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Scale Effects on Wake Distribution of Ships with Bilge Vortices

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Abstract

The scale effects of the boundary layer and wake distribution of ships with bilge vortices are investigated as an extension of the author's previous paper on the same problem without bilge vortices. It is assumed that the flow consists of the ordinary wake portion without bilge vortices and the vortex wake. The characteristics of the latter are discussed firstly with the main purpose of investigating the Reynolds number effects on the location of the vortex center, circulation, velocity and vorticity distributions. Secondly a method of correlating the model and ship's wake is proposed.

1. Introduction

This paper describes the scale effects on the wake distribution of ships with bilge vortices. Here the word "wake" is used, as in the author's previous paper[1], to mean the velocity defect at or near the propeller plane. It is not strictly restricted to the flow field in the downstream of the stern end, but is used more generally to include the boundary layer at the stern.

In the previous paper the author described the scale effects on wake distribution without the occurrence of bilge vortices. More previously Sasajima and the present author[2] wrote a paper on the same problem and proposed a simple method to predict the full scale wake from the model's value. Both methods seem to have been applied for obtaining the wake distribution of full scale ships with bilge vortices without detailed checking about the effect of longitudinal vortices on scale effects of stern flow field.

To the present author, however, it seems to be necessary to further investigate if such boundary-layer-like approaches are applicable to the flow with vortices of longitudinal axis inside the boundary layer. There are some other attempts[3] for obtaining the ship wake distribution from model tests but physical aspects of the problem seem to be still missing. All such circumstances gave the author the motivation for this paper.

In the following, discussions are made for turbulent flow without free surface. Main concern is on the effect of scale, in other words, the effect of Reynolds number on flow characteristics.

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2. Conceptual structure of boundary layer and wake with bilge vortices

Boundary layer and wake of three-dimensional (3D) bodies are expressed as the distributions of transverse and longitudinal vortices to the flow at infinity. In the case without 3D separation both vortices adhere to the surface of the body and do not separate from it. Hence the longitudinal vortices are simply the appearance of a characteristic of the 3D boundary layer and wake. They are not concentrated at a particular point. While, in the bodies with 3D separation, there are concentrations of longitudinal vortices due to separation from the surface, like bilge vortices of ships. For this case, which we have in mind, a simplest model of flow field is that the boundary layer and wake consist of two parts, one the transverse vortices representing the usual boundary layer and wake without separation and the other the longitudinal separating vortices representing bilge vortices. Both vortices orthogonally intersect, so the characteristics of each vortex are probably approximated to be independent of each other, i.e., the existence of longitudinal vortices does not affect the principal nature of the boundary layer and wake and, vice versa, the boundary layer and wake does not change the nature of longitudinal vortices as a first approximation. This is the initial standpoint of this paper. From this, the characteristics of the sole longitudinal vortex are discussed first, because the ones of the boundary layer and wake are already treated before. Combined whole flow field is discussed next with a comparison of the theory and experimental data.

3. Characteristics of longitudinal vortices

3.1 Flow field near bilge part at the stern is schematically shown in Fig. 1. Bilge part at ship's bottom is nearly straight and the flow direction here is nearly along the center line of the bottom, hence oblique to the bilge, which is similar to the flow to a yawed body. Thus, the orthogonal coordinate axes x, y, z are conveniently taken as in the figure. Velocity components in each axis direction are u, v, w respectivly in the boundary layer, U, V, W being their values at the edge of the boundary layer. Variation of flow field in the direction of x is considered to be small. 3D separation occurs due to the separation of lateral flow component w caused by pressure gradient in this direction. Separation point is Z_s. (Fig. 2)

The flow pattern in lateral y-z plane is quite similar to 2D (two-dimensional) case except one

