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Approximate Prediction of Flow Field around Ship Stern by Asymptotic Expansion Method

by Kazuhiro Mori, *Member** and Yasuaki Doi, *Member**

Summary

The separated ship stern flow field is divided into five subregions according to the flow characteristics; potential flow region, boundary layer region, vorticity diffusion region, separated retarding region and viscous sublayer. This is because experimental studies suggest no single approximation of Navier-Stokes equation is valid for the whole flow field.

The asymptotic expansions of velocity or vorticity for each region are assumed by using a small parameter $R_e^{-1/8}$, where R_e is the Reynolds number.

Governing equations for each region are obtained by substituting the asymptotic expansions and picking up the leading terms; vorticity diffusion equation, elliptic type equation including the Reynolds stress terms and laminar boundary layer equation are obtained for the vorticity diffusion region, the separated retarding region and the viscous sublayer respectively.

Numerical calculations are carried out for a flat plate with zero attack angle and a tanker ship model with a simple stern form and compared with experimental data. Promissing results are obtained.

1. Introduction

It is one of the most important problems of ship hydrodynamics to estimate the flow field around ship stern including the affects of separation. Especially it is very urgent, from the practical point of view, to get the wake distribution on propeller disk.

In previous papers[1),2)], the velocity distribution is estimated by solving the vorticity diffusion equation. There well agreements with measurements are obtained except near ship hull, especially at the deep draft. Usually propellers being equipped there, the previous results are not satisfactory for the above mentioned important purpose. It can be pointed out that the vorticity diffusion approximation is not valid near ship hull; more precise prediction is required there.

Navier-Stokes equation (abbreviating *N-S* eq. hereafter) is solely the governing equation for this field. However, it is necessary to simplify it as possible in order to make sure the practical calculations. The possible way to make effective approximations is to follow the characteristics of flow field.

2. Subdivision of Flow Field

In order to get the flow characteristics, experimental studies of stern flow are carried out using 3 m model of MS-02 (the body plan is shown in Fig. 7)[3)]. Figs. 1 and 2 are the typical results of them.

Fig. 1 shows the streamwise velocity profiles on the horizontal plane of 6 cm beneath the free

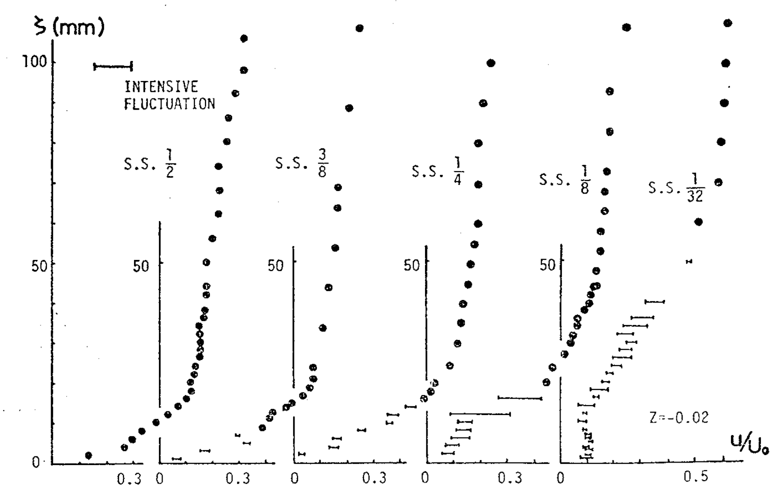

Fig. 1 Velocity profiles near the separation position

* Hiroshima University

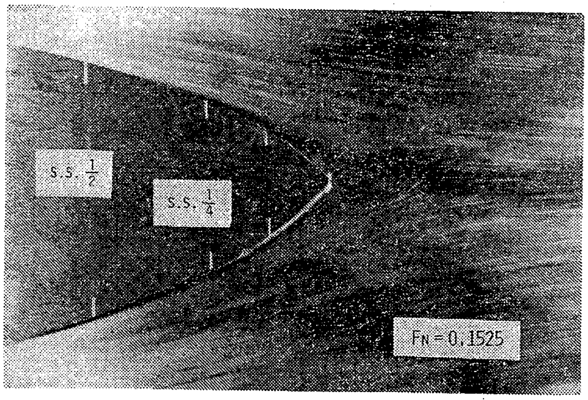

Fig. 2 Free-surface flow near the stern

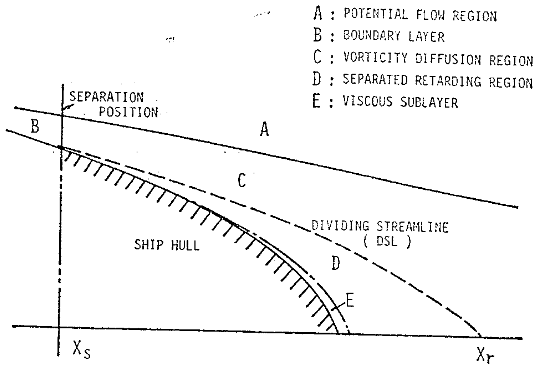

Fig. 3 Subdivision of separated flow near the stern

surface which are measured by a hot anemometer. ζ in Fig. 1 is the normal distance from the hull surface. Although the probe is fixed in uniform flow direction always, flow characteristics can be guessed. The bars mean the velocity is varying between them. There can be supposed a border line outside and inside of which flow characteristics much differ; inside of it flow is very slow and the turbulence is very intensive.

The above situation can be examined more clearly in Fig. 2 which is a free-surface flow picture obtained by the aluminum powder method. Clear differences are observed inside and outside of the border line which may be a kind of dividing streamline. Recirculating flow is existing inside, on the other hand, outside of the dividing streamline, flow is much simpler and almost convective.

It can be mentioned from the above discussions that the flow of separated wake seems to have at least two signifficantly different regions and that no single approximation of *N-S* eq. seems to be uniformly valid for the whole field.

It is proposed to divide the wake flow field into five subregions, as shown in Fig. 3, potential flow region, boundary layer region, vorticity diffusion region, separated retarding region and viscous sublayer region, and it is also proposed to find out the most suitable approximate equations for each subregion.

The characteristics of the subregions are as follows; potential flow region: the region where the viscous terms can be safely neglected and only the displacement effects should be taken into account; boundary layer region: the region where the boundary layer assumption is valid enough and the backward influence of the separation can be neglected; vorticity diffusion region: the region where the vorticity, which has been generated in the boundary layer, is diffused convectively and viscously into the downstream and new vorticity is not generated any more in this region; separated retarding region: the region where the velocity is very small and the turbulence is intensive, because there can be observed a recirculating flow, the governing equation should be elliptic type; viscous sublayer region: the very thin region which just adheres to the hull surface, the molecular viscosity is predominant and the velocity profile should satisfy the no-slip condition on the hull surface.

