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A Study on Free-Surface Flow around Bow of Slowly Moving Full Forms

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Summary

From the flow measurement around the bow of a slowly moving full form it was found that except in the thin boundary layer near the free surface, velocity components agree well with the calculated values from the velocity potential which satisfies rigid-wall free surface condition. In the thin free surface layer the velocity components change depthwise very rapidly. Based on this experimental result a theory to analyze the thin free surface layer around slowly moving full forms was developed. To verify the validity of the present theory wave making resistance of geometrically simpler forms, viz., a vertical, infinite circular cylinder and a semi submerged sphere was calculated in low speed limit. From this calculation it was found that the order of magnitude and the trend of wave making resistance with respect to Froude number are in good agreement with those of conventional full forms determined experimentally.

1. Introduction

Free surface disturbance induced by a slowly moving full form is very small except in the bow region. On the free surface around the blunt bow ripple like short waves are observed. With increase of ship speed the short waves are transformed into breaking waves.

To understand such flow characteristics on the free surface near the bow region of slowly moving full forms, flow measurement by use of a 5-hole pitot tube was conducted in Nagasaki Experimental Tank.

From the flow measurement it was found that except in the thin boundary layer near the free surface, velocity components agree well with the calculated values from the double model velocity potential which satisfies the rigid-wall free sureface condition. In the thin free surface layer, on the other hand, the velocity components differ from the rigid-wall solutions and change depthwise very rapidly until they reach the values of rigid-wall solutions.

Based on this experimental result a theory to analyze the thin free surface layer around slowly moving full forms is developed. The theory supposes that there is a thin free surface layer on the non-uniform flow derived by the rigid-wall solution which is quite accurate everywhere except in the thin layer; here the variables change very rapidly in such a way that the free surface conditions are satisfied.

The characteristics of wave pattern due to a point disturbance on the non-uniform flow was studied by Ursell (1960). The equations for the wave crests are deduced, not rigorously from the equation of motion (as for uniform flow), but from assumptions which appear physically reasonable[1]. In Ursell's theory the velocity components of the non-uniform flow are assumed to vary slowly with space variables. Relating to the wave resistance problem in the low speed limit Ogilvie (1968) studied a two-

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dimensional problem of the thin free surface layer on the non-uniform flow which is determined by the rigid-wall solution. Ogilvie assumed also that the basic non-uniform flow vary slowly with space variables while in the thin layer physical variables such as velocity and wave height are assumed to vary very rapidly[2].

In a recent paper by Hermans wave resistance problem in low speed is treated in the similar manner as Ogilvie and a theory to analyze the free surface flow around a semi submerged horizontal cylinder, perpendicular to the incoming flow is developed[3]. Timman extended Hermans' theory further to study flow around a semi submerged three-axial ellipsoid with its middle axis horizontally on the still water plane so that the free surface meets the bow over a relatively broad part of the front. Then the problem is transformed into two-dimensional one in the plane perpendicular to the middle axis[4]. Keller (1974) studied wave patterns of full ships in low speed by a different manner from those mentioned above, but based on the similar assumption on the water flow, viz., Keller assumed that for small Froude number, the flow consists of the double body flow plus an oscillatory flow which represents the wave motion. The ray methods like those of geometrical optics are applied to the analysis of the wave motion[5].

The present study is a direct extension of Ogilvie's theory to the three-dimensional case with some modifications in determining a solution. Originally Ogilvie did not intend to apply his theory to a surface piercing body. Here, however, a theory for the free surface flow around a floating body in low speed limit is developed, since the result of flow measurements motivated us to apply Ogilvie's theory to our problem.

2. Flow measurement around bow of a slowly moving full form

For the flow measurement by use of a 5-hole pitot tube, a simple hull form M. 2201 as shown in Fig. 1 was used. The principal particulars of the ship model are shown in Table 1. The flow measurements were conducted in two different load conditions. Fig. 2 and 3 show the flow patterns around the bow of shallow draft (200 mm) and deep draft (400 mm) respectively at speed $U=1.089$ m/sec ($U/\sqrt{gL}=0.1420$). With increase of ship speed short waves observed in Fig. 2 and 3 are transformed into rather confused disturbed flow (breaking waves) as shown in Fig. 4 and 5 ($U=1.284$ m/sec, $U/\sqrt{gL}=0.1674$).

