A Rankine Panel Method to Calculate Unsteady Ship Hydrodynamic Forces

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Summary

A panel method to calculate unsteady ship hydrodynamic forces is introduced. First, formulations were made for both steady and unsteady free-surface conditions under the assumption of small perturbation over the double body flow around a ship. A newly derived free-surface condition for unsteady motion makes a theoretical pair with Dawson's steady linearized free-surface condition. Next, a Rankine panel method was applied to solve the equations based on the present unsteady free-surface condition. For radiation condition of waves a numerical damping was introduced into the free-surface condition. Calculations were made of the unsteady hydrodynamic forces such as added mass, damping and wave exciting forces for a two-dimensional submerged cylinder and an ellipsoidal ship hull form. By comparing with other calculations and experiments it was shown that the present numerical method is useful for better understanding of the unsteady free-surface flow problems.

1. Introduction

Ship hydrodynamic forces have been predicted commonly by using strip method, thin ship theory and slender body theory. To take three-dimensionality in more consistent way, singularity distribution methods have also been proposed[2)7)8)9)14)]. These methods are based on the solutions for the classical linearized free-surface condition.

Further, for an improvement of the linearized solution, approaches have been made to deal with the free-surface condition taking steady perturbation flow around a ship into account. For instance, in 1983 Lee considered the double body flow as a basic flow to calculate added resistance of a ship in head waves[15)]. Sakamoto and Baba extended the low speed wave resistance theory[1)] to unsteady free-surface flow problem, and derived an asymptotic solution of added resistance[19)]. Zhao et al. calculated added resistance of a semi-submerged sphere[24)]. Kashiwagi and Ohkusu presented a hybrid method in radiation problem for a half-immersed circular cylinder[13)]. Huijismans and Hermans discussed the effect of steady perturbation flow on drifting forces of ships under the assumption of low speed[6)]. At present, however, it is difficult to deal with general ship hull form with forward velocity, since the computation scheme for solving the basic equations is complicated and the amount of the calculations is considerably large.

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On the other hand, there is Rankine source method[3)4)], a kind of numerical method for solving steady-state free-surface flow. The results show good agreement with experiments and give an improvement over the conventional linearized solution[18)]. The Rankine source method can easily deal with the free-surface condition taking the steady perturbation flow into account. Therefore, recently some attempts are made to apply the Rankine source method to the unsteady free-surface flow problem. Nakos calculated the wave elevation generated by a two-dimensional submerged body with forward and oscillatory motion[17)] by using quadratic B-spline panels[20)]. Takagi proposed a technique to satisfy radiation condition of waves by use of an artificial parameter corresponding to Rayleigh viscosity, and calculated added resistance by solving diffraction problem for a full hull form advancing in waves[21)22)]. In his formulation the double body flow is employed for steady perturbation flow. However a relation with the steady free-surface flow problem is not considered.

In this paper, an investigation was made of panel method for unsteady ship hydrodynamic forces. First, formulations were made for both steady and unsteady free-surface conditions under the assumption of small perturbation over the double body flow. A newly derived free-surface condition for unsteady motion makes a theoretical pair with Dawson's steady linearized free-surface condition[3)]. Next, a Rankine panel method was applied to solve the equations based on the present unsteady free-surface condition. For radiation condition of waves a numerical damping was introduced into the free-surface condition with reference to Dawson[3)] and

Takagi[21]. Calculations were made of unsteady hydrodynamic forces such as added mass, damping and exciting forces for a two-dimensional submerged cylinder and an ellipsoidal ship hull form. By comparing with other calculations and experiments it was shown that the present method is useful to solve numerically the unsteady free-surface flow problems.

2. Problem Formulation

Let us consider two coordinate systems $o'-x'y'z'$ fixed with respect to a ship, and $o-xyz$ moving in steady translation with the forward velocity of a ship as shown in Fig. 1. x-axis coincides with the negative direction of the ship's forward velocity U. $z=0$ plane coincides with the undistrurbed free-surface, and the z-axis is taken positive upward. The $o'-x'y'z'$ coordinate system is defined so as to coincide with the $o-xyz$ coordinate system in steady-state equilibrium.

Supposing a ship is in an inviscid, irrotational, incompressible fluid, the velocity potential Φ, which represents flow around the ship and satisfies Laplace's equation $\nabla^2\Phi=0$, is introduced. With reference to Sakamoto and Baba[19] it is assumed that the total potential Φ can be expressed as the sum of three components as:

$$\Phi(x, y, z, t)=\phi_0(x, y, z)+\phi_1(x, y, z)+\phi_U(x, y, z, t), \tag{1}$$

where

ϕ_0 : velocity potential representing steady double body flow,

ϕ_1 : velocity potential representing steady wavy flow,

ϕ_U : velocity potential representing unsteady wavy flow.

The total potential Φ has to satisfy free-surface condition, hull surface condition and radiation condition of waves.

2.1 Free-Surface Condition

The kinematic and dynamic boundary conditions on free-surface are written respectively as:

$$\Phi_x\zeta_x+\Phi_y\zeta_y-\Phi_z+\zeta_t=0 \quad \text{on } z=\zeta, \tag{2}$$

$$g\zeta+\frac{1}{2}(\nabla\Phi\cdot\nabla\Phi-U^2)+\Phi_t=0 \quad \text{on } z=\zeta, \tag{3}$$

where ζ is wave elevation, g the gravitational acceleration and subscript means partial differential.

