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Wind Effect on Course Stability of Two Towed Vessels

by Katsuro Kijima*, *Member*
Kamlesh Varyani*, *Member*

Summary

Course stability is one important factor while taking into consideration safety of navigation for towed vessels. Recently, the problem of towing operation, towing equipment etc. has been considered at the design stage.

Course stability will depend on various factors and especially in this paper emphasis has been placed upon the effects of wind on the two towed systems. A theoretical approach to the calculations is also elaborated. Calculations for three different types of systems, composed of cargo and tanker vessels, i. e. for the case when the towed vessels are identical, small and large, large and small respectively, are performed and the effects of wind on each system are as follows.

For higher wind velocity, the above mentioned systems have a tendency to become stable, however as the wind direction changes from the against wind to the following wind, there is a tendency for the systems to become unstable. Besides these, the tow line length, the location of the tow and towed points also influence the stability of the systems.

1 Introduction

In the case of machinary damage, grounding or collision, it is often impossible for the ship to move by itself. During these situations, and for safe navigation, these disabled ships generally have to be towed away by tugboats. Recently, International Maritime Organization has also been discussing on the towing of disabled tankers, and attention is also being paid to the towing operation.

In the previous paper[1], one of the authors had shown the calculation method on the course stability of single towed vessel, and the effects of wind on the course stability. The results have shown that it is dominated largely by, the size of tow and towed vessels, length of tow line, wind speed and wind direction.

In this paper, emphasis has been placed upon the effects of wind on the course stability of two towed vessels. The stability of towed vessels under wind pressure will basically depend on the inherent stability of the individual towed vessel. However, vessels which have large exposed area to the wind will be greatly affected by wind, even if the inherent stability of the individual towed vessel is stable.

A theoretical approach to the problem is worked out in this study, and discussions are provided on the various factors which play an important role in determining the course stability of the tow and towed system.

This paper clarifies the characteristics of the course stability of a tow and two towed vessels under the wind pressure by extending the method derived in the previous paper, and it is aimed at safe towing of two towed vessels, taking into consideration the effects of various factors including wind as a significant factor.

* Faculty of Engineering, Kyushu University

2 Basic Formulation

We now refer to such a system in Fig. 1, which is composed of a tow and two towed vessels. The equations of motion of a ship can now be applied to tow and towed vessels, for analyzing the motion of the towed system. The following assumptions are considered in this paper.

(1) The motion is considered in the horizontal plane, and the effect of waves is not considered.

(2) The tension of the tow line is the only coupling term between tow and towed vessels.

(3) The tow line is rigid and straight.

(4) The mass and elasticity of the tow line are neglected.

Referring to the coordinate system shown in Fig. 1, we have the non-dimensionalized equations of motion of tow and towed vessel as follows.

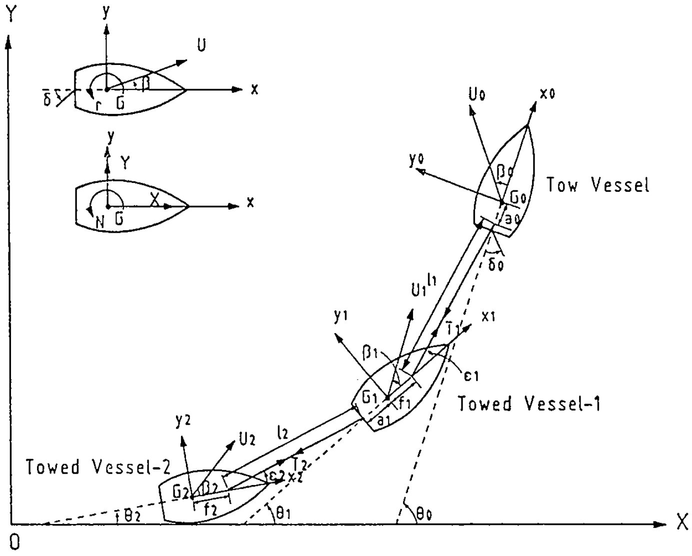


Fig. 1 Coordinate systems in tow and towed vessels

$$\left.\begin{aligned}&(m_i'+m_{xi}')\frac{L_i}{U_i}\left(\frac{\dot{U}_i}{U_i}\cos\beta_i-\dot{\beta}_i\sin\beta_i\right)\\&\qquad-(m_i'+m_{yi}')r_i'\sin\beta_i=X_i'\\&(m_i'+m_{yi}')\frac{L_i}{U_i}\left(\frac{\dot{U}_i}{U_i}\sin\beta_i+\dot{\beta}_i\cos\beta_i\right)\\&\qquad+(m_i'+m_{xi}')r_i'\cos\beta_i=Y_i'\\&(I_{zi}'+i_{zi}')\frac{L_i}{U_i}\left(\frac{\dot{U}_i}{U_i}r_i'+\dot{r}_i'\right)=N_i'\end{aligned}\right\}\quad(1)$$

In these equations, subscript "i" refers to the i-th vessel, where the subscript $0(i=0)$ refers to the tow vessel, 1 $(i=1)$ towed vessel-1 and $2(i=2)$ towed vessel-2, respectively. The superscript "$'$" refers to the non-dimensionalized quantities which are as follows.

$$m_i',\ m_{xi}',\ m_{yi}'=m_i,\ m_{xi},\ m_{yi}\Big/\frac{1}{2}\rho L_i^2 d_i,$$

$$I_{zi}',\ i_{zi}'=I_{zi},\ i_{zi}\Big/\frac{1}{2}\rho L_i^4 d_i$$

$$X_i',\ Y_i'=X_i,\ Y_i\Big/\frac{1}{2}\rho L_i d_i U_i^2,$$

$$N_i'=N_i\Big/\frac{1}{2}\rho L_i^2 d_i U_i^2$$

$$r_i'=L_i r_i/U_i$$

Now let the subscript "P" symbolize propeller, "H" hull, "R" rudder, "W" wind and "T" tension of the tow line. Then the individual component of force and moment due to propeller, rudder and wind, and tension of the tow line in the right hand terms of the equation (1) can be written as follows.

$$\left.\begin{aligned}X_i'&=X_{Pi}'+X_{Hi}'+X_{Ri}'+X_{Wi}'+X_{Ti}'\\Y_i'&=Y_{Hi}'+Y_{Ri}'+Y_{Wi}'+Y_{Ti}'\\N_i'&=N_{Hi}'+N_{Ri}'+N_{Wi}'+N_{Ti}'\end{aligned}\right\}\quad(2)$$

where X_{pi}' is thrust of the i-th vessel. The individual components of tow and towed vessels in this paper are the same as described in the previous paper[1]. From the consideration of balance of the forces in the x-axis direction, we have the equations of the tow vessel as follows.

$$\begin{aligned}X_{P0}'=&-(X_{H0S}'+X_{H1S}'+X_{H2S}')\\&-(X_{R0S}'+X_{R1S}'+X_{R2S}')\\&-(X_{W0S}'+X_{W1S}'+X_{W2S}')\end{aligned}\quad(3)$$

where X_{H0S}', X_{R0S}', X_{W0S}' are the hull, rudder and wind resistances acting on the tow vessels when advancing straight respectively. X_{H1S}', X_{R1S}', X_{W1S}', X_{H2S}', X_{R2S}' and X_{W2S}' are forces in the x-axis direction acting on hull, rudder and due to wind of towed vessels 1 and 2 respectively when advancing straight.

The hydrodynamic force X_{H0}' acting on bare hull of the tow vessel is approximated as follows, which includes the effects of oblique motion.

$$X_{H0}'=-R_0'(1+13\beta_0^2)\quad(4)$$

where R_0' is the resistance when advancing straight.

