（昭和55年5月　日本造船学会春季講演会において講演）

On the Hydrodynamic Forces for Shallow Draft Ships in Shallow Water (3rd Report)

—On the Pitch Hydrodynamic Forces of a Circular Disk—

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Summary

The hydrodynamic forces of pitch mode on a circular disk as a shallow draft ship in shallow water are investigated. The boundary value problem is formulated by the use of the concept of the surface distributed sources so that integral equation for the source densities are obtained. In the case of long waves, the problem is solved analytically. The numerical solution of the integral equation is found. The added moment of inertia, wave damping factor, wave exciting moment, radiation pressures, wave exciting pressures and motions are calculated. The corresponding experiments are carried out and the results of the numerical calculation are in good agreement with those of the experiments.

1. Introduction

In the first and second reports, the hydrodynamic forces of heave mode were investigated experimentally and numerically.[1),2)] The conclusions were that the effectiveness of the numerical method was proved and the shallow water effect and the typical phenomenon on a ringed circular plate were shown. In this paper, the forced oscillation tests and wave excitation tests of the pitch mode and motion tests in waves of a circular disk are carried out. The experimental results of the radiation forces, radiation pressures, wave excitation of the pitch mode, wave exciting pressures, pitching and heaving motions, dynamic pressures of a circular disk oscillating in waves are compared with the corresponding numerical results.

2. Numerical Calculation

2.1 Boundary Value Problem[2)]

A cartesian coordinate system is defined with its origin on a mean free surface of a fluid as shown in Fig. 1. Let $\phi\exp\{-i\omega t\}$ be the radiation potential due to unit velocity amplitude or the diffraction potential which corresponds to an unit amplitude of the incident wave. The corresponding boundary value problem may be written as follows,

$$\nabla^2\phi=0 \quad \text{in fluid} \tag{1.a}$$

$$\phi_z-K\phi=0 \quad \text{at } z=0 \tag{1.b}$$

$$\phi_z=f(P) \quad \text{on } C \tag{1.c}$$

$$\phi_z=0 \quad \text{at } z=-h \tag{1.d}$$

$$\phi\sim\text{radiation condition at } \sqrt{x^2+y^2}\to\infty \tag{1.e}$$

where C represents the surface of a circular disk, deep water wave number $K=\omega^2/g$, g is the gravity acceleration, h is the depth of water, and $f(P)$ is the boundary value on the point P

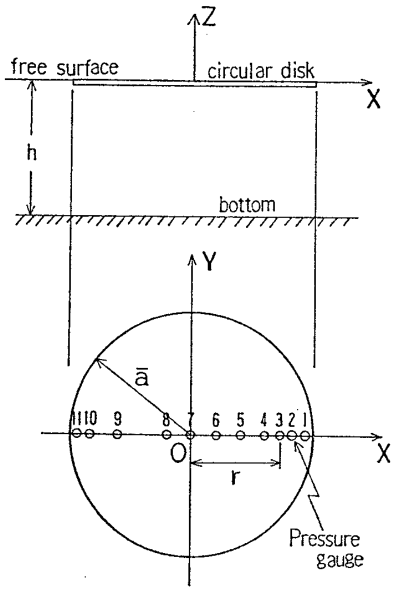

Fig. 1　Coordinate system and pressure gauges

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of C. Let G $(x, y, z; a, b, c)$ be the velocity potential at a point P (x, y, z) of the point source or the so called Green function with its singularity located at a point Q (a, b, c).

$$G=G_c+iG_s \tag{2.a}$$

$$G_c=-2\pi\frac{m_0^2-K^2}{hm_0^2-hK^2+K}\cosh m_0(c+h)\cdot\cosh m_0(z+h)\,Y_0(m_0R)+4\sum_{n=1}^{\infty}\frac{m_n^2+K^2}{hm_n^2+hK^2-K}\cos m_n(c+h)\cdot\cos m_n(z+h)K_0(m_nR) \tag{2.b}$$

$$G_s=2\pi\frac{m_0^2-K^2}{hm_0^2-hK^2+K}\cosh m_0(c+h)\cdot\cosh m_0(z+h)J_0(m_0R) \tag{2.c}$$

where

$$R=\sqrt{(x-a)^2+(y-b)^2} \tag{3}$$

the suffix c and s denote the real and imaginary part of a complex number respectively. The dispersive relations are written as follows.

