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Solutions to Three-dimensional Second-order Diffraction Problems by Means of Simple-source Integral-equation Method

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Summary

Second-order diffraction problems for a three-dimensional body are solved by the use of simple sources and eigen functions. Numerical results of second-order velocity potentials are available as well as second-order wave forces, which are usually computed from first-order velocity potentials only, invoking Green's theorem.

Numerical analyses are done for a vertical circular cylinder in deep water so as to verify the validity of the method. Agreement between the results by the present method and existing results are satisfactory. It is also confirmed that the second-order velocity potentials on the weather side penetrate the water much deeper than the first-order ones, i. e., the second-order velocity potentials surpass the first-order ones in the depth.

1. Introduction

It is not difficult within the extent of linear theories to compute wave forces on and resultant motions of offshore structures ; a lot of results have been reported by the use of finite element method, boundary element method and so on.

Linear theories, however, are not good enough in the case of extreme waves or particular geometries which should be dealt with on the nonlinear basis. Nonlinear theories could be categorized into two groups : partially nonlinear theories by the perturbation method and fully nonlinear ones. Fully nonlinear theories require lots of numerical work, though satisfying nonlinear boundary conditions exactly and completing water-wave problems. On the other hand, partially nonlinear theories by the perturbation method are regarded as complements to linear theories and of wide application. Hence, partially nonlinear theories will be considered here.

Higher-order analysis by the perturbation method cannot be done with great ease for three-dimensional problems ; third or higher-order analyses have not been reported. As far as the second order analyses are concerned, Papanikolau and Nowacki[1] and Kyozuka[2] dealt with two-dimensional problems. Many papers have been published for three-dimensional problems, most of which are not free from incompleteness regarding boundary conditions. Radiation condition for second-order problems, in particular, is a controversy. Molin[3], whose method appears to be acceptable, computed second-order wave forces on the vertically fixed circular cylinder. But second-order pressure is not available in his paper because second-order forces were computed without seeking second-order velocity potentials. Taylor and Hung[4] also gave detailed results for second-order forces on vertical cylinders, modifying Molin's method. Hunt and Baddour[5] gave a second-order velocity potential representation as well as second-order wave forces. Their formulation seems appropriate and their results roughly agree with Taylor and Hung's results though their explanations for second-order radiation condition, i. e., zero radial energy flux, cannot be accepted from a physical viewpoint. Garrison[6] attempted to solve second-order problems for three-dimensional bodies of arbitrary shape by the use of a second-order Green function, failing to satisfy the second-order radiation condition.

As mentioned so far, second-order nonlinear analyses for three-dimensional problems are almost confined to circular cylinders and therefore more versatile studies are needed. A method will be shown, which can perform a second-order nonlinear analysis for three-dimensional bodies and topographies of arbitrary shape by means of simple sources on boundary elements and eigen functions, which were applied by Yeung[7] for linear problems. In this study, a vertically infinite circular cylinder is analyzed to verify the validity of the present

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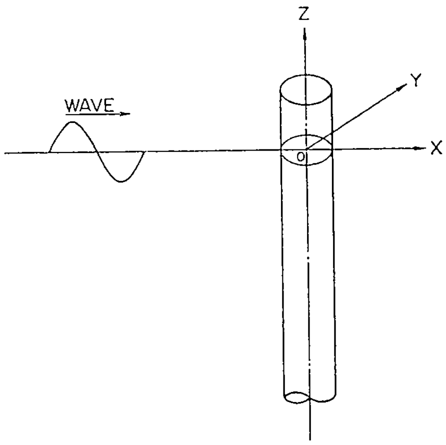

Fig. 1 Coordinate system

method because detailed data are available in Hunt and Baddour's paper.

2. Formulation

2.1 Governing equations

A Cartesian coordinate system o-xyz with oz vertically upward is employed as shown in Fig. 1. We make the usual assumption of irrotational incompressible flow, which allows us to describe the flow by a velocity potential. The velocity potential Φ must satisfy the equations :

$$\left.\begin{array}{ll}
\nabla^2\Phi=0 & \text{in fluid,} \\
\dfrac{\partial\Phi}{\partial n}=0 & \text{on body,} \\
\dfrac{\partial^2\Phi}{\partial t^2}+g\dfrac{\partial\Phi}{\partial z}+\dfrac{\partial}{\partial t}(\nabla\Phi)^2 & \\
\qquad+\dfrac{1}{2}\nabla\Phi\cdot\nabla(\nabla\Phi)^2=0 & \text{on free surface,} \\
\nabla\Phi\rightarrow 0 & \text{as } z\rightarrow-\infty \\
\text{radiation condition} & \text{in far field,}
\end{array}\right\} \tag{1}$$

and wave elevation ζ is described by

$$\zeta=-\frac{1}{g}\left\{\frac{\partial\Phi}{\partial t}+\frac{1}{2}(\nabla\Phi)^2\right\}, \tag{2}$$

where g is acceleration of gravity.

