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On the Shearing Deformation in Cross Section of Pure Car Carriers

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Summary

It is well known that the racking phenomenon is the most important structural design problem for a ship having few transverse bulkhead. This paper reviews the results of study on the method of calculation of the racking deformation in the cross section of such a ship.

First, the authors explain the method of estimation of deviation loads responsible for the racking. The authors define the deviation loads as differences between the actual loads and beam loads. The beam loads are assumed to be acting, proportionately to the resultant shear forces, on the cross section of each longitudinal strength member as an elastic thin walled beam.

Secondly, the authors examine the distribution of racking as the shearing deformations in cross sections of a Pure Car Carrier, which is capable of carrying 3000 motor cars, by somewhat simplified structural idealization.

1. Introduction

The authors consider a ship going through the waves. In this instance, loads including wave loads and inertia forces are acting on an arbitrary cross section of the ship as shown in Fig. 1(a). The loads can be broken down into two components; the one is symmetric in relation to the vertical centre line of a cross section and the other is inversely symmetric, as shown in Fig. 1 (b) and (c), respectively. The former causes the ship hull to bend vertically while the latter causes the hull to bend horizontally and also to twist with respect to the shear centre. Generally, the vertical bending strength of a ship hull is calculated as a beam passing through the centre of gravity of the hull cross section. And also, the horizontal bending and torsional strength of a ship hull are calculated as a beam passing through the shear centre of the hull cross section. In either instance, it is assumed that the cross section of the ship hull does not deform. This assumption can be true only where the external force acting on each longitudinal strength member, such as upper deck, side shell, bottom shell, etc. is proportional to the resultant shearing force in the cross section of the longitudinal strength member evaluated by an ordinary theory of elastic thin walled beam. Actually, however, it generally is the case that the external force does not act on each longitudinal strength member in that manner. Therefore, though a ship hull is generally considered to deform as a beam, each longitudinal strength member undergoes its own deformation which is different from that of a beam. According to this theory, M. Yamakoshi, et. al. proposed a method of calculations of shearing deformation of wing tank.[1),2)]

The authors apply this theory to the study of horizontal bending to calculate what is called racking deformation as shearing deformation of a hull cross section. In this case, the torsion will also be studied at the same time.

J. Yagi, et. al. made valuable researches on the phenomenon of racking of ship hull.[3),4)] They investigated the phenomenon of racking by calculating the structural response of the hull under the loads acting as shown in Fig. 2(c). For that purpose, they converted the loads shown in Fig. 1(c) into the equivalent concentrat-

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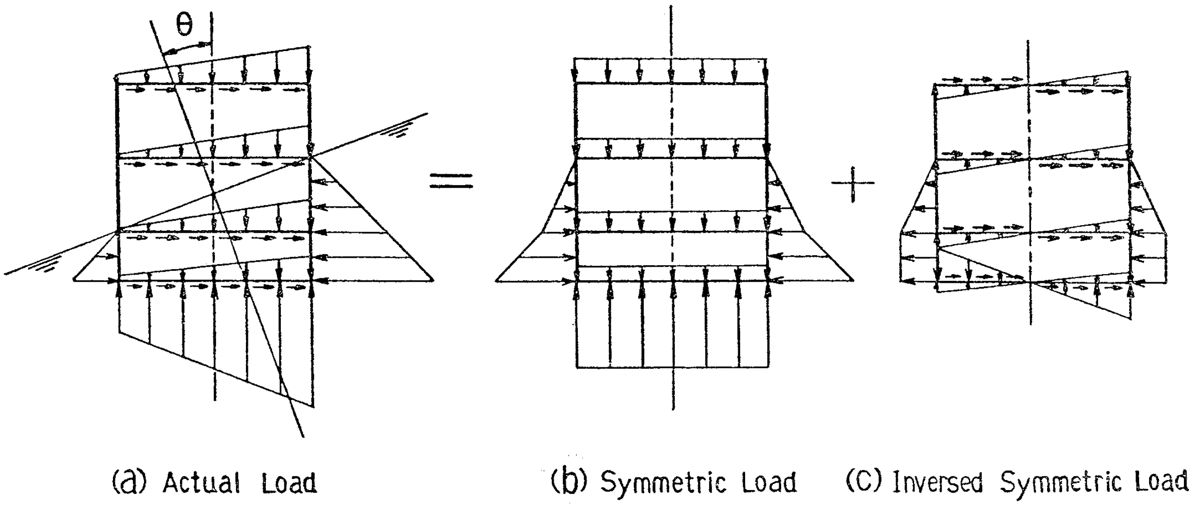

Fig. 1 Actual Load on the Cross Section of a Ship's Hull

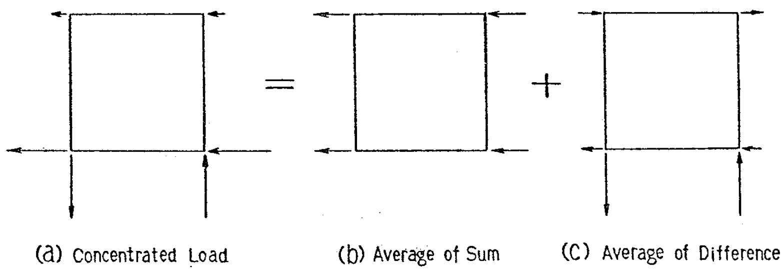

Fig. 2 Resolution of Inversed Symmetric Load

ed loads as shown in Fig. 2(a). And then, they divided the equivalent concentrated loads into two components, i.e, the average of sum as indicated in Fig. 2(b) and the average of difference as shown in Fig. 2(c), respectively, and treated the loads shown in Fig. 2(c) as the loads responsible for the racking.