3. Approximation of *N-S* Equation by Local Asymptotic Expansion

3.1 Basic Equation

The right-hand Cartesian coordinate, $O\text{-}xyz$, and the curvilinear coordinate, $x_1x_2x_3$, are adopted to predict the flow field. The ship is fixed and the uniform flow, whose velocity is U_0, is oncoming to x-direction. The origine of $O\text{-}xyz$ is at the midship and on the waterplane of the fixed ship. x_1 coincides with the potential flow streamlines and x_3 is normal to the hull surface (Fig. 4). The convergences of the curvilinear coordinate, K_1 and K_2 are written by

$$K_1=-\frac{1}{h_1h_2}\frac{\partial h_2}{\partial x_1},\quad K_2=-\frac{1}{h_1h_2}\frac{\partial h_1}{\partial x_2} \tag{1}$$

using the metric coefficients h_i which are defined by

$$h_i^2=\left(\frac{\partial x}{\partial x_i}\right)^2+\left(\frac{\partial y}{\partial x_i}\right)^2+\left(\frac{\partial z}{\partial x_i}\right)^2. \tag{2}$$

If x_3 coordinate can be assumed to be linear, i.e., h_3 being equal to unity, the continuity condition and Reynolds equations are written as follows;

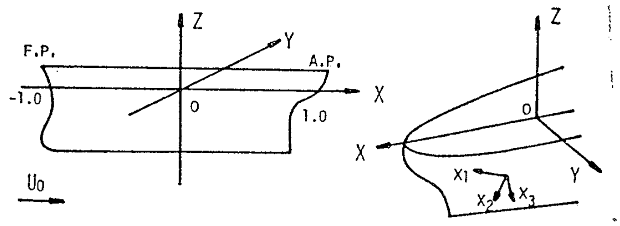

Fig. 4 Coordinate systems

$$\frac{\partial q_1}{h_1\partial x_1}+\frac{\partial q_2}{h_2\partial x_2}+\frac{\partial q_3}{\partial x_3}-K_1q_1-K_2q_2=0\,, \quad (3)$$

$$\frac{q_1}{h_1}\frac{\partial q_1}{\partial x_1}+\frac{q_2}{h_2}\frac{\partial q_1}{\partial x_2}+q_3\frac{\partial q_1}{\partial x_3}-K_2q_1q_2+K_1q_2^2$$
$$=-\frac{\partial}{h_1\partial x_1}\left(\frac{p}{\rho}+q_1'^2\right)-\frac{\partial}{h_2\partial x_2}q_1'q_2'$$
$$-\frac{\partial}{\partial x_3}q_3'q_1'+2K_2q_1'q_2'+K_1(q_1'^2-q_2'^2)$$
$$+\frac{\nu}{h_2}\left\{\frac{\partial}{\partial x_3}(h_2\omega_2)-\frac{\partial\omega_3}{\partial x_2}\right\}, \quad (4a)$$

$$\frac{q_1}{h_1}\frac{\partial q_2}{\partial x_1}+\frac{q_2}{h_2}\frac{\partial q_2}{\partial x_2}+q_3\frac{\partial q_2}{\partial x_3}+K_2q_1^2-K_1q_1q_2$$
$$=-\frac{\partial}{h_2\partial x_2}\left(\frac{p}{\rho}+q_2'^2\right)-\frac{\partial}{\partial x_3}q_2'q_3'$$
$$-\frac{\partial}{h_1\partial x_1}q_1'q_2'-K_2(q_1'^2-q_2'^2)+2K_1q_1'q_2'$$
$$+\frac{\nu}{h_1}\left\{\frac{\partial\omega_3}{\partial x_1}-\frac{\partial}{\partial x_3}(h_1\omega_1)\right\}, \quad (4b)$$

$$\frac{q_1}{h_1}\frac{\partial q_3}{\partial x_1}+\frac{q_2}{h_2}\frac{\partial q_3}{\partial x_2}+q_3\frac{\partial q_3}{\partial x_3}$$
$$=-\frac{\partial}{\partial x_3}\left(\frac{p}{\rho}+q_3'^2\right)-\frac{\partial}{h_1\partial x_1}q_1'q_3'$$
$$-\frac{\partial}{h_2\partial x_2}q_2'q_3'+K_1q_1'q_3'+K_2q_2'q_3'$$
$$+\frac{\nu}{h_1h_2}\left\{\frac{\partial}{\partial x_2}(h_1\omega_1)-\frac{\partial}{\partial x_1}(h_2\omega_2)\right\}, \quad (4c)$$

where q_i and q_i' are time-averaged velocity components and fluctuating parts respectively and ν is the molecular kinematic viscosity, p is the pressure. ω_i is the components of vorticity given by

$$\left.\begin{aligned}\omega_1&=\frac{\partial q_3}{h_2\partial x_2}-\frac{\partial q_2}{\partial x_3}\,,\\ \omega_2&=\frac{\partial q_1}{\partial x_3}-\frac{\partial q_3}{h_1\partial x_1}\,,\\ \omega_3&=\frac{\partial q_2}{h_1\partial x_1}-\frac{\partial q_1}{h_2\partial x_2}+K_2q_1-K_1q_2\,,\end{aligned}\right\} \quad (5)$$

and satisfying

$$\frac{\partial\omega_1}{h_1\partial x_1}+\frac{\partial\omega_2}{h_2\partial x_2}+\frac{\partial\omega_3}{\partial x_3}-K_1\omega_1-K_2\omega_2=0\,. \quad (6)$$

In the reduction of Eqs. (3) and (4) from *N–S* eq., conventional predictions for turbulent components are used; the velocity is assumed to consist of time-averaged terms and fluctuating terms.

On the other hand, if the constant eddy viscosity hypothesis can be assumed, following vorticity diffusion equations are derived from *N–S* eq.;

$$\frac{\partial}{h_2\partial x_2}(\omega_1q_2-\omega_2q_1)+\frac{\partial}{\partial x_3}(\omega_1q_3-\omega_3q_1)$$
$$=\nu_e\left\{\left(\frac{1}{h_2^2}\frac{\partial^2}{\partial x_2^2}+\frac{\partial^2}{\partial x_3^2}\right)\omega_1\right.$$
$$-\frac{\partial}{h_1\partial x_1}\left(\frac{\partial\omega_2}{h_2\partial x_2}+\frac{\partial\omega_3}{\partial x_3}\right)$$
$$-\frac{\partial}{h_2\partial x_2}(K_2\omega_1-K_1\omega_2)$$
$$\left.-K_2\frac{\partial\omega_2}{h_1\partial x_1}+K_{22}\frac{\partial\omega_1}{h_2\partial x_2}\right\}, \quad (7a)$$

$$\frac{\partial}{h_1\partial x_1}(\omega_2q_1-\omega_1q_2)+\frac{\partial}{\partial x_3}(\omega_2q_3-\omega_3q_2)$$
$$=\nu_e\left\{\left(\frac{1}{h_1^2}\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_3^2}\right)\omega_2\right.$$
$$-\frac{\partial}{h_2\partial x_2}\left(\frac{\partial\omega_1}{h_1\partial x_1}+\frac{\partial\omega_3}{\partial x_3}\right)$$
$$-\frac{\partial}{h_1\partial x_1}(K_1\omega_2-K_2\omega_1)$$
$$\left.-K_1\frac{\partial\omega_1}{h_2\partial x_2}+K_{11}\frac{\partial\omega_2}{h_1\partial x_1}\right\}, \quad (7b)$$

$$\left(K_1-\frac{\partial}{h_1\partial x_1}\right)(\omega_1q_3-\omega_3q_1)$$
$$+\left(K_2-\frac{\partial}{h_2\partial x_2}\right)(\omega_2q_3-\omega_3q_2)$$
$$=\nu_e\left\{\left(\frac{1}{h_1^2}\frac{\partial^2}{\partial x_1^2}+\frac{1}{h_2^2}\frac{\partial^2}{\partial x_2^2}\right)\omega_3\right.$$
$$-\frac{\partial}{\partial x_3}\left(\frac{\partial\omega_1}{h_1\partial x_1}+\frac{\partial\omega_2}{h_2\partial x_2}\right)$$
$$+\frac{\partial}{\partial x_3}(K_1\omega_1+K_2\omega_2)-(K_1-K_{11})\frac{\partial\omega_3}{h_1\partial x_1}$$
$$\left.-(K_2-K_{22})\frac{\partial\omega_3}{h_2\partial x_2}\right\}, \quad (7c)$$

where

$$K_{11}=-\frac{1}{h_1^2}\frac{\partial h_1}{\partial x_1}\,,\quad K_{22}=-\frac{1}{h_2^2}\frac{\partial h_2}{\partial x_2}\,, \quad (8)$$

and ν_e is the eddy viscosity.