$$m_0\tanh m_0h=K\,,\qquad m_n\tan m_nh=-K \tag{4}$$

where $m_0=2\pi/\lambda$ is the shallow water wave number and λ is the wave length. $J_0(m_0R)$, $Y_0(m_0R)$, $K_0(m_0R)$ are the Bessel function of the first and second kind of 0-th order, and the modified Bessel function of the second kind of 0-th order respectively. From the boundary condition on C and the free surface condition, the following integral equation is obtained,

$$f(P)=\sigma(P)+\frac{K}{4\pi}\iint_C\sigma(Q)\cdot G(P,Q)dS_Q\,,\qquad P\in C \tag{5}$$

where P and Q refer to points (x, y, z) and (a, b, c) on C respectively, and σ is the density of the distribution. Using the solution σ of (5), the velocity potential ϕ on C can be written as

$$\phi(P)=\frac{f(P)-\sigma(P)}{K}\,,\qquad P\in C \tag{6}$$

The boundary condition of pitch mode on C is written as follows,

$$f(P)=-x=-r\cos\theta\,,\qquad \text{on } C \tag{7}$$

and the density of the distribution is

$$\sigma(P)=\sigma_0(r)\cos\theta\,,\qquad \text{on } C \tag{8}$$

The non dimensional added moment of inertia and wave damping coefficient are now given as

$$\frac{J_x}{\rho\bar{a}^5}=\frac{\pi}{K\bar{a}^5}\int_0^{\bar{a}}(r+\sigma_{0c}(r))\cdot r^2dr \tag{9.a}$$

$$\frac{N_p}{\rho\omega\bar{a}^5}=\frac{\pi}{K\bar{a}^5}\int_0^{\bar{a}}\sigma_{0s}(r)\cdot r^2dr \tag{9.b}$$

where ρ is the density of the fluid, and

$$\sigma_0(r)=\sigma_{0c}(r)+i\sigma_{0s}(r) \tag{10}$$

The pitch component $E_5\exp(-i\omega t)$ of the wave excitation force can be written as

$$e_5=\frac{E_5}{\rho g\zeta_0\bar{a}^3}=\frac{2}{K\bar{a}^3}\cdot\frac{\{h(m_0^2-K^2)+K\}}{m_0^2}\cdot(-P_s+iP_c) \tag{11}$$

The radiation pressure $p(P)$ is written as

$$\frac{p(r)}{\rho g\bar{a}\cdot\bar{\varphi}}=\left\{\frac{r}{\bar{a}}+\frac{\sigma_0(r)}{\bar{a}}\right\}\cos\theta \tag{12}$$

According to the Haskind relation, the following relation is derived,

$$\frac{N_p}{\rho\omega\bar{a}^5}=\frac{1}{K^2\bar{a}^5}(P_c^2+P_s^2)\cdot\frac{h(m_0^2-K^2)+K}{m_0^2}=\frac{\bar{a}}{4}\frac{m_0^2}{h(m_0^2-K^2)+K}\cdot\left[\frac{|E_5|}{\rho g\zeta_0\cdot\bar{a}^3}\right]^2 \tag{13}$$

where ζ_0 is the amplitude of a plane incident wave, $\bar{a}$ is the radius of the circular disk, $\bar{\varphi}$ is the amplitude of forced pitching, θ and r are variables of polar coordinates on a circular disk, and (P_c+iP_s) are the function which are derived later and which correspond to Kochin function.

2.2 Numerical Procedure[1)]

$Q(r', \theta')$ represents an arbitrary point on a circular disk. The boundary condition is given on a point $P(r, \theta)$. The integral equation (5) is reduced to the following two sets of integral equation for unknowns $\sigma_{01}(r)$, $\sigma_{0d}(r)$, taking account of the pitch mode;

$$-r=\sigma_{01}(r)+2\pi\int_0^{\bar{a}}\sigma_{01}(r')\cdot\tilde{G}_c(r,r')r'dr' \tag{14a}$$

$$-J_1(m_0r)=\sigma_{0d}(r)+2\pi\int_0^{\bar{a}}\sigma_{0d}(r')\cdot\tilde{G}_c(r,r')r'dr' \tag{14b}$$

where

$$\tilde{G}_c(r,r')=-\frac{KA}{4\pi}S_1(m_0r)\cdot T_1(m_0r')+\frac{K}{\pi}\sum_{n=1}^{\infty}B_n\cdot V_1(m_nr)\cdot W_1(m_nr') \tag{15}$$

