A_r/A_g	59	27	6·8	3·4	1

Again, from the expression of $\nu=(M\sigma^2/EI)(2g/\pi H\sigma^2)$ and the fact that $\sigma_0^2\propto I$, the decrease of the moment of inertia of the section decreases A_r/A_g. Thus, the decrease of the depth of the ship is doubly effective in the decrease of A_r/A_g.

VI. *Damping due to Structural Force.*

The vibration equation of a bar subjected to inner resistance was obtained by us about ten years ago,(7) its form being

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\xi I\frac{\partial^5 y}{\partial t\partial x^4}=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(55)$$

where ξ is the coefficient of solid viscosity. The values of ξ for ferrous materials etc. obtained by Honda and others(8) are of the order $10^8\sim10^9$ in C. G. S. units, whereas the same values for some of non-ferrous materials determined by us(9) are less than 10^7 C. G. S. units. The above equation may be written

$$M\frac{\partial^2 y}{\partial t^2}+EI\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial x^4}=0 \quad\ldots\ldots\ldots\ldots\ldots\ldots(55')$$

where $\tau=\xi/E$. In the actual structual condition the effective values of ξ are greater than the values above described, owing to the fact that the energy dissipation in material joints as well as the contact surfaces between the ship's structure and the load mass. Contact resistance under dynamic forces is very small and is of such a kind as dissipates much dynamical energy. The values of τ for pure materials

(7) K. Sezawa, *Bull. Earthq. Res. Inst.*, 3 (1927), 50.
(8) K. Honda and K. Konno, *Phil. Mag.*, 42 (1921), 115-123.
(9) K. Sezawa and K. Kubo, "Measurement of Solid Viscosities of Metals through the Flexural Vibrations of a Bar", *Rep. Aeron. Res. Inst.*, No. 89 (1932).

obtained by special experimental conditions is of the order 10^{-3} s, whereas the values of τ for composite materials estimated from the result due to certain engineering tests[10] is of the order 10^{-2} s, notwithstanding that the composite condition under consideration is relatively simple. From this fact we shall now take τ as 10^{-2} s. For example, in the case of steel $E=2.10^{12}$ C. G. S., so that $\tau E=2.10^{10}$ C. G. S.

Writing $y=y_1e^{i\sigma t}$, (55′) reduces to

$$\frac{d^4y_1}{dx^4}=\frac{M\sigma^2(1-i\tau\sigma)}{EI(1+\tau^2\sigma^2)}y_1. \qquad (56)$$

Comparing this with (1), (10) we have

$$\left.\begin{aligned} m^4&=\frac{M\sigma^2(1-i\tau\sigma)}{EI(1+\tau^2\sigma^2)},\\ \mu&=\frac{M\sigma^2}{EI(1+\tau^2\sigma^2)},\\ \nu&=\frac{M\sigma^2}{EI(1+\tau^2\sigma^2)}\tau\sigma. \end{aligned}\right\} \qquad (57)$$

Using the values $\sigma_0=6\,\mathrm{s}^{-1}$, $(M/EI)^{1/4}=0{\cdot}00957\,\mathrm{m}^{-1}\,\mathrm{s}^{1/2}$, we have obtained the

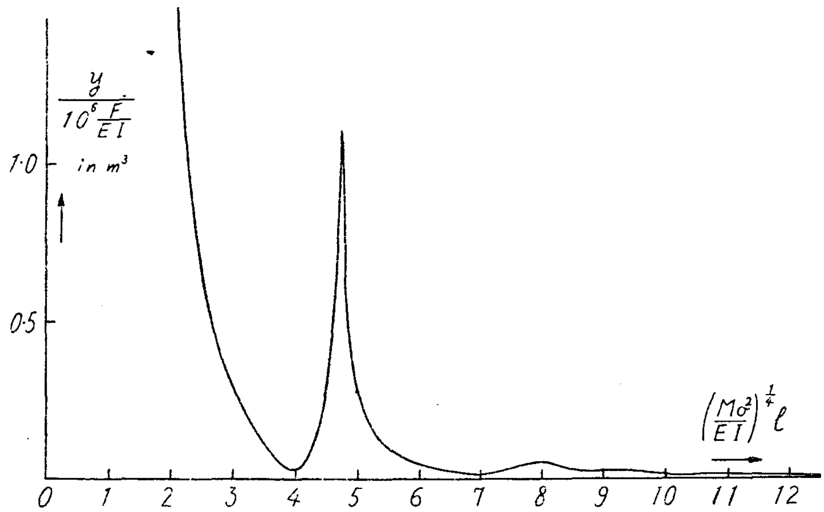

Fig. 7. Resonance Curve in the Case of Structural Damping; F=Constant.

resonance curves for both cases F=constant and f=constant, the result being plotted in Figs. 7, 8.