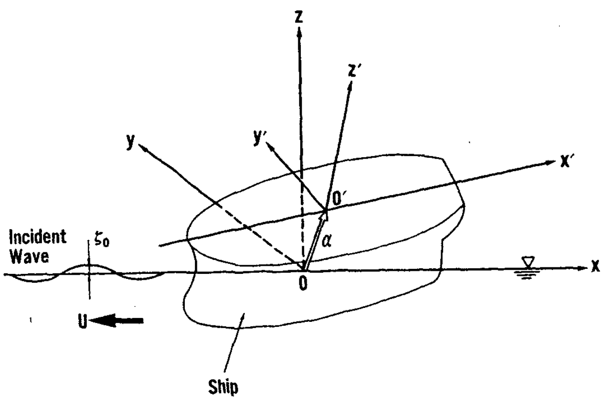

Fig. 1 Coordinate systems

Eliminating ζ from (2) and (3), the exact non-linear free-surface boundary condition is expressed as:

$$\Phi_{tt}+2\nabla\Phi\cdot\nabla\Phi_t+\frac{1}{2}\nabla\Phi\cdot\nabla(\nabla\Phi\cdot\nabla\Phi)+g\Phi_z=0 \quad \text{on } z=\zeta. \tag{4}$$

Eq. (4) has to be satisfied on the elevated free-surface, which is, however, not determined until that the velocity potential is obtained. Following Dawson's way[3] the free-surface condition is assumed to be satisfied on $z=0$. Further ϕ_1 and ϕ_U are assumed small. Substituting (1) into (4) and neglecting higher order terms for ϕ_1 and ϕ_U, a linearized free-surface condition based on the double body flow is derived as:

$$\begin{aligned}&\phi_{Utt}+2\nabla\phi_0\cdot\nabla\phi_{Ut}+\frac{1}{2}\nabla\phi_0\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_0)\\&+\nabla\phi_0\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_1)+\nabla\phi_0\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_U)\\&+\frac{1}{2}\nabla\phi_1\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_0)+\frac{1}{2}\nabla\phi_U\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_0)\\&+g(\phi_{1z}+\phi_{Uz})=0 \quad \text{on } z=0.\end{aligned} \tag{5}$$

An index s which denotes a derivative in the streamline direction of the double body flow is introduced. Using a following relation,

$$(\nabla\phi_0\cdot\nabla)\phi=\phi_{0s}\phi_s, \tag{6}$$

eq. (5) is rewritten as

$$\begin{aligned}&\phi_{Utt}+2\phi_{0s}\phi_{Ust}+\frac{1}{2}\nabla(\phi_0+\phi_1+\phi_U)\cdot\nabla(\phi_{0s}^2)\\&+\phi_{0s}[(\phi_{0s}\phi_{1s})_s+(\phi_{0s}\phi_{Us})_s]\\&+g(\phi_{1z}+\phi_{Uz})=0 \text{ on } z=0.\end{aligned} \tag{7}$$

If the curvature of the streamlines of double body flow is small, we can approximate as[10]:

$$\frac{1}{2}\nabla\phi_U\cdot\nabla(\phi_{0s}^2)\simeq\phi_{0s}\phi_{0ss}\phi_{Us}. \tag{8}$$

Then eq. (7) is expressed as:

$$\begin{aligned}&\phi_{Utt}+2\phi_{0s}\phi_{Ust}+\phi_{0s}^2(\phi_{1ss}+\phi_{Uss})+2\phi_{0s}\phi_{0ss}(\phi_{1s}+\phi_{Us})\\&+g(\phi_{1z}+\phi_{Uz})=-\phi_{0s}^2\phi_{0ss} \quad \text{on } z=0.\end{aligned} \tag{9}$$

Eq. (9) can be separated into two independent linearized free-surface conditions, one for the steady wave-making flow and the other for the time dependent wave motion as follows:

[steady component]

$$\phi_{0s}^2\phi_{1ss}+2\phi_{0s}\phi_{0ss}\phi_{1s}+g\phi_{1z}=-\phi_{0s}^2\phi_{0ss} \quad \text{on } z=0, \tag{10}$$

[unsteady component]

$$\phi_{Utt}+2\phi_{0s}\phi_{Ust}+\phi_{0s}^2\phi_{Uss}+2\phi_{0s}\phi_{0ss}\phi_{Us}+g\phi_{Uz}=0 \quad \text{on } z=0. \tag{11}$$

Eq. (10) coincides with Dawson's free-surface condition[3]. Therefore eq. (11), which is a newly derived in the present study, corresponds to an unsteady linearized free-surface condition which makes a theoretical pair with Dawson's one.

Relations among the present free-surface condition (11), Sakamoto and Baba's condition[19] and the classical linearized condition are explained in Appendix.

2.2 Hull Surface Condition

Exact ship hull surface condition is expressed as:

$$\Phi_n=[\dot{\alpha}+(\nabla\Phi\cdot\nabla)\alpha]\cdot\boldsymbol{n} \quad \text{on } S, \tag{12}$$

where $\boldsymbol{n}$ is the unit normal vector which is defined to point out of the fluid domain, and S the submerged portion of the ship's surface. $\boldsymbol{\alpha}$ is the vector of the oscillatory displacement of a ship (see Fig. 1) and is assumed small quantity. Then total velocity on S, $\nabla\Phi$ is expanded in Taylor series as:

$$\nabla\Phi|_S=\nabla\Phi|_{S_m}+[(\boldsymbol{\alpha}\cdot\nabla)\nabla\Phi]_{S_m}+O[(\boldsymbol{\alpha}\cdot\nabla)^2\nabla\Phi], \quad (13)$$

where S_m means the ship's surface in steady-state position. Substituting (1) and (13) into (12) and neglecting the higher order terms for ϕ_1, ϕ_U and $\boldsymbol{\alpha}$, it follows that

$$\phi_{0n}+\phi_{1n}+\phi_{Un}=[\dot{\boldsymbol{\alpha}}+\nabla\times(\boldsymbol{\alpha}\times\nabla\phi_0)]\cdot\boldsymbol{n} \quad \text{on } S_m. \quad (14)$$

It is noted that the eq. (14) can be derived also by applying the present assumptions to the hull surface condition presented by Timman and Newman[23].