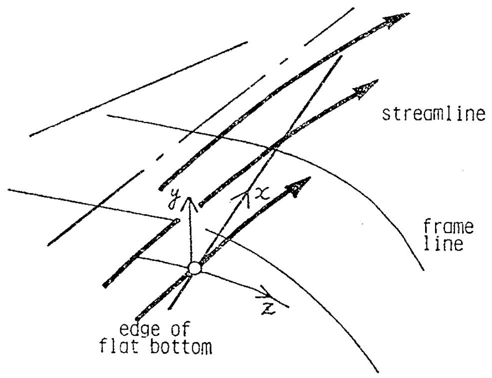


Fig. 1 Flow near the bilge part and coordinate axes

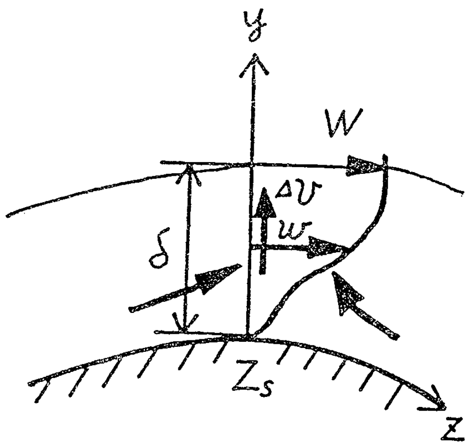


Fig. 2 Schematic flow pattern in transverse section near separation place

important difference that there is a strong flow component u in x direction. Separated x-wise axis vortices have no time to quickly form a big separated zone, because the vortices are carried away by u to downstream immediately after their generation. This suggests the longitudinal vortices generated in the boundary layer do not develop into a strong vortices outside the boundary layer, but stay inside or near the layer, unless they are extremely strong in strength.

3.2 The above discussion seems to mean there is no essential difference between the boundary layer with 3D-separation vortices and the one without them. Here we have to mention one condition which differentiates the two flow fields in theory. Let us consider the flow field at the separation point Z_s of a longitudinal vortex inside the boundary layer. At Z_s, w cannot flow in the direction of z, so, from continuity condition, the flux has to be diverted to the other directions. Since we assume the flow does not change much in the x direction, it is sufficient to consider the flux has to go out in the y direction, normal to the surface of the body.

Next, consider the amount of the flux. In simplest thought, the quantity ΔQ to be separated at the separation point will be the portion of flow which has less kinetic energy, namely the flow inside the boundary layer. Thus,

$$\Delta Q=\int_0^\delta w\,dy \tag{1}$$

where δ is the thickness of the boundary layer. We assume the direction of flow vector at each y inside the boundary layer is the same. Then $w \propto u$, so

$$\Delta Q \propto \int_0^\delta u\,dy \tag{2}$$

u slightly changes its shape inside the boundary layer according to Re, Reynolds number, but, if we neglect it as a first approximation, the effect of Re is represented by δ. Then ΔQ is written as

$$\Delta Q \propto UL\cdot\frac{\delta}{L} \tag{3}$$

As was explained in the previous report, $\delta/L \propto \sqrt{C_F}$, if we assume the flow in consideration is the 2D boundary layer for convenience, where C_F is, exactly speaking, the frictional resistance coefficient of the ship up to the location considering.

C_F, however, is nearly equal to the value corresponding to the overall ship, because the location we consider is anyhow at the stern and C_F does not change appreciably according to the location. Thus,

$$\Delta Q \propto UL\cdot\sqrt{C_F} \tag{4}$$

Since the initial normal velocity Δv is proportional to ΔQ, we obtain the following relation:

$$\Delta v \propto U\sqrt{C_F} \tag{5}$$

This shows Δv at the separation place caused by the lateral flow separation is different accoring to Re and varies in proportion to $\sqrt{C_F}$.

In the above derivation of Δv, we assumed the flux to be influenced by the separation is all the quantity inside the boundary layer. However, there may be arguments on this point. If we assume the pressure gradient along z is not strong and the velocity component u is large, the normal flux occuring as the result of separation will be smaller than the above in quantity. To reflect this into (1) or (2) the upper limit δ of the integral has to be changed to, e.g., δ^* displacement thickness. Then

$$\Delta v \propto \int_0^{\delta^*} u\,dy \tag{6}$$

In this expression the change of u according to Re has a slightly bigger influence on the value of integration because the relative importance of the integrand to the value of the integral is bigger than before due to the facts $\delta^* \ll \delta$. However even in this expression, under the same assumption for deriving (3), it is easily shown the first approximation for Δv is

$$\Delta v \propto U\frac{\delta^*}{L} \tag{7}$$

which leads to

$$\Delta v \propto UC_F \tag{8}$$

It is also shown that, if we choose θ, momentum thickness, as the upper limit of the integral instead of δ^*, Eq. (8) is again obtained as a first approximation.

On the contrary, if we use some length which is bigger than δ as the upper limit of the integral, the effect of Re on the integral will gradually decrease and finally the integral reaches to the value independent of Re. This corresponds to the case where the vortex formation by separation is very big so that the vortex moves well far out of the inside of the boundary layer.