Neglecting the non-linear terms of the lateral force and yaw moment acting on tow vessel, we have the following expressions from reference 2).

$$\left.\begin{aligned}Y_{H0}'&=-Y_{\beta0}'\beta_0+Y_{r'0}'r_0'\\N_{H0}'&=-N_{\beta0}'\beta_0-N_{r0}'r_0'\end{aligned}\right\}\quad(5)$$

Once the body shape of the vessel is known, it is then possible to estimate the hydrodynamic force and moment shown in equation (5).

For estimation of rudder force and moment various methods have been proposed in the past, however in this paper an extension of Fujii's[3] expression has been used, which is as follows.

$$\left.\begin{aligned}X_{R0}'&=-C_{N0}\sin\delta_0\\Y_{R0}'&=C_{N0}\cos\delta_0\\N_{R0}'&=-1/2\,C_{N0}\cos\delta_0\end{aligned}\right\}\quad(6)$$

$$C_{N0}=\frac{6.13k_{R0}}{k_{R0}+2.25}(1+C_R S_0^{1.5})(1-w_0)^2 \times\sin\left\{\delta_0+2S_0-\gamma_0\left(\beta_0-\frac{1}{2}r_0'\right)\right\}A_{R0}'$$

where,

k_{R0} : aspect ratio of rudder.
w_0 : wake fraction coefficient.
C_R : coefficient for starboard and port rudder
S_0 : slip ratio.
A_{R0}' : rudder area ratio.
γ_0 : flow straightening coefficient.

In this paper, it is assumed that the rudder of the tow vessel is controlled by the parameters of the heading angle and the angular velocity as follows.

$$\delta_0=k_{01}\theta_1+k_{02}r_0' \tag{7}$$

where k_{01} and k_{02} are constant.

Furthermore, as shown in Fig. 2, we get the following expressions for wind force and moment.

$$\left.\begin{aligned}X_{W0}'&=\frac{\rho'}{\rho}\cdot\frac{A_{f0}}{L_0d_0}C_{W0X}\times\left\{1+\left(\frac{V}{U_0}\right)^2+\frac{2V}{U_0}\sin(\theta_0+\beta_0-\nu)\right\}\\ Y_{W0}'&=\frac{\rho'}{\rho}\cdot\frac{A_{S0}}{L_0d_0}C_{W0Y}\times\left\{1+\left(\frac{V}{U_0}\right)^2+\frac{2V}{U_0}\sin(\theta_0+\beta_0-\nu)\right\}\\ N_{W0}'&=\frac{\rho'}{\rho}\cdot\frac{A_{S0}}{L_0d_0}C_{W0N}\times\left\{1+\left(\frac{V}{U_0}\right)^2+\frac{2V}{U_0}\sin(\theta_0+\beta_0-\nu)\right\}\end{aligned}\right\} \tag{8}$$

where,

A_{f0} : longitudinal projected area above the water line.
A_{S0} : transverse projected area above the water line.
ρ' : density of air.

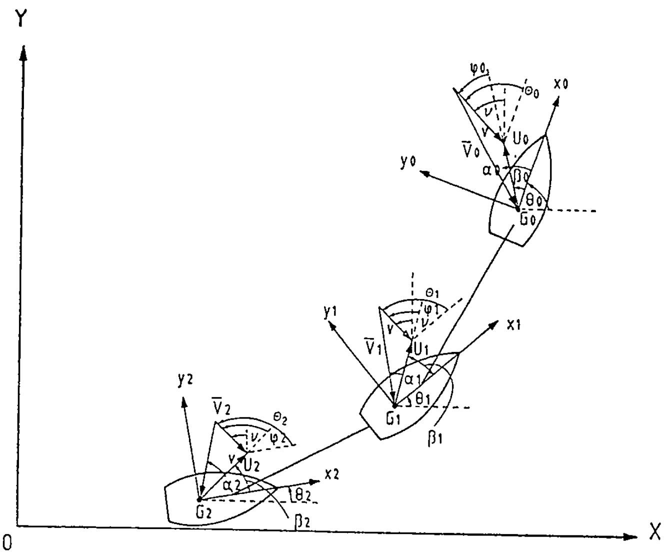


Fig. 2 Wind velocity and wind direction

$C_{W0X}, C_{W0Y}, C_{W0N}$: coefficient of wind force and moment respectively.

The values of the coefficients C_{W0X}, C_{W0Y} and C_{W0N} are determined by using Isherwood's[4] method.

On the tension force and moment of the tow line, let T_{1S}', T_{2S}', R_1', R_2', X_{W1S}', X_{W2S}' be the tension, hull resistance and wind force of the first and second towed vessels when advancing straight respectively. We have

$$T_{1S}'=R_1'+R_2'+X_{W1S}'+X_{W2S}'$$
$$T_{2S}'=R_2'+X_{W2S}'.$$

Considering an oblique towing state, we finally arrive at the following expressions.

$$\left.\begin{aligned}X_{T0}'&=-T_{1S}'\{1+13(\beta_1-\varepsilon_1)^2\}\cos(\theta_0-\theta_1-\varepsilon_1)\\ Y_{T0}'&=T_{1S}'\{1+13(\beta_1-\varepsilon_1)^2\}\sin(\theta_0-\theta_1-\varepsilon_1)\\ N_{T0}'&=-T_{1S}'a_0'\{1+13(\beta_1-\varepsilon_1)^2\}\sin(\theta_0-\theta_1-\varepsilon_1)\end{aligned}\right\} \tag{9}$$

where $a_0'=a_0/L_0$.

Therefore, the external forces for the tow vessel can be estimated by using the formulas as given above. For the first and second towed vessels, the external forces can be estimated by using the method as mentioned above. In this paper, these towed vessels have no propeller thrust and no rudder control.

3 Course Stability

One of the important problem in the towing operation is to know whether the towing system is stable or unstable. Especially during towing operation, the effects of external disturbance such as wind are very prominent. Hence there is the necessity to know the course stability of the system. For simplification of the problem, the following assumptions are made in this paper.

(1) The velocity of tow and towed vessels is kept constant(U).

(2) As the perturbation of θ_i, $\beta_i(i=0,1,2)$, $\varepsilon_i(i=1,2)$ are small, only the linear motion is taken into consideration.

The terms of the wind component in the equation of motion are also simplified and from the consideration of the coordinates, the following relationship is derived.

$$\left.\begin{aligned}\beta_1&=\dot{\beta}_0+\dot{\theta}_0-\dot{\theta}_1-\frac{a_0'L_0}{U}\ddot{\theta}_0-\frac{L_1}{U}(p_1+q_1)\ddot{\theta}_1-\frac{q_1L_1}{U}\ddot{\varepsilon}_1\\ \dot{\beta}_2&=\dot{\beta}_0+\dot{\theta}_0-\dot{\theta}_2-\frac{a_0'L_0}{U}\ddot{\theta}_0-\frac{L_1}{U}(p_1+q_1+a_1')\ddot{\theta}_1-\frac{q_1L_1}{U}\ddot{\varepsilon}_1-\frac{L_2}{U}(p_2+q_2)\ddot{\theta}_2-\frac{q_2L_2}{U}\ddot{\varepsilon}_2\end{aligned}\right\} \tag{10}$$

where $p_1=f_1/L_1$, $p_2=f_2/L_2$, $q_1=l_1/L_1$, $q_2=l_2/L_2$, $a_1'=a_1/L_1$.