The loads shown in Fig. 2(a), however, cause not only the horizontal bending but twisting of the ship hull. It is, therefore, necessary to exclude not only the loads shown in Fig. 2(b) which cause the horizontal bending, but also all the loads which cause the twisting with respect to the shear centre, for the exact evaluation of the racking deformation.

2. Loads Responsible for Racking

The symmetric and inversed symmetric loads per unit length of the hull girder shown in Fig. 1 consist of components which are shown in Table 1. It is obvious that the inversed symmetric components cause the racking. The inversed symmetric components can be divided into four groups as follows.

(1) load components acting on each deck in the transverse direction, namely;
$p_y^I,\ q_{yj}^I,\ \bar{q}_{yj}^I,\ r_{yj}^I,\ \bar{r}_{yj}^I$

(2) moments of the above loads with respect to the shear centre, namely;
$m_{py},\ m_{qyj},\ \bar{m}_{qyj},\ m_{ryj},\ \bar{m}_{ryj}$

(3) load components acting on each deck in direction of depth, namely;
$p_z^I,\ r_{zj}^I,\ \bar{r}_{zj}^I$

(4) moments of the above loads with respect to the shear centre, namely;
$m_{pz},\ m_{rzj},\ \bar{m}_{rzj}$

For convenience, water pressures acting on the ship sides are converted into concentrated forces on the bottom shell and each deck and are denoted with the notations of p_{yj}^I, etc. Also, the moment m_{py} is divided into the components m_{pyj} corresponding to each p_{yj}^I. Then, the resultant forces and moments can be calculated by Eqs. (1).

$$\left.\begin{aligned}
F_y &= \sum_j F_{yj} \\
&= \sum_j (2p_{yj}^I + q_{yj}^I + \bar{q}_{yj}^I + r_{yj}^I + \bar{r}_{yj}^I) \\
F_z &= \sum_j F_{zj} \\
&= p_{z1}^I + \sum_j (r_{zj}^I + \bar{r}_{zj}^I) \\
M_y &= \sum_j M_{yj} \\
&= \sum_j (2m_{pyj} + m_{qyj} + \bar{m}_{qyj} + m_{ryj} + \bar{m}_{ryj}) \\
M_z &= \sum_j M_{zj} \\
&= \sum_j (m_{pz} + m_{rzj} + \bar{m}_{rzj})
\end{aligned}\right\} \quad (1)$$

The resultant force F_y causes the horizontal

Table 1 Details of Symmetric Load and Inversed Symmetric Load

Load	Direction	Notation	Symmetric Load: Notation	Inversed Symmetric Load: Notation	Load Distribution	Moment with respect to S
Water Pressure	Z-Direction (Bottom)	p_z	p_z^S	p_z^I	p_y^I, p_z^I	m_{pz}
	Y-Direction (Side)	p_y	p_y^S	$2p_y^I$		$2m_{py}$
Deck Load	Z-Direction	q_{zj}	q_{zj}^S	—	q_{yj}^I	—
	Y-Direction	q_{yj}	—	q_{yj}^I		m_{qyj}
Hull Weight	Z-Direction	$\bar{q}_{zj}$	$\bar{q}_{zj}^S$	—	$\bar{q}_{yj}^I$	—
	Y-Direction	$\bar{q}_{yj}$	—	$\bar{q}_{yj}^I$		$\bar{m}_{qyj}$
Inertia Force of Deck Load	Z-Direction	r_{zj}	—	r_{zj}^I	r_{zj}^I, r_{yj}^I	m_{rzj}
	Y-Direction	r_{yj}	—	r_{yj}^I		m_{ryj}
Inertia Force of Hull Weight	Z-Direction	$\bar{r}_{zj}$	—	$\bar{r}_{zj}^I$	$\bar{r}_{zj}^I$, $\bar{r}_{yj}^I$	$\bar{m}_{rzj}$
	Y-Direction	$\bar{r}_{yj}$	—	$\bar{r}_{yj}^I$		$\bar{m}_{ryj}$

bending of the ship hull and also the resultant moment M_y, which is produced by F_y, causes the twisting of the ship hull. The resultant force F_z is zero because each component of F_z has no resultant force. It however has the resultant moment because each component of F_z has the resultant moment with respect to the vertical centre line of hull cross section. This moment, denoted with M_z, causes the twisting of the ship hull.

2.1 Deviation Loads with respect to Horizontal Force F_y

Let us consider the deviation loads with respect to F_y. It is well known that the ship hull is bent horizontally and at the same time is twisted when the force F_y acts horizontally on the hull. Let the increments of shearing force acting on the j-th deck per unit length of the hull girder be equal to $(\alpha_j F_y)$ when the ship hull is bent horizontally according to the theory of an elastic thin walled beam. If each component of F_y acting on the j-th deck is distributed in proportion to $(\alpha_j F_y)$, there should be no shearing deformation of the hull cross section but only the horizontal bending deflection.

Next, let the moment of F_y with respect to the shear centre of the cross section be M_y. It can be understood easily that the moment M_y constitutes the elementary torsional moment for this cross section. Let the increments of shearing force acting on the cross section in way of the j-th deck per unit length of the hull girder be equal to $(\beta_j M_y/D)$ when the ship hull is twisted by the torsional moment M_y according to the theory of torsion of an elastic thin walled beam.