Thus, we have many different cases regarding Re dependency of Δv depending on the magnitude of the upper limit of the integral, i.e., the strength of separated vortex. In this paper we restrict ourselves to a slightly limited flow configuration. Namely, let us implicitly suppose a longitudinal vortex is inside or near the boundary layer. This means we omit a very strong 3D separation such as the one at sharp corners of delta wings. Then, in the framework of the boundary layer theory, it seems to us that the possible length to be used as the upper limit in general way will be δ. The reason for this is that δ is the only thickness to designate the flow zone which has less kinetic energy and hence is influenced by adverse pressure gradient.

For the other two cases it seems to be difficult to give them physical necessity for the upper limits, because we can conceive more different distances other than the two. So, let us take the case of Δv proportional to δ/L as a representative one expressing the amount of flow involved in the separation. About the other cases we discuss later again.

Now let us discuss the location of longitudinal vortex for this case. Since we obtained the normal velocity at the separation place, we can estimate the location of longitudinal vortices as follows. Longitudinal velocity u is much larger than Δv. So, even if u suffers a slight change in magnitude according to change in Re, the convective nature of the vortex is the same throughout the Re change. This means the longitudinal vortex flows downstream under the combined action of the longitudinal velocity, which has no Re dependency, and the initial normal velocity to the surface, which is proportional to δ/L or $\sqrt{C_F}$. Therefore, assuming that the difference of the location of the vortex is only caused by initial condition, we can conclude the distance of the longitudinal vortex from the surface varies according to $\sqrt{C_F}$.

It is admitted, however, that some arbitrariness exists in deducing the result only from the initial condition. But, it is also to be noted that, if we consider a consecutive occurrence of 3D separation along a separation line, we have to consider a consecutive distribution of Δv on it. In this case we obtain the same conclusion in a more amenable manner.

Now let us discuss the case where Δv is proportional to C_F. Since this case is obtained by using δ^* or θ as the upper limit of the flow quantity to be involved in 3D separation, this corresponds to the case of very weak separation. Eq. (8) shows that the longitudinal vortex suffers the Re dependency proportional to C_F, instead of $\sqrt{C_F}$, if the separation is very weak.

To look into this result from a slightly different direction, let us consider the, so-to-speak, limiting position of 3D separation vortex. As was referred to in 2, the boundary layer over the body surface without separation can be expressed by transverse and longitudinal vortices distributed over the surface. From the standpoint of the boundary layer theory, vortices existing along the normal to the surface inside the boundary layer can be replaced by a concentrated summed-up vortex placed at the distance δ^* from the surface. If the 3D separation tends to be very weak, the limiting configuration of vortex system will be coincident with the one without separation. This suggests the limiting position of the longitudinal vortex is at δ^*, i.e., on the surface of the equivalent body whose surface is expressed by adding δ^* on the original surface.

As this discussion refers to the location of vortex in limiting condition and the former discussion of Δv proportional to C_F does to the slope of a streamline to carry the vortex to the downstream, both do not necessarily mention the same thing. However, both suggest that the inner limit of the distance of longitudinal vortex from the surface is δ^*/L and proportional to C_F. This nature seems to be noted when the longitudinal vortex is very weak.

A supplementary remark is added here. It is obvious the effect of induced velocity due to image vortices has also to be considered to determine the location of the longitudinal vortex. Therefore the Re dependency also appears in this aspect but this does not matter, at least to the distance of the vortex from the wall. Furthermore, exactly speaking, the final solution to express the location of the vortex is to be determined by considering the balancing condition existing between the location of the vortex, the circulation strength, and the separation place. This condition will be very predominant if the circulation of vortex is very strong and the longitudinal velocity component is very weak. For the analysis of flow field of very full ships with very strong, clear trailing vortices, it may be necessary to consider this feature of separated flow, but in this paper we omit this kind of consideration as the outside of the general framework of the present theory.

3.3 Next let us consider the scale effect of the strength of circulation Γ. As to this the answer is simple. From the discussion above, it is obvious the circulation of the vortex is determined by the lateral component of velocity W at the outside of the boundary layer. W changes its value slightly according to Re because the flow outside the boundary layer is affected by the thickness of the layer, which differs as Re changes, as is well known. But we can neglect this change in the framework of the present paper so it is concluded that Γ of the longitudinal vortex is constant in non-dimensional form, namely in the form of Γ divided by the product of representative speed and length.

