### Free Surface Shock Waves and Methods for Hull Form Improvement

#### (First Report)

# by Masao Nito\*, *Member* Hisashi Kajitani\*\*, *Member* Hideaki Miyata\*\*, *Member* Yoshihiro Tsuchiya\*\*

#### Summary

A new method of designing hull forms of minimum wave resistance is developed in which two major components of wave resistance, i.e., one due to free surface shock waves and the other due to linear waves are taken into account. A procedure of extracting wave profiles due to free surface shock waves is proposed, which is utilized as a quantitative measure of the magnitude of free surface shock waves. The relation between hull form and resistance coefficient due to free surface shock waves is empirically derived and it is incorporated into the wave-analytical procedure of obtaining hull forms of minimum wave pattern resistance, so that the sum of wave pattern resistance (linear wave resistance) and resistance due to free surface shock waves (nonlinear wave resistance) is minimized. The effectiveness of the new method is demonstrated by experiments.

#### 1. Introduction

The characteristics of Free Surface Shock Wave (abbreviated as FSSW) found and investigated at the Experimental Tank of the University of Tokyo are entirely different from those of linear dispersive waves (Kelvin waves). The nonlinear characteristics of FSSW described in the references provide strong barriers against crucial and analytical evaluation of the resistance due to FSSWs, and it may only be achieved by a somewhat tedious numerical procedure in future.

In this paper an empirical procedure of hull form improvement is developed, which is based on experimental results of series ship models and takes the resistance component due to FSSWs into account. Without comprehensive considerations of the resistance component due to FSSWs no satisfactory method of hull form improvement can be achieved, because every ship generates FSSWs, intensely or slightly depending on hull form and speed of advance. This paper is composed mainly of two chapters. In Chapter 3 a method of extracting wave profiles of FSSWs from measured wave records is presented and a new parameter that denotes the magnitude of FSSWs is introduced with some applications to series models. A new design method for the sectional-area curve of minimum wave resistance is described in Chapter 4 together with the results of its application to a container hull form improvement.

#### 2. Nomenclature

- $A_0$  initial height of modeled FSSW profile
- A' obtained height of modeled FSSW profile
- a parameter of modeled FSSW profile
- aoi, a1i, a2i coefficients of quadratic approximation
  - B beam of ship
- $C(\theta), S(\theta)$  amplitude functions
- $C^*(\theta), S^*(\theta)$  weighted amplitude functions
- $C_0^*(\theta), S_0^*(\theta)$  measured weighted amplitude functions of parent model
- $C_{j^*}(\theta), S_{j^*}(\theta)$  do of *j*-th modified model on  $C_p$ -curve series
- $\Delta C_{j}^{*}(\theta), \ \Delta S_{j}^{*}(\theta): \ C_{j}^{*}(\theta) C_{o}^{*}(\theta), \ S_{j}^{*}(\theta) S_{o}^{*}(\theta), \ (\theta), \ \text{respectively}$ 
  - $C_p(x)$  optimum sectional-area coefficient

<sup>\*</sup> Graduate School, The University of Tokyo (present affiliation is Mitsui Engineering and Shipbuilding Co., Ltd.)