If each component of F_y acting on the j-th deck, which produces the elementary torsional moment M_y, were distributed in proportion to $(\beta_j M_y/D)$, then resultant moment of F_y would only cause the twisting of the hull cross section but no shearing deformation. Actually, however, the external force F_{yj} acts on the j-th deck. Let differences between the actual load acting on the j-th deck F_{yj} and the resultants of $(\alpha_j F_y)$ and $(\beta_j M_y/D)$ be the deviation loads on the j-th deck. Then the deviation load H_j^y and H_s^y acting on each longitudinal strength member are given by Eqs. (2) (refer to the Fig. 3(a), (b), (c) and (d)).

$$\left.\begin{aligned} &\text{for the } j\text{-th deck;}\\ &\quad H_j^y = F_{yj} - (\alpha_j F_y + \beta_j M_y/D)\\ &\text{for the side shell;}\\ &\quad H_s^y = -\beta_s M_y/B \text{ (starboard side)}\\ &\qquad\;\; = +\beta_s M_y/B \text{ (port side)} \end{aligned}\right\} \tag{2}$$

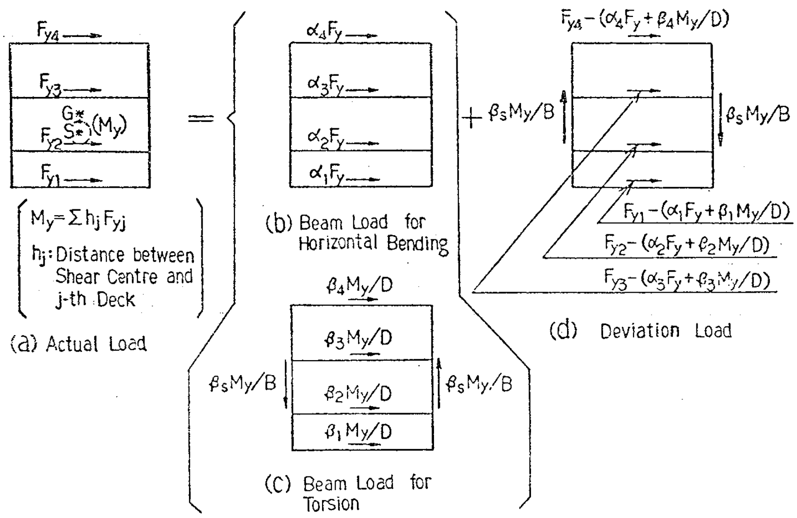

Fig. 3 Resolution of Horizontal Actual Load

The coefficient α_j is the ratio of the shearing force on the j-th deck to the total shearing force in the hull cross section when the ship hull is bent horizontally according to the theory of an elastic thin walled beam. The method of calculation of α_j is shown in Appendix 1. And also the coefficient β_j and β_s are the ratios of the shearing force on the j-th deck and side shell to the arbitrary torsional moment M divided by D or B, respectively when the ship hull is twisted with respect to its shear centre according to the theory of an elastic thin walled beam. The methods of calculation of β_j and β_s are shown in Appendix 2.

2.2 Deviation Loads with respect to Vertical Force F_z

Let us consider the deviation loads with respect to F_z. It is obvious that the load components p_z^I, r_{zj}^I and $\bar{r}_{zj}^I$ have no resultant forces but resultant moments with respect to the shear centre of the hull cross section (i.e. with respect to the vertical centre line of the hull cross section). The inversed symmetric loads p_z^I, r_{zi}^I and $\bar{r}_{zj}^I$ are converted into the concentrated forces F_{pzsj}, F_{rzsj} and $\bar{F}_{rzsj}$ acting on each side shell, respectively. Let the total resultant moments be M_z. This M_z constitutes the elementary torsional moment for the ship hull. As discussed in **2.1**, let the increments of shearing force acting on the j-th deck per unit length of the hull girder be equal to $(\beta_j M_z/D)$ when the ship hull is twisted according to the theory of an elastic thin walled beam. If each component of F_{zs} acting on the j-th deck, which produces the moment M_z, were distributed in proportion to $(\beta_j M_z/D)$, then resultant moment of F_{zs} would only cause the twisting of the hull cross section and no shearing deformation. In this case, the deviation load H_j^z and H_s^z acting on each longitudinal strength member are given by Eqs. (3) (refer to the Fig. 4(a), (b) and (c)).

$$\left.\begin{array}{l}\text{for the } j\text{-th deck;}\\ \quad H_j^z=-\beta_j M_z/D\\ \text{for the side shell;}\\ \quad H_s^z=F_{zs}-\beta_s M_z/B \ \text{(starboard side)}\\ \qquad =-F_{zs}+\beta_s M_z/B \ \text{(port side)}\end{array}\right\} \quad (3)$$

where

$$M_z=B\cdot F_{zs} \quad (4)$$

$$F_{zs}=\sum_j F_{zsj} \quad (5)$$

$$F_{zsj}=F_{pzsj}+F_{rzsj}+\bar{F}_{rzsj} \quad (6)$$

$$\left.\begin{array}{l}F_{pzsj}=m_{pzj}/B\\ F_{rzsj}=m_{rzj}/B\\ \bar{F}_{rzsj}=\bar{m}_{rzj}/B\end{array}\right\} \quad (7)$$

The deviation loads with respect to the horizontal bending load F_y and the torsional moment M_y and M_z can be obtained as outlined above. The differences between the actual loads shown in Fig. 3(a) and Fig. 4(a) and the loads on an elastic thin walled beam under horizontal bending and torsion shown in Fig. 3(b), 3(c) and Fig. 4(b) constitute the deviation loads as shown in Fig. 3-(d) and Fig. 4(c). It is obvious that the deviation loads cause the racking of the hull cross section. For a ship having 4 decks the deviation load H_j and H_s acting on each longitudinal strength member are given by Eqs. (8). In this case, the double bottom is considered as the 1-st deck.