the sets of (S_1, T_1), (V_1, W_1) correspond to the sets of (J_1, Y_1), (I_1, K_1) respectively or vice versa, according to $r>r'$ or $r<r'$, and

$$A=2\pi\frac{m_0^2-K_2}{hm_0^2-hK^2+K}\cosh m_0(c+h)\cdot\cosh m_0(z+h)\Big|_{z=c=0} \tag{16a}$$

$$B_n=\frac{m_n+K^2}{hm_n^2+hK^2-K}\cos m_n(c+h)\cdot\cos m_n(z+h)\Big|_{z=c=0} \tag{16b}$$

Let the interval $[0, \bar{a}]$ consist of N small segments, that is

$$[0, \bar{a}]=\bigcup_{\nu=1}^{N}[a_{\nu-1}, a_\nu] \tag{17.a}$$

where

$$0=a_0<a_1<a_2<\cdots<a_\nu<\cdots<a_N=\bar{a} \tag{17.b}$$

We assume that the source density is constant at the each segment $[a_{\nu-1}, a_\nu]$. Then the following approximation is hold.

$$\int_0^{\bar{a}}\sigma(r')\cdot\bar{G}_c(r, r')r'dr'$$

$$\approx\sum\sigma\left(\frac{a_{\nu-1}+a_\nu}{2}\right)\int_0^{\bar{a}}\bar{G}(r, r')\cdot r'dr' \tag{18}$$

If we introduce

$$r_\mu=\frac{a_{\mu-1}+a_\mu}{2} \tag{19}$$

and

$$D_{\mu\nu}=\int_{a_{\nu-1}}^{a_\nu}\bar{G}_c(r_\mu, r')\cdot r'dr' \tag{20}$$

then the equation (15) and (20) yields the following expression;

a) when $a_\nu<r_\mu$

$$D_{\mu\nu}=-\frac{KA}{4\pi}Y_1(m_0r_\mu)\int_{a_{\nu-1}}^{a_\nu}J_1(m_0r')r'dr'$$
$$+\frac{K}{\pi}\sum_{n=1}^{\infty}B_n\cdot K_1(m_nr_\mu)\int_{a_{\nu-1}}^{a_\nu}I_1(m_nr')r'dr' \tag{21.a}$$

b) when $r_\mu<a_{\nu-1}$

$$D_{\mu\nu}=-\frac{KA}{4\pi}\cdot J_1(m_0r_\mu)\int_{a_{\nu-1}}^{a_\nu}Y_1(m_0r')r'dr'$$
$$+\frac{K}{\pi}\sum_{n=1}^{\infty}B_n\cdot I_1(m_nr_\mu)\int_{a_{\nu-1}}^{a_\nu}K_1(m_nr')r'dr' \tag{21.b}$$

c) when $a_{\nu-1}<r_\mu<a_{\nu-1}$

$$D_{\mu\nu}=-\frac{KA}{4\pi}Y_1(m_0r_\mu)\int_{a_{\nu-1}}^{r_\mu}J_1(m_0r')r'dr'$$
$$+\frac{K}{\pi}\sum_{n=1}^{\infty}B_n\cdot K_1(m_nr_\mu)\int_{a_{\nu-1}}^{r_\mu}I_1(m_nr')r'dr'$$
$$-\frac{KA}{4\pi}J_1(m_0r_\mu)\int_{r_\mu}^{a_\nu}Y_1(m_0r')r'dr'$$
$$+\frac{K}{\pi}\sum_{n=1}^{\infty}B_n\cdot I_1(m_nr_\mu)\int_{r_\mu}^{a_\nu}K_1(m_nr')r'dr' \tag{21.c}$$

The integral equation (14) can now be approximated by the following simultaneous linear equations for unknowns $\sigma_{01}(r_\mu)$, $\sigma_{0d}(r_\mu)$ $(\mu=1, 2, \cdots, N)$:

$$\left.\begin{matrix}-r_\mu\\-J_1(m_0r_\mu)\end{matrix}\right\}=\sum_{\nu=1}^{N}\left\{\begin{matrix}\sigma_{01}(r_\mu)\\\sigma_{0d}(r_\mu)\end{matrix}\right\}(2\pi D_{\mu\nu}+\delta_{\mu\nu}), \tag{22.a}$$

$$\mu=1, 2, \cdots, N \tag{22.b}$$

where

$$\delta_{\mu\nu}=\begin{cases}1 & (\mu=\nu)\\0 & (\mu\neq\nu)\end{cases} \tag{23}$$