Performing a perturbation expansion for Φ and ζ by the use of small parameter ε, we obtain

$$\left.\begin{array}{l}
\Phi=\Phi^{(1)}(\varepsilon)+\Phi^{(2)}(\varepsilon^2)+\cdots\cdots, \\
\zeta=\zeta^{(1)}(\varepsilon)+\zeta^{(2)}(\varepsilon^2)+\cdots\cdots.
\end{array}\right\} \tag{3}$$

Furthermore, Φ or its derivatives on the free surface can be expanded in a Taylor series about $z=0$: Φ, e. g., is

$$\begin{aligned}
\Phi(x,y,z,t)&=\Phi(x,y,o,t)\\
&\quad+\frac{\partial\Phi(x,y,o,t)}{\partial z}\zeta(x,y,t)+\cdots\cdots\\
&=\Phi^{(1)}(x,y,o,t)\\
&\quad+\left\{\Phi^{(2)}(x,y,o,t)+\frac{\partial\Phi^{(1)}(x,y,o,t)}{\partial z}\right.\\
&\quad\cdot\zeta^{(1)}(x,y,t)+\cdots\cdots.
\end{aligned} \tag{4}$$

Substituting Eqs. (2) through (4) into Eq. (1) and rearranging them in terms of the same order, we obtain governing equations for first and second-order problems as follows :

1 st-order

$$\left.\begin{array}{ll}
\nabla^2\Phi^{(1)}=0 & \text{in fluid,} \\
\dfrac{\partial\Phi^{(1)}}{\partial n}=0 & \text{on body,} \\
\dfrac{\partial^2\Phi^{(1)}}{\partial t^2}+g\dfrac{\partial\Phi^{(1)}}{\partial z}=0 & \text{on } z=0, \\
\nabla\Phi^{(1)}\rightarrow 0 & \text{as } z\rightarrow-\infty, \\
\text{radiation condition} & \text{in far field,} \\
\zeta^{(1)}=-\dfrac{1}{g}\dfrac{\partial\Phi^{(1)}}{\partial t} & \text{on } z=0,
\end{array}\right\} \tag{5}$$

2 nd-order

$$\left.\begin{array}{ll}
\nabla^2\Phi^{(2)}=0 & \text{in fluid,} \\
\dfrac{\partial\Phi^{(2)}}{\partial n}=0 & \text{on body,} \\
\dfrac{\partial^2\Phi^{(2)}}{\partial t^2}+g\dfrac{\partial\Phi^{(2)}}{\partial z}=-\dfrac{\partial}{\partial t}(\nabla\Phi^{(2)})^2+\dfrac{\partial\Phi^{(1)}}{\partial t} & \\
\qquad\times\dfrac{\partial}{\partial z}\left(\dfrac{\partial\Phi^{(1)}}{\partial z}+\dfrac{1}{g}\dfrac{\partial^2\Phi^{(1)}}{\partial t^2}\right) & \text{on } z=0, \\
\nabla\Phi^{(2)}\rightarrow 0 & \text{as } z\rightarrow-\infty, \\
\text{radiation condition} & \text{in far field,} \\
\zeta^{(2)}=-\dfrac{1}{g}\left\{\dfrac{\partial\Phi^{(2)}}{\partial t}+\dfrac{1}{2}(\nabla\Phi^{(1)})^2\right. & \\
\qquad\left.+\dfrac{\partial^2\Phi^{(1)}}{\partial z\cdot\partial t}\zeta^{(1)}\right\} & \text{on } z=0,
\end{array}\right\} \tag{6}$$

where $\partial/\partial n$ denotes the derivative in the direction of the unit normal directed out of the fluid domain.

Under the assumption of harmonic motions such that

$$\left.\begin{array}{l}
\Phi^{(1)}=\phi^{(1)}e^{i\omega t}, \\
\Phi^{(2)}=\phi_0^{(2)}e^{2i\omega t}+\phi_s^{(2)}, \\
\zeta^{(1)}=\eta^{(1)}e^{i\omega t}, \\
\zeta^{(2)}=\eta_0^{(2)}e^{2i\omega t}+\eta_s^{(2)},
\end{array}\right\} \tag{7}$$

where subscripts o and s denote oscillatory and steady component, respectively, governing equations (5) and (6) are described as follows :

1 st-order

$$\left.\begin{array}{ll}
\nabla^2\phi^{(1)}=0 & \text{in fluid,} \\
\dfrac{\partial\phi^{(1)}}{\partial n}=0 & \text{on body,} \\
-\omega^2\phi^{(1)}+g\dfrac{\partial\phi^{(1)}}{\partial z}=0 & \text{on } z=0, \\
\nabla\phi^{(1)}\rightarrow 0 & \text{as } z\rightarrow-\infty, \\
\text{radiation condition} & \text{in far field,} \\
\eta^{(1)}=-\dfrac{i\omega}{g}\phi^{(1)} & \text{on } z=0,
\end{array}\right\} \tag{8}$$