Eq. (14) can be also separated into two independent hull surface conditions, one for the steady wave-making flow and the other for the time dependent wave motion as follows:

[steady component]

$$\phi_{0n}+\phi_{1n}=0 \quad \text{on } S_m, \quad (15)$$

[unsteady component]

$$\phi_{Un}=[\dot{\boldsymbol{\alpha}}+\nabla\times(\boldsymbol{\alpha}\times\nabla\phi_0)]\cdot\boldsymbol{n} \quad \text{on } S_m. \quad (16)$$

2.3 Wave Elevation and Hydrodynamic Forces

Wave elevation is represented by a linearized form of (3) as:

$$\zeta=\frac{1}{2g}[U^2-\phi_{0x}^2-\phi_{0y}^2-2\phi_{0x}(\phi_{1x}+\phi_{Ux}) -2\phi_{0y}(\phi_{1y}+\phi_{Uy})-2\phi_{Ut}]. \quad (17)$$

Eq. (17) can be separated into two components as follows:

[steady component]

$$\zeta_S=\frac{1}{2g}[U^2-\phi_{0x}^2-\phi_{0y}^2-2\phi_{0x}\phi_{1x}-2\phi_{0y}\phi_{1y}], \quad (18)$$

[unsteady component]

$$\zeta_U=-\frac{1}{g}[\phi_{0x}\phi_{Ux}+\phi_{0y}\phi_{Uy}+\phi_{Ut}]. \quad (19)$$

Pressure on ship hull surface p_H is represented by Bernoulli's equation as:

$$p_H=-\rho\left[\frac{1}{2}(\nabla\Phi\cdot\nabla\Phi-U^2)+\Phi_t\right]-\rho gz \quad \text{on } S. \quad (20)$$

Neglecting the higher order terms for ϕ_1, ϕ_U and $\boldsymbol{\alpha}$, p_H is represented as:

$$p_H=-\rho\left[\frac{1}{2}(\nabla\phi_0\cdot\nabla\phi_0-U^2)+\nabla\phi_0\cdot\nabla\phi_1+\nabla\phi_0\cdot\nabla\phi_U +\phi_{Ut}+\frac{1}{2}(\boldsymbol{\alpha}\cdot\nabla)(\nabla\phi_0\cdot\nabla\phi_0)\right]-\rho gz \quad \text{on } S_m. \quad (21)$$

Integrating the pressure p_H over the ship hull surface, the hydrodynamic forces F_{Hi} acting on a ship can be obtained as:

$$\begin{aligned}F_{Hi}&=-\iint_{Sm}p_H n_i dS\\ &=\rho\iint_{Sm}\left[\frac{1}{2}(\nabla\phi_0\cdot\nabla\phi_0-U^2)+\nabla\phi_0\cdot\nabla\phi_1\right]n_i dS\\ &\quad+\rho\iint_{Sm}\left[\nabla\phi_0\cdot\nabla\phi_U+\phi_{Ut} +\frac{1}{2}(\boldsymbol{\alpha}\cdot\nabla)(\nabla\phi_0\cdot\nabla\phi_0)\right]n_i dS\\ &\quad+\rho g\iint_{Sm}zn_i dS, \end{aligned}\quad (22)$$

where

$$(n_1, n_2, n_3)=\boldsymbol{n}, \quad (23)$$

$$(n_4, n_5, n_6)=\boldsymbol{r}\times\boldsymbol{n}. \quad (24)$$

Here subscript i denotes i-th motion, where 1, 2, 3, 4, 5 and 6 mean surge, sway, heave, roll, pitch and yaw respectively. $\boldsymbol{r}$ is the vector of coordinate of the ship hull surface. The first and second integration terms in (22) mean the steady and unsteady hydrodynamic forces respectively, and the third integration term the hydrostatic forces.

3. A Rankine Panel Method for Unsteady Ship Hydrodynamic Forces in Regular Waves

In this section, a panel method is presented to solve the equations about the unsteady components as described above, and calculate hydrodynamic forces acting on a ship advancing in regular waves. In the present study Dawson's Rankine panel method was applied to the unsteady free-surface problems.

3.1 Velocity Potential and Boundary Conditions

A ship moving with constant mean forward velocity in regular sinusoidal waves is considered. It is assumed that the ship motions are linear and harmonic. Then the vector of the oscillatory displacement of the ship $\boldsymbol{\alpha}$ is expressed as:

$$\boldsymbol{\alpha}=Re[(\boldsymbol{l}+\boldsymbol{\Omega}\times\boldsymbol{r})e^{-i\omega t}], \quad (25)$$

where

$$\boldsymbol{l}=(\xi_1, \xi_2, \xi_3),$$

$$\boldsymbol{\Omega}=(\xi_4, \xi_5, \xi_6).$$

Here $\boldsymbol{l}$ and $\boldsymbol{\Omega}$ denote the oscillatory translation and rotation of the ship, relative to the origin o'. ω means the frequency of encounter. Re denotes to take the real part of a complex number.

The velocity potential representing the unsteady free-surface flow ϕ_U is expressed in the same form as (25):

$$\phi_U(x, y, z, t)=Re[\phi(x, y, z)e^{-i\omega t}], \quad (26)$$

where the potential $\phi(x, y, z)$, which is to be solved, is given in complex form. The potential is expressed in the following form:

$$\phi=\sum_{j=1}^{6}\xi_j\phi_{Rj}+\zeta_0(\phi_D+\phi_I), \quad (27)$$

where

ϕ_{Rj} : radiation potential per unit motion of j-th direction,

ϕ_D : diffraction potential per unit incident-wave height,

ϕ_I : incident-wave potential,

ζ_0 denotes the amplitude of incident-wave.

Substituting (25) through (27) into the boundary conditions (11) and (16) and dropping the term of $e^{-i\omega t}$, the boundary conditions for radiation and diffraction problems are obtained as follows:

(a) radiation problem

$$[\text{F}] \quad \phi_{0s}^2\phi_{Rssj}+2\phi_{0s}(\phi_{0ss}-i\omega)\phi_{Rsj}+g\phi_{Rzj}-\omega^2\phi_{Rj}=0 \quad \text{on } z=0, \quad (28)$$

$$[\text{H}] \quad \phi_{Rnj}=-i\omega n_j+m_j \quad \text{on } S_m, \quad (29)$$

where

$$(m_1, m_2, m_3) = -(\boldsymbol{n}\cdot\nabla)\nabla\phi_0, \quad (30)$$

$$(m_4, m_5, m_6) = -(\boldsymbol{n}\cdot\nabla)(\boldsymbol{r}\times\nabla\phi_0), \quad (31)$$