<sup>\*\*</sup> Department of Naval Architecture, The University of Tokyo

- $C_{p0}(x)$  sectional-area coefficient of parent model
- $C_{pj}(x)$  do of *j*-th modified model
- $\Delta C_{pj}(x) \quad C_{pj}(x) C_{p0}(x)$ 
  - $C_w$  wave resistance coefficient obtained from towing test
  - $C_{wp}$  wave-pattern resistance coefficient obtained from wave analysis
  - $C_{ws}$  FSSW resistance coefficient given by  $C_{w}-C_{wp}$
- $\Delta C_{wp}$  increment of wave-pattern resistance
- $\Delta C_{\boldsymbol{w}}$  do of wave resistance
- d draft
- E(x) error due to asymptotic expression at y=0
  - $F_n$  Froude number based on L
- $h(x) \quad \zeta_1^* \zeta_0^*$ 
  - $\Delta h$  magnitude of FSSW obtained from measured wave records
  - $k_0$  principal wave number
  - L ship length (between perpendiculars) l 1/2 L
  - M number of tested ship models
- N number of modified models
- $P(\theta), Q(\theta)$  amplitude functions
  - $\gamma$  correlation coefficient
  - U velocity of uniform flow
- x, x' axis parallel to centerline of ship, aftward positive
  - y axis parallel to ship's beam, positive to starboard side
  - z vertical axis, upward positive
  - y<sub>0</sub> y-coordinate of base-line for longitudinal-cut
  - $y_1$  do of test-line
  - $\alpha_j$  coefficients
  - $\beta$  shock angle of FSSWs
  - $\eta_i$  y-coordinate of hull on load-water-line at *i*-th s.s., nondimensionalized by a half of beam length
  - $\eta_{i0}$   $\eta_i$  of parent model
  - $\eta_{ij}$   $\eta_i$  of *j*-th model
  - $\Delta \eta_i \quad \eta_{ij} \eta_{i0}$ 
    - $\zeta$  wave elevation
  - $\zeta_0$  calculated wave height on the test-line





derived from measured amplitude functions obtained on the base-line

(length including wave height is nondimensionalized by *l* unless otherwise defined.)

### 3. Extraction of FSSW profiles from measured wave records

It is important to clarify the relations between hull form and resistance due to FSSW experimentally at the first step of hull form improvement. Parameters indicating the magnitude of FSSWs must be introduced for quantitative treatments. The shock angle  $\beta$ , which varies with speed of advance, draft and entrance angle<sup>4)5)6)</sup>, has been proposed as one of those parameters. However, shock angle alone does not always represent the magnitude of FSSWs satisfactorily in case the differences of hull form are relatively Another parameter which analogically small. corresponds to the jump of free surface across a shock front of shallow water shock wave comes to be important as a new parameter by which the magnitude of FSSWs can be expressed more precisely than shock angle.

3.1 Procedure of extraction

The procedure of extracting wave height due to FSSWs is shown in Fig. 2. There exist free wave, local-disturbance and FSSWs in the vicinity of a ship whereas at far away from the ship only free-wave exists. In order to exclude the local-disturbance, the method of longitudinalcut wave analysis such as Newman-Sharma's method (abbreviated as N-S method) is available.

The procedure for extracting wave height due to FSSWs is as follows:

1) Choose two lines parallel to the centerline of ship as shown in Fig. 2. One is at a small distance from the centerline of the ship  $(y=y_1)$ where the magnitudes are to be evaluated (called test-line hereafter), and the other is at a sufficient distance from the ship  $(y=y_0)$  where the amplitude functions of free-wave are solely obtained (called base-line).

2) Calculate wave profiles on  $y = y_1$  from the amplitude functions obtained on the two-different y-line, which are denoted by  $\zeta_0^*$  and  $\zeta_1^*$  as in Fig. 2.  $\zeta_1^*$  contains free-wave and FSSWs while  $\zeta_0^*$  contains only free-wave.

3) If h(x) is defined by  $\zeta_1^* - \zeta_0^*$ , then h(x) gives the approximate profile of FSSWs.

On calculating wave heights from amplitude functions, equivalent singularity distributions are not introduced, but the asymptotic expression at  $y \rightarrow \infty$ , that is,

$$\zeta = \int_{0}^{\pi/2} [C(\theta) \cos\{k_0 \sec^2 \theta(x \cos \theta + y \sin \theta) + S(\theta) \sin\{k_0 \sec^2 \theta(x \cos \theta + y \sin \theta)\}] d\theta \quad (1.1)$$

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Fig. 2 Procedure of extracting wave profile of FSSW

is used, though it generates some errors in h(x) because actually y is not large enough. The property of the error term is examined in the following section.