The procedure to get σ_{0c} and σ_{0s} from σ_{01} and σ_{0d} is as follows,

(I)

$$\left\{\begin{matrix}P_1\\P_d\end{matrix}\right\}=\frac{KA}{2}\int_0^{\bar{a}}\left\{\begin{matrix}\sigma_{01}(r')\\\sigma_{0d}(r')\end{matrix}\right\}J_1(m_0r')r'dr' \qquad \begin{matrix}(24.a)\\(24.b)\end{matrix}$$

(II)

$$\begin{cases}P_c=P_1/(1+P_d^2) & (25.a)\\P_s=P_c\cdot P_d & (25.b)\end{cases}$$

(III)

$$\begin{cases}\sigma_{0c}(r)=\sigma_{01}(r)-P_s\cdot\sigma_{0d}(r) & (26.a)\\\sigma_{0s}(r)=P_c\cdot\sigma_{0d}(r) & (26.b)\end{cases}$$

2.3 Long Wave Approximations[5)]

The procedure in pitch mode is as same as that in heave mode. The only difference is the boundary condition on a circular disk. The velocity potential ϕ may be expressed as

$$\phi=\begin{cases}\phi_e, & r>\bar{a} \qquad (27.a)\\\phi_i, & \bar{a}>r>0 \qquad (27.b)\end{cases}$$

ϕ_e and ϕ_i must then satisfy the following equations:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\phi_e+m_0^2\phi_e=0, \quad r\geqq\bar{a} \tag{28.a}$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right)\phi_i=-\frac{\cos\theta}{h}\cdot r, \quad \bar{a}>r\geqq0 \tag{28.b}$$

The axi-assymetric solution of (28) with an outgoing progressive wave is given as

$$\phi_i=\left[\frac{1}{4h}\cdot\frac{\bar{a}}{m_0}\cdot\frac{H_1^{(2)}(m_0\bar{a})}{H_2^{(2)}(m_0\bar{a})}\cdot r+\frac{1}{8h}(\bar{a}^2-r^2)r\right]\cos\theta, \quad 0\leqq r<\bar{a} \tag{29.a}$$

$$\phi_e=\frac{\bar{a}^3}{4h}\cdot\frac{1}{m_0\bar{a}}\cdot\frac{H_1^{(2)}(m_0\bar{a})}{H_2^{(2)}(m_0\bar{a})}\cdot\cos\theta, \quad r\geqq\bar{a} \tag{29.b}$$

The added moment of inertia J_x and the wave damping N_p may then be derived as

$$\left.\begin{matrix}\dfrac{J_x}{\rho\bar{a}^5}\\\dfrac{N_p}{\rho\omega\bar{a}^5}\end{matrix}\right\}=\frac{1}{\bar{a}^5}\left\{\begin{matrix}R_e\\I_m\end{matrix}\right\}\left[\int_0^{2\pi}\int_0^{\bar{a}}\phi_i r^2\cos\theta d\theta dr\right]$$

$$=\begin{cases}\left(\dfrac{\bar{a}}{h}\right)\dfrac{\pi}{16}\left[\dfrac{1}{m_0\bar{a}}\cdot\dfrac{J_1(m_0\bar{a})J_2(m_0\bar{a})+Y_1(m_0\bar{a})Y_2(m_0\bar{a})}{\{J_2(m_0\bar{a})\}^2+\{Y_2(m_0\bar{a})\}^2}+\dfrac{1}{6}\right] & (30.a)\\[2ex]\left(\dfrac{\bar{a}}{h}\right)\dfrac{1}{8}\cdot\dfrac{1}{(m_0\bar{a})^2}\cdot\dfrac{1}{\{J_2(m_0\bar{a})\}^2+\{Y_2(m_0\bar{a})\}^2} & (30.b)\end{cases}$$

where R_e and I_m mean to take the real and imaginary parts of the corresponding quantity. The equation (30) indicates that $J_x/\rho\bar{a}^5\cdot(h/\bar{a})$ and $N_p/\rho\omega\bar{a}^5\cdot(h/\bar{a})$ are functions of $m_0\bar{a}$ alone.