Using simplifications and assuming $\gamma_0=1.0$, $\gamma_1=1.0$, $\gamma_2=1.0$, we have the following equations.

$$\frac{L_0}{U}\left\{M_{x0}'-Y_{r0}'-C_0\left(k_{02}+\frac{1}{2}\right)\right\}\dot{\theta}_0-(C_0k_{01}+W_0C_{W0Y}+T_{1S}')\theta_0+M_{y0}'\frac{L_0}{U}\dot{\beta}_0$$
$$+(Y_{\beta0}'+C_0-W_0C_{W0Y})\beta_0+T_{1S}'(\theta_1+\varepsilon_1)-\frac{\rho'}{\rho}\frac{A_{S0}}{L_0d_0}C_{W0Y}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (11)$$

$$I_{z0}'\left(\frac{L_0}{U}\right)^2\ddot{\theta}+\frac{L_0}{U}\left\{N_{r0}'+\frac{C_0}{2}\left(k_{02}+\frac{1}{2}\right)\right\}\dot{\theta}_0+\left(\frac{C_0}{2}k_{01}-W_0C_{W0N}+T_{1S}'a_0'\right)\theta_0$$
$$+\left(N_{\beta0}'-\frac{C_0}{2}-W_0C_{W0N}\right)\beta_0-T_{1S}'a_0'(\theta_1+\varepsilon_1)-\frac{\rho'}{\rho}\frac{A_{S0}}{L_0d_0}C_{W0N}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (12)$$

$$-M_{y1}'\frac{L_0L_1}{U^2}a_0'\ddot{\theta}_0+\frac{L_1}{U}\left\{M_{y1}'-\frac{L_0}{L_1}a_0'(Y_{\beta1}'+C_1-W_1\cdot C_{W1Y})\right\}\dot{\theta}_0+(Y_{\beta1}'+C_1-W_1\cdot C_{W1Y})\theta_0$$
$$+M_{y1}'\frac{L_1}{U}\dot{\beta}_0+(Y_{\beta1}'+C_1-W_1C_{W1Y})\beta_0-M_{y1}'\left(\frac{L_1}{U}\right)^2(p_1+q_1)\ddot{\theta}_1$$
$$+\frac{L_1}{U}\left\{-M_{y1}'+M_{x1}'-Y_{r1}'-\frac{C_1}{2}-(p_1+q_1)(Y_{\beta1}+C_1-W_1C_{W1Y})\right\}\dot{\theta}_1$$
$$-(Y_{\beta1}'+C_1+T_{2S}')\theta_1-M_{y1}'\cdot q_1\left(\frac{L_1}{U}\right)^2\ddot{\varepsilon}_1-q_1\frac{L_1}{U}(Y_{\beta1}'+C_1-W_1C_{W1Y})\dot{\varepsilon}_1$$
$$-T_{1S}'\varepsilon_1+T_{2S}'(\theta_2+\varepsilon_2)-\frac{\rho'}{\rho}\frac{A_{S1}}{L_1d_1}C_{W1Y}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (13)$$

$$\frac{a_0'L_0}{U}\left(-N_{\beta1}'+\frac{C_1}{2}+W_1\cdot C_{W1N}\right)\dot{\theta}_0+\left(N_{\beta1}'-\frac{C_1}{2}-W_1\cdot C_{W1N}\right)\theta_0+\left(N_{\beta1}'-\frac{C_1}{2}-W_1C_{W1N}\right)\beta_0$$
$$+I_{z1}'\left(\frac{L_1}{U}\right)^2\ddot{\theta}_1+\frac{L_1}{U}\left\{N_{r1}'+\frac{C_1}{4}+(p_1+q_1)\left(-N_{\beta1}'+\frac{C_1}{2}+W_1C_{W1N}\right)\right\}\dot{\theta}_1$$
$$+\left\{-N_{\beta1}'+\frac{C_1}{2}+T_{2S}'a_1'\right\}\theta_1+\frac{q_1L_1}{U}\left\{-N_{\beta1}'+\frac{C_1}{2}+W_1C_{W1N}\right\}\dot{\varepsilon}-T_{1S}'P_1\varepsilon_1$$
$$-T_{2S}'a_1'(\theta_2+\varepsilon_2)-\frac{\rho'}{\rho}\frac{A_{S1}}{L_1d_1}C_{W1N}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (14)$$

$$-M_{y2}'\frac{L_0L_2}{U^2}a_0'\ddot{\theta}_0+\left\{M_{y2}'\frac{L_2}{U}-\frac{L_0a_0'}{U}(Y_{\beta2}'+C_2-W_2C_{W2Y})\right\}\dot{\theta}_0$$
$$+(Y_{\beta2}'+C_2-W_2C_{W2Y})\theta_0-M_{y2}'\cdot\frac{L_1L_2}{U^2}(p_1+q_1+a_1')\ddot{\theta}_1$$
$$+\left\{(p_1+q_1+a_1')\frac{L_1}{U}(-Y_{\beta2}'-C_2+W_2C_{W2Y})\right\}\dot{\theta}_1-M_{y2}'\left(\frac{L_2}{U}\right)^2(p_2+q_2)\ddot{\theta}_2$$
$$+\left[\left(\frac{L_2}{U}\right)\left\{-M_{y2}'+M_{x2}'-Y_{r2}'-\frac{C_2}{2}-(p_2+q_2)(Y_{\beta2}'+C_2-W_2C_{W2Y})\right\}\right]\dot{\theta}_2-(Y_{\beta2}'+C_2)\theta_2$$
$$-M_{y2}'q_2\left(\frac{L_2}{U}\right)^2\ddot{\varepsilon}^2-\frac{q_2L_2}{U}(Y_{\beta2}'+C_2-W_2C_{W2Y})\dot{\varepsilon}_2-T_{2S}'\varepsilon_2-M_{y2}'q_1\frac{L_1L_2}{U^2}\ddot{\varepsilon}_1$$
$$-\frac{q_1L_1}{U}(Y_{\beta2}'+C_2-W_2C_{W2Y})\dot{\varepsilon}_1+M_{y2}'\frac{L_2}{U}\dot{\beta}_0+(Y_{\beta2}'+C_2-W_2\ C_{W2Y})\beta_0$$
$$-\frac{\rho'}{\rho}\frac{A_{S2}}{L_2d_2}C_{W2Y}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (15)$$

$$\frac{L_0a_0'}{U}\left(-N_{\beta2}'+\frac{C_2}{2}+W_2C_{W2N}\right)\dot{\theta}_0+\left(N_{\beta2}'-\frac{C_2}{2}-W_2\cdot C_{W2N}\right)\theta_0$$
$$+(p_1+q_1+a_1')\frac{L_1}{U}\left(-N_{\beta2}'+\frac{C_2}{2}+W_2C_{W2N}\right)\dot{\theta}_1+I_{z2}'\left(\frac{L_2}{U}\right)^2\ddot{\theta}_2$$
$$+\frac{L_2}{U}\left\{N_{r2}'+\frac{C_2}{4}+(P_2+q_2)\left(-N_{\beta2}'+\frac{C_2}{2}+W_2C_{W2N}\right)\right\}\dot{\theta}_2-\left(N_{\beta2}'-\frac{C_2}{2}\right)\theta_2$$
$$+\frac{q_1L_1}{U}\left(-N_{\beta2}'+\frac{C_2}{2}+W_2C_{W2N}\right)\dot{\varepsilon}_1+\frac{q_2L_2}{U}\left(-N_{\beta2}'+\frac{C_2}{2}+W_2C_{W2N}\right)\dot{\varepsilon}_2-T_{2S}'\cdot p_2\cdot\varepsilon_2$$
$$+\left(N_{\beta2}'-\frac{C_2}{2}-W_2C_{W2N}\right)\beta_0-\frac{\rho'}{\rho}\cdot\frac{A_{S2}}{L_2d_2}\cdot C_{W2N}\left\{1+\left(\frac{V}{U}\right)^2-\frac{2V}{U}\sin\nu\right\}=0 \quad (16)$$

where $M_{x0}'\equiv(m_0'+m_{x0}')$, $M_{y0}'\equiv(m_0'+m_{y0}')$, $M_{x1}'\equiv(m_1'+m_{x1}')$,

$$M_{y1}'\equiv(m_1'+m_{y1}'),\quad M_{x2}'\equiv(m_2'+m_{x2}'),\quad M_{y2}'\equiv(m_2'+m_{y2}')$$

$$W_0\equiv\frac{\rho'}{\rho}\frac{A_{S0}}{L_0d_0}\frac{2V}{U}\cos\nu,\quad W_1\equiv\frac{\rho'}{\rho}\frac{A_{S1}}{L_1d_1}\frac{2V}{U}\cos\nu,\quad W_2\equiv\frac{\rho'}{\rho}\frac{A_{S2}}{L_2d_2}\frac{2V}{U}\cos\nu,$$

and

$$C_i\equiv\frac{6.13k_{Ri}}{k_{Ri}+2.25}(1+C_RS_i^{1.5})(1-w_i)^2\cdot\frac{A_{Ri}}{L_id_i},\quad(i=0,1,2).$$

These parameters for towed vessel 1 and 2 correspond to the parameters of tow vessel shown in equation (6).