4) h(x) is also directly obtained from difference of amplitude functions of the two different ylines,  $y_0$  and  $y_1$ , from Eq. (1.1). These differences of the amplitude functions are regarded as the amplitude functions due to FSSWs. They mostly appear in the high- $\theta$  region both in the case of modeled FSSWs and of actual ones.

5) Let  $\Delta h$  be the magnitude of pulse-like shape of h(x) and be introduced as a new parameter in addition to the shock angle  $\beta$ .

#### 3.2 Error due to asymptotic expression

The approximate method of extraction abovedescribed is based on the following bold assumptions.

1) The interactions of free-waves with FSSWs are ignored,

2) Free-waves propagate linearly from the test-line  $(y=y_1)$  to the base-line  $(y=y_0)$ .

The error which may arise through the process of calculating wave profiles on the test-line by amplitude functions at the base-line is considered small for our practical purposes, which is empirically studied in the following section.

In this section effects of some other errors are examined. Let the Cartesian coordinate system be fixed in a ship as shown in Fig. 1. At a great distance sideward from the ship, that is  $y \rightarrow \infty$ , the wave elevation can be expressed asympottically as,

$$\zeta = \frac{k_0 l}{\pi} \int_0^{\pi/2} [P(\theta) \cos\{k_0 l \sec^2 \theta(x \cos \theta + y \sin \theta) + Q(\theta) \sin\{k_0 l \sec^2 \theta(x \cos \theta + y \sin \theta)] \sec^3 \theta d\theta$$
(1.2)

Through Fourier transformation of  $\zeta$  in Eq. (1.2)

with respect to x, the relation between  $\zeta$  and amplitude functions can be rewritten as follows:

$$P(\theta) + iQ(\theta) = \cos\theta \sin\theta \, e^{ik_0 ly \tan\theta \sec\theta} \\ \times \int_{-\infty}^{\infty} \zeta(x, y) \, e^{ik_0 lx \sec\theta} \, dx \quad (1.3)$$

Let us examine the behavior of the error which may arise when Eq. (1.2) which is valid at infinity, is made use of at a certain finite y-coordinate. For simplicity assume that y=0 where the maximum error is to arise. Let  $\zeta^*$  be a calculated wave profile on y=0 by substituting Eq. (1.3) into Eq. (1.2),  $\zeta^*$  can be written as follows by changing variables in the way as  $\omega = k_0 l \sec \theta$ ,

$$\begin{aligned} \zeta^* &= \frac{1}{\pi} \left[ \int_{k_0 l}^{\infty} d\omega \cos\left(\omega x\right) \int_{-\infty}^{\infty} \zeta(x', y) \cos\left(\omega x'\right) dx' \right. \\ &+ \int_{k_0 l}^{\infty} d\omega \sin\left(\omega x\right) \int_{-\infty}^{\infty} \zeta(x', y) \sin\left(\omega x'\right) dx' \\ &= \frac{1}{\pi} \left[ \int_{0}^{\infty} d\omega \int_{-\infty}^{\infty} \zeta(x', y) \cos\{\omega(x - x')\} dx' \right. \\ &- \int_{0}^{k_0 l} d\omega \int_{-\infty}^{\infty} \zeta(x', y) \cos\{\omega(x - x')\} dx' \\ &= \zeta - \frac{1}{\pi} \int_{0}^{k_0 l} d\omega \int_{-\infty}^{\infty} \zeta(x', y) \cos\{\omega(x - x')\} dx' \end{aligned}$$

$$(1.4)$$

where Fourier double integral is considered. The 2nd term of R.H.S. of Eq. (1.4) is the error term caused by applying Eq. (1.2) on y=0. It is clear from Eq. (1.4) that

$$\overset{\mathsf{Y}^*}{\overset{\mathsf{Y}^*}{\rightarrow} 0} \quad \text{if} \ k_0 l \rightarrow 0 (F_n \rightarrow \infty)$$

Thus the error vanishes at the high speed limit whereas at the low speed limit, it has the same distribution as  $\zeta$  with the opposite sign.