3.4 Next the viscous aspects in characteristics of a longitudinal vortex are to be discussed. As was explained in the preceding section, the location of the vortex center and the strength of circulation were obtained corresponding to Re. Next items to be investigated are the velocity and vorticity distribution in viscous, turbulent longitudinal vortex. As is written in 2, the method in this paper is to assume that the characteristics of longitudinal vortex are independently separated from the ordinary, transverse boundary layer components. Therefore, if we obtain the characteristics of a sole longitudinal vortex in uniform axial flow, we can simply add them to the characteristics of the ordinary boundary layer. A reason for this is that the velocity distribution where the longitudinal vortex is located can be approximated as uniform as a first approximation, although it is obvious that it is more plausible to take into account of many detailed aspects of each basic flow and their interaction. Batchelor[4] wrote a paper about a similar flow in laminar condition. Now, with a longitudinal vortex in the boundary layer in mind, we consider the analogy between the laminar and turbulent cases.

If we consider a longitudinal vortex placed in a uniform flow, a possible method to correlate the laminar solution with the turbulent one is to replace kinematic viscosity ν with eddy viscosity ν_T. In principle ν_T slightly changes its value according to the scale, configuration and characteristics of the flow. But, as is well known, the change is gentle and this method of correlation was proved to be effective in many boundary layer and wake problems. Therefore we try to apply this method to the present problem.

Clauser found a useful expression for ν_T in 2D boundary layer, i.e.:

$$\frac{U\delta^*}{\nu_T}=56\equiv\alpha \text{ (constant)} \tag{9}$$

The process to obtain the Re dependency of turbulent solution is to use ν_T instead of ν in the laminar solution obtained by Batchelor, then to use the relation $\delta^*/L \propto C_F$.

In this subsection different notations from preceding subsections are used as shown in Fig. 3. A longitudinal vortex is placed in a uniform flow. x axis is at the center of the vortex whose origin coincides with the origin of the vortex and r axis expresses the distance from the center. All things are axially symmetric. u, v, w are the velocity components along x, r, and circumferential directions respectively. U is the x wise velocity at $r\rightarrow\infty$. Batchelor's solution is written as follows in downstream.

x-wise velocity:

$$u=U-\frac{\Gamma^2}{32\pi^2\nu x}\left(\log\frac{Ux}{\nu}\right)e^{-\eta} \\ +\text{Higher Order Terms} \tag{10}$$

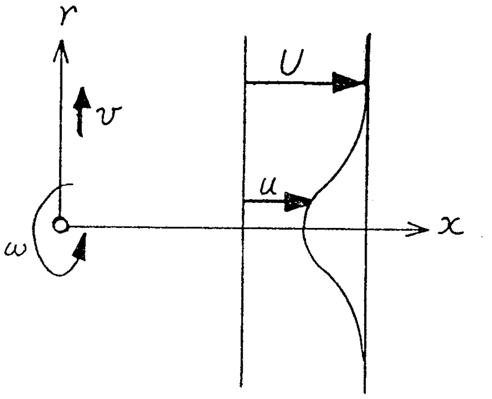


Fig. 3 Coordinate axes and velocity components for longitudinal vortex

where

$$\eta\equiv\frac{Ur^2}{4\nu x}, \tag{11}$$

and Γ is the circulation of the vortex at $r\rightarrow\infty$. Circulation:

$$2\pi rw=\Gamma(1-e^{-\eta}) \tag{12}$$

Longitudinal vorticity component having x-wise axis:

$$\omega=\frac{U\Gamma}{4\pi\nu x}e^{-\eta} \tag{13}$$

All these solutions are derived under the boundary layer approximation:

$$u\ll U, \quad v\ll u, \quad \frac{\partial}{\partial x}\ll\frac{\partial}{\partial r}.$$

These solutions show us many interesting and important characteristics of the longitudinal vortex such as x-wise or r-wise velocity distributions. However, in the following, we concentrate our attention to the effect of Re on r-wise distribution in velocity and vorticity. Namely, we compare various quantites between two Res corresponding to model and ship conditions at the same value of x/L, where L is the length of model or ship.