3.2 Local Asymptotic Expansions

In order to get appropriate approximations of the basic equations for the five subregions, the local asymptotic expansions of related quantities are made using small parameter ϵ defined by

$$\epsilon=R_e^{-1/8}\,, \quad (9)$$

where R_e is the Reynolds number related to the streamwise full length.

The quantity ϵ is first introduced by Stewartson[4] and can be a small parameter in case of large Reynolds number.

(a) Vorticity diffusion region (*C*-region)

For the vorticity diffusion region, the constant eddy viscosity is assumed; Eq. (7) is used as the basic equation.

Introducing non-dimensional curvilinear small line segments $\delta\xi$, $\delta\eta$, $\delta\zeta$, we represent the differentiations with respect to x_1, x_2, x_3 as follows;

$$\frac{\partial}{h_1\partial x_1}=\frac{1}{L\epsilon}\frac{\partial}{\partial\bar{\xi}}, \quad \frac{\partial}{h_2\partial x_2}=\frac{1}{L\epsilon}\frac{\partial}{\partial\bar{\eta}}, \quad \frac{\partial}{\partial x_3}=\frac{1}{L\epsilon^2}\frac{\partial}{\partial\bar{\zeta}}. \tag{10}$$

Here we assume the derivatives by the new variables are all 0(1), i.e.,

$$\frac{\partial}{\partial\bar{\xi}}=0(1), \quad \frac{\partial}{\partial\bar{\eta}}=0(1), \quad \frac{\partial}{\partial\bar{\zeta}}=0(1). \tag{11}$$

The origin of new variables coincides with that of $x_1x_2x_3$ but $\bar{\xi}=0$ corresponds to the position of separation.

We assume that the velocity and the vorticity of the present region can be expanded asymptotically as follows;

$$\left.\begin{aligned} q_1/U_0&=\bar{u}_0(\bar{\xi},\bar{\eta},\bar{\zeta})+\epsilon\bar{u}_1(\bar{\xi},\bar{\eta},\bar{\zeta})+\epsilon^2\bar{u}_2(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots,\\ q_2/U_0&=\epsilon\bar{v}_1(\bar{\xi},\bar{\eta},\bar{\zeta})+\epsilon^2\bar{v}_2(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots,\\ q_3/U_0&=\epsilon^2\bar{w}_1(\bar{\xi},\bar{\eta},\bar{\zeta})+\epsilon^3\bar{w}_2(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots, \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \frac{\omega_1}{U_0/L}&=\frac{1}{\epsilon}\bar{\omega}_{\xi1}(\bar{\xi},\bar{\eta},\bar{\zeta})+\bar{\omega}_{\xi2}(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots,\\ \frac{\omega_2}{U_0/L}&=\frac{1}{\epsilon^2}\bar{\omega}_{\eta1}(\bar{\xi},\bar{\eta},\bar{\zeta})+\frac{1}{\epsilon}\bar{\omega}_{\eta2}(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots,\\ \frac{\omega_3}{U_0/L}&=\frac{1}{\epsilon}\bar{\omega}_{\zeta1}(\bar{\xi},\bar{\eta},\bar{\zeta})+\bar{\omega}_{\zeta2}(\bar{\xi},\bar{\eta},\bar{\zeta})+\cdots. \end{aligned}\right\} \tag{13}$$

All the quantities appearing in Eqs. (12) and (13) are assumed to be 0(1).

Moreover we introduce non-dimensional variables k_1, k_2, k_{11}, k_{22} defined by

$$k_1=L\cdot K_1, \quad k_2=L\cdot K_2, \quad k_{11}=L\cdot K_{11}, \quad k_{22}=L\cdot K_{22} \tag{14}$$

whose orders are 0(1) for all the regions; L is the body length.

Substituting Eqs. (10), (12) and (13) into Eqs. (3) and (6), we get

$$\frac{\partial\bar{u}_0}{\partial\bar{\xi}}=0, \tag{15}$$

$$\frac{\partial\bar{\omega}_{\eta1}}{\partial\bar{\eta}}+\frac{\partial\bar{\omega}_{\zeta1}}{\partial\bar{\zeta}}=0, \tag{16}$$

as leading terms. Substituting them again into Eq. (7), we get,

$$\frac{\partial}{\partial\bar{\eta}}(\bar{\omega}_{\eta1}\bar{u}_0)+\frac{\partial}{\partial\bar{\zeta}}(\bar{\omega}_{\zeta1}\bar{u}_0)=0, \tag{17a}$$

$$\frac{1}{\epsilon^3}\frac{\nu_e}{U_0L}\frac{\partial^2\bar{\omega}_{\eta1}}{\partial\bar{\zeta}^2}-\frac{\partial}{\partial\bar{\xi}}(\bar{\omega}_{\eta1}\bar{u}_0)=0, \tag{17b}$$

$$\frac{1}{\epsilon^3}\frac{\nu_e}{U_0L}\frac{\partial^2\bar{\omega}_{\zeta1}}{\partial\bar{\zeta}^2}-\frac{\partial}{\partial\bar{\xi}}(\bar{\omega}_{\zeta1}\bar{u}_0)=0, \tag{17c}$$

as the leading-terms-governing equations. In order to exist for the viscous diffusion terms, ν_e/U_0L should be $0(\epsilon^3)$ at least.

We have obtained four equations, Eqs. (16) and (17), for three unknowns, $\bar{u}_0$, $\bar{\omega}_{\eta1}$ and $\bar{\omega}_{\zeta1}$, but it can be easily examined one of them is not independent.

Changing the variables back into the original ones, we get the following equations as the governing equations for C-region as far as the leading terms are concerned;

$$\frac{\partial}{h_2\partial x_2}(\omega_2q_1)+\frac{\partial}{\partial x_3}(\omega_3q_1)=0, \tag{18a}$$

$$\nu_e\frac{\partial^2\omega_2}{\partial x_3^2}-\frac{\partial}{h_1\partial x_1}(\omega_2q_1)=0, \tag{18b}$$

$$\nu_e\frac{\partial^2\omega_3}{\partial x_3^2}-\frac{\partial}{h_1\partial x_1}(\omega_3q_1)=0. \tag{18c}$$

The terms of order $0(1/\epsilon^2)$ are neglected in Eq. (18). Eq. (18) is almost the same as the equation of ref. 1).