Provided  $\zeta$  is the local-disturbance, it is desirable that  $\zeta^*=0$  because it must be excluded for

the present purpose. On the other hand, it is desired that  $\zeta^* = \zeta$  when  $\zeta$  is fully due to FSSWs. Therefore, there must be a certain limited region of Froude number for satisfactory extraction of FSSWs with a negligible influence of local disturbance<sup>7)8)</sup>. Some numerical experiments clarified that the influence of local disturbance on N-S method is sufficiently small in the range from  $k_0L = 10$  to 20, particularly in the high- $\theta$  region of amplitude functions where those due to FSSWs are mainly distributed.

Now, suppose that  $\zeta$  represents a profile of FSSWs and assume the following simplified model of FSSW profile in order to examine the effects of longitudinal distribution of FSSW profile and of Froude number on the error term.

$$\zeta = A_0 e^{-a^2 (x - x_0)^2} \tag{1.5}$$

At  $x = x_0$  the error term,  $E(x_0)$ , can be written as follows:

$$E(x_{0}) = -\frac{A_{0}k_{0}l}{\sqrt{\pi} a} \left\{ 1 - \frac{1}{3 \cdot 2 \cdot 1} \left( \frac{k_{0}l}{2a} \right)^{2} + \frac{4}{5 \cdot 2^{2} \cdot 2} \left( \frac{k_{0}l}{2a} \right)^{4} - \frac{8}{7 \cdot 2^{3} \cdot 3!} \left( \frac{k_{0}l}{2a} \right)^{6} + \cdots \right\}$$
(1.6)

If the longitudinal distribution of a FSSW profile is narrower than about  $1/5L_{pp}$  and  $k_0L=10\sim20$ , the error can be expressed approximately as

$$\frac{|E(x_0)|}{A_0} \propto \frac{k_0 l}{a} \tag{1.7}$$







which shows that large  $F_n$  and/or narrow distribution of a FSSW profile give small value of error. The error term produces 'hollow parts' just beside the peak of FSSW profile as shown in Fig. 3. Let A' be the height of reproduced FSSW profile whose real height is  $A_0$ , the error in the height which is expressed by  $1-A'/A_0$ , is given as in Fig. 4. Comparisons of the height of FSSWs in the low speed range should be avoided because of the rapid growth of error according to the decrease of  $F_n$ . In the following analyses of measured wave records, A' is written as  $\Delta h$  and regarded as the height of FSSW on a y-line of interest.

3.3 Applications

(1) WM2

WM2 is a wall-sided model with parabolic waterlines given by

$$y = \frac{B}{2}(1-x^2)$$
,  $B = 1/5L$  (1.8)

The maximum radius of bilge circle is 5 cm at midship. It is observed from wave pattern pictures shown in Fig. 5 that the pattern of FSSWs varies considerably with the change of draft.

Comparisons of  $\zeta_1^*$  and  $\zeta_0^*$  in case of d=20 cm at  $F_n=0.26$  are shown in Fig. 6. Though three different base-lines are chosen in this case, they give similar  $\zeta_0^*$ , which means that waves propagate linearly on those lines and that any of the three lines is suitable for the base-line. On the other hand, a great difference between  $\zeta_1^*$  and  $\zeta_0^*$  is observed in the limited region near FP. It is considered to be caused by the existence of FSSWs and  $h(x) = \zeta_{1}^* - \zeta_{0}^*$  gives a profile of FSSW at  $y=y_1$  with 'hollows' which may be attributed to the error term.