The limiting values for long wave approximations are derived from the equation (30) as follows,

at $m_0\bar{a}\to 0$

$$\frac{J_x}{\rho\bar{a}^5}\cdot\left(\frac{h}{\bar{a}}\right)\sim\frac{5\pi}{192} \tag{31.a}$$

$$\frac{N_p}{\rho\omega\bar{a}^5}\cdot\left(\frac{h}{\bar{a}}\right)\sim\frac{\pi^2(m_0\bar{a})^2}{128} \tag{31.b}$$

at $m_0\bar{a}\to\infty$

$$\frac{J_x}{\rho\bar{a}^5}\cdot\left(\frac{h}{\bar{a}}\right)\sim\frac{\pi}{96} \tag{32.a}$$

$$\frac{N_p}{\rho\omega\bar{a}^5}\cdot\left(\frac{h}{\bar{a}}\right)\sim\frac{\pi}{16m_0\bar{a}} \tag{33.b}$$

3. Experiments[2)]

In order to check the numerical results which depend on linear theory, three kinds of experiments are carried out. These are the forced pitching tests, wave excitation tests and the motion tests in waves. In the forced pitching tests, the kinds of measurement are the added moment of inertia, the wave damping factor and the radiation pressure which include the pressure due to varying static pressure. In the wave excitation tests, the wave excitation of heave and pitch mode, and the wave excitation pressure. In the motion tests in waves, the amplitude of heaving and pitching and the pressure when the circular disk oscilates in waves.

The experiments are carried out in the seakeeping basin of the University of Tokyo. The detail of the false bottom is mentioned in the 2nd report.

The dimension of the model is that 900 mm of the diameter, 100 mm of the depth. The draft of 20, 31 and 50 mm correspond to the weight of 12.7, 19.7 and 31.8 kg, and the moment of inertia of 0.0585, 0.0821 and 0.0778 kg–m² respectively. The locations of the pressure gauges r are as follows, $r/\bar{a}=\pm 0.93$, ± 0.83, $+0.73$, ± 0.6, $+0.4$, ± 0.2 and 0.

4. Numerical and Experimental Results

4.1 Forced Pitching Test of a Circular Disk

The added moment of inertia J_x and the wave damping coefficient N_p of the pitching are shown in Fig. 2 as those of non-dimensional form which depend on the equations (9.a) and (9.b). The pressures measured in the forced oscillation tests include varied ones of static pressure $p_{ps}=\rho g r\cos\theta\cdot\bar{\varphi}$ besides the radiation pressures p_{pr}[6)]. Therefore according to the equation (12), the non-dimensional pressure is represented as follows,

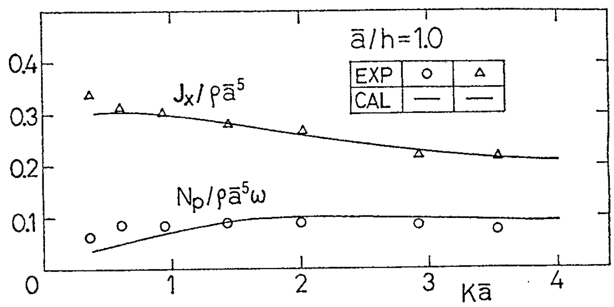

Fig. 2 Added moment of inertia and wave damping coefficient of a circular disk ($\bar{a}/h=1.0$)

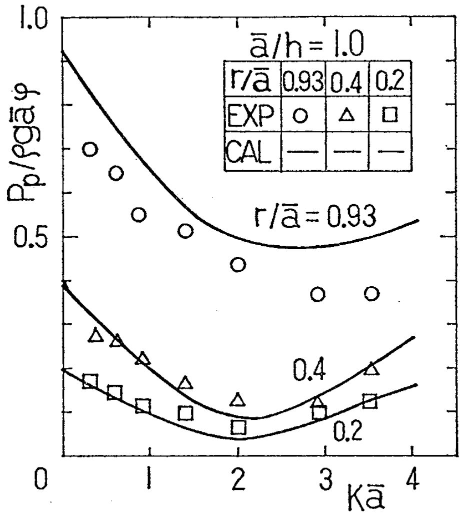

Fig. 3 Radiation pressure of pitch mode of a circular disk ($\bar{a}/h=1.0$)