Once the vorticity distributions are obtained throughout the boundary layer and the wake, say V, the viscous component of velocity, $\boldsymbol{q}_v$, can be calculated as induced velocity of vorticity by invoking Biot-Savart's law;

$$\boldsymbol{q}_v(x,y,z)=\nabla\times\frac{1}{4\pi}\iiint_V\left(\frac{\boldsymbol{\omega}'}{r}-\frac{\boldsymbol{\omega}_1'}{r_1}\right)dx'dy'dz', \tag{19}$$

where $\boldsymbol{\omega}_1'$ is the mirror image of the vorticity vector $\boldsymbol{\omega}'$ whose components are ω_x', ω_y', $-\omega_z'$ in x, y and z directions and

$$\left.\begin{aligned} r^2&=(x-x')^2+(y-y')^2+(z-z')^2,\\ r_1^2&=(x-x')^2+(y-y')^2+(z+z')^2. \end{aligned}\right\} \tag{20}$$

Adding the potential component to $\boldsymbol{q}_v$, we can obtain the velocity distributions of C-region.

(b) Separated retarding region (D-region)

Introducing non-dimensional variables, $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ for the present region in the same manner as C-region, the orders of differentiations with respect to $\hat{\xi}$, $\hat{\eta}$, $\hat{\zeta}$ are assumed

$$\left.\begin{aligned} &\frac{\partial}{h_1\partial x_1}=\frac{1}{L}\frac{\partial}{\epsilon\partial\hat{\xi}}, \quad \frac{\partial}{h_2\partial x_2}=\frac{1}{L}\frac{\partial}{\epsilon\partial\hat{\eta}}, \quad \frac{\partial}{\partial x_3}=\frac{1}{L}\frac{\partial}{\epsilon^3\partial\hat{\zeta}},\\ &\frac{\partial}{\partial\hat{\xi}}=0(1), \quad \frac{\partial}{\partial\hat{\eta}}=0(1), \quad \frac{\partial}{\partial\hat{\zeta}}=0(1). \end{aligned}\right\} \tag{21}$$

Velocity and pressure are assumed to be expanded asymptotically,

$$\left.\begin{aligned} q_1/U_0&=\epsilon(\hat{u}_1+\hat{u}_1')+\epsilon^2(\hat{u}_2+\hat{u}_2')+\cdots,\\ q_2/U_0&=\epsilon(\hat{v}_1+\hat{v}_1')+\epsilon^2(\hat{v}_2+\hat{v}_2')+\cdots,\\ q_3/U_0&=\epsilon^3(\hat{w}_1+\hat{w}_1')+\epsilon^4(\hat{w}_2+\hat{w}_2')+\cdots, \end{aligned}\right\} \tag{22}$$

$$(p-p_\infty)/\rho U_0^2=\epsilon p_1+\epsilon^2p_2+\cdots, \tag{23}$$

where $\hat{u}_1$, $\hat{u}_2$, ... are all time-averaged variables and $\hat{u}_1'$, $\hat{u}_2'$, ... are fluctuating. Here the fluctuating terms of pressure are omitted because they do not appear in the basic equations.

The vorticity can be also expanded asymptotically,

$$\left.\begin{aligned}\frac{\omega_1}{U_0/L}&=-\frac{1}{\epsilon^2}\frac{\partial\hat{v}_1}{\partial\hat{\zeta}}-\frac{1}{\epsilon}\frac{\partial\hat{v}_2}{\partial\hat{\zeta}}+0(1)\,,\\ \frac{\omega_2}{U_0/L}&=\frac{1}{\epsilon^2}\frac{\partial\hat{u}_1}{\partial\hat{\zeta}}+\frac{1}{\epsilon}\frac{\partial\hat{u}_2}{\partial\hat{\zeta}}+0(1)\,,\\ \frac{\omega_3}{U_0/L}&=\frac{\partial\hat{v}_1}{\partial\hat{\xi}}-\frac{\partial\hat{u}_1}{\partial\hat{\eta}}+0(\epsilon)\,.\end{aligned}\right\}\tag{24}$$

Under these assumptions, the leading terms of the continuity equation are written,

$$\frac{\partial\hat{u}_1}{\partial\hat{\xi}}+\frac{\partial\hat{v}_1}{\partial\hat{\eta}}+\frac{\partial\hat{w}_1}{\partial\hat{\zeta}}=0\,,\tag{25}$$

and the governing equations are

$$\begin{aligned}&\hat{u}_1\frac{\partial\hat{u}_1}{\partial\hat{\xi}}+\hat{v}_1\frac{\partial\hat{u}_1}{\partial\hat{\eta}}+\hat{w}_1\frac{\partial\hat{u}_1}{\partial\hat{\zeta}}+\epsilon\left\{\frac{\partial}{\partial\hat{\xi}}(\hat{u}_1\hat{u}_2)\right.\\ &\quad+\left(\hat{v}_1\frac{\partial}{\partial\hat{\eta}}+\hat{w}_1\frac{\partial}{\partial\hat{\zeta}}\right)\hat{u}_2+\left(\hat{v}_2\frac{\partial}{\partial\hat{\eta}}+\hat{w}_2\frac{\partial}{\partial\hat{\zeta}}\right)\hat{u}_1\\ &\quad\left.-k_2\hat{u}_1\hat{v}_1+k_1\hat{v}_1^2\right\}=-\frac{1}{\epsilon}\frac{\partial p_1}{\partial\hat{\xi}}-\left\{\frac{\partial}{\partial\hat{\xi}}(p_2+\hat{u}_1'^2)\right.\\ &\quad+\frac{\partial}{\partial\hat{\eta}}(\hat{u}_1'\hat{v}_1')+\frac{\partial}{\partial\hat{\zeta}}(\hat{u}_1'\hat{w}_1')-\epsilon\left\{\frac{\partial}{\partial\hat{\xi}}(p_3+2\hat{u}_1'\hat{u}_2')\right.\\ &\quad+\frac{\partial}{\partial\hat{\eta}}(\hat{u}_1'\hat{v}_2'+\hat{u}_2'\hat{v}_1')+\frac{\partial}{\partial\hat{\zeta}}(\hat{u}_1'\hat{w}_2'+\hat{u}_2'\hat{w}_1')\\ &\quad\left.-2k_2\hat{u}_1'\hat{v}_1'-k_1(\hat{u}_1'^2-\hat{v}_1'^2)\right\}\\ &\quad+\frac{1}{\epsilon^6}\frac{\nu}{U_0L}\frac{\partial^2\hat{u}_1}{\partial\hat{\zeta}^2}+0(\epsilon^2)\,,\end{aligned}\tag{26a}$$

$$\begin{aligned}&\hat{u}_1\frac{\partial\hat{v}_1}{\partial\hat{\xi}}+\hat{v}_1\frac{\partial\hat{v}_1}{\partial\hat{\eta}}+\hat{w}_1\frac{\partial\hat{v}_1}{\partial\hat{\zeta}}+\epsilon\left\{\frac{\partial}{\partial\hat{\eta}}(\hat{v}_1\hat{v}_2)\right.\\ &\quad+\left(\hat{u}_1\frac{\partial}{\partial\hat{\xi}}+\hat{w}_1\frac{\partial}{\partial\hat{\zeta}}\right)\hat{v}_2+\left(\hat{u}_2\frac{\partial}{\partial\hat{\xi}}+\hat{w}_2\frac{\partial}{\partial\hat{\zeta}}\right)\hat{v}_1\\ &\quad\left.+k_2\hat{u}_1^2-k_1\hat{u}_1\hat{v}_1\right\}=-\frac{1}{\epsilon}\frac{\partial p_1}{\partial\hat{\eta}}-\left\{\frac{\partial}{\partial\hat{\xi}}(\hat{u}_1'\hat{v}_1')\right.\\ &\quad+\frac{\partial}{\partial\hat{\eta}}(p_2+\hat{v}_1'^2)+\frac{\partial}{\partial\hat{\zeta}}(\hat{v}_1'\hat{w}_1')\\ &\quad-\epsilon\left\{\frac{\partial}{\partial\hat{\xi}}(\hat{u}_1'\hat{v}_2'+\hat{u}_2'\hat{v}_1')+\frac{\partial}{\partial\hat{\eta}}(p_3+2\hat{v}_1'\hat{v}_2')\right.\\ &\quad+\frac{\partial}{\partial\hat{\zeta}}(\hat{v}_1'\hat{w}_2'+\hat{v}_2'\hat{w}_1')+k_2(\hat{u}_1'^2-\hat{v}_1'^2)\\ &\quad\left.-2k_1\hat{u}_1'\hat{v}_1'\right\}+\frac{1}{\epsilon^6}\frac{\nu}{U_0L}\frac{\partial^2\hat{v}_1}{\partial\hat{\zeta}^2}+0(\epsilon^2)\,,\end{aligned}\tag{26b}$$