2 nd-order

$$\begin{array}{ll}
\nabla^2\phi_0^{(2)}=\nabla^2\phi_s^{(2)}=0 & \text{in fluid,} \\
\dfrac{\partial\phi_0^{(2)}}{\partial n}=\dfrac{\partial\phi_s^{(2)}}{\partial n}=0 & \text{on body,} \\
-4\omega^2\phi_0^{(2)}+g\dfrac{\partial\phi_0^{(2)}}{\partial z}+i\omega(\nabla\phi^{(1)})^2 & \\
-\dfrac{i\omega}{2}\phi^{(1)}\left(\dfrac{\partial^2\phi^{(1)}}{\partial z^2}-\dfrac{\omega^2}{g}\dfrac{\partial\phi^{(1)}}{\partial z}\right)=0 & \text{on } z=0, \\
g\dfrac{\partial\phi_s^{(2)}}{\partial z}-\dfrac{\omega}{2}\left(i\phi^{(1)},\dfrac{\partial^2\phi^{(1)}}{\partial z^2}\right) &
\end{array}$$

$$
\left.\begin{aligned}
&+\frac{\omega^3}{2g}\left(i\phi^{(1)}, \frac{\partial \phi^{(1)}}{\partial z}\right)=0 && \text{on } z=0,\\
&\eta_0^{(2)}=-\frac{1}{g}\left\{2i\omega\phi_0^{(2)}+\frac{1}{4}(\nabla\phi^{(1)})^2\right.\\
&\left.\qquad+\frac{\omega^2}{2g}\frac{\partial \phi^{(1)}}{\partial z}\phi^{(1)}\right\} && \text{on } z=0,\\
&\eta_S^{(2)}=-\frac{1}{g}\left\{\frac{1}{4}|\nabla\phi^{(1)}|^2\right.\\
&\left.\qquad-\frac{\omega^2}{2g}\left(\frac{\partial \phi^{(1)}}{\partial z}, \phi^{(1)}\right)\right\} && \text{on } z=0,\\
&\nabla\phi_0^{(2)}, \nabla\phi_S^{(2)} \to 0 && \text{as } z\to-\infty,\\
&\text{radiation condition} && \text{in far field,}
\end{aligned}\right\} \tag{9}
$$

where

$$
\Phi_A\Phi_B=\frac{1}{2}\phi_A\phi_B e^{2i\omega t}+\frac{1}{2}(\phi_A, \phi_B) \tag{10}
$$

is used. (ϕ_A, ϕ_B) denotes an inner product of ϕ_A and ϕ_B.

Once a velocity potential is obtained, pressure P can be computed by

$$
P=-\rho\left\{gz+\frac{\partial \Phi}{\partial t}+\frac{1}{2}(\nabla\Phi)^2\right\} \tag{11}
$$

where ρ is the fluid density. In the same manner as Φ, P is split into four parts :

$$
\left.\begin{aligned}
&P=p^{(0)}(1)+p^{(1)}(\varepsilon)e^{i\omega t}\\
&\qquad+\{p_0^{(2)}(\varepsilon^2)e^{2i\omega t}+p_S^{(2)}(\varepsilon^2)\},\\
&p^{(0)}=-\rho g z,\\
&p^{(1)}=-i\omega\rho\phi^{(1)},\\
&p_0^{(2)}=-2i\omega\rho\phi_0^{(2)}-\frac{\rho}{4}(\nabla\phi^{(1)})^2,\\
&p_S^{(2)}=-\frac{\rho}{4}|\nabla\phi^{(1)}|^2.
\end{aligned}\right\} \tag{12}
$$

As clearly shown in Eqs. (9) and (12), the second-order steady potential $\phi_S^{(2)}$ has no contribution to steady wave elevation $\eta_S^{(2)}$ or steady pressure $p_S^{(2)}$. It is, therefore, concluded that $\phi_S^{(2)}$ has no meaning for the second-order problems.

Horizontal wave forces acting on a circular cylinder, the radius of which is a, is given in terms of pressure integration :

$$
\left.\begin{aligned}
&F_x=a\int_0^{2\pi}d\theta\int_{-\infty}^{\zeta}Pn_x dz=f_x^{(1)}e^{i\omega t}\\
&\qquad+f_{0x}^{(2)}e^{2i\omega t}+f_{Sx}^{(2)},\\
&f_x^{(1)}=-i\rho a\omega\int_0^{2\pi}d\theta\int_{-\infty}^{0}\phi^{(1)}n_x dz,\\
&f_{0x}^{(2)}=-2i\rho a\omega\int_0^{2\pi}d\theta\int_{-\infty}^{0}\phi_0^{(2)}n_x dz\\
&\qquad-\frac{\rho a}{4}\int_0^{2\pi}d\theta\int_{-\infty}^{0}(\nabla\phi^{(1)})^2 dz\\
&\qquad-\frac{\rho a\omega^2}{4g}\int_0^{2\pi}(\phi^{(1)})^2\Big|_{z=0}n_x d\theta,\\
&f_{Sx}^{(2)}=-\frac{\rho a}{4}\int_0^{2\pi}d\theta\int_{-\infty}^{0}|\nabla\phi^{(1)}|^2 n_x dz\\
&\qquad+\frac{\rho a\omega^2}{4g}\int_0^{2\pi}\left|\phi^{(1)}\right|^2_{z=0}n_x d\theta,
\end{aligned}\right\} \tag{13}
$$

where n_x is x component of the unit normal directed out of the fluid domain.