(b) diffraction problem

$$[\mathrm{F}] \quad \phi_{0s}^2\phi_{Dss} + 2\phi_{0s}(\phi_{0ss} - i\omega)\phi_{Ds} + g\phi_{Dz} - \omega^2\phi_D = -[\phi_{0s}^2\phi_{Iss} + 2\phi_{0s}(\phi_{0ss} - i\omega)\phi_{Is} + g\phi_{Iz} - \omega^2\phi_I] \quad \text{on } z=0, \quad (32)$$

$$[\mathrm{H}] \quad \phi_{Dn} = -\phi_{In} \quad \text{on } S_m. \quad (33)$$

Incident-wave potential ϕ_I is expressed as:

$$\phi_I = -\frac{ig}{\omega_0} e^{Kz} e^{i(-Kx\cos\chi + Ky\sin\chi)} \quad (34)$$

where ω_0 is the incident-wave frequency, K the wavenumber and χ the angle of incident-wave direction (see Fig. 2). Eq. (34) satisfies the classical free-surface condition. Substituting (34) into (19), the incident-wave elevation ζ_{IU} is expressed as:

$$\zeta_{IU} = Re[\zeta_I e^{-i\omega t}], \quad (35)$$

where

$$\zeta_I = \frac{\zeta_0}{g}(\phi_{0x}\phi_{Ix} + \phi_{0y}\phi_{Iy} - i\omega\phi_I) = \frac{\zeta_0}{\omega_0}[K(\phi_{0x}\cos\chi - \phi_{0y}\sin\chi) + \omega] e^{i(-Kx\cos\chi + Ky\sin\chi)}. \quad (36)$$

If the perturbation of the steady flow field due to a ship is neglected in (36), $\phi_{0x} = U$ and $\phi_{0y} = 0$, and thus

$$\zeta_I = \zeta_0 e^{i(-Kx\cos\chi + Ky\sin\chi)}. \quad (37)$$

3.2 Unsteady Hydrodynamic Forces

Substituting (25) and (26) into the second integration term of (22), the unsteady hydrodynamic forces F_{Uj} can be written as:

$$F_{Uj} = Re[F_j e^{-i\omega t}], \quad (38)$$

where

$$F_j = \rho \iint_{S_m} \left[\nabla\phi_0\cdot\nabla\phi - i\omega\phi + \frac{1}{2}\{(\boldsymbol{l} + \boldsymbol{\Omega}\times\boldsymbol{r})\cdot\nabla\}(\nabla\phi_0\cdot\nabla\phi_0)\right] n_j dS. \quad (39)$$

(a) added mass and damping

The unsteady hydrodynamic pressure forces associated with the added mass and damping can be expressed as:

$$F_i = \sum_{j=1}^{6} \xi_j T_{ij} \equiv -\sum_{j=1}^{6}(\omega^2 A_{ij} + i\omega B_{ij}). \quad (41)$$

Here A_{ij} and B_{ij} are respectively the added mass and damping coefficients associated with the force in the i-th direction due to the j-th mode of motion. T_{ij} is expressed as:

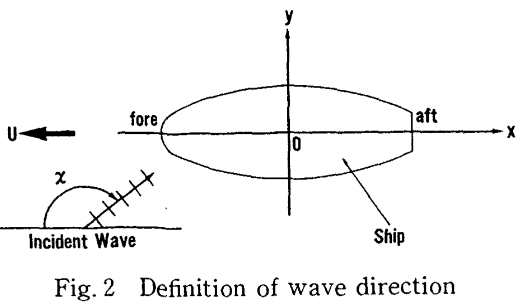

Fig. 2 Definition of wave direction

$$T_{ij} = \rho \iint_{S_m} \left[\nabla\phi_0\cdot\nabla\phi_{Rj} - i\omega\phi_{Rj} + \frac{1}{2}(\boldsymbol{\beta}_j\cdot\nabla)(\nabla\phi_0\cdot\nabla\phi_0)\right] n_i dS, \quad (42)$$

where

$$\left.\begin{aligned} \boldsymbol{\beta}_j &= \boldsymbol{e}_j \quad (j=1,2,3), \\ \boldsymbol{\beta}_j &= \boldsymbol{\beta}_{j-3} X\boldsymbol{r} \quad (j=4,5,6), \end{aligned}\right\} \quad (43)$$

$\boldsymbol{e}_j$: unit vector with respect to j-th direction.

(b) wave exciting forces

The wave exciting force E_j can be expressed as:

$$E_j = E_{FKj} + E_{Dj}, \quad (44)$$

where

$$E_{FKj} = \rho\zeta_0 \iint_{S_m} (\nabla\phi_0\cdot\nabla\phi_I - i\omega\phi_I) n_j dS, \quad (45)$$

$$E_{Dj} = \rho\zeta_0 \iint_{S_m} (\nabla\phi_0\cdot\nabla\phi_D - i\omega\phi_D) n_j dS. \quad (46)$$

Here E_{FKj} is Froude-Krylov force and E_{Dj} the wave exciting force due to diffraction potential. Here it is noted that the E_{FKj} includes the effect of steady perturbation.

3.3 Radiation Condition of waves

Waves due to oscillatory singularity with forward speed are composed of two components: one is the deformed Kelvin waves due to oscillation and the other is the deformed Ring waves due to forward velocity. Following Naito et al. the former is called k_1 waves and the latter k_2 waves[16]. Therefore radiation conditions corresponding to these waves should be considered. This can be guessed from the following two-dimensional linearized free-surface condition with Rayleigh viscosity μ:

$$U^2\phi_{xx} - 2iU\omega\phi_x + g\phi_z - \omega^2\phi - \mu(i\omega\phi - U\phi_x) = 0 \quad \text{on } z=0. \quad (47)$$

In case of $\omega=0$ eq. (47) coincides with the free-surface condition for steady-state wave-making problems. Then term of $\mu U\phi_x$ has the role of the radiation of the k_1 waves (Kelvin waves) only in the downstream direction. On the other hand, in case of $U=0$, eq. (47) coincides with the free-surface condition for unsteady motion. Then term of $-i\mu\omega\phi$ has the role of the radiation of the k_2 waves (Ring waves) in the outward direction of the ship.

It has been known that the radiation condition of k_1 waves is ensured by an upstream finite difference operator for ϕ_{xx} in (47) (ϕ_{ss} in $3D$ case)[3], and that the truncation error in the upstream finite difference operator is equivalent to the Rayleigh viscosity term $\mu U\phi_x$ in (47)[11]. That is, by the use of the upstream finite difference, the term $\mu U\phi_x$ can be omitted.