$$\left.\begin{array}{l}\text{for the 4-th deck;}\\ \quad H_4=F_{y4}-\{\alpha_4 F_y+\beta_4(M_y+M_z)/D\}\\ \text{for the 3-rd deck;}\\ \quad H_3=F_{y3}-\{\alpha_3 F_y+\beta_3(M_y+M_z)/D\}\\ \text{for the 2-nd deck;}\\ \quad H_2=F_{y2}-\{\alpha_2 F_y+\beta_2(M_y+M_z)/D\}\\ \text{for the 1-st deck;}\\ \quad H_1=F_{y1}-\{\alpha_1 F_y+\beta_1(M_y+M_z)/D\}\\ \text{for the side shell in starboard side;}\\ \quad H_{s1}=F_{zs}-\beta_s(M_y+M_z)/B\\ \text{for the side shell in port side;}\\ \quad H_{s2}=-F_{zs}+\beta_s(M_y+M_z)/B\end{array}\right\} \quad (8)$$

It can be proved easily that the deviation loads given by Eqs. (8) have no resultant forces and resultant moments. Thus, the authors can calculate the distribution of shearing deformation, or the racking, of hull cross sections, using the three dimensional structural model under the deviation loads.

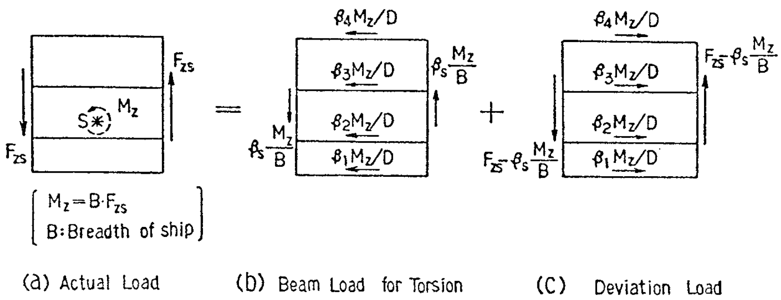

Fig. 4 Resolution of Vertical Actual Load

3. Example of Numerical Calculation

3.1 Ship Subjected to Study

In order to examine the characteristics of the racking deformation, the authors carried out the numerical calculations using a structural model of Pure Car Carrier, assuming that the model is subjected to deviation loads which cause the racking deformation as described in 2. The rough midship section of the ship is shown in Fig. 5. The principal dimensions are as follows;

$L \times B \times D - d$: 160 m × 25.6 m × 22.72 m − 7.22 m

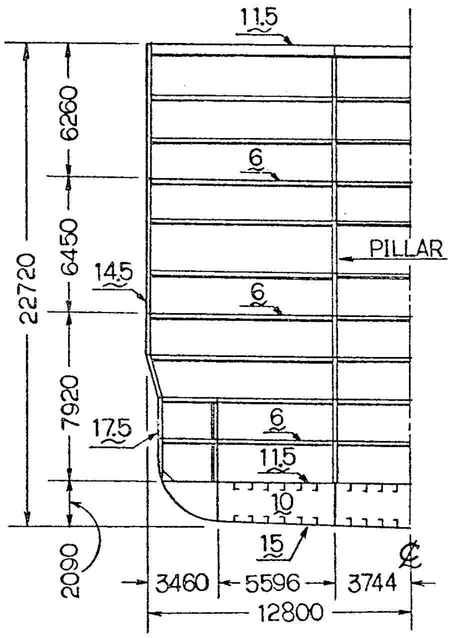

Fig. 5 Rough Midship Section of Pure Car Carrier

3.2 Idealization of Structure

The authors idealized a port half of the ship as a three dimensional finite element model as shown in Fig. 6. Deck plates, side shells and transverse bulkheads were idealized as plate elements. Further, deck beams, deck girders, side frames and pillars were idealized as frame elements. The finite element idealization of the midship section is shown in Fig. 7. For simplicity, eleven decks in the actual ship were considered to be four for the finite element idealization. The authors determined the equivalent rigidities of side frames, after comparing the rigidities of the eleven-deck-structure with those of the four-deck-structure by the in-plane frame calculations. In the numerical calculations, as it was our purpose to roughly investigate the behaviour of racking, the authors made the simplified structural idealizations.

As to the boundary conditions, the inversed symmetric conditions were given along the

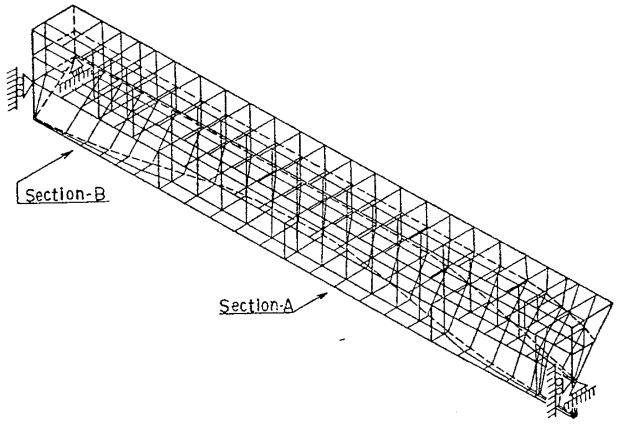

Fig. 6 Three Dimensional Finite Element Idealization

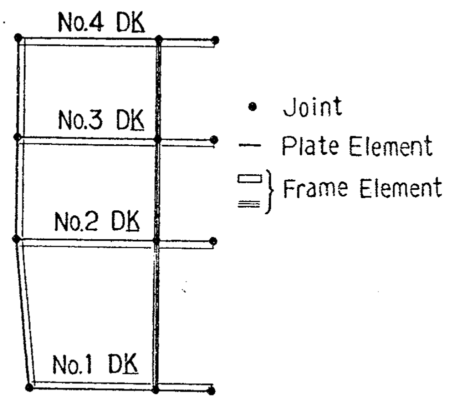

Fig. 7 Idealization of Midship Section

centre line section. Next, the transverse bulkheads at fore and aft ends were assumed to be supported simply in both transverse and vertical directions as shown in Fig. 6.