2.2 Integral equations

If we take $1/r$ as a kernel function, Green's theorem is shown as

$$
\begin{pmatrix}\phi\\0\end{pmatrix}=\frac{1}{2\pi}\int\left(\frac{\partial\phi}{\partial n}\frac{1}{r}-\phi\frac{\partial}{\partial n}\left(\frac{1}{r}\right)\right)ds \quad \begin{pmatrix}\text{on boundary}\\ \text{in outer domain}\end{pmatrix}, \tag{14}
$$

where the integration area includes all the boundaries. It is beneficial, however, to consider as a boundary a fictitious cylinder, whose radius R_0 is large enough, leading to three different boundaries : the body surface boundary S_B, the free surface boundary S_F and the fictitious cylinder boundary S_R. Note that S_F is no longer infinite and that the velocity potential ϕ is 0 in the outer domain.

Substitutions of Eqs. (8) and (9) into Eq. (14) give integral equations for the first and second-order velocity potentials, respectively.

1 st-order

Dividing the first-order potential $\phi^{(1)}$ into an incident wave potential $\phi_I^{(1)}$ and a diffraction potential $\phi_D^{(1)}$, let us seek integral equations with respect to $\phi_D^{(1)}$:

$$
\begin{aligned}
&2\pi\begin{pmatrix}\phi_D^{(1)}\\0\end{pmatrix}+\int\phi_D^{(1)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_B\\
&\quad+\int\phi_D^{(1)}\left\{\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{\omega^2}{g}\frac{1}{r}\right\}dS_F\\
&\quad+\int\left\{\phi_D^{(1)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{\partial\phi_D^{(1)}}{\partial n}\frac{1}{r}\right\}dS_R\\
&\quad=-\int\frac{\partial\phi_I^{(1)}}{\partial n}\frac{1}{r}dS_B,
\end{aligned} \tag{15}
$$

where the boundary condition

$$
\frac{\partial\phi_D^{(1)}}{\partial n}+\frac{\partial\phi_I^{(1)}}{\partial n}=0 \quad \text{on body} \tag{16}
$$

is used. In addition, $\phi_I^{(1)}$ is given by

$$
\phi_I{}^{(1)}=i\frac{\zeta_A\omega}{K}e^{Kz}e^{-iKx}, \tag{17}
$$

where ζ_A is a wave amplitude and K is a wave number. $\phi_D^{(1)}$ is represented on the fictitious boundary in terms of eigen-function expansion such that

$$
\phi_D{}^{(1)}=e^{Kz}\sum_{m=0}^{M-1}A_m^{(1)}H_m^{(2)}(KR_0)\cos m\theta, \tag{18}
$$

where we take the usual polar coordinate system, 0-$R\theta z$ and $A_m^{(1)}$'s are unknown coefficients of Hankel function of the second kind, $H_m^{(2)}$. Therefore Eq. (15) is rewritten

$$
\begin{aligned}
&2\pi\begin{pmatrix}\phi_D^{(1)}\\0\end{pmatrix}+\int\phi_D^{(1)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_B\\
&\quad+\int\phi_D^{(1)}\left\{\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{K}{r}\right\}dS_F\\
&\quad+R_0\sum_{m=0}^{M-1}A_m^{(1)}\left\{H_m^{(2)}(KR_0)\int_0^{2\pi}\cos m\theta d\theta\right.\\
&\quad\times\int_{-\infty}^{0}e^{Kz}\frac{\partial}{\partial R}\left(\frac{1}{r}\right)dz-\frac{\partial H_m^{(2)}(KR_0)}{\partial R}
\end{aligned}
$$

$$\times\int_0^{2\pi}\cos m\theta d\theta\int_{-\infty}^{0} e^{Kz}\frac{1}{r}dz\Big\}$$
$$=-\int\frac{\partial\phi_I^{(1)}}{\partial n}\frac{1}{r}dS_B. \tag{19}$$

We discretize the body boundary and the free surface boundary with N boundary elements. Under the assumption that $\phi_D^{(1)}$ is constant on each element, Eq. (19) reduces to a set of simultaneous equations with N unknowns for $\phi_D^{(1)}$ and M unknowns for $A_m^{(1)}$. Namely, M-boundary condition points are necessary other than those on each element. We impose M boundary conditions on the free surface in the outer domain. Note that the first term in Eq. (19) is 0 in this case.