Now, putting (9) into (11), we obtain the expression of η for turbulent flow as follows.

$$\eta=\frac{Ur^2}{4\nu_T x}=\frac{\alpha}{4}\frac{r^2}{x\delta^*}=\mathrm{const}\times\frac{(r/L)^2}{x/L\cdot\delta^*/L} \tag{14}$$

We take two ηs, one in model scale and the other in ship scale. Suffixes m and s are used to signify the respective cases. Let us consider the two points where $\eta_m=\eta_s$ and call them the corresponding points each other. For these points, the relationship between $(r/L)_m$ and $(r/L)_s$ is given as follows, remembering that we compare the points with the same x/L:

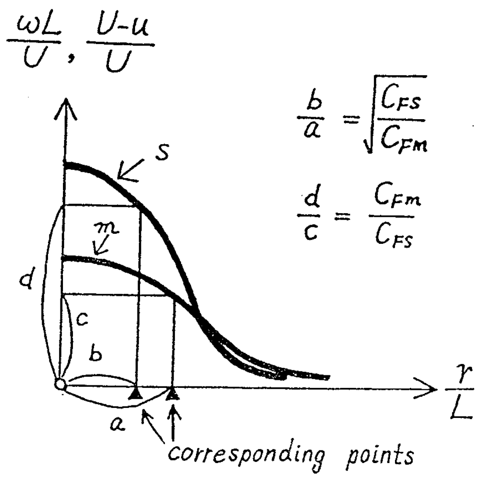


Fig. 4 Correlation method for vorticity and wake of longitudinal vortex between model and ship

$$\left(\frac{r}{L}\right)_s=\left(\frac{r}{L}\right)_m\times\sqrt{\frac{(\delta^*/L)_s}{(\delta^*/L)_m}}=\left(\frac{r}{L}\right)_m\times\sqrt{\frac{C_{Fs}}{C_{Fm}}} \tag{15}$$

To derive the last term from the middle in Eq. (15), the relation $\delta^*/L\propto C_F$ was used as before.

The corresponding points play a crucial role in the correlation of velocity and vorticity distributions between model and ship, because the r-wise distributions of them are all uniquely decided by η. As shown in Eq. (15), if we contract the model abscissa $(r/L)_m$ in the ratio of $\sqrt{C_{Fs}/C_{Fm}}$, we obtain the ship abscissa $(r/L)_s$ to achieve the similarity in this direction. In other words, the radius of the vortex in ship scale is smaller than the model in the ratio of $\sqrt{C_{Fs}/C_{Fm}}$.

Next, let us check the vorticity distribution. From eq. (13) we obtain the following:

$$\frac{\omega L}{U}=\mathrm{const}\times\frac{\Gamma}{UL}\frac{1}{\delta^*/L\cdot x/L}e^{-\eta} \tag{16}$$

As was explained before $\Gamma/UL=$const., therefore

$$\frac{\omega L}{U}\propto\frac{1}{\delta^*/L}e^{-\eta} \tag{17}$$

at the same value of x/L. Therefore, at the corresponding points between model and ship

$$\left(\frac{\omega L}{U}\right)_s=\left(\frac{\omega L}{U}\right)_m\times\frac{(\delta^*/L)_m}{(\delta^*/L)_s}=\left(\frac{\omega L}{U}\right)_m\times\frac{C_{Fm}}{C_{Fs}} \tag{18}$$

The ordinate of $\omega L/U$ in ship scale is C_{Fm}/C_{Fs} times the value of $\omega L/U$ in model scale, i.e. the ship's vorticity in non-dimensional form is larger than the model's value. Fig. 4 is the illustration of the correlation of vorticity between model and ship.

For the wake $(U-u)/U$, we obtain

$$\frac{U-u}{U}=\text{const}\times\left(\frac{\Gamma}{UL}\right)^2\frac{1}{x/L}\frac{1}{\delta^*/L}\times\log\left(\frac{x}{L}\frac{\alpha}{\delta^*/L}\right)e^{-\eta}+\text{Higher Order Terms}. \tag{19}$$