Substitute $\dot\theta_0=\phi_1$, $\dot\theta_1=\phi_2$, $\dot\theta_2=\phi_3$, $\dot\varepsilon_1=\phi_4$, $\dot\varepsilon_2=\phi_5$ into the equation from (11) to (16) and rewriting the equations for $\dot\phi_1, \dot\phi_2, \dot\phi_3, \dot\phi_4, \dot\phi_5, \dot\beta_0$, we have

$$\left.\begin{aligned}
\dot\phi_1&=H(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)\\
\dot\phi_2&=I(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)\\
\dot\phi_3&=J(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)\\
\dot\phi_4&=K(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)\\
\dot\phi_5&=L(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)\\
\dot\beta_0&=M(\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \beta_0,\ \theta_0,\ \theta_1,\ \theta_2,\ \varepsilon_1,\ \varepsilon_2)
\end{aligned}\right\}\tag{17}$$

From the equilibrium condition of the parameters $\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \beta_0, \theta_0, \theta_1, \theta_2, \varepsilon_1$ and ε_2 if we bring about small variations of the order $\Delta\phi_1, \Delta\phi_2, \Delta\phi_3, \Delta\phi_4, \Delta\phi_5, \Delta\beta_0, \Delta\theta_0, \Delta\theta_1, \Delta\theta_2, \Delta\varepsilon_1$ and $\Delta\varepsilon_2$, in each of the parameters, and from equation (17) and the relation $\phi_1=\dot\theta_0$, $\phi_2=\dot\theta_1$, $\phi_3=\dot\theta_2$, $\phi_4=\dot\varepsilon_1$, and $\phi_5=\dot\varepsilon_2$, we have as follows.

$$\begin{aligned}
\frac{d}{dt}(\Delta\phi_1)&=H_{\phi_1}\cdot\Delta\phi_1+H_{\phi_2}\cdot\Delta\phi_2+H_{\phi_3}\cdot\Delta\phi_3+H_{\phi_4}\cdot\Delta\phi_4+H_{\phi_5}\cdot\Delta\phi_5+H_{\beta_0}\cdot\Delta\beta_0\\
&\quad+H_{\theta_0}\cdot\Delta\theta_0+H_{\theta_1}\cdot\Delta\theta_1+H_{\theta_2}\cdot\Delta\theta_2+H_{\varepsilon_1}\cdot\Delta\varepsilon_1+H_{\varepsilon_2}\cdot\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\phi_2)&=I_{\phi_1}\cdot\Delta\phi_1+I_{\phi_2}\cdot\Delta\phi_2+I_{\phi_3}\cdot\Delta\phi_3+I_{\phi_4}\cdot\Delta\phi_4+I_{\phi_5}\cdot\Delta\phi_5+I_{\beta_0}\cdot\Delta\beta_0\\
&\quad+I_{\theta_0}\cdot\Delta\theta_0+I_{\theta_1}\cdot\Delta\theta_1+I_{\theta_2}\cdot\Delta\theta_2+I_{\varepsilon_1}\cdot\Delta\varepsilon_1+I_{\varepsilon_2}\cdot\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\phi_3)&=J_{\phi_1}\cdot\Delta\phi_1+J_{\phi_2}\cdot\Delta\phi_2+J_{\phi_3}\cdot\Delta\phi_3+J_{\phi_4}\cdot\Delta\phi_4+J_{\phi_5}\cdot\Delta\phi_5+J_{\beta_0}\cdot\Delta\beta_0\\
&\quad+J_{\theta_0}\cdot\Delta\theta_0+J_{\theta_1}\cdot\Delta\theta_1+J_{\theta_2}\cdot\Delta\theta_2+J_{\varepsilon_1}\cdot\Delta\varepsilon_1+J_{\varepsilon_2}\cdot\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\phi_4)&=K_{\phi_1}\cdot\Delta\phi_1+K_{\phi_2}\cdot\Delta\phi_2+K_{\phi_3}\cdot\Delta\phi_3+K_{\phi_4}\cdot\Delta\phi_4+K_{\phi_5}\cdot\Delta\phi_5+K_{\beta_0}\cdot\Delta\beta_0\\
&\quad+K_{\theta_0}\cdot\Delta\theta_0+K_{\theta_1}\cdot\Delta\theta_1+K_{\theta_2}\cdot\Delta\theta_2+K_{\varepsilon_1}\cdot\Delta\varepsilon_1+K_{\varepsilon_2}\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\phi_5)&=L_{\phi_1}\cdot\Delta\phi_1+L_{\phi_2}\cdot\Delta\phi_2+L_{\phi_3}\cdot\Delta\phi_3+L_{\phi_4}\cdot\Delta\phi_4+L_{\phi_5}\cdot\Delta\phi_5+L_{\beta_0}\cdot\Delta\beta_0\\
&\quad+L_{\theta_0}\cdot\Delta\theta_0+L_{\theta_1}\cdot\Delta\theta_1+L_{\theta_2}\cdot\Delta\theta_2+L_{\varepsilon_1}\cdot\Delta\varepsilon_1+L_{\varepsilon_2}\cdot\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\beta_0)&=M_{\phi_1}\cdot\Delta\phi_1+M_{\phi_2}\cdot\Delta\phi_2+M_{\phi_3}\cdot\Delta\phi_3+M_{\phi_4}\cdot\Delta\phi_4+M_{\phi_5}\cdot\Delta\phi_5+M_{\beta_0}\cdot\Delta\beta_0\\
&\quad+M_{\theta_0}\cdot\Delta\theta_0+M_{\theta_1}\cdot\Delta\theta_1+M_{\theta_2}\cdot\Delta\theta_2+M_{\varepsilon_1}\cdot\Delta\varepsilon_1+M_{\varepsilon_2}\cdot\Delta\varepsilon_2\\
\frac{d}{dt}(\Delta\theta_0)&=\Delta\phi_1,\quad\frac{d}{dt}(\Delta\theta_1)=\Delta\phi_2,\quad\frac{d}{dt}(\Delta\theta_2)=\Delta\phi_3,\quad\frac{d}{dt}(\Delta\varepsilon_1)=\Delta\phi_4,\quad\frac{d}{dt}(\Delta\varepsilon_2)=\Delta\phi_5
\end{aligned}\tag{18}$$

where $H_{\phi_1}=\partial H/\partial\phi_1$, $H_{\phi_2}=\partial H/\partial\phi_2$ etc.