Table 1 Principal particulars of M. 2201

Load	Deep	Shallow
Lpp (m)	6.000	6.000
B (mm)	1000.00	1000.00
d (mm)	400.00	200.00
Δn (kg)	2004.7	958.68
Lpp/B	6.00	6.00
B/d	2.50	5.00
Cb	0.8353	0.7989
Cp	0.8359	0.7999
Cm	0.9993	0.9987

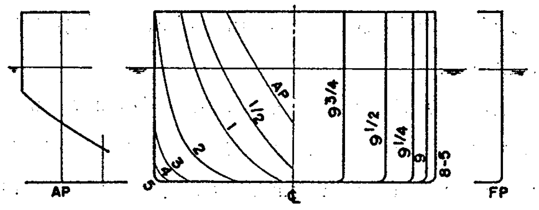

Fig. 1 Lines of M. 2201

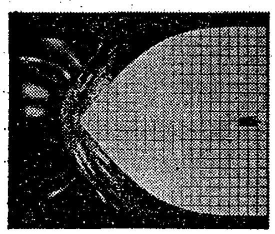

Fig. 2 Flow pattern around bow of a full form at $U=1.089$ m/sec ($U/\sqrt{gL}=0.1420$) in shallow draft

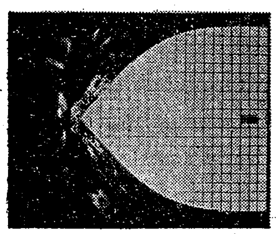

Fig. 3 $U=1.089$ m/sec in deep draft

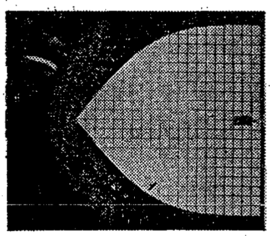

Fig. 4 $U=1.248$ m/sec ($U/\sqrt{gL}=0.1674$) in shallow draft

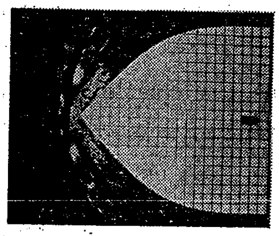

Fig. 5 $U=1.248$ m/sec in deep draft

Since there are rather confused short waves on the free surface, only the mean elevation of free surface from the still water level were measured first (Fig. 6). Then the velocity components (u, v, w) were measured up to the free surface by use of a 5-hole spherical pitot tube of 7 mmϕ. The measured results are shown in Fig. 7 through 10 comparing with the velocity components calculated numerically from the double model velocity potential which satisfies the rigid-wall free surface condition. In those figures $z=0$ corresponds to the free surface. The rigid-wall solutions are extrapolated up to the free surface from the values on the still water level, since they have very little change in depthwise.