2 nd-order

$$2\pi\begin{pmatrix}\phi_0^{(2)}\\ 0\end{pmatrix}+\int\phi_0^{(2)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_B$$
$$+\int\phi_0^{(2)}\left\{\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{4K}{r}\right\}dS_F$$
$$+\int\left\{\phi_0^{(2)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{\partial\phi_0^{(2)}}{\partial n}\frac{1}{r}\right\}dS_R$$
$$=-\frac{i\omega}{g}\int(\nabla\phi^{(1)})^2\frac{1}{r}dS_F \tag{20}$$

Note that one of the inhomogeneous terms in the free surface condition Eq. (9) has no contribution to the cylindrical body presently considered.

On the fictitious boundary, $\phi_0^{(2)}$ is composed of the following two potentials, i.e., a free wave potential $\phi_F^{(2)}$ and a locked wave potential $\phi_L^{(2)}$ (see Appendix) :

$$\left.\begin{aligned}
&\phi_0^{(2)}=\phi_F^{(2)}+\phi_L^{(2)}\\
&\phi_F^{(2)}=e^{4Kz}\sum_{m=0}^{M-1}A_m^{(2)}H_m^{(2)}(4KR_0)\cos m\theta\\
&\phi_L^{(2)}=-K\zeta_A\sqrt{\frac{2}{\pi KR}}\sum_{m=0}^{M-1}A_m^{(1)}e^{i\frac{2m+1}{4}\pi}\\
&\qquad\times\sum_{l=0}^{1}F_l(\theta,m)e^{\kappa_l(\theta,m)z},
\end{aligned}\right\} \tag{21}$$

where

$$\left.\begin{aligned}
&\kappa_l(\theta,m)=\sqrt{K^2(2+2\cos\theta)+\left(\frac{m}{R_0}\right)^2 -(-1)^l 2K\frac{m}{R_0}\sin\theta},\\
&F_l(\theta,m)=\frac{e^{-iKR_0(1+\cos\theta)}}{4K-\kappa_l(\theta,m)}\Big\{K(1-\cos\theta)\\
&\qquad+(-1)^l\frac{m}{R_0}\sin\theta\Big\}\{\cos m\theta-i(-1)^l\sin m\theta\}.
\end{aligned}\right\} \tag{22}$$

Hence, we deduce the following from Eq. (20) :

$$2\pi\begin{pmatrix}\phi_0^{(2)}\\ 0\end{pmatrix}+\int\phi_0^{(2)}\frac{\partial}{\partial n}\left(\frac{1}{r}\right)dS_B$$
$$+\int\phi_0^{(2)}\left\{\frac{\partial}{\partial n}\left(\frac{1}{r}\right)-\frac{4K}{r}\right\}dS_F$$
$$+R_0\sum_{m=0}^{M-1}A_m^{(2)}\Big\{H_m^{(2)}(4KR_0)\int_0^{2\pi}\cos m\theta d\theta\int_{-\infty}^{0}e^{4Kz}$$
$$\times\frac{\partial}{\partial R}\left(\frac{1}{r}\right)dz-\frac{\partial H_m^{(2)}(4KR_0)}{\partial R}\int_0^{2\pi}\cos m\theta d\theta$$
$$\times\int_{-\infty}^{0}e^{4Kz}\frac{1}{r}dz\Big\}=-\frac{i\omega}{g}\int(\nabla\phi^{(1)})^2\frac{1}{r}dS_F$$
$$+K\zeta_A\sqrt{\frac{2}{\pi KR_0}}R_0\sum_{m=0}^{M-1}A_m^{(1)}e^{i\frac{2m+1}{4}\pi}$$
$$\times\int_0^{2\pi}\sum_{l=0}^{1}F_l(\theta,m)d\theta\int_{-\infty}^{0}e^{\kappa_l(\theta,m)z}\frac{\partial}{\partial R}\left(\frac{1}{r}\right)dz$$
$$+iK\zeta_A\sqrt{\frac{2}{\pi KR_0}}KR_0\sum_{m=0}^{M-1}A_m^{(1)}e^{i\frac{2m+1}{4}\pi}$$
$$\times\int_0^{2\pi}\sum_{l=0}^{1}(1+\cos\theta)F_l(\theta,m)d\theta\int_{-\infty}^{0}\frac{e^{\kappa_l(\theta,m)z}}{r}dz. \tag{23}$$

3. Numerical Results

In order to verify the validity of the present method, computations are conducted for the second-order wave forces on a vertical circular cylinder of infinite length, for which existing data are available.