3.3 Load Conditions

The authors divided the full load displacement in proportion to the ratios of numbers of the simplified decks and distributed the divided full load displacement as cargo and hull weights on each deck uniformly along the whole hull length. As a result, the deck load was 0.6 ton/m^2 on the fourth deck, 0.9 ton/m^2 on the third and second decks, and 1.5 ton/m^2 on the first deck, respectively. The authors assumed that the ship was rolling in still water fully loaded, with rolling angle θ of 20 degrees and rolling period T of 20 seconds.

Under the above-mentioned load conditions, the authors calculated the actual loads acting on twenty five cross sections in the finite element idealization. These actual loads consisted of inclined and inertia components of the deck loads and also of still water pressures. Next, the authors determined the shear force coefficients for horizontal force and torsional moment for all the cross sections. Using the Eqs. (8), the authors

Table 2 Resolution of Inversed Symmetric Load on the Cross Section (Section-A, -B of Fig. 6).

Section in Fig.6	Direction of Force	Deck No.	Actual Load (ton)	Deviation Load (ton)	Shear Force Coeffts. for Horiz. Shear Force (α_j, α_s)		Shear Force Coeffts. for Torsion (β_j, β_s)	
Sect.-A	Horizontal Force	DK.4	21.9	49.4	α_4	0.2649	β_4	0.01011
		DK.3	30.8	46.1	α_3	0.1958	β_3	0.00281
		DK.2	−43.1	−31.8	α_2	0.2042	β_2	−0.00137
		DK.1	−72.0	−63.7	α_1	0.3351	β_1	−0.01154
	Vertical Force	DK.4	1.0	−2.4	α_{s4}	−0.0060	β_{s4}	−0.00247
		DK.3	1.5	−5.3	α_{s3}	−0.0047	β_{s3}	−0.00573
		DK.2	20.0	14.6	α_{s2}	0.0240	β_{s2}	−0.00773
		DK.1	138.8	136.7	α_{s1}	0.0226	β_{s1}	−0.00448
Sect.-B	Horizontal Force	DK.4	11.0	14.3	α_4	0.3070	β_4	0.01265
		DK.3	15.4	17.6	α_3	0.2218	β_3	0.00280
		DK.2	−13.5	−10.9	α_2	0.3053	β_2	−0.00749
		DK.1	−22.3	−21.0	α_1	0.1659	β_1	−0.00795
	Vertical Force	DK.4	0.5	0.3	α_{s4}	−0.0054	β_{s4}	−0.00309
		DK.3	0.7	0.5	α_{s3}	0.0004	β_{s3}	−0.00699
		DK.2	30.5	31.1	α_{s2}	0.0488	β_{s2}	−0.00864
		DK.1	33.3	34.0	α_{s1}	0.0431	β_{s1}	−0.00475

calculated the deviation loads acting on the cross sections. In this case, the distributed loads were converted into the concentrated loads acting on the intersections of the deck plates and the side shell. This is the reason why the authors avoided the intricacy of allowing for equilibrium of moments along the inclined side shell.

The actual loads and deviation loads acting on the section-A and -B shown in Fig. 6 are listed in Table 2. The shear force coefficients for horizontal force and torsional moments of section-A and -B are also listed in Table 2.

3.4 Result of Calculations

The results of calculations of racking deformations for section-A and -B are shown in Fig. 8. For more general expression of the racking as the shearing deformation of a hull cross section (due to the deviation loads) shown in Fig. 9, the authors defined the value of the racking γ by Eqs. (9).

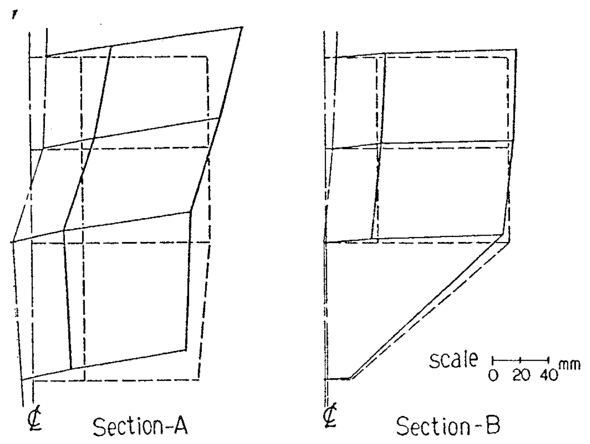

Fig. 8 Racking Deformation of Cross Sections (Section-A, -B of Fig. 6)

$$\left.\begin{aligned} \gamma &= \gamma_V + \gamma_H \\ \gamma_V &= \delta_H / D \\ \gamma_H &= \delta_V / B \end{aligned}\right\} \tag{9}$$

where

$$\delta_H = \delta_{H1} + \delta_{H2} \tag{10}$$

$$\delta_V = 2 \cdot \delta_{V2} \tag{11}$$

Here, γ_V and γ_H denote the relative deflections divided by the span for the vertical and horizontal members, respectively. Displacements δ_{H1}, δ_{H2} and δ_{V2} are shown in Fig. 9.