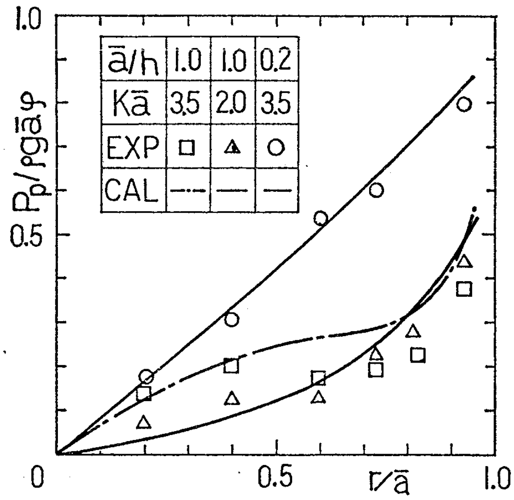

Fig. 4 Radiation pressure of pitch mode of a circular disk

$$\frac{p_p}{\rho g\bar{a}\cdot\bar{\varphi}}=\frac{p_{pr}}{\rho g\bar{a}\cdot\bar{\varphi}}+\frac{p_{ps}}{\rho g\bar{a}\cdot\bar{\varphi}}$$
$$=\left(\frac{r}{\bar{a}}+\frac{\sigma_0(r)}{\bar{a}}\right)\cos\theta+\frac{r}{\bar{a}}\cos\theta \qquad (33)$$

The non-dimensional pressures as functions of non-dimensional frequency $K\bar{a}$ are shown in Fig. 3 and those as functions of the pressure location $r/\bar{a}$ are in Fig. 4.

4.2 Wave Excitation Test of a Circular Disk

The wave excitation moment of the pitch mode is shown in Fig. 5. The wave exciting pressure p_w is non-dimensionalized by the amplitude of the incident wave ζ_0 as $p_w/\rho g\zeta_0$. The wave exciting pressures as functions of $K\bar{a}$ are shown in Fig. 6 and those as functions of $r/\bar{a}$ in Fig. 7.

4.3 Motion Test in Waves of a Circular Disk

The amplitudes of heaving $|z|/\zeta_0$ and pitching $|\varphi|/K\zeta_0$ of a circular disk are shown in Fig. 8. In the theory, the cross term between heave and pitch of the equation of motion is neglected. The surge is also ignored. The hydrodynamic forces of heave mode depend on those of the first report.[1),4)] The pressure of a circular disk which oscillates in head waves is non-dimensionalized as $p/\rho g\zeta_0$. The pressure is deduced by the summation of the pressures of heave radiation pressure, pitch radiation pressure, wave exciting pressure, and

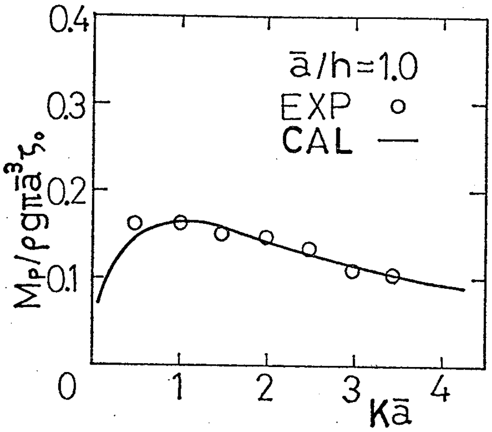

Fig. 5 Wave exciting moment of pitch mode of a circular disk

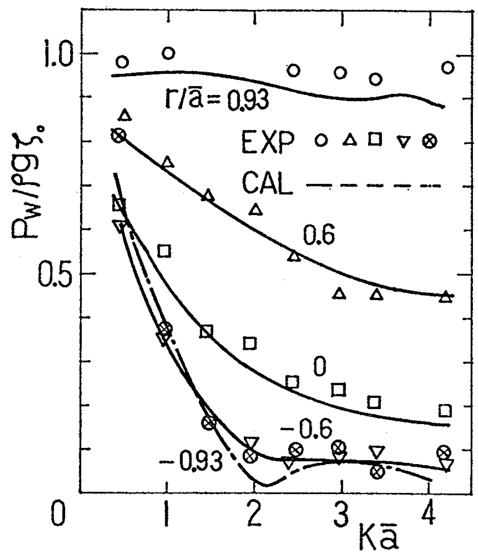

Fig. 6 Wave exciting pressure of a circular disk ($\bar{a}/h=1.0$)