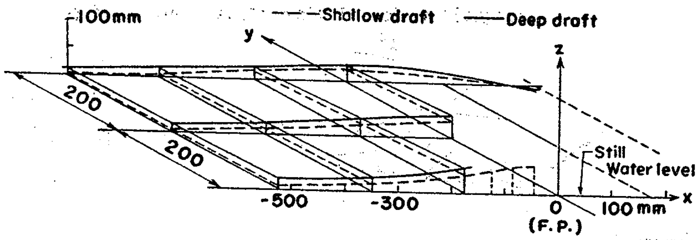

Fig. 6 Measured wave heights at $U=1.089$ m/sec

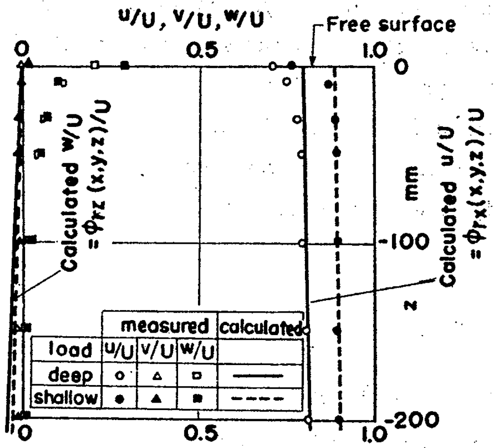

Fig. 7 Velocity components at $x=-350$ mm, $y=0$

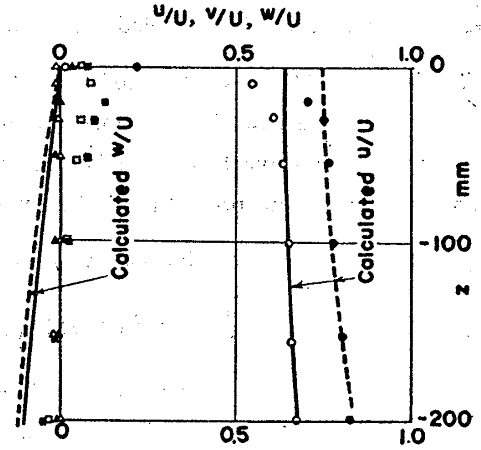

Fig. 8 Velocity components at $x=-175$ mm, $y=0$

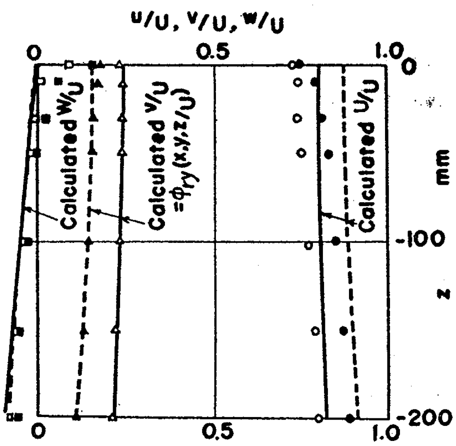

Fig. 9 Velocity components at $x=0$, $y=400$ mm

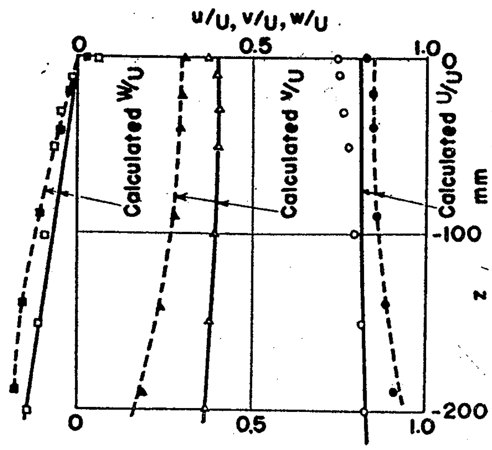

Fig. 10 Velocity components at $x=175$ mm, $y=400$ mm

From the comparison it is found that except in the thin boundary layer near the free surface (the thickness is less than 0.1λ, where $\lambda=2\pi U^2/g$, g is the acceleration of gravity) the measured velocity components agree well with the calculated values based on the rigid-wall solution. In the thin free surface layer measured velocity components show rapid changes with increase of depth while the calculated values show very little change.

3. Theoretical study

From the results of flow measurements we may assume that the rigid-wall solution is quite accurate everywhere except in the thin layer near the free surface.

Taking the rectangular coordinate system fixed on the body with the origin on the still water plane, we set x-axis directing to the uniform flow U and z-axis directing upwards as shown in Fig. 11. Supposing a ship floating on an inviscid, irrotational, incompressible fluid, we consider the velocity potential for the free surface problem as the sum of two parts:

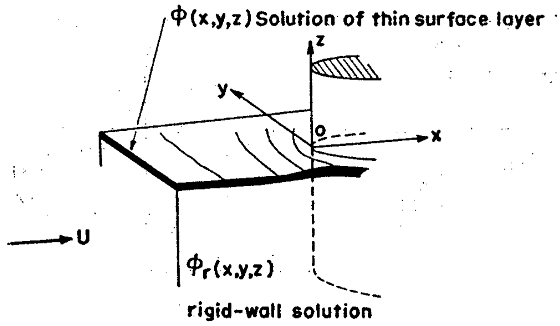

Fig. 11 Scheme of the modeled phenomenon

$$\Phi(x,y,z)=\phi_r(x,y,z)+\phi(x,y,z), \tag{1}$$

where $\phi_r(x,y,z)$ is the potential for the rigid-wall problem, and $\phi(x,y,z)$ is an additional potential to $\phi_r(x,y,z)$ so that the sum satisfies the free surface conditions. Fig. 11 shows the modeled scheme of the present problem.

Although we are considering short waves appeared in front of the blunt bow of a ship, the surface tension effects on wave formation is neglected for the simplicity of treatment.

The boundary value problem for the present study is written as follows.

[L] $$0=\Phi_{xx}+\Phi_{yy}+\Phi_{zz}, \tag{2}$$

[A] $$\frac{1}{2}U^2=gH(x,y)+\frac{1}{2}[\Phi_x^2+\Phi_y^2+\Phi_z^2], \quad \text{on} \quad z=H(x,y), \tag{3}$$

[B] $$0=H_x\Phi_x+H_y\Phi_y-\Phi_z, \quad \text{on} \quad z=H(x,y), \tag{4}$$

[N] $$0=\Phi_n, \quad n \text{ is a normal unit vector on body surface,} \tag{5}$$

[R] $$\Phi-Ux=\begin{cases} O(1/\sqrt{x^2+y^2}) & \text{for} \quad x>0, \\ & \text{as} \quad x^2+y^2\longrightarrow\infty \\ o(1) & \text{for} \quad x<0. \end{cases} \tag{6}$$

The last one is the condition insuring that waves only follow the ship.

According to Ogilvie we assume that wave height $H(x,y)$ is expressed as the sum of two parts:

$$H(x,y)=\zeta_r(x,y)+\zeta(x,y),$$

where $\zeta_r(x,y)$ is the wave height due to the rigid-wall potential, viz.,

$$\zeta_r(x,y)=\frac{1}{2g}[U^2-\phi_{rx}^2(x,y,0)-\phi_{ry}^2(x,y,0)], \tag{7}$$

since $\phi_{rz}(x,y,0)=0$, and $\zeta(x,y)$ is a superposed wave on $\zeta_r(x,y)$.

In the low speed limit we introduce the following assumptions about orders of magnitude. The detail reasoning of the assumptions is found in Ogilvie's paper.