Therefore, in the present calculation, the upstream finite difference operator is applied for the radiation condition of k_1 waves, and for the radiation condition of k_2 waves Rayleigh viscosity μ is employed[21]. Namely, the following condition with the term of $-i\mu\omega\phi$ is dealt with:

$$\phi_{0s}^2\phi_{ss} + 2\phi_{0s}(\phi_{0ss} - i\omega)\phi_s + g\phi_z - \omega^2\phi - i\mu\omega\phi = 0 \quad \text{on } z=0. \quad (48)$$

The value of μ has to be determined by parametric study. The optimum value is searched from the series calculations of μ as explained later.

3.4 Numerical Procedure

Now the potential ϕ, where $\phi=\phi_{Rj}$ for radiation problem with respect to j-th motion and $\phi=\phi_D$ for diffraction problem, is expressed as a sum of two parts:

$$\phi=\phi_F+\phi_H, \tag{49}$$

where

ϕ_F : potential representing free-surface,

ϕ_H : potential representing ship hull.

ϕ_F and ϕ_H are expressed respectively as follows:

$$\phi_F(P)=\iint_{S_F}\sigma_F(Q')G_F(P,Q')dx'dy', \tag{50}$$

$$\phi_H(P)=\iint_{S_m}\sigma_H(Q)G_H(P,Q)dS, \tag{51}$$

where

$$G_F(P,Q')=1/\sqrt{(x-x')^2+(y-y')^2+z^2}, \tag{52}$$

$$G_H(P,Q)=1/\sqrt{(x-x_1)^2+(y-y_1)^2+(z-z_1)^2}+1/\sqrt{(x-x_1)^2+(y-y_1)^2+(z+z_1)^2}, \tag{53}$$

P : field point (x, y, z),

Q : source point of the ship hull (x_1, y_1, z_1),

Q' : source point of the free-surface $(x', y', 0)$,

σ_F : strength of the source distribution on the free-surface,

σ_H : strength of the source distribution on the ship hull.

Here σ_F and σ_H are complex variable.

Substituting (50) and (51) into the free-surface and hull surface conditions, integral equations are obtained as follows:

$$\phi_{0s}^2\left[\iint_{S_F}\sigma_F\frac{\partial^2 G_F}{\partial s^2}dx'dy'+\iint_{S_m}\sigma_H\frac{\partial^2 G_H}{\partial s^2}dS\right]+2\phi_{0s}(\phi_{0ss}-i\omega)\left[\iint_{S_F}\sigma_F\frac{\partial G_F}{\partial s}dx'dy'+\iint_{S_m}\sigma_H\frac{\partial G_H}{\partial s}dS\right]-(\omega^2+i\mu\omega)\left[\iint_{S_F}\sigma_F G_F dx'dy'+\iint_{S_m}\sigma_H G_H dS\right]-2\pi g\sigma_F=-E_0 \quad \text{on } z=0, \tag{54}$$

$$\iint_{S_F}\sigma_F\frac{\partial G_F}{\partial n}dx'dy'+\iint_{S_m}\sigma_H\frac{\partial G_H}{\partial n}ds=-H_o \quad \text{on } S_m, \tag{55}$$

where

(a) radiation problem

$$\left.\begin{array}{l}E_0=0,\\ H_0=i\omega n_j-m_j,\end{array}\right\} \tag{56}$$

(b) diffraction problem

$$\left.\begin{array}{l}E_0=\phi_{0s}^2\phi_{Iss}+2\phi_{0s}(\phi_{0ss}-i\omega)\phi_{Is}+g\phi_{Iz}-\omega^2\phi_I,\\ H_0=\phi_{In}.\end{array}\right\} \tag{57}$$

Discretizing (54) and (55) by using the method presented by Hess and Smith[5] and Dawson[3], the simultaneous equations with respect to the source strength are composed. By solving the equations the σ_F and σ_H are obtained.

4. Results and Discussions

4.1 Results for a Two-Dimensional Submerged Cylinder

Calculations were made of the unsteady hydrodynamic forces for a two-dimensional submerged cylinder. The immersed depth is 2a, where a is radius of the cylinder as shown in Fig. 3. The cylinder surface is divided into 72 segments, and the free-surface region of $-2.5\lambda\leq x\leq 2.5\lambda$, where λ is wave length, is divided into 150 segments.

First, calculations were made in case of zero speed. In this case the present free-surface condition coincides with the classical linearized condition, and the exact solution of the unsteady hydrodynamic forces has been obtained. By comparing the present solution with the exact solution, the effect of artificial parameter μ for the radiation condition was investigated.

It is indicated by Takagi that the optimum value for μ' $(=\mu/\omega)$ is about 0.10[21]. For the verification of the value, calculations with three different values as $\mu'=$ 0.08, 0.10 and 0.12 were carried out. Figs. 4, 5 and 6 show the comparison of calculated results for surge motion. The discrepancy among three added mass coefficients for different μ' is small and these results show good agreement with the exact solution. With increase of μ', damping coefficient becomes larger and is a little smaller than the exact solution near $\omega^2 a/g=0.5$. It seems that the calculated results for the radiation problem has sufficient accuracy. Wave exciting forces show very good agreement with the exact solution in all μ'. Thus in case of zero speed it was verified that the present method has sufficient accuracy for μ' values considered. From these results, 0.10 as μ' was selected for the present calculations.