In the present case the damping under the resonance condition is fairly large. The ratio of the vibration amplitude under resonance, A_r, to the greatest one at the frequencies outside and below the resonance, A_g, is about 18. The same ratio

(10) A. B. Eason, *The Prevention of Vibration and Noise* (London, 1923).

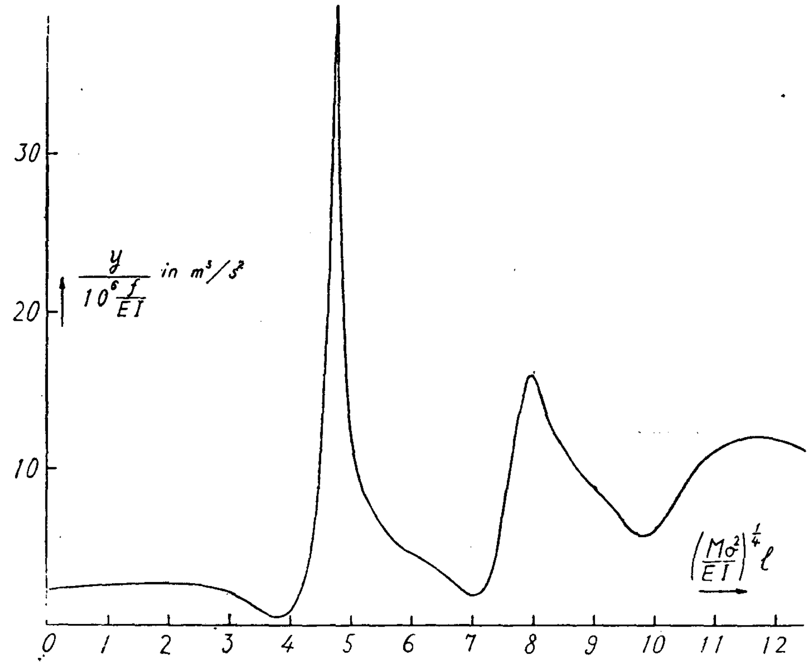

Fig. 8. Resonance Curve in the Case of Structural Damping; f = Constant.

assumes the following approximate values in accordance with the difference in the value of τ.

τ	10^{-2}	10^{-1}	10^{-3}
A_r/A_g	18	1·8	180

The largeness of the effective value of τ is greatly probable to be present, in the case of full load in the ship, namely in the case of deep draught.

VII. *General Summary and Conclusion.*

From the result of the investigation shown in every preceding section it will be seen that the principal causes of damping in ship's vibrations are the energy losses due to the structural forces as well as the energy dissipation in the form of surface waves. The viscous friction in the water, even should the value for eddy viscosity be used in the constant of resistance, would not much participate in the ship's damping. The generation of pressure waves also does not dissipate the vibrational energy in usual frequencies of forced vibrations.

In the case of a ship of shallow draught the generation of surface waves is the primary cause, whereas in the case of a ship with full load and deep draught, the effective structural damping resistance becomes the primary cause under consideration.

A suggestion for increasing the damping under resonance condition is to fit such flat surfaces like bilge keels or Motora's stabilizer[11] to the ship as possibly as near the water surface. This increases the generation of surface waves due to ship's vibrations provided the area of the surface of the plate is suitably large. It appears rather improbable to expect higher damping resistance from the structural condition. Inglis's suggestion[12] of putting an oscillator on board a ship seems rather hopeless, owing to the reason that the temporary adjustment of damper and frequency of the massive oscillator is practically impossible, and that one of th frequency of the coupled vibrations is so near that of the useful natural oscillation of the oscillator alone that the vibrating system is liable to encounter resonance-like condition.