(a) $\phi_r(x,y,z)=O(U)$,

(b) $\zeta_r(x,y)=O(U^2)$,

(c) $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial z}=O(1)$ when operating on $\phi_r(x, y, z)$ or $\zeta_r(x, y)$,

(d) $\phi(x, y, z)=O(U^5)$,

(e) $\zeta(x, y)=O(U^4)$,

(f) $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, $\frac{\partial}{\partial z}=O(U^{-2})$ when operating on $\phi(x, y, z)$ or $\zeta(x, y)$.

These assumptions are interpreted physically as follows. To an observer at the free surface around a slowly moving full form, it appears that there is a uniform stream below him which has speed equal to the local value $\sqrt{\phi_{rx}^2+\phi_{ry}^2}$. This observer can not see the free surface disturbance represented by $\zeta_r(x, y)$, for it varies on a length scale which is too large. Only the small, superposed waves, represented by $\zeta(x, y)$, will be visible to him.

Under the above assumptions about orders of magnitude, the lowest order terms of the free surface boundary conditions [A] and [B] are rewritten respectively:

$$\text{[A]}\qquad g\zeta(x, y)+\phi_{rx}(x, y, 0)\phi_x(x, y, z)+\phi_{ry}(x, y, 0)\phi_y(x, y, z)=0, \quad \text{on} \quad z=\zeta_r(x, y). \tag{8}$$

$$\text{[B]}\qquad \phi_z(x, y, z)-\zeta_x(x, y)\phi_{rx}(x, y, 0)-\zeta_y(x, y)\phi_{ry}(x, y, 0)$$
$$=\frac{\partial}{\partial x}[\zeta_r(x, y)\phi_{rx}(x, y, 0)]+\frac{\partial}{\partial y}[\zeta_r(x, y)\phi_{ry}(x, y, 0)], \quad \text{on} \quad z=\zeta_r(x, y). \tag{9}$$

Eliminating $\zeta(x, y)$ from (8) and (9), and taking the lowest order terms, which are of $O(U^3)$, we have the following free surface condition.

$$\frac{1}{g}\phi_{rx}^2(x, y, 0)\phi_{xx}(x, y, z)+\frac{2}{g}\phi_{rx}(x, y, 0)\phi_{ry}(x, y, 0)\phi_{xy}(x, y, z)+\frac{1}{g}\phi_{ry}^2(x, y, 0)\phi_{yy}(x, y, z)$$
$$+\phi_z(x, y, z)=D(x, y), \quad \text{on} \quad z=\zeta_r(x, y), \tag{10}$$

where

$$D(x, y)=\frac{\partial}{\partial x}[\zeta_r(x, y)\phi_{rx}(x, y, 0)]+\frac{\partial}{\partial y}[\zeta_r(x, y)\phi_{ry}(x, y, 0)]. \tag{11}$$

In general, we have to solve the equation (10) on $z=\zeta_r(x, y)$. Here, however, for the simplicity of treatment we introduce a following non-conformal transformation of coordinates.

$$x'=x, \quad y'=y, \quad z'=z-\zeta_r(x, y).$$

The Laplace equation for $\phi(x', y', z')$ is written as

$$0=\frac{\partial^2\phi}{\partial x'^2}+\frac{\partial^2\phi}{\partial y'^2}+\frac{\partial^2\phi}{\partial z'^2}-2\frac{\partial\zeta_r}{\partial x'}\frac{\partial^2\phi}{\partial x'\partial z'}-2\frac{\partial\zeta_r}{\partial y'}\frac{\partial^2\phi}{\partial y'\partial z'}+O(U^5).$$
$$[U] \qquad [U^3] \qquad [U^3]\text{; order}$$

Thus, to leading order, ϕ satisfies the usual Laplace equation in terms of the new variables x', y', z'. Taking the lowest order terms and dropping the primes on the new variables, our boundary value problem is written as:

$$\phi_{xx}(x, y, z)+\phi_{yy}(x, y, z)+\phi_{zz}(x, y, z)=0, \quad z<0 \tag{12}$$

$$\frac{1}{g}\phi_{rx}^2(x, y, 0)\phi_{xx}(x, y, 0)+\frac{2}{g}\phi_{rx}(x, y, 0)\phi_{ry}(x, y, 0)\phi_{xy}(x, y, 0)$$
$$+\frac{1}{g}\phi_{ry}^2(x, y, 0)\phi_{yy}(x, y, 0)+\phi_z(x, y, 0)=D(x, y), \tag{13}$$

$$\zeta(x, y)=-\frac{1}{g}\{\phi_{rx}(x, y, 0)\phi_x(x, y, 0)+\phi_{ry}(x, y, 0)\phi_y(x, y, 0)\}, \tag{14}$$

$$\phi(x, y, z)=\begin{cases} O(1/\sqrt{x^2+y^2}) & \text{for} \quad x>0 \\ & \text{as} \quad x^2+y^2\longrightarrow\infty \\ o(1) & \text{for} \quad x<0. \end{cases} \tag{15}$$

In addition to these conditions the body boundary condition must be added. Here, howeve, this condition is omitted, since $\phi(x, y, z)$ is considered as the solution of the first step of iteration cycle.