Fig. 2 shows a sample of element division on the body and the free surface boundaries. Elements in a great depth, which are insignificant through a sense of the first-order problem, cannot be neglected for the second-order problem. Second-order potential penetrates the water much deeper than the first-order potential ; Taylor and Hung investi-

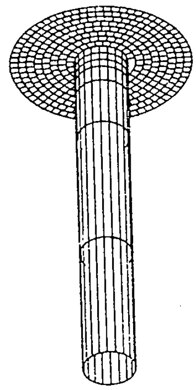

Fig. 2 Element division

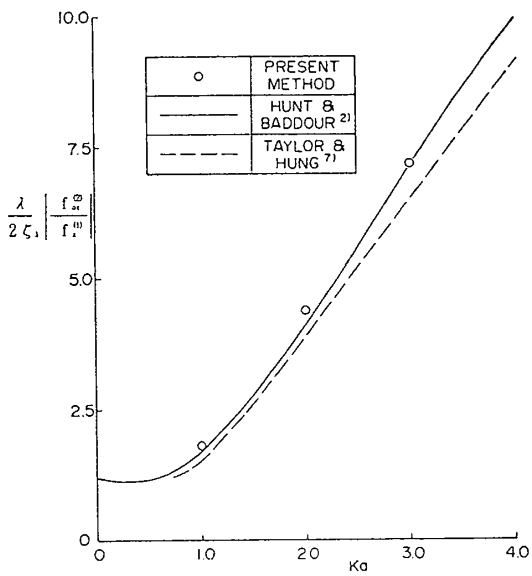

Fig. 3 Oscillatory second-order wave force

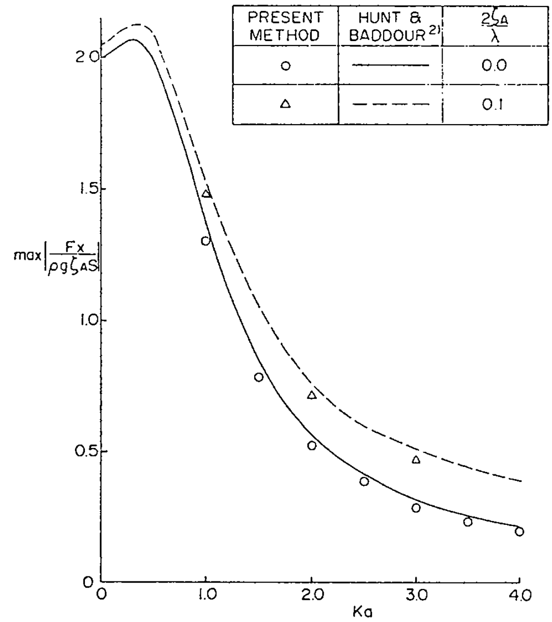

Fig. 4 Maximum total wave force

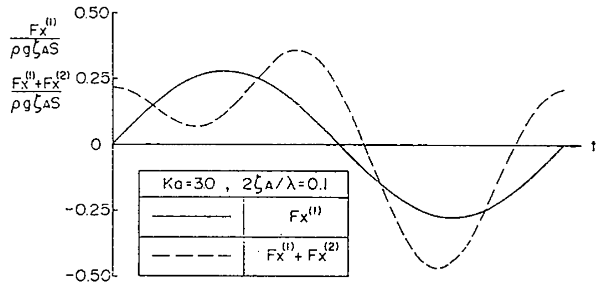

Fig. 5 Variations of first-order wave force and total wave force with time

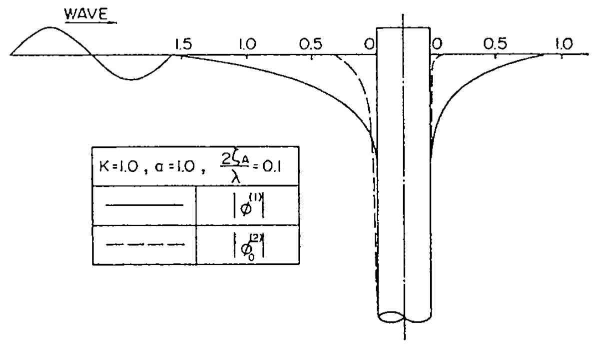

Fig. 6 Variations in z direction of oscillatory first-order and second-order potentials on weather and lee sides ($Ka=1.0$)

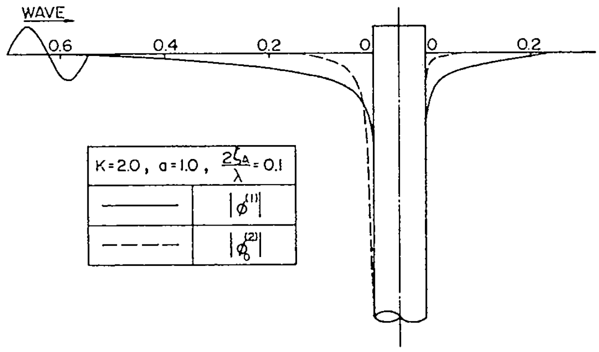

Fig. 7 Variations in z direction of oscillatory first-order and second-order potentials on weather and lee sides ($Ka=2.0$)

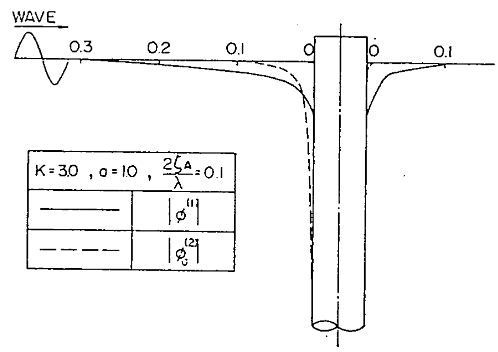

Fig. 8 Variations in z direction of oscillatory first-order and second-order potentials on weather and lee sides ($Ka=3.0$)

gated that the second-order force on the cylinder whose draft-radius ratio d/r is 10, is still as much as 5 percent smaller than that on the infinitely long cylinder. The element division is done here so that r/d is more than 10.