$$\frac{\partial p_1}{\partial\hat{\zeta}}+\epsilon\frac{\partial p_2}{\partial\hat{\zeta}}+0(\epsilon^2)=0\,.\tag{26c}$$

The leading terms of Eq. (26) yield,

$$p_1(\hat{\xi},\hat{\eta},\hat{\zeta})=\text{const.}\,.\tag{27}$$

Eq. (27) means that the pressure is almost constant throughout the present region as far as $0(\epsilon)$ is concerned. This is very natural because D-region is a kind of the dead flow region.

Now the second terms of Eq. (26) yield,

$$\begin{aligned}&\hat{u}_1\frac{\partial\hat{u}_1}{\partial\hat{\xi}}+\hat{v}_1\frac{\partial\hat{u}_1}{\partial\hat{\eta}}+\hat{w}_1\frac{\partial\hat{u}_1}{\partial\hat{\zeta}}=-\frac{\partial}{\partial\hat{\xi}}(p_2+\hat{u}_1'^2)\\ &\quad-\frac{\partial}{\partial\hat{\eta}}(\hat{u}_1'\hat{v}_1')-\frac{\partial}{\partial\hat{\zeta}}(\hat{u}_1'\hat{w}_1')\end{aligned}\tag{28a}$$

$$\begin{aligned}&\hat{u}_1\frac{\partial\hat{v}_1}{\partial\hat{\xi}}+\hat{v}_1\frac{\partial\hat{v}_1}{\partial\hat{\eta}}+\hat{w}_1\frac{\partial\hat{v}_1}{\partial\hat{\zeta}}=-\frac{\partial}{\partial\hat{\xi}}(\hat{u}_1'\hat{v}_1')\\ &\quad-\frac{\partial}{\partial\hat{\eta}}(p_2+\hat{v}_1'^2)-\frac{\partial}{\partial\hat{\zeta}}(\hat{v}_1'\hat{w}_1')\end{aligned}\tag{28b}$$

$$\frac{\partial p_2}{\partial\hat{\zeta}}=0\,.\tag{28c}$$

In Eq. (28), the molecular viscosity disappears but its affects are still existing indirectly through the turbulence.

Eq. (28c) is the so-called boundary layer approximation. However, because there exist cross terms of turbulent components in Eqs. (28a) and (28b), Eq. (28) does not always yield the same type as the boundary layer equations which can not predict such a recirculating flow as observed.

(c) Viscous sublayer (*E*-region)

In the present region, the no-slip condition should be satisfied on the hull surface. Here the intensity of turbulence is quite small and all the turbulent terms in the Reynolds equations vanish infinitesimally small.

After the Blasius solution, following asymptotic expansions are assumed,

$$\left.\begin{aligned}q_1/U_0&=\epsilon u_1^*+\epsilon^2u_2^*+\cdots\,,\\ q_2/U_0&=\epsilon v_1^*+\epsilon^2v_2^*+\cdots\,,\\ q_3/U_0&=\epsilon^4w_1^*+\epsilon^5w_2^*+\cdots\,,\end{aligned}\right\}\tag{29}$$

where orders of each term are all $0(1)$.

The derivatives are represented by

$$\left.\begin{aligned}&\frac{\partial}{h_1\partial x_1}=\frac{1}{L}\frac{\partial}{\epsilon\partial\xi}\,,\qquad \frac{\partial}{h_2\partial x_2}=\frac{1}{L}\frac{\partial}{\epsilon\partial\eta}\,,\\ &\frac{\partial}{\partial x_3}=\frac{1}{L}\frac{\partial}{\epsilon^4\partial\zeta^*}\,,\end{aligned}\right\}\tag{30}$$

and their orders are

$$\frac{\partial}{\partial\xi}=0(1)\,,\quad\frac{\partial}{\partial\eta}=0(1)\,,\quad\frac{\partial}{\partial\zeta^*}=0(1)\,.\tag{31}$$

Substituting the above assumptions into Eq. (4), the leading terms are obtained as follows in original variables;

$$\begin{aligned}&q_1\frac{\partial q_1}{h_1\partial x_1}+q_2\frac{\partial q_1}{h_2\partial x_2}+q_3\frac{\partial q_1}{\partial x_3}\\ &\quad=-\frac{1}{\rho}\frac{\partial p}{h_1\partial x_1}+\nu\frac{\partial^2q_1}{\partial x_3^2}\,,\end{aligned}\tag{32a}$$

$$\begin{aligned}&q_1\frac{\partial q_2}{h_1\partial x_1}+q_2\frac{\partial q_2}{h_2\partial x_2}+q_3\frac{\partial q_2}{\partial x_3}\\ &\quad=-\frac{1}{\rho}\frac{\partial p}{h_2\partial x_2}+\nu\frac{\partial^2q_2}{\partial x_3^2}\,,\end{aligned}\tag{32b}$$

$$\frac{\partial p}{\partial x_3}=0\,. \tag{32c}$$

The continuity equation is

$$\frac{\partial q_1}{h_1\partial x_1}+\frac{\partial q_2}{h_2\partial x_2}+\frac{\partial q_3}{\partial x_3}=0\,. \tag{33}$$

Here the quantities of $0(\epsilon^2)$ are omitted.

Eq. (32) is the boundary layer equation itself and it can be matched to the outer flow, the solution of D-region, in the quite same manner as the conventional method of the boundary layer calculation.

3.3 Boundary Conditions and Matching Conditions

The boundary conditions are

$$u^*(\bar\xi,\bar\eta,0)=0\,,\quad v^*(\bar\xi,\bar\eta,0)=0\,,\quad w^*(\bar\xi,\bar\eta,0)=0 \tag{34}$$

on the hull surface which are imposed to the solution of E-region; and in the wake,

$$v(x,0,z)=0 \tag{35}$$

where v is the y-direction component. Because the governing equation of E-region is the boundary layer equation itself, the governing equations of D-region should be solved under the condition

$$\boldsymbol{q}\cdot\boldsymbol{n}=0\,, \tag{36}$$

where $\boldsymbol{n}$ is the unit normal vector of hull surface.

Far downstream, the perturbation velocity must die away.

The matching conditions are as follows.