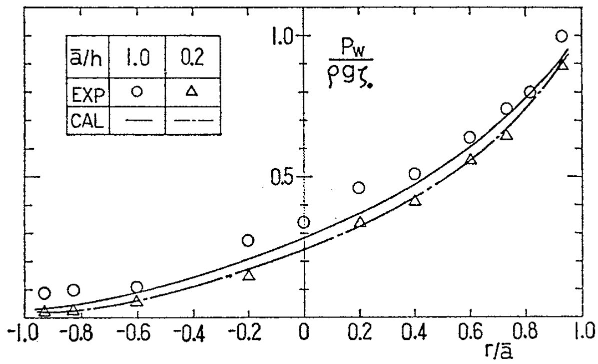

Fig. 7 Wave exciting pressure of a circular disk ($K\bar{a}=2.0$)

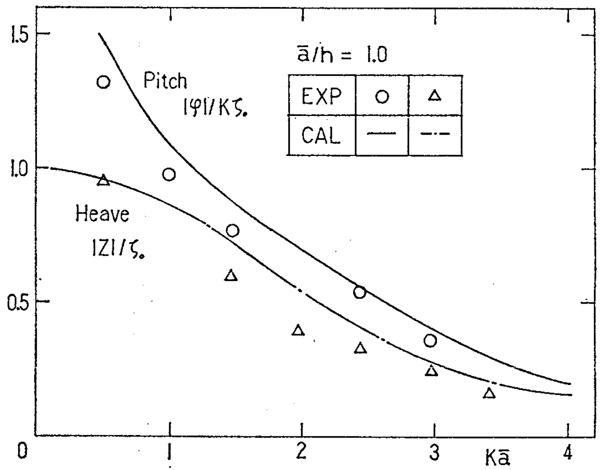

Fig. 8 Heaving and pitching amplitude of a circular disk ($\bar{a}/h=1.0$)

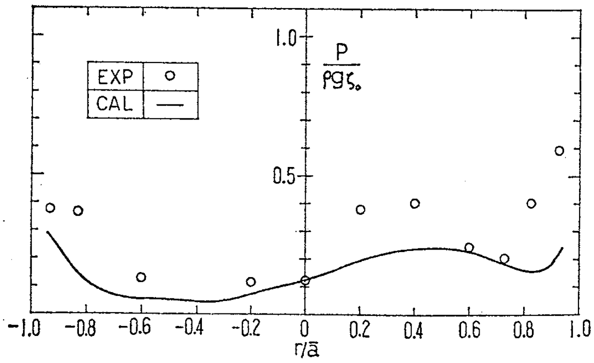

Fig. 9 Pressure distributions on a circular disk which oscillates in head waves ($\bar{a}/h=1.0$ and $K\bar{a}=2.0$)

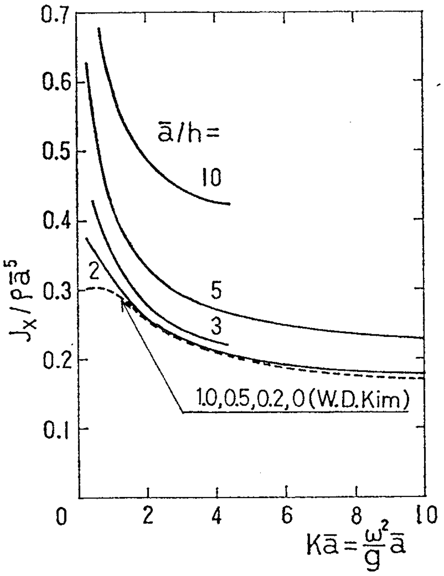

Fig. 10 Added moment of inertia of pitch mode of a circular disk as functions of frequency $K\bar{a}$

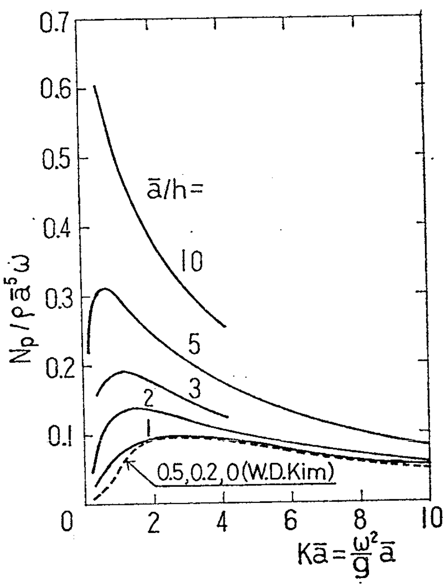

Fig. 11 Wave damping factor of pitch mode of a circular disk as functions of frequency $K\bar{a}$