When dropping the terms including ϕ_{ry}, we have a free surface condition which coincides with the Ogilvie's two-dimensional expression. Ogilvie obtained an explicit solution by use of complex functions. In our three-dimensional problem, Fourier's method is used with slight modification.

First we introduce a double layer potential $\Psi(x, y, z)$ which equals to $D(x, y)$ at $z=0$ as

$$\Psi(x, y, z)=\frac{1}{4\pi^2}\int_0^\infty kdk\int_{-\pi}^{\pi} d\theta e^{kz+ik\{(x-\xi)\cos\theta+(y-\eta)\sin\theta\}}\iint_{-\infty}^{\infty} d\xi d\eta D(\xi, \eta). \tag{16}$$

Then, we assume the form of velocity potential $\phi(x, y, z)$ as

$$\phi(x, y, z)=\frac{1}{4\pi^2}\int_0^\infty kdk\int_{-\pi}^{\pi} d\theta e^{kz+ik(x\cos\theta+y\sin\theta)}F(x, y, k, \theta), \tag{17}$$

where $F(x, y, k, \theta)$ is determined in such a way that $\phi(x, y, z)$ satisfies Laplace equation and the free surface condition (13). From (17) we may consider that $F(x, y, k, \theta)$ is relating to the wave amplitude which depends on the intensity of disturbance acting on the free surface. In our problem $D(x, y)$, the right hand side of (13), is considered as disturbance acting on the free surface. Then we may assume intuitively that $F(x, y, k, \theta)$ is expressed in terms of the rigid-wall potential. This assumption means that the operation $\partial/\partial x$ or $\partial/\partial y$ on F does not change the order of magnitude due to the assumption (c). $\phi(x, y, z)$ is already assumed to be of $O(U^5)$ in the thin boundary layer near the free surface where $x, y, z=O(U^2)$. Therefore, from (17), we may deduce the following orders for k and $F(x, y, k, \theta)$:

$$k=O(U^{-2}), \qquad F(x, y, k, \theta)=O(U^9).$$

Then,

$$\phi_{xx}(x, y, z)=\frac{1}{4\pi^2}\int_0^\infty kdk\int_{-\pi}^{\pi} d\theta\, e^{kz+ik(x\cos\theta+y\sin\theta)}\Big\{\underset{[U^5]}{-k^2\cos^2\theta F}+\underset{[U^7]}{2ik\cos\theta\frac{\partial F}{\partial x}}+\underset{[U^9]}{\frac{\partial^2 F}{\partial x^2}}\Big\}. \tag{18}$$

The second and third terms in the above bracket are higher order than the first term. The same rule is applied to ϕ_{yy} and ϕ_{xy}. When taking the lowest order terms, we see first that ϕ satisfies Laplace equation. Then, substituting them into the free surface condition (13), $F(x, y, k, \theta)$ is determined as follows.

$$F(x, y, k, \theta)=\frac{k_0(x, y, \theta)}{k\{k_0(x, y, \theta)-k\}}\iint_{-\infty}^{\infty} d\xi d\eta\, e^{-ik(\xi\cos\theta+\eta\sin\theta)}D(\xi, \eta), \tag{19}$$

where

$$k_0(x, y, \theta)=g/\{\phi_{rx}(x, y, 0)\cos\theta+\phi_{ry}(x, y, 0)\sin\theta\}^2. \tag{20}$$

We see that $F(x, y, k, \theta)$ is expressed in terms of rigid-wall solution. Thus F satisfies the previously assumed conditions, viz., $\partial F/\partial x$, $\partial F/\partial y=O(F)$.

Substituting (19) into (17), and taking the radiation condition (15) into account, we have the following expression of our velocity potential:

$$\phi(x, y, z)=\frac{1}{2\pi^2}\iint_{-\infty}^{\infty} d\xi\, d\eta\, D(\xi, \eta)\int_{-\pi/2}^{\pi/2} d\theta\, k_0(x, y, \theta)\, P.V.\int_0^\infty dk\frac{e^{kz}\cos(k\tilde{w})}{k_0(x, y, \theta)-k}$$
$$+\frac{1}{2\pi}\iint_{-\infty}^{\infty} d\xi\, d\eta\, D(\xi, \eta)\int_{-\pi/2}^{\pi/2} k_0(x, y, \theta)e^{k_0(x,y,\theta)z}\sin\{k_0(x, y, \theta)\tilde{w}\}d\theta, \tag{21}$$

where $\tilde{w}=(x-\xi)\cos\theta+(y-\eta)\sin\theta$.