In this eq. Re dependency through δ^* appears in two places, at the outside of log and the inside of it. Re dependency from the latter makes the discussion slightly obscure, so let us make some approximation here. If we consider a little far downstream from the origin of the vortex, say $x/\delta^*=O(10)$, the difference of δ^*/L according to Re does not much change the value of log. The situation is more and more true if we consider farther downstream. It is however, quite obvious that there is a question whether or not the location of flow field we are now considering really corresponds to the flow far downstream. But, for the purpose of obtaining a good insight into a fundamental characteristic about the Re dependency, let us neglect the effect from the log-term as secondary. Then, we obtain the next equation:

$$\frac{U-u}{U}\propto\frac{1}{\delta^*/L}e^{-\eta} \tag{20}$$

at the same value of x/L, which is again the same as eq. (17). Therefore, like $\omega L/U$,

$$\left(\frac{U-u}{U}\right)_s=\left(\frac{U-u}{U}\right)_m\times\frac{(\delta^*/L)_m}{(\delta^*/L)_s}=\left(\frac{U-u}{U}\right)_m\times\frac{C_{Fm}}{C_{Fs}} \tag{21}$$

at the corresponding points of model and ship. Thus the similarity law of the wake distribution is the same as the vorticity in non-dimensional form. Fig. 4 is also serves as the illustration of wake correlation between model and ship.

4. Scale effects of the boundary layer and wake with longitudinal vortices

In this section we superpose the characteristics of the boundary layer and wake obtained in the earlier paper and the ones of the longitudinal vortex explained in this paper, attempting to obtain the correlation law between the model and ship scale. Explanation is made assuming that we consider such parts of the boundary layer and vortex as are located very close to the end of the stern.

In the previous paper, the author explained the correlation laws for the boundary layer and wake, in which there were several cases according as the flow is 2D or 3D and the boundary layer or the wake. Here, for convenience sake, let us take the case of the 2D boundary layer and wake. (Henceforth the boundary layer and wake is called wake only for the sake of convenience.) According to the previous result, the non-dimensional thickness of the wake δ/L is proportional to $\sqrt{C_F}$ and the non-dimensional velocity defect in viscous flow portion, $(U-u)/U$, is proportional to $\sqrt{C_F}$ at the same non-dimensional distance from the wall, y/δ (or in other words, at the corresponding point). In this correlation method, inviscid, potential flow portion should not be counted in velocity defect.

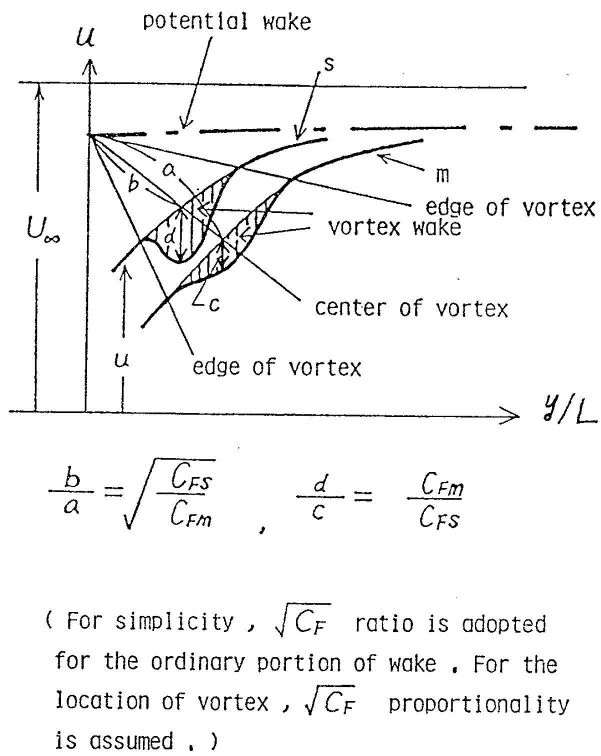


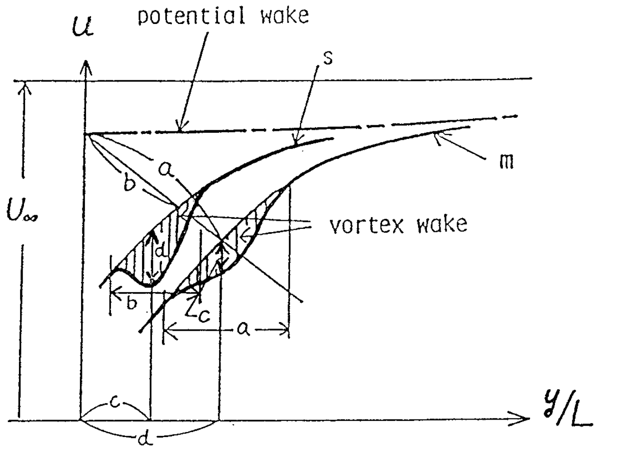
Fig. 5 Correlation method for wake with longitudinal vortex between model and ship ($\sqrt{C_F}$ contraction method for the location of vortex).