Distribution of γ and γ_H along the ship hull length are shown in Fig. 10. In Fig. 10, the value of γ are very small at the fore and aft ends, because of the presence of full transverse bulkheads at the both ends.

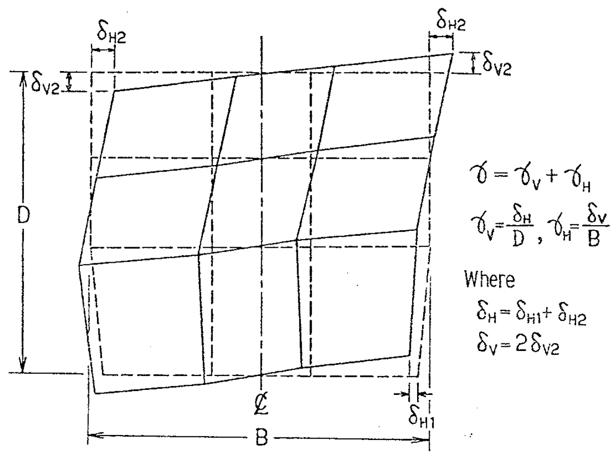

Fig. 9 Definition of Racking as Shearing Deformation of a Cross Section

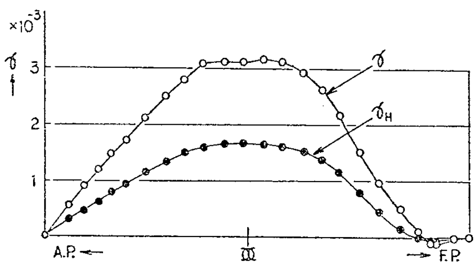

Fig. 10 Distribution of Racking Deformation along the Ship Length

4. Conclusive Remarks

In this paper, the authors made it clear that the racking was induced by the deviation loads with respect to the horizontal bending load and to the loads which cause the twisting of ship hull, and proposed a method of calculation of the deviation loads. The authors are of the opinion that the racking should be defined as the shearing deformations. Applying their theory to somewhat simplified structural model of Pure Car

Carrier, which is capable of carrying 3000 motor cars, the authors carried out the three dimensional strength calculation by the finite element method and showed the distribution of shearing deformations in hull cross sections.

From the results of the investigation, some of the characteristics of the racking were clarified. In the numerical calculations, the authors used the Bredt-Batho's Formula to evaluate the deviation loads with respect to the loads which cause the twisting, but did not take into considerations the warping torsion. Regarding these problems, the authors would like to further continue the investigations including the full scale measurements.

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Appendix 1

Method of Calculation of Shear Force Coefficient α_j, α_{sj} for a Horizontal Force

Shear force coefficients α_j and α_{sj} can be estimated by the theory of elastic thin walled beam.[A1] For convenience of explanation, let us consider the elastic thin walled beam whose cross section is as shown in Fig. A1.

This is the statically indeterminable structure of the third order because of having two bulkheads. Then, it is necessary to determine the shear flow q_1, q_2 and q_3, each being statically indeterminable. Cut off the structure at points A, C and F, then the shear flow q at an arbitrary

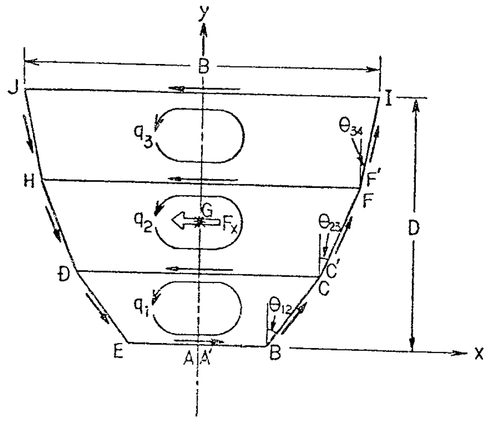

Fig. A1 Symbols of Cross Section for Horizontal Shear Force

point can be given by Eqs. (A-1).