Numerical results for oscillatory second-order force are plotted in Fig. 3 against nondimensional incident-wave frequency, Ka, in comparison with Hunt and Baddour's and Taylor and Hung's. They are nondimensionalized by wave steepness, $2\zeta_A/\lambda$, where λ is the incident wave length, and first-order wave force amplitude $f^{(1)}$. Fig. 3 shows that the oscillatory second-order forces computed by the three different methods agree with one another and importance of the oscillatory second-order force increases for higher-frequency incident waves.

Maximum values of total wave forces including oscillatory first-order force, oscillatory second-order force and steady second-order force are shown in Fig. 4, where values by the present method is in good agreement with those by Hunt and Baddour. The solid line and the circles correspond to the linear theory. On the other hand, the broken line and the triangles are for wave steepness of 0.1. Contributions of the second-order force are significant for higher-frequency waves ; say, for more than 2.0 of Ka, increase of the force from the first-order one is over 30%.

Fig. 5 obtained by the present method shows how the total wave force changes with time. While the first-order force changes sinusoidally, the total force including the second-order forces changes quite differently. It is possible to understand that maximum wave force acts in the direction opposite to the wave propagation.

Figs. 6 through 8, obtained by the present method, give the idea how differently the oscillatory second-order potential varies in z direction from the first-order one. For $Ka=1.0$, the second-order potential decays more slowly in z direction than the first-order one but its contribution is still small. Contribution of the second-order potential on the weather side is of great importance for $Ka=2.0$, but the second-order potential on the lee side decays faster in z direction rather than the first-order one. It is interesting to see in Fig. 8 that the second-order potential does not decay completely in z direction up to the depth several times deeper than the wave length.

4. Conclusions

A numerical method has been developed in order to obtain complete solutions to the second-order velocity potentials for the three-dimensional diffraction problem. Computations are made for a vertical circular cylinder in deep water so as to verify the validity of the present method.

In comparison with existing results by other methods, the present method is proved useful. Moreover, it is reconfirmed that the second-order potential on the weather side does not decay so fast as the first-order one.

Although the vertical circular cylinder in deep water is dealt with in this study, the present method can be applied with small modifications to three-dimensional bodies and topographies of arbitrary shape.

Acknowledgements

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References

1) Papanikolaou, A. and Nowacki, H.: Second-order Theory of Oscillating Cylinders in a Regular Steep Wave, Proceedings of 13 th Symposium on Naval Hydrodynamics, Tokyo, (1980).
2) Kyozuka, Y. : Non-Linear Hydrodynamic Forces Acting on Two-Dimensional Bodies (1 st Report, Diffraction Problem), Journal of the Society of Naval Architects of Japan, Vol. 148, (1980).
3) Molin, B.: Second-order Diffraction Loads upon Three-dimensional Bodies, Applied Ocean Research, Vol. 11, No. 4, (1979).
4) Taylor, R. E. and Hung, S. M.: Second Order Diffraction Forces on a Vertical Cylinder in Regular Waves, Unpublished, (1986).
5) Hunt, J. N. and Baddour, R. E.: The Diffraction of Nonlinear Progressive Waves by a Vertical Cylinder, Quarterly Journal of Mechanics and Applied Mathematics, Vol. 33, Part 3, (1980).
6) Garrison, C. J.: Nonlinear Wave Loads on Large Structures, Proceedings of 3 rd International OMAE Symposium, Vol. 2, (1984).
7) Yeung, R. W. : A Singularity-distribution Method for Free-surface Flow Problems with an Oscillating Body, University of California, Berkeley, California College Engineering Report, NA 73-6.
8) Newman, J. N.: Distributions of Sources and Normal Dipoles over a Quadrilateral Panel, Journal of Engineering Mathematics 20, (1986).