With this result and the characteristics of a vortex at hand, we can predict the full scale wake distribution with a longitudinal vortex from the model's data as follows. (See Fig. 5.) Here the explanation is made only for the longitudinal component of wake distribution.

For the sake of convenience, the representative case of $\sqrt{C_F}$ proportionality in Δv and thus in the location of longitudinal vortex is explained first.

1. Suppose the wake distribution in model scale is available as in the figure. Separate the distribution into the ordinary wake portion without dent and the dent portion which shows the vortex effect.

2. The ordinary wake portion is transferred to ship scale following the previous law.



$$\frac{b}{a}=\sqrt{\frac{C_{FS}}{C_{Fm}}}\ ,\quad \frac{d}{c}=\frac{C_{Fm}}{C_{FS}}$$

(For simplicity , $\sqrt{C_F}$ ratio is adopted for the ordinary portion of wake . For the location of vortex , C_F proportionality , the inner limit, is assumed .)

Fig. 6 Correlation method for wake with longitudinal vortex between model and ship (C_F contraction method for the location of vortex, the inner limit).

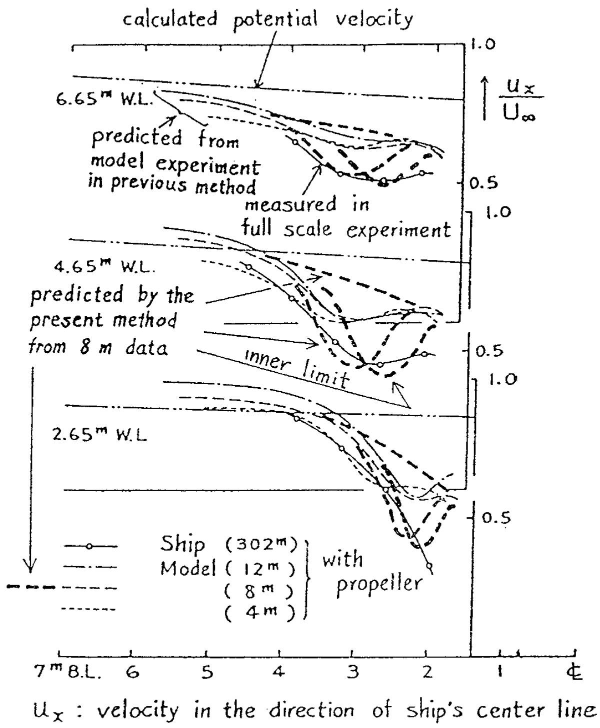


Fig. 7 Comparison of velocity distributions between the measured full scale experiment and the predicted distributions from model experiments (Niizuru Maru, SR 107)

3. Vortex center is transferred to the new position in ship scale according to the result that

$$\left(\frac{h}{L}\right)_s=\left(\frac{h}{L}\right)_m\times\sqrt{\frac{C_{Fs}}{C_{Fm}}} \tag{22}$$

where h is the distance of the vortex center from the surface.

4. The distribution of the additional wake due to vortex is similar in shape between model and ship, but the distribution should be contracted in radius direction and be magnified in the value of vortex wake. The ratio of the contraction is $\sqrt{C_{Fs}/C_{Fm}}$ and the ratio of the magnification is C_{Fm}/C_{Fs}.

As a reference it may be worthwhile to describe another case, the inner limit, of the location of a vortex. In this case the procedure to obtain the ship's value from the model's one is illustrated in Fig. 6. The steps to be taken are only different from the above at the following point:

3′ Vortex center is transferred to the new position in ship scale according to the result that

$$\left(\frac{h}{L}\right)_s=\left(\frac{h}{L}\right)_m\times\frac{C_{Fs}}{C_{Fm}} \tag{23}$$

This concludes the steps to be taken to predict the ship wake with vortex from the model's value. The method is based on a very simplified flow model that the 3D separation vortex sheet in complicated, deformed shape is replaced by a single vortex of longitudinal axis. Therefore there may be some ambiguous or illogical portions in applying this method to the actual, measured data. However, such things are inevitable in this kind of approaches.