The wave height $\zeta(x, y)$ due to potential $\phi(x, y, z)$ is determined from (14) when taking the lowest order terms of derivatives of ϕ with respect x and y (Note that the operation $\partial/\partial x, \partial/\partial y=O(1)$ when operating on $k_0(x, y, \theta)$).

$$\zeta(x, y)=\frac{1}{2\pi^2}\iint_{-\infty}^{\infty} d\xi\, d\eta\, D(\xi, \eta)\int_{-\pi/2}^{\pi/2}\frac{d\theta}{\{\phi_{rx}(x, y, 0)\cos\theta+\phi_{ry}(x, y, 0)\sin\theta\}}P.V.\int_0^{\infty}\frac{k\sin(k\tilde{w})dk}{k_0(x, y, \theta)-k}$$
$$-\frac{g}{2\pi}\iint_{-\infty}^{\infty} d\xi\, d\eta\, D(\xi, \eta)\int_{-\pi/2}^{\pi/2}\frac{\cos\{k_0(x, y, \theta)\tilde{w}\}d\theta}{\{\phi_{rx}(x, y, 0)\cos\theta+\phi_{ry}(x, y, 0)\sin\theta\}^3}. \quad (22)$$

In the far downstream where $\phi_{rx}\longrightarrow U$, $\phi_{ry}\longrightarrow 0$ the wave height is expressed as

$$\zeta(x, y)=\int_{-\pi/2}^{\pi/2} d\theta\left[S(\theta)\sin\left\{\frac{g}{U^2}\sec^2\theta\,(x\cos\theta+y\sin\theta)\right\}+C(\theta)\cos\left\{\frac{g}{U^2}\sec^2\theta\,(x\cos\theta+y\sin\theta)\right\}\right], \quad (23)$$

where

$$C(\theta)+iS(\theta)=-\frac{g}{\pi U^3}\sec^3\theta\iint_{-\infty}^{\infty} d\xi\, d\eta\, D(\xi, \eta)\exp\left\{i\frac{g}{U^2}\sec^2\theta\,(\xi\cos\theta+\eta\sin\theta)\right\}. \quad (24)$$

Then the wave making resistance R_w is expressed as

$$R_w=\pi\rho U^2\int_0^{\pi/2}|C(\theta)+iS(\theta)|^2\cos^3\theta\, d\theta, \quad (25)$$

where ρ is the density of water.

In principle, we can calculate wave making resistance of any body shape, since $D(x, y)$ is determined from the rigid-wall solutions which are obtainable numerically or analytically.

4. Calculation of wave making resistance of a vertical, infinite, circular cylinder and a semi submerged sphere at low speed

Calculation of wave pattern around bow of a ship by use of the present theory makes it possible to compare directly with the observation. However, this calculation needs lengthy numerical work even for a geometrically simple form. For the preliminary stage to verify the validity of the present theory, it may be enough to calculate wave making resistance of simple forms.

For the convenience of analytical work, a vertical, infinite circular cylinder and a semi submerged sphere are considered as the examples of full forms. The rigid-wall velocity potentials are written as follows.

$$\phi_r(x, y, z)=\phi_r(x, y)=U\left(x+\frac{a^2x}{x^2+y^2}\right) \quad \text{for a vertical, circular cylinder of radius } a, \quad (26)$$

$$\phi_r(x, y, z)=U\left(x+\frac{a^3x}{2(x^2+y^2+z^2)^{3/2}}\right) \quad \text{for a semi submerged sphere.} \quad (27)$$

From (11) we have the following non-dimensional expression for $D(x, y)$.

$$\frac{D(s, \beta)}{4UF_n^2}=\frac{1}{s}P(s)\cos\beta+\frac{1}{s}Q(s)\cos 3\beta, \qquad \text{for} \quad s\geq 1, \quad (28)$$

where the origin is fixed at the center of the crossplane of the cylinder or the sphere and the still water plane, and $F_n=U/\sqrt{2ga}$, $x/a=s\cos\beta$, $y/a=s\sin\beta$, $s=\sqrt{x^2+y^2}/a$,

$$P(s)=2s^{-4}-s^{-6}, \qquad Q(s)=-s^{-2} \quad \text{for a vertical, circular cylinder,}$$

$$P(s)=-\frac{9}{16}s^{-3}+\frac{87}{32}s^{-6}-\frac{201}{128}s^{-9}, \quad Q(s)=-\frac{15}{16}s^{-3}+\frac{33}{32}s^{-6}-\frac{39}{128}s^{-9} \quad \text{for a semi submerged sphere.}$$

Substituting $D(s, \beta)$ into (24), we have

$$C(\theta)+i\,S(\theta)=-i\,4a\sec^3\theta\left\{\cos\theta\int_1^{\infty}P(s)J_1\left(\frac{s\sec^2\theta}{2F_n^2}\right)ds-\cos 3\theta\int_1^{\infty}Q(s)J_3\left(\frac{s\sec^2\theta}{2F_n^2}\right)ds\right\},$$

where J_1, J_3 are Bessel functions, and the cross area of a body and the still water plane is excluded from the integral range, since $D(s, \beta)=0$ inside the body.