Next, calculation accuracy of the present method was examined in case of non-zero speed (Froude number F_{na} $(=U/\sqrt{ga})=0.4$). The calculations using two different free-surface conditions were made: one is the present free-surface condition and the other the classical free-surface condition. The present method of solving problem is applied for the classical free-surface condition (we call Linear calculation) to compare with the linear solutions calculated by Kashiwagi et al. [12] for the case that (30) and (31) are omitted. Figs. 7, 8 and 9 show the

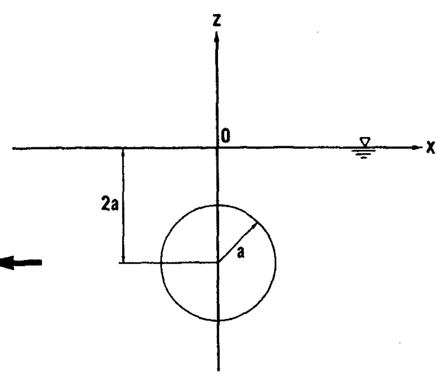

Fig. 3 Coordinate system and notation for the problem of a two-dimensional submerged cylinder

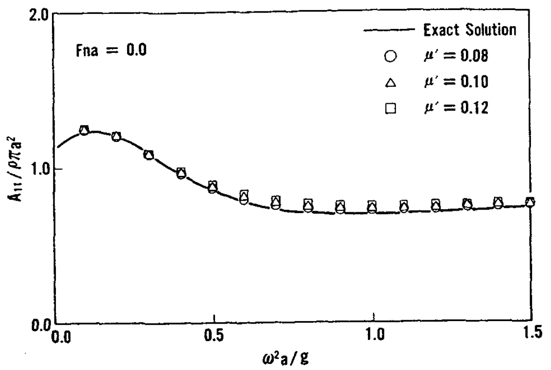

Fig. 4 Comparison of added mass coefficient for various μ' (F_{na}=0.0)

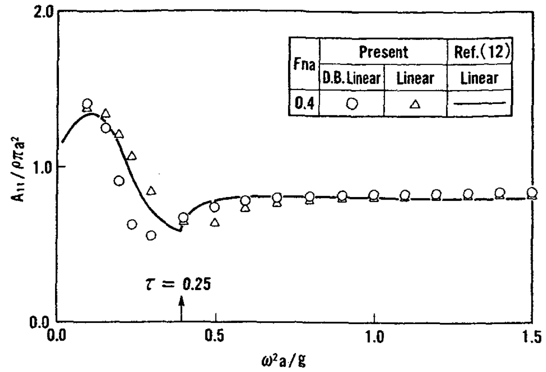

Fig. 7 Comparison of added mass coefficient (F_{na}=0.4)

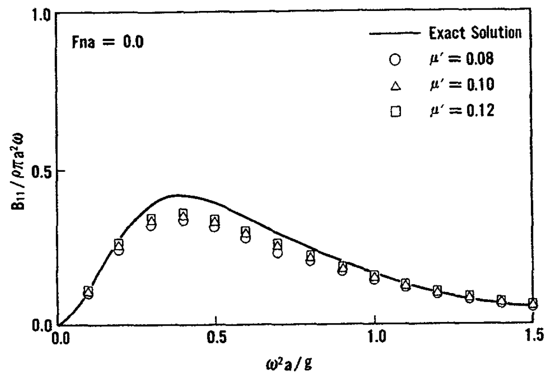

Fig. 5 Comparison of damping coefficient for various μ' (F_{na}=0.0)

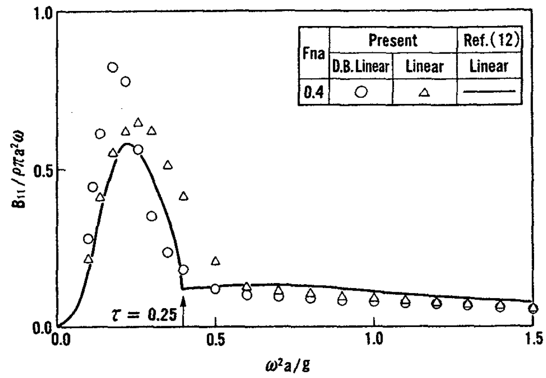

Fig. 8 Comparison of damping coefficient (F_{na}=0.4)

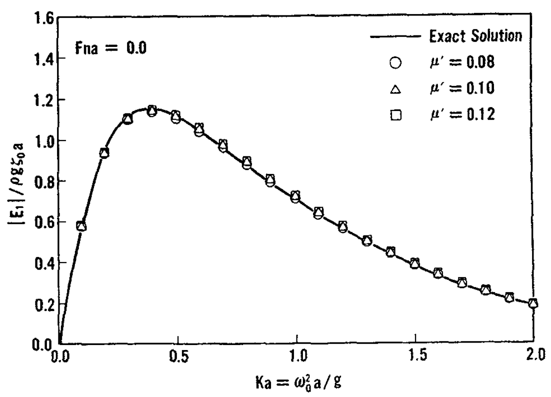

Fig. 6 Comparison of wave exciting force for various μ' (F_{na}=0.0)

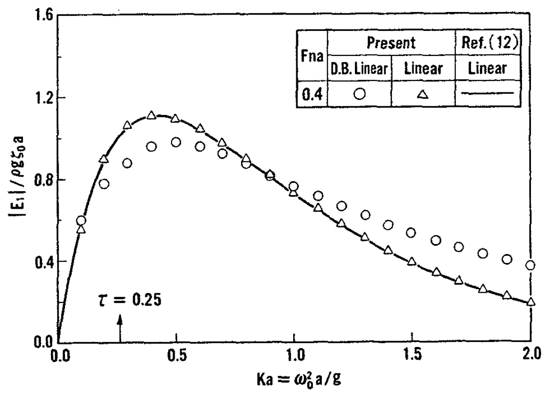

Fig. 9 Comparison of wave exciting force (F_{na}=0.4)

comparison with calculated results for surge motion. Added mass and damping coefficient for Linear calculation show good agreement with Kashiwagi's results except near τ $(=\omega U/g)=0.25$. It is difficult to hold the discontinuity of hydrodynamic coefficients at $\tau=0.25$ in the classical linearized theory, since radiation condition of waves is dealt with approximately. The solution for the present free-surface condition (we call D. B. Linear calculation) are almost same with the Linear calculations except smaller wavenumber ($\omega^2 a/g$). Wave exciting forces in head waves for Linear calculation show very good agreement with Kashiwagi's result. D. B. Linear calculation is larger than Linear calculation at large wavenumber and is smaller at small wavenumber. For the diffraction problem the influence of steady perturbation flow appears more remarkably when it is compared with the results of the radiation problem.

4.2 Results for an Ellipsoidal Ship Hull Form

Calculations were made of diffraction problem for an ellipsoidal ship hull form representing as:

$$\left(\frac{x}{L/2}\right)^2+\left(\frac{y}{B/2}\right)^2+\left(\frac{z}{d}\right)^2=1,$$

$$L/B=4,\quad B/d=2.5, \tag{58}$$

where L is ship length, B the breadth and d the ship draft. This form was used by Kobayashi[14] for investigation of forward speed effect on hydrodynamic forces in waves.