Now we put

$$\begin{aligned}
&H_{\phi_i}\equiv k_{1,i}\ (i=1\sim5),\ H_{\beta_0}\equiv k_{1,6},\ H_{\theta_i}\equiv k_{1,i+7}\ (i=0\sim2),\ H_{\varepsilon_1}\equiv k_{1,10},\ H_{\varepsilon_2}\equiv k_{1,11}\\
&I_{\phi_i}\equiv k_{2,i}\ (i=1\sim5),\ I_{\beta_0}\equiv k_{2,6},\ I_{\theta_i}\equiv k_{2,i+7}\ (i=0\sim2),\ I_{\varepsilon_1}\equiv k_{2,10},\ I_{\varepsilon_2}\equiv k_{2,11}\\
&J_{\varphi_i}\equiv k_{3,i}\ (i=1\sim5),\ J_{\beta_0}\equiv k_{3,6},\ J_{\theta_i}\equiv k_{3,i+7}\ (i=0\sim2),\ J_{\varepsilon_1}\equiv k_{3,10},\ J_{\varepsilon_2}\equiv k_{3,11}\\
&K_{\phi_j}\equiv k_{4,i}\ (i=1\sim5),\ K_{\beta_0}\equiv k_{4,6},\ K_{\theta_i}\equiv k_{4,i+7}\ (i=0\sim2),\ K_{\varepsilon_1}\equiv {}_{4,10},\ K_{\varepsilon_2}\equiv k_{4,11}\\
&L_{\phi_i}\equiv k_{5,i}\ (i=1\sim5),\ L_{\beta_0}\equiv k_{5,6},\ L_{\theta_i}\equiv k_{5,i+7}\ (i=0\sim2),\ L_{\varepsilon_1}\equiv k_{5,10},\ L_{\varepsilon_2}\equiv k_{5,11}\\
&M_{\phi_i}\equiv k_{6,i}\ (i=1\sim5),\ M_{\beta_0}\equiv k_{6,6},\ M_{\theta_i}\equiv k_{6,i+7}\ (i=0\sim2),\ M_{\varepsilon_1}\equiv k_{6,10},\ M_{\varepsilon_2}\equiv k_{6,11}.
\end{aligned}\tag{19}$$

Let us assume the solution of equation (17) as follows.

$$\left.\begin{aligned}
&\Delta\phi_i=A_ie^{\lambda t}\quad(i=1\sim5)\\
&\Delta\beta_0=A_6e^{\lambda t},\quad\Delta\theta_0=A_7e^{\lambda t},\quad\Delta\theta_1=A_8e^{\lambda t},\quad\Delta\theta_2=A_9e^{\lambda t}\\
&\Delta\varepsilon_1=A_{10}e^{\lambda t},\quad\Delta\varepsilon_2=A_{11}e^{\lambda t}
\end{aligned}\right\}\tag{20}$$

where the coefficient A_i is a constant.

From the equation (19) and (20), we have as follows.

$$\left.\begin{array}{ll} \lambda A_i=\sum k_{i,j}\cdot A_j & (i=1\sim 6,\ j=1\sim 11) \\ \lambda A_{i+6}=A_i & (i=1\sim 5) \end{array}\right\} \qquad (21)$$

Equation (21) is a set of simultaneous equation with A_i. For this set of equations to have solutions other than $A_i=0$, the following determinant must be satisfied.

$$\begin{vmatrix} k_{1,1}-\lambda & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} & k_{1,6} & k_{1,7} & k_{1,8} & k_{1,9} & k_{1,10} & k_{1,11} \\ k_{2,1} & k_{2,2}-\lambda & k_{2,3} & k_{2,4} & k_{2,5} & k_{2,6} & k_{2,7} & k_{2,8} & k_{2,9} & k_{2,10} & k_{2,11} \\ k_{3,1} & k_{3,2} & k_{3,3}-\lambda & k_{3,4} & k_{3,5} & k_{3,6} & k_{3,7} & k_{3,8} & k_{3,9} & k_{3,10} & k_{3,11} \\ k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4}-\lambda & k_{4,5} & k_{4,6} & k_{4,7} & k_{4,8} & k_{4,9} & k_{4,10} & k_{4,11} \\ k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5}-\lambda & k_{5,6} & k_{5,7} & k_{5,8} & k_{5,9} & k_{5,10} & k_{5,11} \\ k_{6,1} & k_{6,2} & k_{6,3} & k_{6,4} & k_{6,5} & k_{6,6}-\lambda & k_{6,7} & k_{6,8} & k_{6,9} & k_{6,10} & k_{6,11} \\ 1 & 0 & 0 & 0 & 0 & 0 & -\lambda & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\lambda & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\lambda \end{vmatrix}=0 \qquad (22)$$

Hence solving this equation, the course stability on the towed systems can be determined.

4 Characteristics of the Towed System

4.1 Course stability

By using the method mentioned in the previous section, the numerical calculations for course stability of the towed system are carried out. In this paper, three different types of two towed vessels systems are considered for calculation, and they are System-A, System-B and System-C, as shown in Fig. 3, which are classified as follows.

System-A : Tug followed by identical towed vessels.

System-B : Tug followed by small and large towed vessels.

System-C : Tug followed by large and small towed vessels.

All the vessels in the various systems are in their loaded condition, and their principal particulars are shown in Table 1. The course stability of these systems is investigated on the following parameters, i. e. wind speed V, wind direction φ_1, q_1, p_1, q_2 and p_2, where φ_1, described the wind direction measured from the actual forward direction, is particularly used. The aft tow point of tow vessel (tugboat) is located at $a'_0=0.1$, and the aft tow point of towed vessel-1 is located at $a'_1=0.5$. For the wind coefficients Isherwood's[4] calculation method is used.

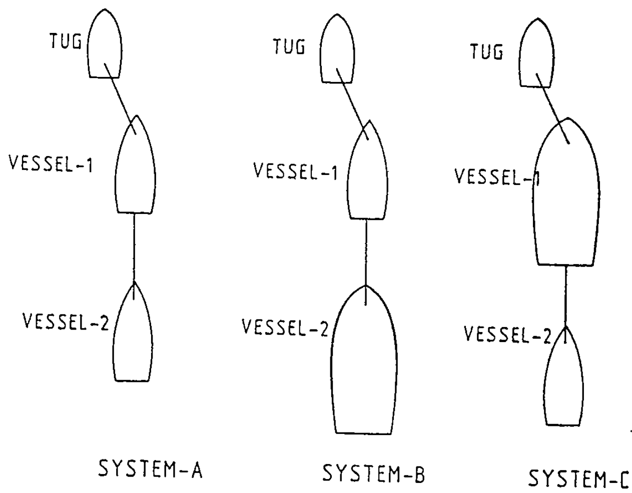


Fig. 3 Systems composed of a tow and two towed vessels

Table 1 Main particulars of a tow and two towed vessels

VESSEL	L(m)	B(m)	d(m)	Disp:(ton)	Cb	L/B	Ka	Ks	Af	As	Δ	Remarks
TUG	24.0	8.0	2.30	271.6	0.600	3.00	0.192	0.017	0.829	1.231	-0.0165	
CARGO	128.0	20.0	7.63	15040.0	0.751	6.40	0.119	0.016	0.342	0.931	-0.0021	Midship bridge
TANKER	242.0	37.2	14.86	111737.0	0.815	6.51	0.123	0.016	0.306	0.814	-0.0032	Aft bridge

Ka : aspect ratio of ship
Ks : rudder area ratio
Af : non-dimensionalized longitudinal projected area above water line
As : non-dimensionalized transverse projected area above water line
Δ : course stability criteria of single vessel