The change of wave profile of FSSWs with the change of draft is well expressed by  $\zeta_0^*$  and  $\zeta_1^*$  as shown in Fig. 7. The backward shift of the longitudinal position of Wave-A according to the decrease of draft implies the decrease of the shock angle of Wave-A. The changes of h(x)





Fig. 5 Wave pattern pictures of WM2 at  $F_n=0.26$ 

due to the change of y-coordinate of the test-line are shown in Fig. 8 exemplifing the case of d =6 cm at  $F_n = 0.26$ , in which one can see gradual diminishing of FSSWs with the increase of



distance from the model.

(2) M46 sectional-area curve series

M46 sectional-area curve ( $C_p$  curve) series is composed of four ship models, i.e., the parent ship model M46, two other models M47 and M48 with modified  $C_p$  curves and M49 which was designed at  $F_n=0.27$  to have minimum wavepattern resistance by the method of Akashi Ship Model Basin<sup>9)</sup> (abbreviated as ASMB).  $C_p$  curves of the fore-body are compared in Figs. 9 and 10. Aft-bodies are common to all models. They are semisymmetric models of a container hull form without bow bulb. The frame-lines are kept unvaried.

Entrance angles of M46~M49 are 11.4°, 8.1°, 13.1° and 11.0° respectively, if they are defined as  $\tan^{-1} y/(\frac{1}{20} L_{pp})$ , where y is the offset on load water line at s.s. 9<sup>1</sup>/<sub>2</sub>. The obtained profiles of h(x) of these models at Fn=0.27 are shown in Fig. 11. M48 with the largest entrance angle among the four generates Wave-A and Wave-B conspicuously, whereas they are so weak on M47 that Wave-B is almost indistinguishable. Although M49 has the  $C_p$  curve of minimum wave-

Mođel		M46	M47	M48	M49	M50	WM2		
rbb	(m)		L	2.0		1		2.4	·····
в	(m)		-	0.3077			0.48		
đ	(m)			0.1047			0.20	0.10	0.06
L/B				6.5	· · · · ·			5.0	
Cm			·	0.984			0.996	0.993	0.988
СЬ				0.542	· · ·		0.665	0.663	0,660
Cp				0.551				0.668	
∇	(m³)	0.03482	0.03482	0.03483	0.3484	0.3483	0.1532	0.0764	0.0456
s	(m²)	0.6951	0.6961	0.6959	0.6963	0.6963	1.6645	1.1759	0.9839

Table 1 Principal particulars

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Fig. 8 h(x) of WM2 at  $F_n = 0.22$ , d = 6 cm

pattern resistance, FSSWs are not reduced compared with those of the parent model M46, which suggests that the design method for the reduction of wave-pattern resistance is not sufficient for the reduction of wave-making resistance that includes FSSW resistance.

It is quite important that the profile of the FSSWs of M49 is similar to that of M46, since hull forms near the bow (FP $\sim$ s.s.9) are almost the same. Thus the local hull form near the bow gives dicisive effect on the generation of FSSWs.

3.4 Relation between  $\Delta h$  and  $C_{w-}C_{wp}$ The new parameter  $\Delta h$  has intimate relation



with the resistance due to FSSWs because it is an approximate strength of discontinuity. On the other hand, the resistance due to FSSW can be approximately derived by substracting  $C_{wp}$ from  $C_w$ , if superposition of resistance components is admitted. The relation between  $\Delta h$  of the foremost FSSW and  $C_{w}-C_{wp}$  is shown in Fig. 12, in which the correlation is simple and linear for series models with comparatively





Fig. 12 Relation between  $\Delta h$  and  $C_{w}-C_{wp}$ 

small modifications. This is very advantageous for  $C_{w}-C_{wp}$  to be considered as wave resistance due to FSSWs. In the following chapter this value is considered to express FSSW resistance for practical purposes. However, it should be noted that  $C_{wp}$  suffers scale effect, and consequently  $C_{w}-C_{wp}$ . This unfavorable effect would better be avoided by making use of larger model ships.