Now let us apply this method to the result of experiments conducted by using three geosim models and a full scale ship by Panel SR107, Ship Research Association of Japan[5]. This data was used in Fig. 4 in the author's previous paper. In that, the wake distribution was scaled following the previous correlation method without paying attention to the dent in the distribution. This time we follow the above four steps to predict full scale value from model tests. For the purpose of comparison we show here first of all the same figure as before, i.e. Fig. 4 in the previous paper. (See Fig. 7) In the figure it is noticed that there are no big difference between the previous predictions from three geosim models data. Therefore, as a representative, the data of 8 m model with propeller is used to predict the ship's value by the present correlation method. The result is shown in the figure by

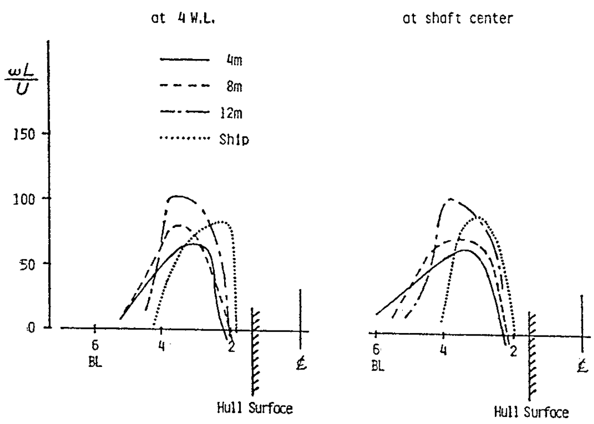


Fig. 8 Calculated vorticity distributions from measured velocity distributions (Niizuru Maru, SR 107)

three thick dotted lines, the first the ordinary wake portion, the second the final value corrected for the effect of vortex following $\sqrt{C_F}$ contraction law, and the third the inner limit case. Generally speaking, the comparison seems to be favorable, although we cannot put any priority between the two predictions.

As a complementary comparison the calculated vorticity distribution from measured velocity distribution is compared between the models and ship. This is shown in Fig. 8 for two waterline sections. The result shows a similar tendency to the prediction by the present method for models, but the ship's value is smaller than the prediction. The reason for this is not clear.

5. Conclusions

The scale effects on the boundary layer and wake distribution of ships with bilge vortices have been investigated as an extension of the author's previous report. Some of the main ideas and findings are as follows.

1. It seems to be possible to investigate the characteristics of the boundary layer and wake by dividing them into two portions, one the ordinary portion without bilge vortex and the other the vortex portion.

2. The ordinary portion obeys the correlation law explained in the previous report.

3. The vortex portion obeys a similar but partly different correlation law as follows.

a. The circulation of vortex is not affected by Re.

b. The distance of vortex center from the hull surface is proportional to $\sqrt{C_F} \sim C_F$ under the condition of item a. Simplest, straight-forward reasoning gives $\sqrt{C_F}$ proportionality, while C_F proportionality is derived as the inner limit of the location of the longitudinal vortex.

c. The wake of bilge vortex increases its magnitude in inverse proportion to C_F, under the condition of item a, but its radius decreases in proportion to $\sqrt{C_F}$.

d. The vorticity distribution of vortex has the shape proportional to the wake of the vortex under the condition of item a, both in magnitude and in radius.

4. In analysis it is assumed the kinematic viscosity in Batchelor's solution can be replaced by the eddy viscosity. This seems to have been effective.

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References

1) Tanaka, I.: Scale Effects on Wake Distribution and Viscous Pressure Resistance of Ships, Jour. Soc. Nav. Arch. Japan, No. 146 (1979)
2) Sasajima, H. and Tanaka, I.: On the Estimation of Wake of Ships, Proc. 11th ITTC (1966)
Sasajima, H., Tanaka, I. and Suzuki, T.: Wake distribution of Full Ships, Jour. Soc. Nav. Arch. Japan, No. 120 (1966) (in Japanese)
3) For example:
Hoekstra, M.: Prediction of Full Scale Wake Characteristics Based on Model Wake Survey, I.S.P., Vol. 22 (1975)
Dyne, G.: A study of Scale Effect on Wake, Propeller Cavitation, and Vibratory Pressure at Hull of Two Tanker Models, Trans. SNAME, Vol. 82 (1974)
4) Batchelor, G. K.: Axial flow in Trailing Line Vortices, Jour. Fluid Mech., Vol. 20 (1964)
5) Rep. SR107: Investigation into the Speed Measurement and Improvement of Accuracy in Powering of Full Ships, Rep 73 (1973) (in Japanese)