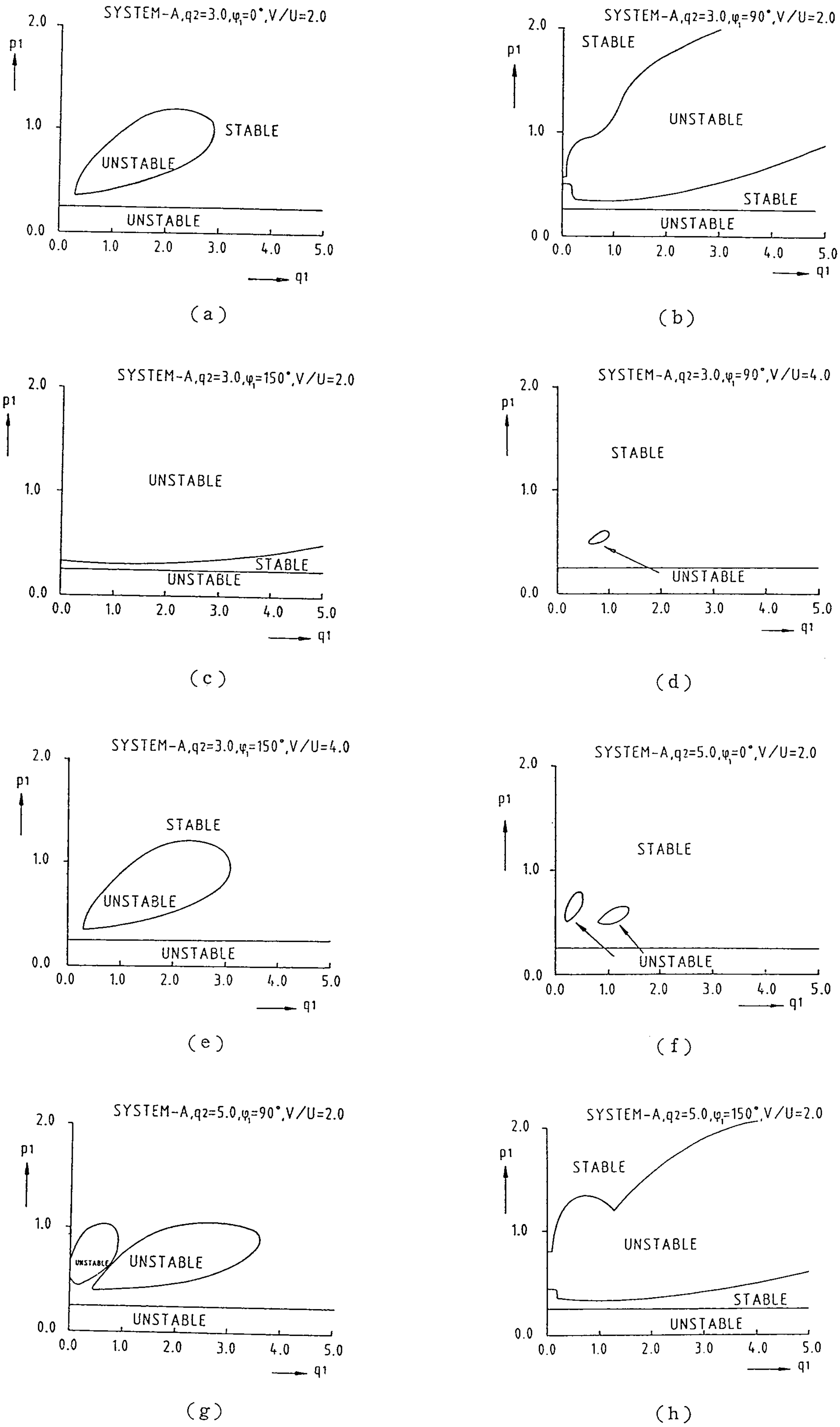


Fig. 4 Stable and unstable zones on the course stability of System-A as function of p_1 and q_1

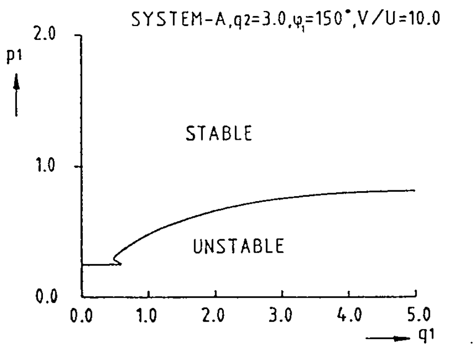
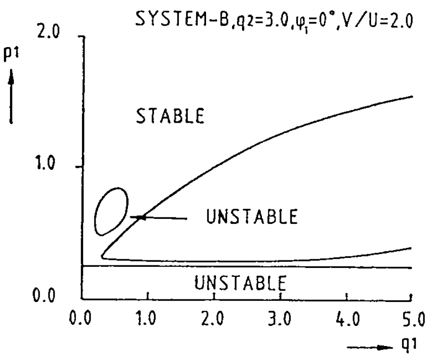
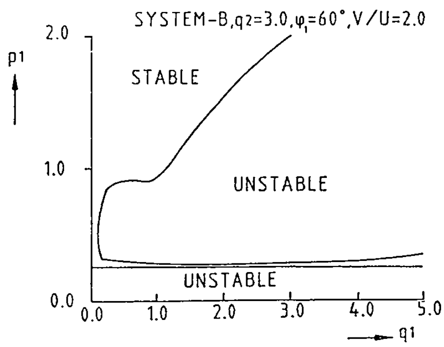


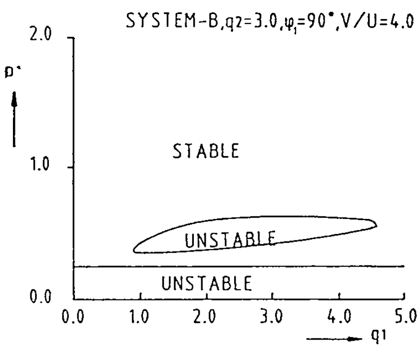
Fig. 4 (i)



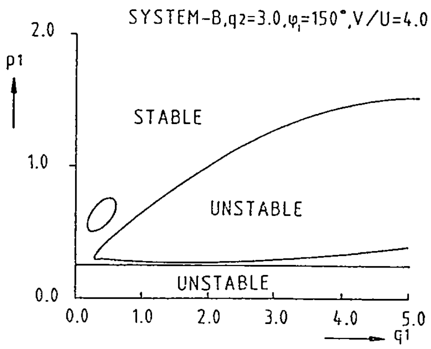
(a)



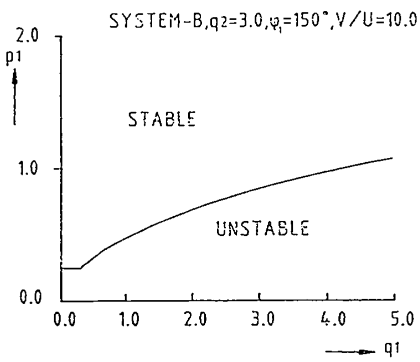
(b)



(c)



(d)



(e)

Fig. 5 Stable and unstable zones on the course stability of System-B as function of p_1 and q_1