#### 4. Method of optimizing $C_p$ curves

#### 4.1 Applications of the ASMB method<sup>12)</sup>

The ASMB method<sup>9</sup> recently developed is based on wave analysis and aims at giving the optimum  $C_p$  curve of minimum wave-pattern resistance at a given  $F_n$ . Being different from other similar methods<sup>10),11)</sup>, its theoretical construction seems to be simple without any ambiguous correction functions which will require a considerable amount of accumulated experimental data. This method is applied to the optimization of the  $C_p$ -curve of the fore-body of a container hull form and M49 is obtained at  $F_n=0.27$  as shown in Fig. 10.

 $C_w$  from towing test (Schoenherr friction lines are used) and  $C_{wp}$  from N-S method of M46~ M49 are shown in Fig. 13. M49 shows remarkable reduction in  $C_{wp}$  around design  $F_n$ . The measured weighted amplitude functions of M49 coincide well with the prediction, which ensures the propriety of the fundamental assumptions of the ASMB method. Nevertheless,  $C_w$  of M49 at  $F_n=0.27$  remains almost the same with M47 whose  $C_{wp}$  is more than the double of M49. The reason is now clearly attributed to the resistance component due to FSSWs. M49 could not surpass M47 in  $C_w$  which has very weak FSSWs. Thus, in the second step, attempts should be



Fig. 13  $C_w$  and  $C_{wp}$  curves of M46~M49

made for the finding of the optimum  $C_p$  curve of minimum  $C_w$  instead of  $C_{wp}$ .

### 4.2 Correlations between hull form and FSSW resistance

The effect of bulbs is not studied in this report but only that of  $C_p$  curves are discussed.  $C_p$ curves of M46 series straightly reflect on L.W.L. curves because of the invariability of their framelines. Let  $\eta_{ij}$  be offset on L.W.L. at *i*-th s.s. of *j*-th model and examine the relations between  $\eta_i$  near FP and FSSW resistance  $C_{ws}$  in terms of correlation coefficients assuming that  $C_{ws}$  is given by  $C_w-C_{wp}$ . Correlation coefficient of  $C_{ws}$ and  $\eta_i$  is defined as,

$$\gamma = D_1 / \sqrt{D_2 \cdot D_3} \tag{4.1}$$

where

$$D_{1} = \sum_{j=1}^{M} (C_{ws,j} - \bar{C}_{ws})(\eta_{ij} - \bar{\eta}_{i})$$

$$D_{2} = \sum_{j=1}^{M} (C_{ws,j} - \bar{C}_{ws})^{2}$$

$$D_{3} = \sum_{j=1}^{M} (\eta_{ij} - \bar{\eta}_{i})^{2}$$

$$\bar{C}_{ws} = \frac{1}{M} \sum_{j=1}^{M} C_{ws,j} \quad \bar{\eta}_{i} = \frac{1}{M} \sum_{j=1}^{M} \eta_{ij}$$

$$(4.2)$$

As shown in Fig. 14-(a), r is very close to 1 at any  $F_n$  within the region from s.s. 9 to FP. In other words, fine entrance reduces  $C_{ws}$  at any speed of advance, which is quite reasonable from the general properties of 'shock waves'. On the other hand, correlations between  $C_{wp}$ and  $\eta_i$ , which are obtained from Eq. (4.1) by replacing  $C_{ws}$  by  $C_{wp}$ , vary considerably depending on  $F_n$  as shown in Fig. 14(b). In low and





medium speed range, i.e,  $F_n = 0.20 \sim 0.26$ , hull forms having fine-entrance and hard-shoulder give smaller  $C_{wp}$ , while at  $F_n = 0.27$  and 0.28, better results are expected by a little fuller entrance, and at  $F_n = 0.30$  the hull form of fullentrance and easy-shoulder is desired for smaller  $C_{wp}$ . (It is noted that full-entrance gives easyshoulder and vise versa since the displacement is assumed constant.) Such tendency is well known in terms of linear wave resistance theory. It should be noted that two relations,  $C_{ws}$  vs.  $\eta_i$  and  $C_{wp}$  vs.  $\eta_i$ , have the same tendency of reducing resistance when  $F_n$  is smaller than 0.26 but that they conflict each other in the higher speed range where the optimization should be carried out so as to minimize the sum of both components i.e.  $C_{w}$ . 1.12