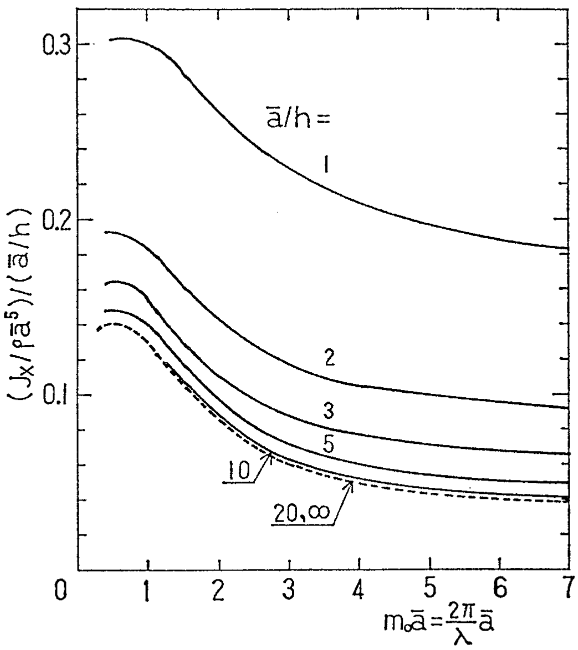

Fig. 12 Added moment of inertia of pitch mode of a circular disk as functions of shallow water wave number $m_0\bar{a}$

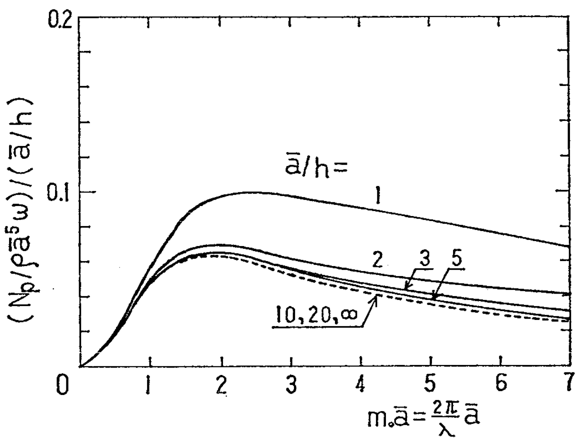

Fig. 13 Wave damping factor of pitch mode of a circular disk as functions of shallow water wave number $m_0\bar{a}$

varied pressure of static pressure by heave and pitch. The pressure distribution is shown in Fig. 9.

4.4 Shallow Water Effect on a Circular Disk

The added moment of inertia and wave damping coefficient as functions of $K\bar{a}$ for several values of the shallow water parameter $\bar{a}/h$ are shown in Fig. 10 and 11 respectively. The shallow water parameter $\bar{a}/h=0.2$ corresponds to infinite water depth $\bar{a}/h=0.0$.[3] In Fig. 12 and 13, the added moment of inertia and wave damping coefficient are plotted as functions of non-dimensional shallow water wave number $m_0\bar{a}$ together with comparison curve for long wave approximations ($\bar{a}/h\rightarrow\infty$), given by the equations (30.a) and (30.b).

5. Conclusions

i) The present numerical method which depends on characteristics of a circular disk is proved to be a good prediction method for hydrodynamic forces of pitch mode, at least in the range of the shallow water parameter less than $\bar{a}/h=1.0$ due to Fig. 2 and 5. The computer time by this method is about 1/10 of that by usual 3-D source distribution method.

ii) The present method is also effecive for the prediction of pressures. Because the results of calculation for radiation and diffraction pressures are in good agreement with those of the corresponding experiments according to Fig. 3, 4, 6 and 7 except those at the end $r/\bar{a}=\pm 0.93$. Only the numerical results for the diffraction pressure depend on the usual 3-D source distribution method.

iii) The shallow water effect for hydrodynamic forces of pitch mode is almost as same as that of heave mode. The shallow water parameter $\bar{a}/h$ less than 1.0 seems to correspond to the deep water. And the shallow water parameter $\bar{a}/h$ larger than 5.0 may be approximated by long wave.

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