$$
\left.
\begin{aligned}
&\mathrm{A'}\sim\mathrm{B}: \quad q=q_1-\frac{F_x}{I_y}\int_{\mathrm{A'}\to} x\,dA\\
&\mathrm{B}\sim\mathrm{C}: \quad q=q_1-\frac{F_x}{I_y}\int_{\mathrm{A'}\to\mathrm{B}\to} x\,dA\\
&\mathrm{C}\sim\mathrm{D}: \quad q=q_1-q_2-\frac{F_x}{I_y}\int_{\mathrm{A'}\to\mathrm{B}\to\mathrm{C}\to} x\,dA\\
&\mathrm{D}\sim\mathrm{E}: \quad q=q_1-\frac{F_x}{I_y}\int_{\mathrm{A'}\to\mathrm{B}\to\mathrm{C}\to\mathrm{D}\to} x\,dA\\
&\qquad\qquad -\frac{F_x}{I_y}\int_{\mathrm{C'}\to\mathrm{F}\to\mathrm{H}\to\mathrm{D}} x\,dA\\
&\qquad\qquad -\frac{F_x}{I_y}\int_{\mathrm{F'}\to\mathrm{I}\to\mathrm{J}\to\mathrm{H}} x\,dA\\
&\mathrm{E}\sim\mathrm{A}: \quad q=q_1-\frac{F_x}{I_y}\int_{\mathrm{A'}\to\mathrm{B}\to\mathrm{C}\to\mathrm{D}\to\mathrm{E}} x\,dA\\
&\qquad\qquad -\frac{F_x}{I_y}\int_{\mathrm{C'}\to\mathrm{F}\to\mathrm{H}\to\mathrm{D}} x\,dA\\
&\qquad\qquad -\frac{F_x}{I_y}\int_{\mathrm{F'}\to\mathrm{I}\to\mathrm{J}\to\mathrm{H}} x\,dA\\
&\mathrm{C'}\sim\mathrm{F}: \quad q=q_2-\frac{F_x}{I_y}\int_{\mathrm{C'}\to} x\,dA\\
&\mathrm{F}\sim\mathrm{H}: \quad q=q_2-q_3-\frac{F_x}{I_y}\int_{\mathrm{C'}\to\mathrm{F}\to} x\,dA\\
&\mathrm{H}\sim\mathrm{D}: \quad q=q_2-\frac{F_x}{I_y}\int_{\mathrm{C'}\to\mathrm{F}\to\mathrm{H}\to} x\,dA\\
&\qquad\qquad -\frac{F_x}{I_y}\int_{\mathrm{F'}\to\mathrm{I}\to\mathrm{J}\to\mathrm{H}} x\,dA\\
&\mathrm{F'}\sim\mathrm{I}: \quad q=q_3-\frac{F_x}{I_y}\int_{\mathrm{F'}\to} x\,dA\\
&\mathrm{I}\sim\mathrm{J}: \quad q=q_3-\frac{F_x}{I_y}\int_{\mathrm{F'}\to\mathrm{I}\to} x\,dA\\
&\mathrm{J}\sim\mathrm{H}: \quad q=q_3-\frac{F_x}{I_y}\int_{\mathrm{F'}\to\mathrm{I}\to\mathrm{J}\to} x\,dA
\end{aligned}
\right\} \quad \text{(A-1)}
$$

where, dA denotes the effective elemental area for bending, and the integral $\int_{\mathrm{A'}\to}$ denotes, for example, the integration from point A′ to an arbitrary point between A′ and B. F_x denotes the horizontal force and I_y denotes the moment of inertia of the cross section with respect to the y-axis. There must be no discrepancy between points A and A′, points C and C′ and points F and F′, respectively, so Eqs. (A-2) can be obtained from which the shear flow q_1, q_2 and q_3 can be determined.

$$\left.\begin{array}{l} \displaystyle\int_{A'\to B\to C\to E\to A} \frac{q}{t}ds=0 \\ \displaystyle\int_{C'\to F\to H\to D\to C} \frac{q}{t}ds=0 \\ \displaystyle\int_{F'\to I\to J\to H\to F} \frac{q}{t}ds=0 \end{array}\right\} \qquad (A\text{–}2)$$

By substituting q of Eqs. (A-1) into Eqs. (A-2), the simultaneous equations for q_1, q_2 and q_3 can be obtained. q_1, q_2 and q_3 are determined by solving Eqs. (A-2), and then the shear flow distribution with respect to F_x can be obtained by substituting these values into Eqs. (A-1).

Thus, coefficient $\bar{\alpha}_j$ is obtained by integrating the shear flow q_j in the j-th deck across the whole width of this deck, and then dividing by F_x for normalization as given by Eq. (A-3).

$$\bar{\alpha}_j=\int_j q_j ds/F_x \quad (j=\sim 1\sim 4) \qquad (A\text{–}3)$$

Similarly, coefficients α_{BC} etc. can be obtained by integrating the shear flow q_s along the side shell, and then dividing by F_x for normalization as given Eqs. (A-4).

$$\left.\begin{array}{l} \alpha_{BC}=\displaystyle\int_{B\to C} q_s ds/F_x \\ \alpha_{C'F}=\displaystyle\int_{C'\to F} q_s ds/F_x \\ \alpha_{F'I}=\displaystyle\int_{F'\to I} q_s ds/F_x \end{array}\right\} \qquad (A\text{–}4)$$

And also, α_{sj} is given by Eqs. (A-5).

$$\left.\begin{array}{l} \alpha_{s1}=\alpha_{BC}\cdot\cos\theta_{12}/2 \\ \alpha_{s2}=(\alpha_{BC}\cos\theta_{12}+\alpha_{C'F}\cos\theta_{23})/2 \\ \alpha_{s3}=(\alpha_{C'F}\cos\theta_{23}+\alpha_{F'I}\cos\theta_{34})/2 \\ \alpha_{s4}=\alpha_{F'I}\cos\theta_{34}/2 \end{array}\right\} \qquad (A\text{–}5)$$

where angles θ_{12} etc. are shown in Fig. A1.

Next, coefficient α_j can be given by Eqs. (A-6).

$$\alpha_j=\bar{\alpha}_j+\alpha'_{sj} \quad (j=1\sim 4) \qquad (A\text{–}6)$$

where $\bar{\alpha}_j$ is given by Eq. (A–3) and α'_{sj} is given by Eqs. (A-7).

$$\left.\begin{array}{l} \alpha'_{s1}=\alpha_{BC}\sin\theta_{12}/2 \\ \alpha'_{s2}=(\alpha_{BC}\sin\theta_{12}+\alpha_{C'F}\sin\theta_{23})/2 \\ \alpha'_{s3}=(\alpha_{C'F}\sin\theta_{23}+\alpha_{F'I}\sin\theta_{34})/2 \\ \alpha'_{s4}=\alpha_{F'I}\sin\theta_{34}/2 \end{array}\right\} \qquad (A\text{–}7)$$

Appendix 2

Method of Calculation of Shear Force Coefficients β_j, β_{sj} for a Torsional Moment

Shear force coefficients β_j and β_{sj} can be estimated by the theory of torsion of an elastic thin walled beam.[A2)] For convenience of explanation, let us consider the elastic thin walled beam whose cross section is as shown in Fig. A2. Let the shear flow in the 1-st deck, 2-nd deck, 3-rd deck and the 4-th deck be f_1, f_2, f_3 and f_4 respectively, and the shear flow in the side shell between point C and D be f_{23}, and also let the torsional angle per unit length be θ when the torsional moment is T. Then, from the condition of continuity of shear flow at points C and D, Eqs. (A-8) are obtained.