The informations obtained here explain the quantitative difference of  $C_w$  curves and  $C_{wp}$  curves which can be seen in Fig. 13. It is frequently observed in tank test results of  $C_p$  curve series models that the model with a fine entrance keeps superiority in  $C_w$  in the wider range of advance speed, whereas its superiority in  $C_{wp}$  ceases at a comparatively low advance speed. In Fig. 13, M47 shows smallest  $C_w$  in the range  $F_n < 0.27$ , while it is overcome in  $C_{wp}$  in the range  $F_n > 0.23$  by M49. Since fine-entrance reduces FSSW resistance at any  $F_n$ , it plays a role of suppressing the rise of  $C_w$  curves and consequently brings forth the above tendency.

## 4.3 Method of minimizing the sum of linear and nonlinear wave resistance

For the sum of wave pattern resistance (linear wave resistance) and resistance due to FSSW (nonlinear wave resistance) to be minimized, the correlation between hull form and resistance component due to FSSW ( $C_{ws}$ ) is incorporated into the ASMB method as follows.

1) Obtain the relations between  $C_{ws}$  and  $\eta_i$  at several s.s. near F.P. from experimental results on  $C_p$  curve series and approximate the relations in the following quadratic form.

$$C_{ws} = a_{0i} + a_{1i}\eta_i + a_{2i}\eta_i^2 \tag{4.3}$$

Coefficients  $a_{0i}$ ,  $a_{1i}$  and  $a_{2i}$  are determined by the method of least square. For M46 series, quadratic approximations are given at four s.s. as shown in Fig. 15 in which M43E3 has similar particulars to those of M46 series but with the entrance angle of 15°.

2) Express the optimum  $C_p$  curve at a certain  $F_n$  as,

$$C_{p}(x) = C_{p0}(x) + \sum_{j=1}^{N} \alpha_{j} \varDelta C_{pj}(x)$$
 (4.4)

Since frame-lines are not modified,  $\eta_i$  is given as



$$\eta_i = \eta_{i0} + \sum_{j=1}^N \alpha_j \varDelta \eta_{ij} \tag{4.5}$$

Then the increment of  $C_w$  is expressed as

$$\Delta C_{w} = 4\pi \int_{0}^{\pi/2} \left[ \sum_{j=1}^{N} \alpha_{j} \left( \frac{C_{0}^{*}}{L} \cdot \frac{\Delta C_{j}^{*}}{L} + \frac{S_{0}^{*}}{L} \right) \right] d\theta + 2\pi \int_{0}^{\pi/2} \left[ \left( \sum_{j=1}^{N} \alpha_{j} \frac{\Delta C_{j}^{*}}{L} \right)^{2} + \left( \sum_{j=1}^{N} \alpha_{j} \frac{\Delta S_{j}^{*}}{L} \right)^{2} \right] d\theta + a_{1i} \sum_{j=1}^{N} \alpha_{j} \Delta \eta_{ij} + a_{2i} (\eta_{i0} + \sum_{j=1}^{N} \alpha_{j} \Delta \eta_{ij})^{2} - 2a_{2i} \eta_{0}^{2} \quad (4.6)$$

The first and second terms are for  $\Delta C_{wp}$  in the ASMB method.  $\alpha_j$  of minimum  $C_w$  is determined by,

$$\frac{\partial \Delta C_w}{\partial \alpha_j} = 0 \qquad j = 1, N \tag{4.7}$$

which leads to the following simultaneous equations.