The asymptotic expansion of the amplitude function in the low speed is expressed as

$$C(\theta)+i\,S(\theta)\sim -i\frac{16a}{\sqrt{\pi}}F_n^3[P(1)\cos\theta+Q(1)\cos 3\theta]\cos\left(\frac{1}{2F_n^2}\sec^2\theta-\frac{\pi}{4}\right)+O(F_n^5), \tag{29}$$

where $P(1)=1$, $Q(1)=-1$ for a cylinder, and $P(1)=75/128$, $Q(1)=-27/128$ for a semi submerged sphere.

The wave making resistance coefficient C_w in low speed limit is expressed as follows.

$$C_w=R_w\Big/\frac{1}{2}\rho U^2(2a)^2\sim\frac{8192}{315}F_n^6+O(F_n^8)\quad\text{for a vertical, circular cylinder,} \tag{30}$$

$$C_w\sim\frac{633}{70}F_n^6+\frac{9}{2}\sqrt{\pi}\,F_n^7\sin\left(\frac{1}{F_n^2}+\frac{\pi}{4}\right)+O(F_n^8)\quad\text{for a semi submerged sphere.} \tag{31}$$

C_w values are shown in Fig. 12 comparing with experimental values of a simple ship model M. 2201 used for the flow measurement in section 2 and of a conventional full form of $C_b=0.84$. It is found that the order of magnitude of theoretical values agrees with that of experimental values which are determined by Hughes' method ($C_w=C_{\text{total}}-(1+k)C_{f\,\text{Hughes}}$, k is the form factor). It is also shown hat the trend of theoretical C_w curves with respect to Froude number resembles that of experimental curves.

This result encourages us to apply the present theory to the study of free surface flow around slowly moving full forms.

For the comparison with other theories we calculated wave making resistance of a vertical, circular cylinder by two other methods. The first method is to calculate wave making resistance due to the surface source distribution on the cylinder which is determined by the so-called zero-Froude-number approximation[6]. This is the conventional method of calculating wave making resistance. The second method is the Brard's one which includes the contribution of the line singularity around the intersection of the body and the still water plane in addition to the above surface source distribution[7].

In the conventional method the amplitude function due to the surface source distribution for a vertical, circular cylinder ($b/a=\epsilon\longrightarrow 1$ in (39) of Brard's paper together with $C=-U$) is written as

$$B_s(\theta)+iA_s(\theta)=-i4a\,J_1\left(\frac{1}{2F_n^2}\sec^2\theta\right). \tag{32}$$

The asymptotic form of C_w at low speed limit is expressed as

$$C_w\sim\frac{128}{15}F_n^2-8\sqrt{\pi}\,F_n^3\sin\left(\frac{1}{F_n^2}+\frac{\pi}{4}\right)+O(F_n^4). \tag{33}$$

The order of magnitude of C_w is different from (30) by F_n^4. Therefore it is evident that this formula gives practically unacceptable high values in low speed.

In the second method, the amplitude function due to the line singularity is derived as follows for a vertical circular cylinder ($\epsilon\longrightarrow 1$ in (52) of Brard's paper):

$$\delta B_s(\theta)+i\delta A_s(\theta)=i\,4a\,J_1\left(\frac{1}{2F_n^2}\sec^2\theta\right)+i\,8aF_n^2\sec\theta\cos 3\theta\,J_0\left(\frac{1}{2F_n^2}\sec^2\theta\right)$$
$$-i\,32aF_n^4\cos\theta\cos 3\theta\,J_1\left(\frac{1}{2F_n^2}\sec^2\theta\right). \tag{34}$$

It should be noted that the first term cancels the amplitude function due to the surface source distribution. The asymptotic form of the sum of both amplitude functions (32) and (34) in low speed is written as

$$B_s(\theta)+i\,A_s(\theta)+\delta B_s(\theta)+i\,\delta As(\theta)\sim i\frac{16a}{\sqrt{\pi}}F_n^3\cos 3\theta\cos\left(\frac{1}{2F_n^2}\sec^2\theta-\frac{\pi}{4}\right)+O(F_n^5). \tag{35}$$

This is quite similar to the amplitude function (29) derived by the present theory. The order of magnitude of both expressions agrees each other, viz., $O(F_n^3)$. The asymptotic form of C_w at low speed limit is expressed as

$$C_w \sim \frac{6656}{315} F_n{}^6 + 32\sqrt{\pi}\, F_n{}^7 \sin\left(\frac{1}{F_n{}^2} + \frac{\pi}{4}\right) + O(F_n{}^8). \tag{36}$$