(i) for upstream;

$$\left.\begin{aligned}&\lim_{\bar\xi\to-\infty}\bar u_0(\bar\xi,\bar\eta,\bar\zeta)=u_B\,,\quad \lim_{\bar\xi\to-\infty}\bar v_1(\bar\xi,\bar\eta,\bar\zeta)=v_B\,,\\ &\lim_{\bar\xi\to-\infty}\bar w_1(\bar\xi,\bar\eta,\bar\zeta)=w_B\end{aligned}\right\} \tag{37}$$

where u_B, v_B and w_B are the velocity components in boundary layer as to x_1,x_2,x_3 directions respectively.

(ii) far departing from the hull surface; the solutions should be matched with the solution of A-region, potential flow.

(iii) between C- and D-region;

$$\left.\begin{aligned}&\bar u_0(\bar\xi,\bar\eta,0)=0\,,\\ &\bar u_1(\bar\xi,\bar\eta,0)=\lim_{\hat\zeta\to\infty}\left\{\hat u_1(\bar\xi,\bar\eta,\hat\zeta)-\hat\zeta\frac{\partial}{\partial\bar\zeta}\bar u_0(\bar\xi,\bar\eta,\bar\zeta)|_{\bar\zeta=0}\right\}\end{aligned}\right\} \tag{38a}$$

$$\hat v_1(\bar\xi,\bar\eta,0)=\lim_{\hat\zeta\to\infty}\hat v_1(\bar\xi,\bar\eta,\hat\zeta)\,, \tag{38b}$$

$$\left.\begin{aligned}&\bar w_1(\bar\xi,\bar\eta,0)=0\,,\\ &\bar w_2(\bar\xi,\bar\eta,0)=\lim_{\hat\zeta\to\infty}\left\{\hat w_1(\bar\xi,\bar\eta,\hat\zeta)-\hat\zeta\frac{\partial}{\partial\bar\zeta}\bar w_1(\bar\xi,\bar\eta,\bar\zeta)|_{\bar\zeta=0}\right\}\,.\end{aligned}\right\} \tag{38c}$$

(iv) between D- and E-region;

$$\hat u_1(\bar\xi,\bar\eta,0)=\lim_{\zeta^*\to\infty}u_1{}^*(\bar\xi,\bar\eta,\zeta^*)\,, \tag{39a}$$

$$\hat v_1(\bar\xi,\bar\eta,0)=\lim_{\zeta^*\to\infty}v_1{}^*(\bar\xi,\bar\eta,\zeta^*)\,, \tag{39b}$$

$$\left.\begin{aligned}&\hat w_1(\bar\xi,\bar\eta,0)=0\,,\\ &\hat w_2(\bar\xi,\bar\eta,0)=\lim_{\zeta^*\to\infty}\left\{w_1{}^*(\bar\xi,\bar\eta,\zeta^*)-\zeta^*\frac{\partial}{\partial\hat\zeta}\hat w_1(\bar\xi,\bar\eta,\hat\zeta)|_{\hat\zeta=0}\right\}\,.\end{aligned}\right\} \tag{39c}$$

3.4 Auxiliary Equations for Turbulence

The governing equation for the separated retarding region, Eq. (28), includes the time-fluctuating parts in it. Some auxiliary equations are required.

Representing them by the Reynolds stress tensor,

$$\begin{bmatrix}\sigma_1 & \tau_{12} & \tau_{23}\\ \tau_{12} & \sigma_2 & \tau_{23}\\ \tau_{13} & \tau_{23} & \sigma_3\end{bmatrix} =-\rho U_0{}^2\begin{bmatrix}\epsilon^2(p_2+\hat u_1{}'^2) & \epsilon^2\hat u_1{}'\hat v_1{}' & \epsilon^4\hat u_1{}'\hat w_1{}'\\ \epsilon^2\hat u_1{}'\hat v_1{}' & \epsilon^2(p_2+\hat v_1{}'^2) & \epsilon^4\hat v_1{}'\hat w_1{}'\\ \epsilon^4\hat u_1{}'\hat w_1{}' & \epsilon^4\hat v_1{}'\hat w_1{}' & \epsilon^2 p_2+\epsilon^8\hat w_1{}'^2\end{bmatrix}, \tag{40}$$

the orders of each component should be equal to those of fluctuating terms;

$$\left.\begin{aligned}&\sigma_1,\quad \sigma_2,\quad \tau_{12}=0(\epsilon^2)\\ &\tau_{13},\quad \tau_{23}=0(\epsilon^4)\end{aligned}\right\}\,. \tag{41}$$

Now let's assume, analogically to the case of laminar flow, each element of the stress tensor is related to the gradients of the averaged velocity, namely,

$$\left.\begin{aligned}&\frac{\sigma_1}{\rho U_0{}^2}=\epsilon^2 p_2+2\kappa_1\frac{\partial\hat u_1}{\partial\bar\xi}\,,\quad \frac{\sigma_2}{\rho U_0{}^2}=\epsilon^2 p_2+2\kappa_1\frac{\partial\hat v_1}{\partial\bar\eta}\,,\\ &\frac{\tau_{12}}{\rho U_0{}^2}=\kappa_1\left(\frac{\partial\hat v_1}{\partial\bar\xi}-\frac{\partial\hat u_1}{\partial\bar\eta}\right),\\ &\frac{\tau_{13}}{\rho U_0{}^2}=\frac{1}{\epsilon^2}\kappa_2\left(\frac{\partial\hat u_1}{\partial\hat\zeta}+\epsilon^4\frac{\partial\hat w_1}{\partial\bar\xi}\right),\\ &\frac{\tau_{23}}{\rho U_0{}^2}=\frac{1}{\epsilon^2}\kappa_2\left(\frac{\partial\hat v_1}{\partial\hat\zeta}+\epsilon^4\frac{\partial\hat w_1}{\partial\bar\eta}\right),\end{aligned}\right\} \tag{42}$$

where κ_1 and κ_2 are the constants which can be supposed to be the components of eddy viscosity vector. They should be $\kappa_1=O(\epsilon^2)$ and $\kappa_2=O(\epsilon^6)$ in orders in D-region. The fact that the order of κ_2 is less than that of κ_1 means the fluctuation of normal direction is less than tangential, which is compatible with the Klebanoff's experiments[5].

Substituting Eq. (42) into Eqs. (28a) and (28b), we get

$$\hat u_1\frac{\partial\hat u_1}{\partial\bar\xi}+\hat v_1\frac{\partial\hat u_1}{\partial\bar\eta}+\hat w_1\frac{\partial\hat u_1}{\partial\hat\zeta} =-\frac{\partial p_2}{\partial\bar\xi}+\kappa_1\left(\frac{\partial^2\hat u_1}{\partial\bar\xi^2}+\frac{\partial^2\hat u_1}{\partial\bar\eta^2}-\frac{\partial^2\hat w_1}{\partial\bar\xi\partial\hat\zeta}\right)+\kappa_2\frac{\partial^2\hat u_1}{\partial\hat\zeta^2}\,, \tag{43a}$$

$$\hat{u}_1\frac{\partial \hat{v}_1}{\partial \hat{\xi}}+\hat{v}_1\frac{\partial \hat{v}_1}{\partial \hat{\eta}}+\hat{w}_1\frac{\partial \hat{v}_1}{\partial \hat{\zeta}}$$
$$=-\frac{\partial p_2}{\partial \hat{\eta}}+\kappa_1\left(\frac{\partial^2 \hat{v}_1}{\partial \hat{\xi}^2}+\frac{\partial^2 \hat{v}_1}{\partial \hat{\eta}^2}-\frac{\partial^2 \hat{w}_1}{\partial \hat{\eta}\partial \hat{\zeta}}\right)+\kappa_2\frac{\partial^2 \hat{v}_1}{\partial \hat{\zeta}^2}\,. \tag{43b}$$

Because Eq. (43) is closure and elliptic type, it can represent the separated retarding flow observed in D-region.