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1N} \\ A_{21} & A_{22} & \cdots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \cdots & A_{NN} \end{bmatrix}^{-1} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_N \end{pmatrix} (4.8)$$

where

$$A_{kj} = 4\pi \int_{0}^{\pi/2} \left( \frac{\Delta C_{k}^{*}}{L} \cdot \frac{\Delta C_{j}^{*}}{L} + \frac{\Delta S_{k}^{*}}{L} \cdot \frac{\Delta S_{j}^{*}}{L} \right) d\theta$$
$$+ 2a_{2i}\Delta\eta_{ik}\eta_{ij} \qquad (4.9a)$$
$$B_{k} = -4\pi \int_{0}^{\pi/2} \left( \frac{C_{0}^{*}}{L} \cdot \frac{\Delta C_{k}^{*}}{L} + \frac{S_{0}^{*}}{L} \cdot \frac{\Delta S_{k}^{*}}{L} \right) d\theta$$
$$- 2a_{1i}\Delta\eta_{ik} - 2a_{2i}\eta_{i0} \qquad (4.9b)$$

3) When the relation between  $C_{ws}$  and  $\eta_i$  (Eq. (4.3)) is obtained at four square stations, for instance, four sets of  $\{\alpha_j\}$  and sixteen estimated values of  $C_{ws}$  are obtained. Thus sixteen values of  $C_w$  are estimated which is the



sum of  $C_{ws}$  and  $C_{wp}$ , though  $C_{wp}$  has four different estimations. Among several sets of  $\{\alpha_J\}$ , one should choose one set which gives as a whole small values of  $C_w$ . Actually, however, little difference is found among the obtained optimum  $C_p$  curves. It is helpful for this choice to make use of the correlation ratio around quadratic of Eq. (4.3).

#### 4.4 Optimum $C_p$ Curves

Optimization schemes for  $C_p$  curves were carried out by both the ASMB method and the present one at several design Froude numbers from 0.20 to 0.32. Three of these are exemplified in Fig. 16. The optimum  $C_p$  curve of minimum  $C_w$  is less influenced by the change of design speed and keeps a tendency of fine-entrance with hard-shoulder up to the high-speed region because of the consideration of FSSW resistance. These results are expected from the correlations shown in Fig. 14 as well.

Based on the optimum  $C_p$  curve of minimum  $C_w$  at  $F_n=0.27$ , M50 was prepared to ascertain the availability of the present method.

#### 4.5 Tank test results of M50

 $C_w$  and  $C_{wp}$  curves of M50 are shown in Fig. 17 together with those of M49. M50 achieves a



Fig. 17  $C_w$  and  $C_{wp}$  curves of M47~M50



Fig. 18 Wave pattern pictures of M46 series at  $F_n = 0.27$ 

9.4% reduction of  $C_w$  compared to M49 at  $F_n = 0.27$ , which is just within the prediction of from 2.6% to 12.8%. Relatively small  $C_{wp}$  is simultaneously obtained at  $F_n = 0.27$  which is predicted with a considerable degree of accuracy. These facts indicate the effectiveness of the present method to reduce  $C_w$ , as well as the effectiveness of the ASMB method to reduce  $C_{wp}$ .

Wave-pattern pictures of M46 series are shown in Fig. 18. The magnitude of FSSWs of M50 should be smaller than M49 and larger than M47, which is observed well in these pictures.

#### 5. Conclusion

Principal conclusions are as follows:

- 1) A method for the extraction of FSSW profiles from measured wave records is developed, wherein a new parameter  $\Delta h$  that indicates the magnitude of FSSW is introduced.
- 2) Local hull form near the bow has dominant influences on FSSWs.
- 3) A new design method for hull forms of minimum wave resistance is developed by incorporating the correlation between hull form and resistance due to FSSW into the ASMB method. The effectiveness of this method is demonstrated by the application to hull form of a container carrier.
- 4) In general, the superiority of fine-entrance with hard-shoulder holds up to the highspeed region when the resistance component due to FSSW is taken into consideration.

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