C_w values expressed by (36) are also shown in Fig. 12. C_w curve by Brard's theory is rather oscillatory compared with that by the present theory where the oscillatory term is disappeared as a higher order

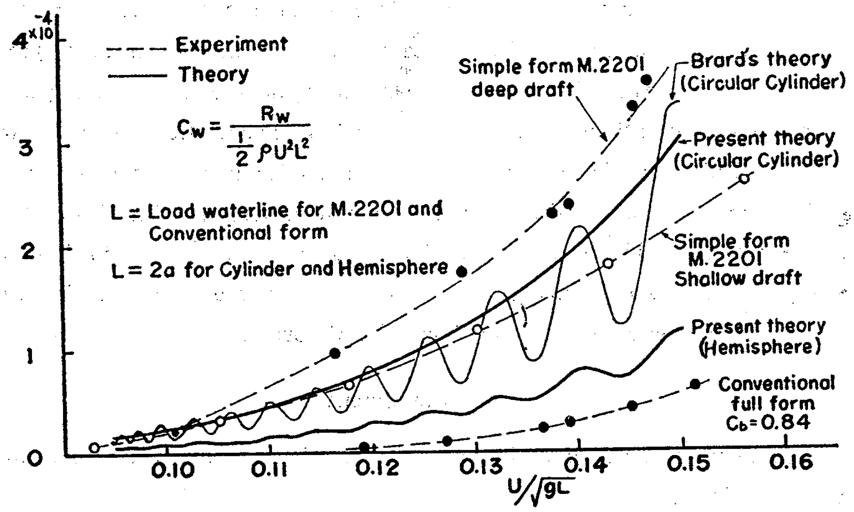

Fig. 12 Comparison of wave making resistance

quantity for the case of a virtical, circular cylinder. The reason for the strong oscillatory property is due to the linearization of the problem in Brard's theory as explained in the followings.

For a vertical, circular cylinder $D(x, y)$ is rewritten from (11) as

$$D(x, y) = (U + \varphi_{rx})\left\{-\frac{U}{g}\varphi_{rxx} - \frac{1}{2g}\frac{\partial}{\partial x}(\varphi_{rx}^2 + \varphi_{ry}^2)\right\} + \varphi_{ry}\left\{-\frac{U}{g}\varphi_{rxy} - \frac{1}{2g}\frac{\partial}{\partial y}(\varphi_{rx}^2 + \varphi_{ry}^2)\right\}, \tag{37}$$

where φ_r is the perturbation velocity potential of the double model approximation:

$$\varphi_r(x, y) = \phi_r(x, y) - Ux = \frac{Ua^2 x}{x^2 + y^2} \quad \text{for a vertical, circular cylinder.}$$

Neglecting the square and the cubic terms of the perturbation velocities, we have

$$D(x, y) = -\frac{U^2}{g}\varphi_{rxx}(x, y) = \frac{2U^3 a^2}{g}\left[\frac{-x^3 + 3xy^2}{(x^2 + y^2)^3}\right].$$

The non-dimensional expression of $D(x, y)$ in the polar coordinate system is written as

$$\frac{D(s, \beta)}{4UF_n{}^2} = -s^{-3}\cos 3\beta. \tag{38}$$

This expression coincides with the second term of the right hand side of (28), viz., $P(s)=0$. Therefore the asymptotic form of the amplitude function in the low speed limit is expressed from (29) as

$$C(\theta) + i\,S(\theta) \sim i\frac{16a}{\sqrt{\pi}} F_n{}^3 \cos 3\theta \cos\left(\frac{1}{2F_n{}^2}\sec^2\theta - \frac{\pi}{4}\right) + O(F_n{}^5), \tag{39}$$

which is exactly the same as Brard's amplitude function (35). This theoretical result indicates that the reason for the strong oscillatory property in Brard's theory is due to the linearization of the problem. In the low speed limit, as Brard suspected, the linearized free surface condition becomes less and less accurate. Then we may say that the present higher order theory is one of the ways to overcome the lack of accuracy arising in the low speed problem with the linearized free surface condition.

5. Concluding remarks

The present theoretical study was motivated by the results of flow measurement around bow of a slowly moving full form. By the extension of Ogilvie's wave resistance theory in low speed limit, a theory to analyze the thin boundary layer near the free surface around full forms was developed. From the calculation of wave making resistance of a vertical, infinite, circular cylinder and a semi submerged sphere it was found that the present theory is applicable to the analytical study of the free surface flow around conventional full forms in the low speed. Calculation of wave making resistance for conventional full forms is left as a future work. Further the calculation of wave pattern around bow of full forms is also awaited for the direct comparison with observations. As Ogilvie stated, for reasonable determination of the level of viscous-resistance curve in low speed limit, the present theory is hoped to be used. Further the present theory is expected to give a new understanding of complex free surface phenomena around bow of full forms.

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