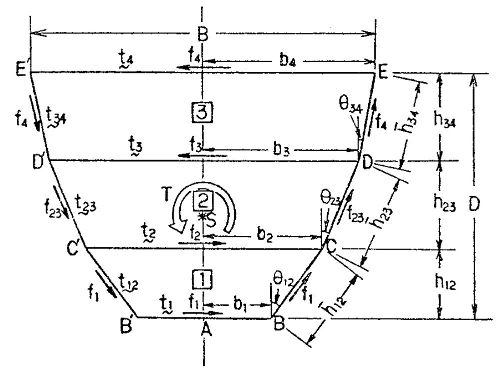

Fig. A2 Symbols of Cross Section for Torsion

$$\left.\begin{array}{l} f_1+f_2-f_{23}=0 \\ f_{23}-f_3-f_4=0 \end{array}\right\} \qquad (A\text{–}8)$$

According to the theory of torsion of elastic thin walled section, Eqs. (A-9) are obtained.

$$\left.\begin{array}{l} f_1\left(\dfrac{2b_1}{t_1}+\dfrac{2\bar{h}_{12}}{t_{12}}\right)-f_2\dfrac{2b_2}{t_2}=2G\theta A_1 \\ f_2\dfrac{2b_2}{t_2}+f_{23}\dfrac{2\bar{h}_{23}}{t_{23}}+f_3\dfrac{2b_3}{t_3}=2G\theta A_2 \\ -f_3\dfrac{2b_3}{t_3}+f_4\left(\dfrac{2\bar{h}_{34}}{t_{34}}+\dfrac{2b_4}{t_4}\right)=2G\theta A_3 \end{array}\right\} \qquad (A\text{–}9)$$

And from the condition of equilibrium of shear flow and the external torsional moment, Eq. (A-10) is obtained.

$$T=2(A_1f_1+A_2f_{23}+A_3f_4) \qquad (A\text{–}10)$$

where, A_1, A_2 and A_3 denote the enclosed areas $\boxed{1}$, $\boxed{2}$ and $\boxed{3}$ in Fig. A2, respectively.

Solving the simultaneous equations (A-8), (A-9) and (A-10), all the unknowns of f_1, f_2, f_3, f_4, f_{23} and θ can be determined easily. Thus, the shear force coefficient $\bar{\beta}_j$ can be obtained by integrating shear flow in the j-th deck across the whole width of this deck, and then dividing by T/D for the normalization as given by Eq. (A-11).

$$\bar{\beta}_j=\int_j f_j ds/(T/D) \quad (j=1\sim4) \tag{A-11}$$

Similarly, the shear force coefficient β_s can be obtained by integrating shear flow f_j in the j-th member along the side shell and then dividing by T/B for the normalization as given by Eqs. (A-12).

$$\left.\begin{aligned}\beta_{BC}&=\int_{B\to C} f_1 ds/(T/B)\\ \beta_{CD}&=\int_{C\to D} f_{23} ds/(T/B)\\ \beta_{DE}&=\int_{D\to E} f_4 ds/(T/B)\end{aligned}\right\} \tag{A-12}$$

And also, β_{sj} is given by Eqs. (A-13).

$$\left.\begin{aligned}\beta_{s1}&=\beta_{BC}\cos\theta_{12}/2\\ \beta_{s2}&=(\beta_{BC}\cos\theta_{12}+\beta_{CD}\cos\theta_{23})/2\\ \beta_{s3}&=(\beta_{CD}\cos\theta_{23}+\beta_{DE}\cos\theta_{34})/2\\ \beta_{s4}&=\beta_{DE}\cos\theta_{34}/2\end{aligned}\right\} \tag{A-13}$$

where angles θ_{12} etc. are shown in Fig. A2.

Next, coefficient β_j can be given by Eqs. (A-14).

$$\beta_j=\bar{\beta}_j+\beta'_{sj} \quad (j=1\sim4) \tag{A-14}$$

where $\bar{\beta}_j$ is given by Eq. (A-11) and β'_{sj} is given by Eqs. (A-15).

$$\left.\begin{aligned}\beta'_{s1}&=\beta_{BC}\sin\theta_{12}/2\\ \beta'_{s2}&=(\beta_{BC}\sin\theta_{12}+\beta_{CD}\sin\theta_{23})/2\\ \beta'_{s3}&=(\beta_{CD}\sin\theta_{23}+\beta_{DE}\sin\theta_{34})/2\\ \beta'_{s4}&=\beta_{DE}\sin\theta_{34}/2\end{aligned}\right\} \tag{A-15}$$

References

A1) M. Ōgushi: Riron Senpaku Kōgaku (Chūkan), Kaibundō, 1966.

A2) K. Terazawa: Sentai Kōzō Rikigaku, Kaibundō, 1974.