Table 1 Particulars of free-surface panel arrangement

λ/L	Panel Region	Minimum Panel Size	Number of Panels
0.4, 0.8	$\lvert x/L\rvert \leq 1.72$, $\lvert y/L\rvert \leq 0.83$	0.018L×0.012L	732
1.2, 1.6	$\lvert x/L\rvert \leq 3.17$, $\lvert y/L\rvert \leq 0.83$	0.018L×0.012L	948
2.0	$\lvert x/L\rvert \leq 4.15$, $\lvert y/L\rvert \leq 0.83$	0.018L×0.012L	1044
3.0	$\lvert x/L\rvert \leq 6.23$, $\lvert y/L\rvert \leq 0.83$	0.018L×0.012L	1188

Fig. 10 shows an example of the panel arrangement for free-surface and hull surface. The free-surface panel region has almost quadruple length of incident-wave length. Table 1 shows the particulars of the free-surface panel arrangement. The number of the hull surface panels is 184.

Figs. 11, 12 and 13 show the comparison of wave exciting forces in head waves for surge, heave and pitch motions respectively. For comparison with the present calculation, experiments and calculated results by using the singularity distribution method presented by Kobayashi[14] are plotted in the figures. The exciting forces for surge and heave motions show good agree-

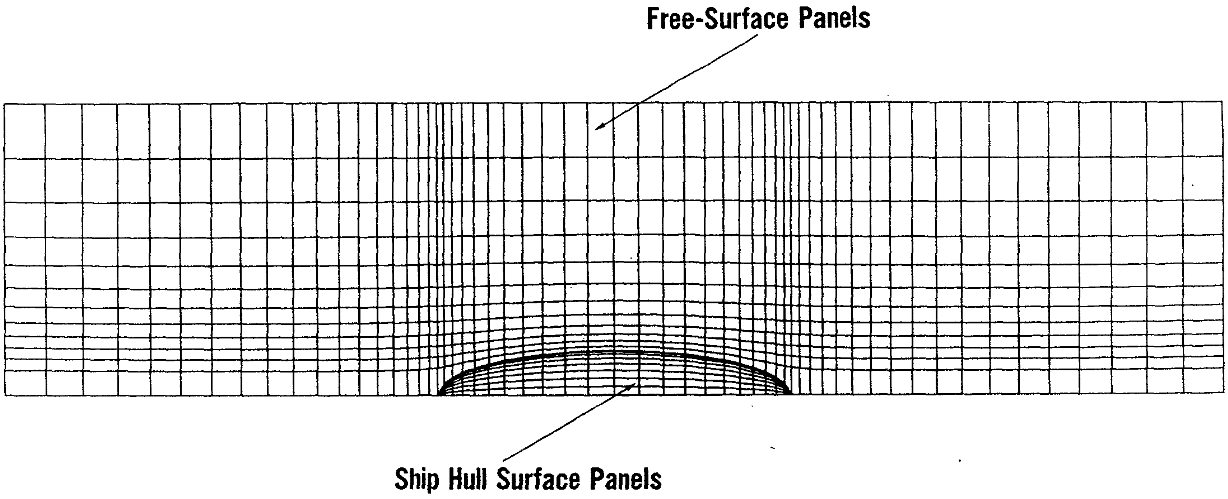

Fig. 10 Panel arrangement of free-surface and ship hull surface

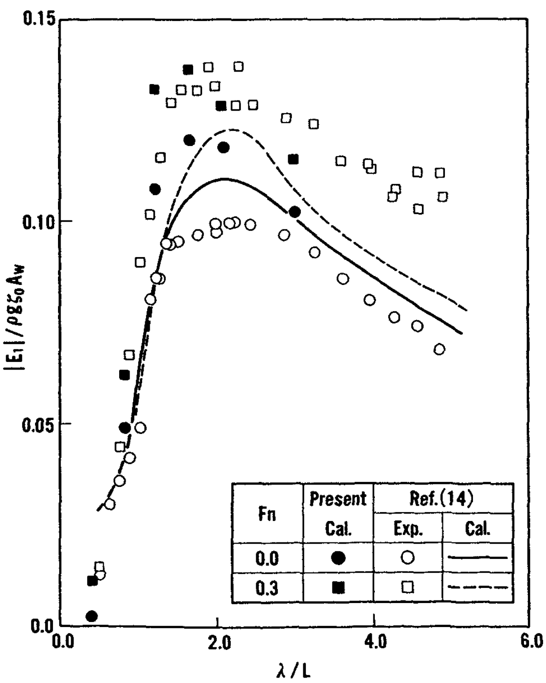

Fig. 11 Comparison of wave exciting force for surge motion

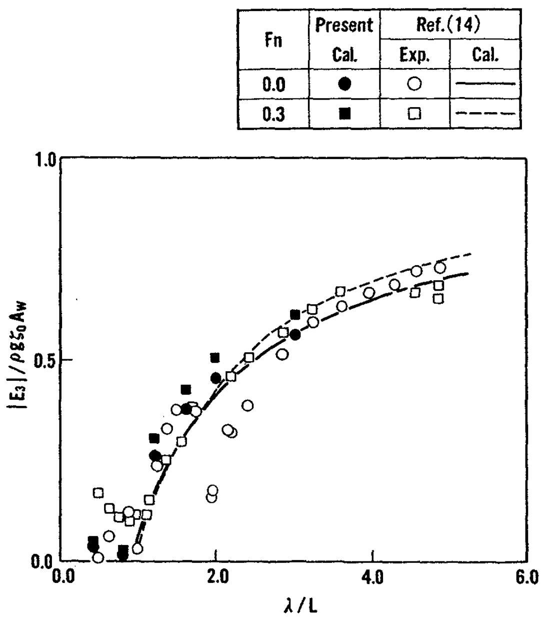

Fig. 12 Comparison of wave exciting force for heave motion

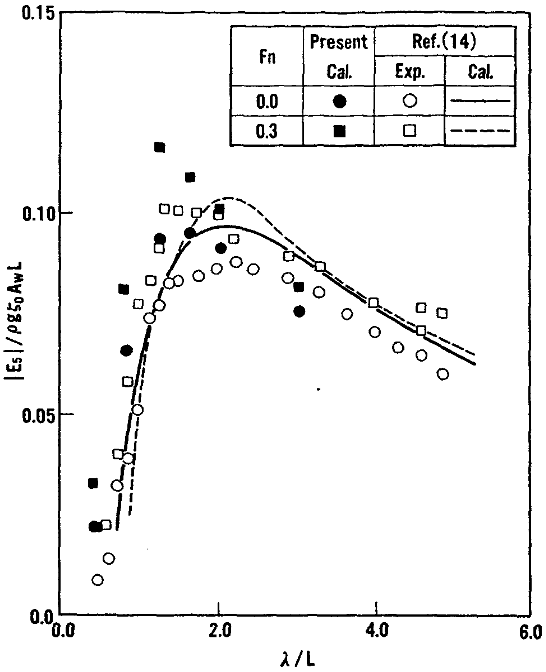

Fig. 13 Comparison of wave exciting force for pitch motion