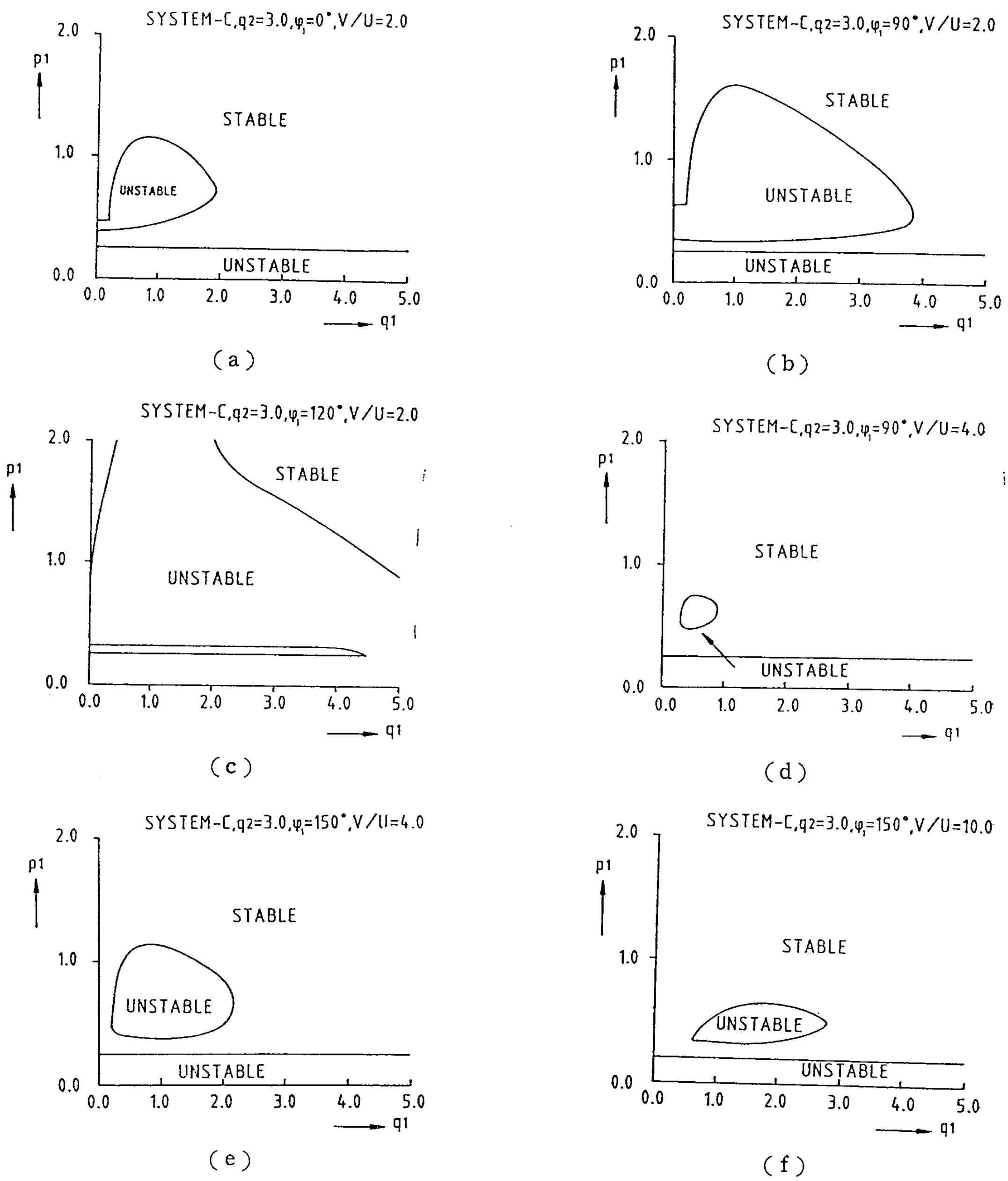


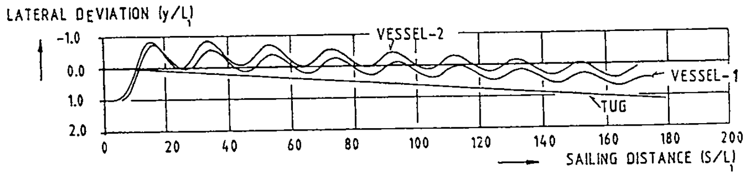
Fig. 6 Stable and unstable zones on the course stability of System-C as function of p_1 and q_1

Arranging the calculated results of the non-dimensionalized forward tow point p_1 and tow line length q_1, we have the following figures for the stable and unstable zones of the course stability of these systems. The course stability of System-A is shown in Fig. 4, System-B in Fig. 5 and System-C in Fig. 6.

At first, for $V/U=2.0$ and for the beam and the following winds, the unstable zone of the system will expand. However, as the value of V/U increases the system has a tendency to become stable. From the consideration of the effect of wind direction, and for $V/U=2.0$ and $V/U=4.0$ as the wind changes from the against wind to the following wind, the system becomes unstable. Furthermore, on the effect of length of tow line on course stability, a large number of variations will have to be investigated, but in this paper the calculations are performed for $q_2=3.0$ and 5.0 respectively. From these results, long tow line makes the system stable.

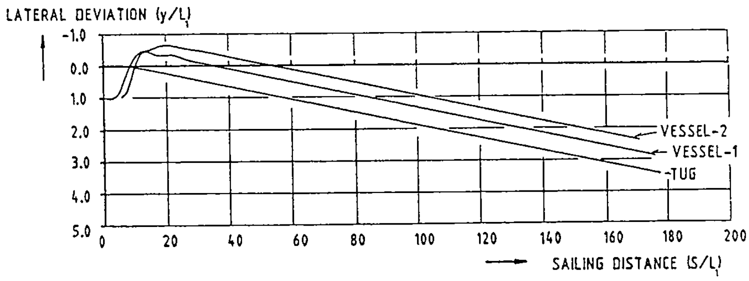
In System-B, from the consideration of the effect of wind velocity, as the wind velocity increases in the case of against wind, the system becomes considerably stable. For the beam wind the system is totally unstable for $V/U=2.0$ from view point of the effect of wind direction. For $V/U=4.0$, the system is fully stable for the against wind, and it gradually becomes unstable as the wind direction changes to the following wind. It seems that the System-B has the same characteristics as that of System-A on the effect of length of tow

SYSTEM-A, ν=90°, V/U=2.0, q2=3.0



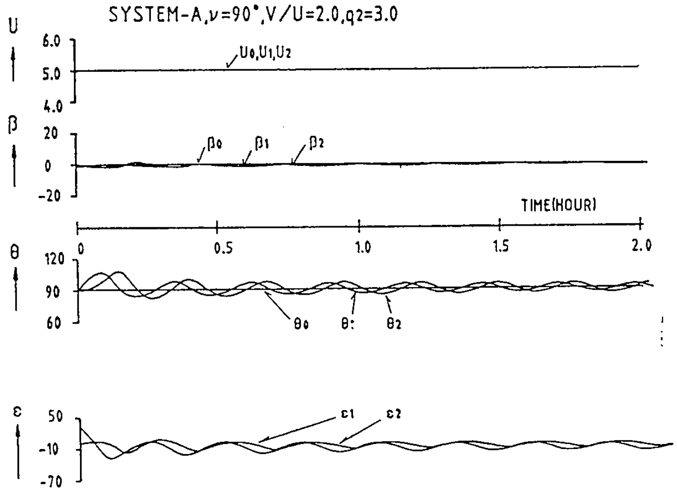
(a)

SYSTEM-A, ν=90°, V/U=4.0, q2=3.0

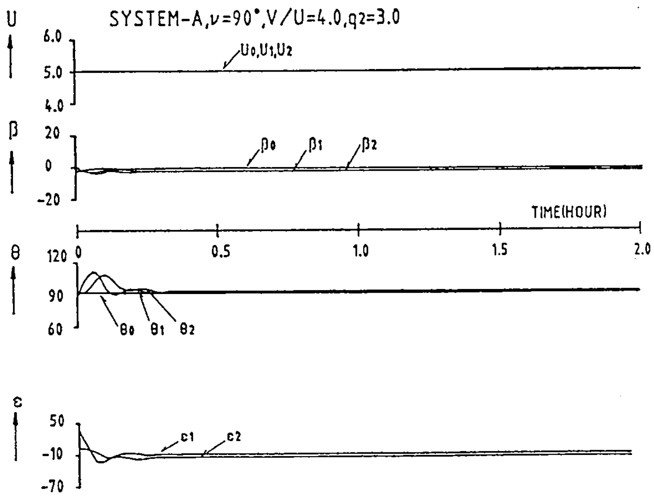


(b)

Fig. 7 Trajectory of a tow and two towed vessels in System-A



(a)



(b)

Fig. 8 Time history of the parameters U (knot), β (degree), θ (degree), ε (degree) of a tow and two towed vessels in System-A