4. Numerical Calculations and Discussions

When the yielded equations are solved numerically, the matching conditions are not automatically satisfied. However, if the surface consisting of dividing streamlines (DSL) where matching conditions between C- and D-regions are imposed, are given a priori, the difficulties can be removed.

Here attempts are made to obtain the velocity distribution in wake for two cases; a flat plate with zero attack angle and C-region flow of a tanker model.

By invoking Eq. (15), Eqs. (18b) and (18c) are transformed into tridiagonal linear equations for $k\geq 2$;

$$\omega_2(i,j,k-1)-2C(i,j,k)\omega_2(i,j,k)+\omega_2(i,j,k+1)=A_2(i,j,k)\,, \tag{44a}$$
$$\omega_3(i,j,k-1)-2C(i,j,k)\omega_3(i,j,k)+\omega_3(i,j,k+1)=A_3(i,j,k)\,, \tag{44b}$$

where ω_2 (i,j,k) etc. denote those values at $x_1=x_{1i}$, $x_2=x_{2j}$ and $x_3=x_{3k}$ and

$$\left.\begin{aligned}A_2(i,j,k)=&-\omega_2(i-1,j,k-1)\\&+2\omega_2(i-1,j,k)\left\{1-\frac{\Delta\zeta^2}{\nu_e\Delta\xi}q_1(i-1,j,k)\right\}\\&-\omega_2(i-1,j,k+1)\,,\\A_3(i,j,k)=&-\omega_3(i-1,j,k-1)\\&+2\omega_3(i-1,j,k)\left\{1-\frac{\Delta\zeta^2}{\nu_e\Delta\xi}q_1(i-1,j,k)\right\}\\&-\omega_3(i-1,j,k+1)\,,\end{aligned}\right\} \tag{45a}$$

$$C(i,j,k)=1+\frac{\Delta\zeta^2}{\nu_e\Delta\xi}q_1(i,j,k)\,, \tag{45b}$$

and $\Delta\xi$, $\Delta\zeta$ are short segments in x_1, x_3 directions.

Eq. (44) can be solved by the forward marching procedure if the velocity profile of q_1 is given at the separation position.

(a) Flat plate with zero attack angle

In this case, D- and E-regions being not existing, the wake flow can be definitely determined by Eq. (18b), i.e., by Eq. (44a), where x_1, x_3 correspond to x, y and ω_2 can be written ω_z. The origine is chosen at the trailing edge of plate. All quantities are made dimensionless by the flat plate length and U_0. 30-times molecular viscosity is used as the eddy viscosity. The boundary values of ω_z at $y=0$, i.e., the values of ω_2 for $k=1$ in Eq. (44a), are made zero for $x>0$. Because the vorticity obtained from the boundary layer calculation gets infinite at $y=0$, the value of $y=0.0001$ is used instead as the initial value at the separation position. $\Delta\xi$ is 0.002 and $\Delta\zeta$ is 0.0005 for $0<y\leq 0.0045$ and 0.0015 for $y>0.0045$.

The boundary layer calculation is carried out by the integral method where Ludwieg-Tillman's formula for the skin friction is used and the shape factor is made constant which is equal to 1.4.

In case of ν_e/q_1 is constant, the solution of Eq. (18b) is given in integral form,

$$\omega_z(x,y)=\frac{1}{2}\sqrt{\frac{q_1}{\nu_e x}}\int_0^\infty \omega_z(0,y')\times\left\{\exp\left(-\frac{q_1}{4\nu_e x}(y-y')^2\right)-\exp\left(-\frac{q_1}{4\nu_e x}(y+y')^2\right)\right\}dy'\,. \tag{46}$$

In order to check the present numerical scheme, the calculated results by Eq. (44) for $q_1=1$ are compared with the results of Eq. (46). Comparisons are shown in Fig. 5. Both results agree well with each other, which guarantees the present numerical scheme.

In Fig. 5 calculated results, representing the initial profile of q_1 by a quadratic function of y which is equal to the uniform flow at the outer edge of boundary layer and 2/3 at $y=0$, are compared with experimental results of Chevray and Kovasznay[6]. Differences due to the initial velocity profiles are not so significant. The quadratic expression seems to yield the final result by less repetitions of iteration.

It may be possible to use the velocity profile at the separation position. However, because such a velocity profile which is equal to zero at

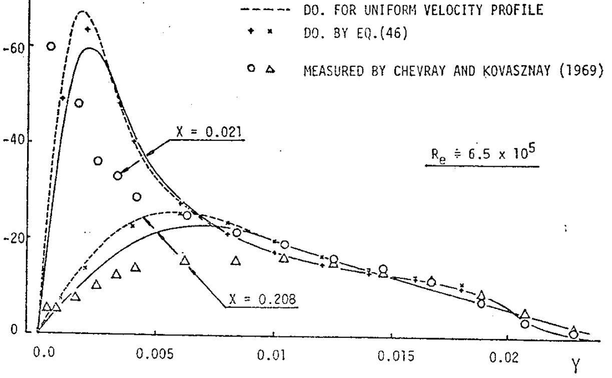

Fig. 5 Comparison of vorticity distributions of flat plate

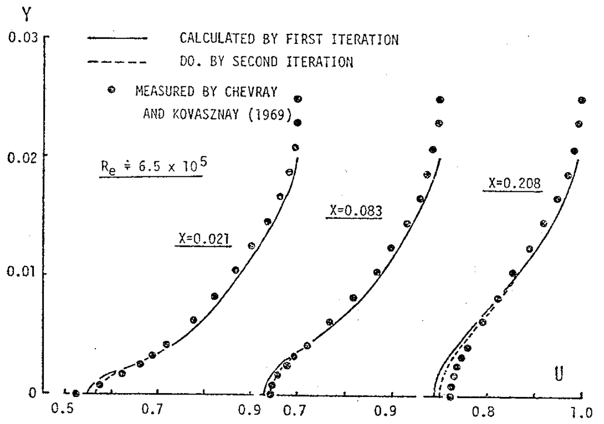

Fig. 6 Comparison of velocity profiles in wake of flat plate

$y=0$ yields infinite conductive coefficient and vorticity is diffused at once there, it is hopeless to get reasonable results even if many iterations are repeated.

In Fig. 6, calculated velocity profiles of x-direction are compared with Chevray-Kovasznay's experimental data. The calculation of second iteration is also carried out; the quadratic expression of the velocity profile is used for the first iteration and the velocity field obtained by the first iteration is used for the second iteration. The integral region, V in Eq. (19), is covering $x'=x\pm0.2$. The integral intervals are 0.02 in x-direction and the same value as $\Delta\zeta$ in y-direction.