Appendix
Second-order Velocity Potential in the Far Field

The second-order velocity potential in the far field must satisfy Laplace equation and the free surface boundary condition :

$$\nabla^2\phi_0^{(2)}=0 \qquad \text{in fluid,} \qquad \text{(A-1)}$$

$$4K\phi_0^{(2)}-\frac{\partial\phi_0^{(2)}}{\partial z}=\frac{i\omega}{g}(\nabla\phi^{(1)})^2 \quad \text{on } z=0. \qquad \text{(A-2)}$$

We assume that $\phi_0^{(2)}$ consists of a free wave potential $\phi_F^{(2)}$ and a locked wave potential $\phi_L^{(2)}$:

$$\phi_0^{(2)}=\phi_F^{(2)}+\phi_L^{(2)}, \qquad \text{(A-3)}$$

where $\phi_F^{(2)}$, satisfying Eq. (A-1), is a general solution to the homogeneous part of Eq. (A-2) and can be described as a summation of Hankel function of the second kind :

$$\phi_F^{(2)}=e^{4Kz}\sum_{m=0}^{M-1}A_m^{(2)}H_m^{(2)}(4KR_0)\cos m\theta. \qquad \text{(A-4)}$$

$\phi^{(1)}$ is given on the free surface intersecting the fictitious cylinder by

$$\phi^{(1)}=i\frac{\zeta_A\omega}{K}e^{Kz}e^{-iKR_0\cos\theta}+\sqrt{\frac{2}{\pi KR_0}}e^{Kz}\sum_{m=0}^{M-1}A_m^{(1)}e^{-i\left(KR_0-\frac{2m+1}{4}\pi\right)}\cos m\theta. \qquad \text{(A-5)}$$

Derivatives of $\phi^{(1)}$ are as follows :

$$\left.\begin{aligned}
\frac{\partial\phi^{(1)}}{\partial R}&=\zeta_A\omega\cos\theta e^{-iKR_0\cos\theta}-iK\sqrt{\frac{2}{\pi KR_0}}\sum_{m=0}^{M-1}A_m^{(1)}e^{-i\left(KR_0-\frac{2m+1}{4}\pi\right)}\cos m\theta,\\
\frac{\partial\phi^{(1)}}{\partial\theta}&=-\zeta_A\omega R_0\sin\theta e^{-iKR_0\cos\theta}-\sqrt{\frac{2}{\pi KR_0}}\sum_{m=0}^{M-1}mA_m^{(1)}e^{-i\left(KR_0-\frac{2m+1}{4}\pi\right)}\sin m\theta,\\
\frac{\partial\phi^{(1)}}{\partial z}&=i\zeta_A\omega e^{-iKR_0\cos\theta}+K\sqrt{\frac{2}{\pi KR_0}}\sum_{m=0}^{M-1}A_m^{(1)}e^{-i\left(KR_0-\frac{2m+1}{4}\pi\right)}\cos m\theta.
\end{aligned}\right\} \qquad \text{(A-6)}$$

Note that $KR_0 \gg 1$ and hence, the right-hand side of Eq. (A-2) reduces to

$$\frac{i\omega}{g}(\nabla\phi^{(1)})^2=\frac{2i\zeta_A\omega^2}{g}\sqrt{\frac{2}{\pi KR_0}}e^{-iKR_0(1+\cos\theta)} \times\sum_{m=0}^{M-1}A_m^{(1)}e^{i\frac{2m+1}{4}\pi}\left\{iK(1-\cos\theta)\cos m\theta +\frac{m}{R_0}\sin\theta\sin m\theta\right\} \tag{A-7}$$

where the m/KR_0 term is kept.

Look for $\phi_L^{(2)}$ in the form :

$$\phi_L^{(2)}=\sum_{m=0}^{M-1}e^{\beta z}e^{-iKR_0(1+\cos\theta)}(\alpha_c\cos m\theta+\alpha_s\sin m\theta) +\sum_{m=0}^{M-1}e^{\delta z}e^{-iKR_0(1+\cos\theta)}(\gamma_c\cos m\theta+\gamma_s\sin m\theta), \tag{A-8}$$

where α_c, α_s, β, γ_c, γ_s and δ are unknowns. Substituting Eq. (A-8) to Eq. (A-2), we obtain

$$\phi_L^{(2)}=-K\zeta_A\sqrt{\frac{2}{\pi KR_0}}\sum_{m=0}^{M-1}A_m^{(1)}e^{i\frac{2m+1}{4}\pi} \times\sum_{l=0}^{1}F_l(\theta,m)e^{\kappa_l(\theta,m)z}, \tag{A-9}$$

where

$$\left.\begin{aligned}\kappa_l(\theta,m)&=\sqrt{K^2(2+2\cos\theta)+\left(\frac{m}{R_0}\right)^2-(-1)^l2K\frac{m}{R_0}\sin\theta},\\ F_l(\theta,m)&=\frac{e^{-iKR_0(1+\cos\theta)}}{4K-\kappa_l(\theta,m)}\\&\quad\times\left\{K(1-\cos\theta)+(-1)^l\frac{m}{R_0}\sin\theta\right\}\\&\quad\times\{\cos m\theta-i(-1)^l\sin m\theta\}.\end{aligned}\right\} \tag{A-10}$$

$\phi_L^{(2)}$ is a particular solution to Eq. (A-2) and can be determined in terms of $\phi^{(1)}$. Hence, it is concluded that the radiation condition for the second-order problem should be imposed only to the free wave potential $\phi_F^{(2)}$ and is the same as the first-order one except that the wave number is quadrupled. This implies that energy radiates from the body to infinity even for the second-order problem, which is different from Hunt and Baddour's explanation.