It is obvious that the nature of the problem shown for the case of the flexural vibration of the ship is naturally present even in the case of its torsional vibration.

Although in the present paper we have separately studied the damping phenomena due to different causes, the actual state is rather under the simultaneous damping action of various forces. The equation of the vibratory motion under the action of different forces is expressed by

$$M\frac{\partial^2 y}{\partial t^2}+EI\frac{\partial^4 y}{\partial x^4}+\rho gBy-2H\mu_1\left(\frac{\partial u}{\partial z}\right)_{z=0}$$
$$+c\rho\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{\partial\phi}{\partial t}\right)_{r=0}\frac{B}{2}d\theta+B\rho\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\frac{\partial\phi_1}{\partial t}\right)_{w=0}d\left(\frac{z}{R}\right)+\xi I\frac{\partial^5 y}{\partial t\partial x^4}=0. \quad \ldots.(58)$$

The nature of the problem under such a condition, however, would not be much different even should the natures of the respective kinds of damping forces be studied before the simultaneous comparison of the respective cases.

The present paper is merely a preliminary note of the investigation which we have recently attempted under Professor Hiraga's kind encouragement. We are now going to make some idealized model experiments as well as the vibration observation on board an actual ship. Since, however, we have devoted ourselves to the research of some science problems during last eleven years, we have been almost impossible to study the ship's problem to any satisfying extent notwithstanding our continual interest in the same problem. Recently, nevertheless, it has kindly been decided by the Council of Kondô Memorial Marine Foundation that the same Foundation will assist us in our investigation on the problem of ship's vibrations, and we wish to express our thanks to the Council of that Foundation, with whose aid the present investigation is being greatly developed.

(11) S. Motora, *Journ. Soc. Nav. Arch.*, **32** (1923), 75-84; **36** (1925), 109-117.

(12) C. E. Inglis, "A Suggested Method for Minimizing Vibrations in Ships", *T. I. N. A.*, **75** (1933), 252-267.

討 論

○座長(平賀讓君) 唯今の御講演に對して御質問又は御意見が御座いますならば御述べを願ひます。御座いませんければ私が一寸お伺ひし度いと思ひます。resonanceがひどくなつた時はstructureはどうなるですか。

○妹澤克惟君 structure は之れで計算出來ますが、夫れは波の上で受ける stress と全く同樣です。之れは stress が振幅に比例するからであります。resonance curve の Fig. 2 を見ますと resonance でない所は、例へば或る與へられた點で abscissa 2 の所の amplitude 即ち stress が 2 と致しますと、resonance の所の amplitude 即ち stress は 2407 となります。

○平賀讓君 其の 2407 と言ふ resonanceになる前に何かゞ抵抗してさうさせぬ、夫れが其の structure の抵抗ですか。

○妹澤克惟君 御説の通りでありまして銘々の frequency で本文にあるやうな抵抗が働いて始終 damp してゐると、resonance のときの amplitude が低くなります。普通ならば鋭い resonance curve は是等の damping を適當に adjust して置くと、resonance になりかけようとする時に其の場合の resonance curve が平らになり決して stress が大きくなりませぬ。元良式の樣に active type ですと resonance になる前にもう damp しかけるからよいが、船の anti-rolling tank 等では調整がうまく行かぬと resonance curve が鋭く昇ります。vibration では寧ろ簡單で、色々な frequency の amplitude が常に鋭くならぬ樣に adjust が出來るので幾らかましと思ひます。

○座長(平賀讓君) どなたか外に御座いませんか。御座いませんければ、一言御挨拶申上げます。妹澤博士は此の前の御講演でも振動に關する大變新しい極めて有益な御意見を述べられました。今日は又進んで振動の非常に難しい問題に入られて玆に始めて其の所見を述べられました。餘り新しく解析されてゐるので猶我々の了解困難な點もありますが、非常に有益な御發表であると信じます。博士の云はれた樣に近藤財團の補助で續いて御研究されるとのことでありますから、どうか益々御研究になり、續いて本協會に御發表下さいます樣に御願ひ致します。諸君と共に拍手を以て感謝の意を表し度いと思ひます。(一同拍手)