The results of the second iteration are slightly improved compared with those of the first iteration especially near the center plane. However,

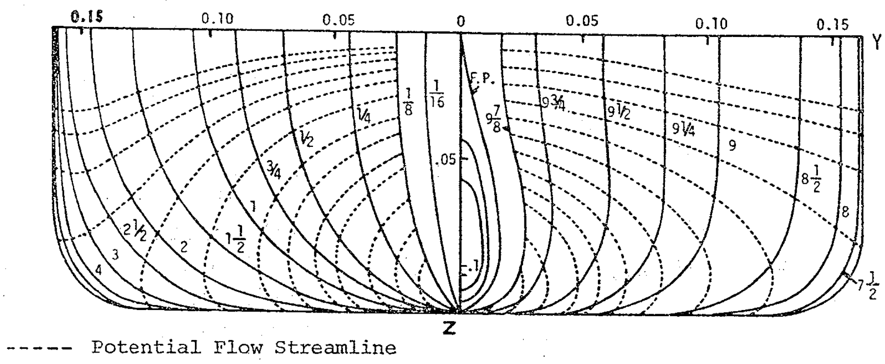

Fig. 7 Body plan of MS-02 and potential flow streamlines

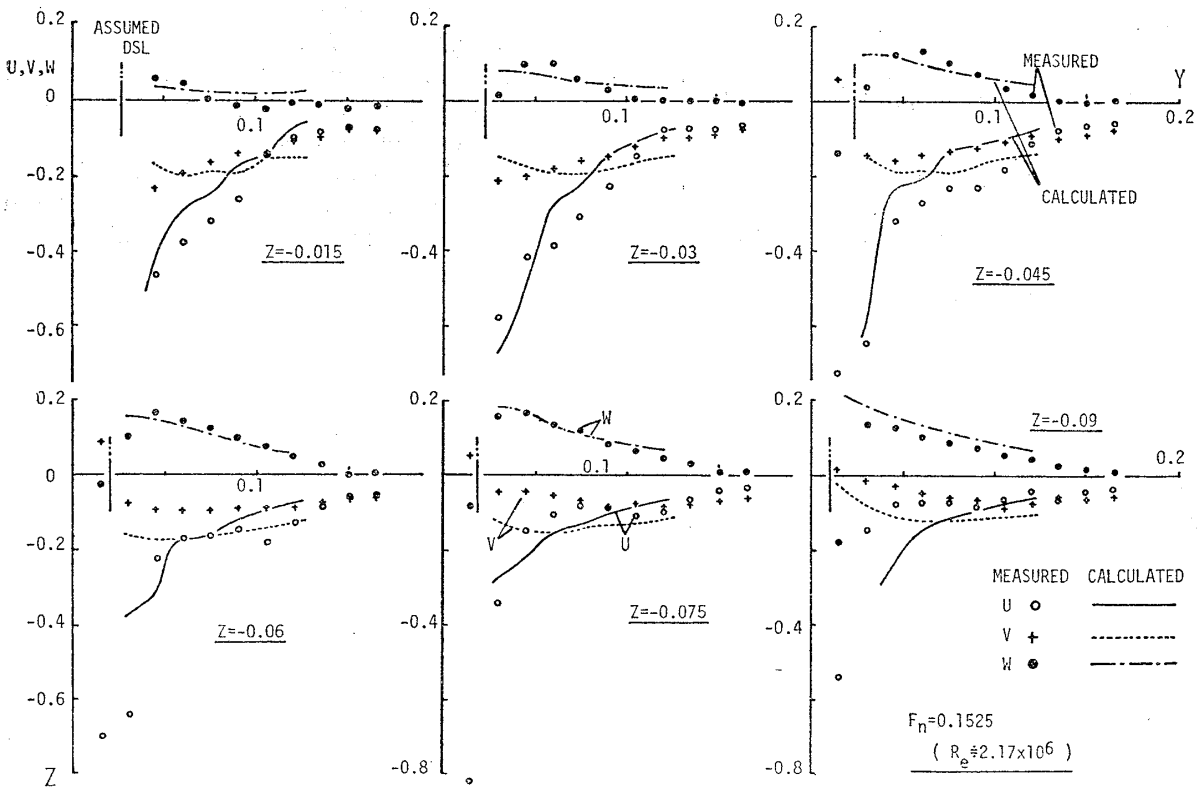

Fig. 8 Perturbation velocity distributions in wake at 1/80 L aft from A.P.

because the differences between them are not so signifficant, the first iteration seems to be enough for our purpose.

(b) *C*-region flow of MS-02

C-region flow of MS-02 is calculated by the same manner as the previous case. MS-02 is a tanker model with a simple stern form whose body plan and potential flow streamlines, along which boundary layer calculations are carried out, are shown in Fig. 7. In this case length scales are made dimensionless by the half ship length, $L/2=1.5$ m and u, v, w are the perturbation components in x, y, z-directions.

DSL is determined a priori for the first iteration; it is consisting of line segments departing from S.S.1/2 and reattaching at $1/20 \cdot L$ aft from A.P. ν_e is chosen 300-times molecular kinematic viscosity according to the experimental studies[2]. The boundary layer calculation is carried out by the integral method as Ref. 7). As to the boundary values of vorticity, values at $k=1$ in Eq. (44) are made equal to those at $k=2$. $\Delta\xi$ is almost equal to 0.005 and $\Delta\zeta$ is 0.0025 for $0 \leq x_3 \leq 0.04$ and 0.005 for $x_3 > 0.04$. Potential velocity components are calculated by the surface-source method and added to the induced velocity, q_v. No attentions are paid to the flow of *D*- and *E*-regions and they are excluded from the integral region, *V*, which is covering from $x=0.8$ to 1.2 for the present calculation. The integral intervals are 0.025 for $0.9 \leq x \leq 1.1$ and 0.05 otherwise in *x*-direction and 0.01 in *y*- and *z*-directions.

Calculations are carried out at $F_n=0.1525$, where F_n is the Froude number based on the ship length. Calculated results are shown in Figs. 8 and 9; in Fig. 8 perturbation velocity distributions are compared with measured and in Fig. 9 the wake contuor is shown. As far as *C*-region is concerned, satisfactory results are obtained. *v* components are always underestimated; this is because the hull surface condition, Eq. (36), is not satisfied in the present calculations.

It is expected further improvements can be attained by taking into account for the *D*- and *E*-regions which are left for the future works.

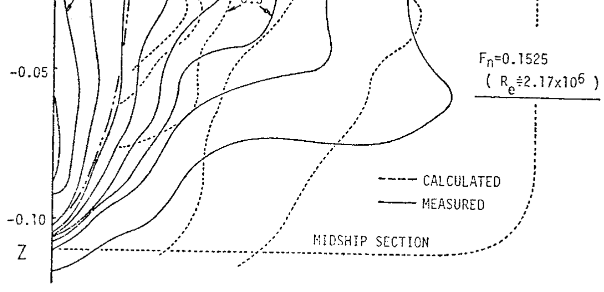

Fig. 9 Wake distribution at 1/80 *L* aft from A.P.

5. Concluding Remarks

An attempt is made to predict the separated ship stern flow by the local asymptotic expansion method following the experimental studies. This is because the flow field is consisting of, at least, two regions and it is not effective to predict the flow of such regions by a single governing equation. Numerical calculations are carried out for the yielded equations and satisfactory results are obtained.

Some important assumptions are made in order to approximate the Navier-Stokes equation; the orders of relevant quantities and derivatives, the relations between the Reynolds' stress and average velocity and so on. However, as far as their treatments are consistent and the yielded equations can predict the existing phenomena, no one can say they are not correct. Of course they must be examined by more precise experiments not only for simple hull forms but for practical ships.

From the practical point of view, the most important problem to predict the *D*-region flow is left for the future task. However, because the governing equations for it are much simpler than the original Navier-Stokes equation, the total prediction of the wake flow will appear soon.

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