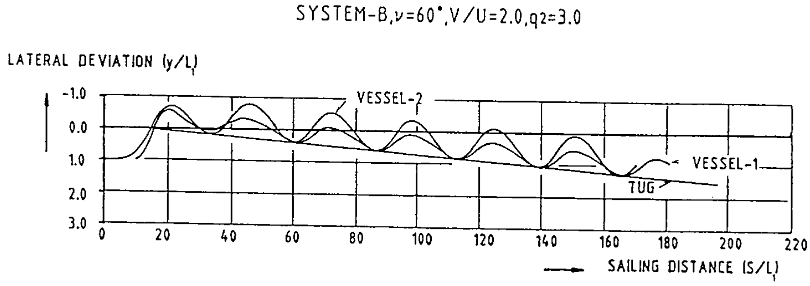


Fig. 9 Trajectory of a tow and two towed vessels in System-B

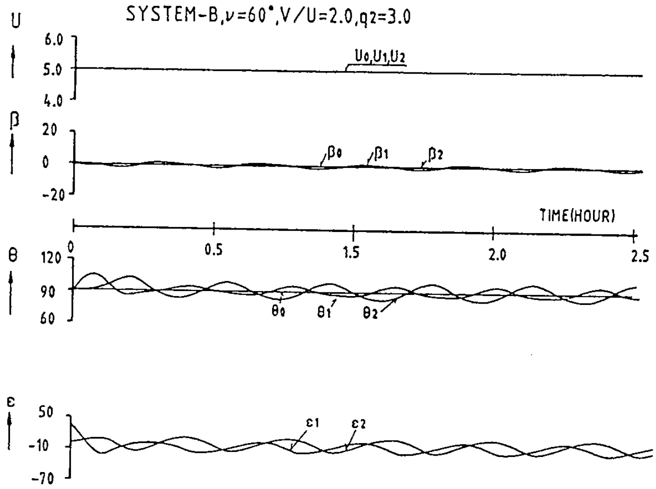


Fig. 10 Time history of the parameters U (knot), β (degree), θ (degree), ε (degree) of a tow and two towed vessels in System-B

line.

Finally in System-C, it can be said that the system is the same as the above systems from the consideration of the effects of length of tow line, wind velocity and wind direction.

On comparison of these systems it is found that for the beam wind and following wind System-A is better than the other systems on the effects of wind velocity on the course stability. From the effects of wind direction, it is found that for $V/U=2.0$ and as the wind direction changes from the against wind to the following wind, System-A shows better results than the other systems.

4.2 Trajectory of the towed vessels

The trajectory calculation for the towed vessels is also performed as a check on the course stability. After having plotted the stable and unstable zones, a certain point is chosen for example within a certain zone and the trajectory for that particular condition is determined. The motion of the tow vessel is on a reasonably straight course as comparatively high values of constant k_{01} and k_{02} for control parameter are chosen. Along with the trajectory, the time history of the various parameters, i. e. speed, drift angle, heading angle etc. is also plotted. In this paper, $p_1=0.5$, $q_1=2.0$, $a'_0=0.1$, $p_2=2.0$, $a'_1=0.5$, $k_{01}=20.0$, $k_{02}=1.0$ generally are kept constant for all the systems, and

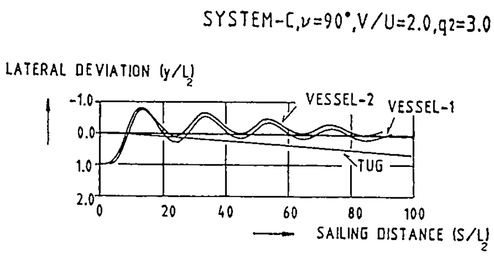


Fig. 11 Trajectory of a tow and two towed vessels in System-C

the parameters that are varied are the wind velocity and the wind direction. The trajectory calculation is performed when the velocity of the tow and towed vessel is kept constant. After giving an initial displacement, a state of the system is investigated by the trajectory calculations. The trajectory and time history of the various parameters of System-A are shown in Fig. 7 and 8, System-B in Figs. 9 and 10 and System-C in Figs. 11 and 12, where "y" is lateral deviation, and "s" sailing distance.

It is also clear from these results that the systems move from the unstable state to stable state as the velocity of the wind increases. This is clearly noticed in the trajectory as fluctuations of the towed vessels begin to decrease. As mentioned in the discussion about the course stability, as the

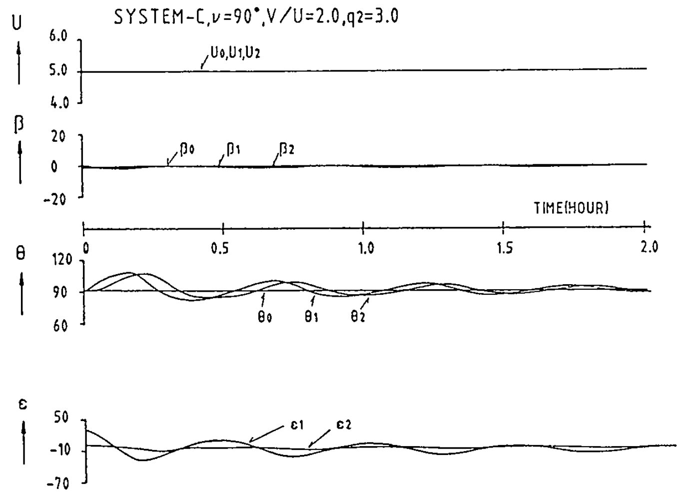


Fig. 12 Time history of the parameters U (knot), β (degree), θ (degree), ε (degree) of a tow and two towed vessels in System-C

wind change from the against wind to the following wind, unstable zone only increases. This principle is also noticed in the trajectory as the fluctuations of the towed vessels increase.

5 Concluding Remarks

On the basis of the various calculations of the course stability and checks on the trajectory of the system used, it seems that System-A and System-B are better than System-C. In general it can be said that System-B, i. e. tugboat followed by small and large vessels, is useful.

The effect of wind which plays an important role in determining the course stability of the tow and towed vessel system can be summarised as follows.

The course stability of the two towed vessels system becomes poorer in comparison with the single towed vessel system. In all practical cases long tow line generally makes the system stable. For the same wind velocity as the steady velocity of the system, the course stability becomes very poor. However when the wind velocity is several times that of the steady velocity of the system, stable characteristics is shown. For the against wind, the course stability is good, but for the following wind the course stability is very poor.

Acknowledgement

The authors wish to express their gratitudes to Mr. Y. Nakiri of Kyushu University for his help in carrying out the outlay of the drawings.

Nomenclature

L_i : Length of i-th vessel
d_i : Draft of i-th vessel
U_i : Velocity of i-th vessel
m_i : Mass of i-th vessel
m_{xi}, m_{yi} : Added mass of i-th vessel in the x and y axis direction
J_{zi} : Moment of inertia of mass of i-th vessel about center of gravity
i_{zi} : Added moment of inertia of mass of i-th vessel about center of gravity
β_i : Drift angle of i-th vessel
r_i : Angular velocity of i-th vessel
X_i, Y_i : External force of i-th vessel in x and y axis direction
N_i : Yaw moment about center of gravity of i-th vessel
ρ : Density of fluid
θ_i : Heading angle of i-th vessel
V : Absolute wind velocity
ν : Absolute wind direction
a_i : Distance between center of gravity and tow point of i-th vessel
ε_i : Angle between tow line and longitudinal center line of i-th towed vessel
f_i : Distance between towed point and center of gravity of i-th towed vessel
l_1, l_2 : Length of tow line between tow vessel and towed vessel-1, and between towed vessel-1 and towed vessel-2
δ_0 : Rudder angle
γ_i : Flow straightening coefficient of i-th vessel

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