ment with experiments. Especially an improvement of the accuracy can be observed in the present solution of the exciting force for surge motion at $F_n=0.3$ when it is compared with Kobayashi's calculation. It seems that the improvement is due to the consideration of steady perturbation flow in the present method. However the exciting force for pitch motion is a little larger than the experiments. Further, change of the results due to different panel arrangement was observed in case of smaller panel region for free-surface. There is a room of improvement for the present.

Fig. 14 shows the configuration of wave components around the ellipsoidal ship advancing in waves for diffraction problem in $Fn=0.3$, $\lambda/L=0.4$ and $\zeta_0/L=0.01$. In the figure the wave height is magnified by 1.5 or 10 times, and steady waves and cosine components of the unsteady waves are shown. Steady waves are obtained such as we can see in the towing test as usual. Incident-waves component is included the effect of steady perturbation flow (double body flow), but the effect is small. Diffraction waves component is smaller than the other components (magnification for the diffraction waves is

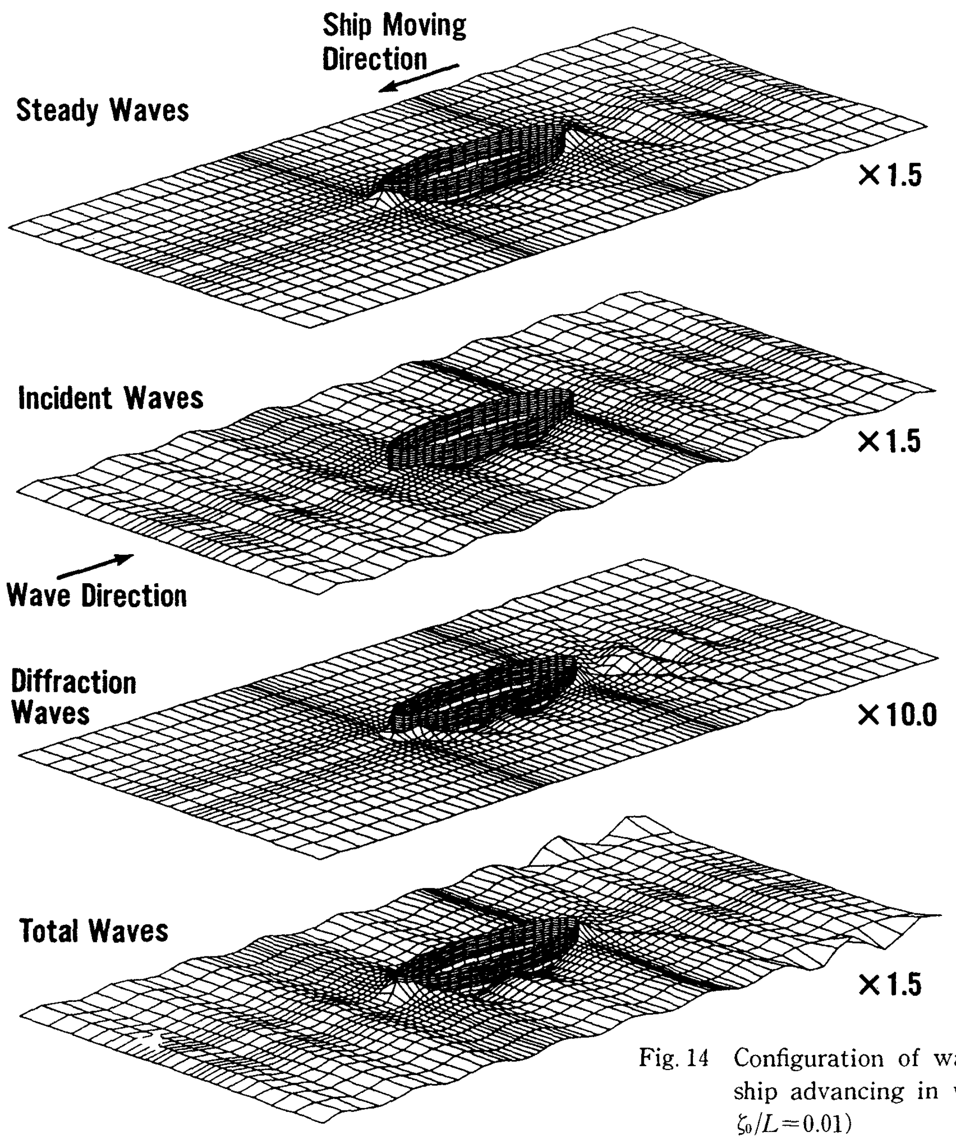

Fig. 14 Configuration of wave components around a ship advancing in waves ($F_n=0.3$, $\lambda/L=0.4$, $\zeta_0/L=0.01$)

10). The wave configuration around a ship advancing in regular waves (we call Total waves) is represented as the sum of steady waves, incident-waves and diffraction waves components. Total waves become larger at the bow and stern parts of the ship due to the superimpose of steady and unsteady waves components.

Fig. 15 shows the change of profile of the Total waves in a period of encounter. It can be observed that at 1/8T, where T is a period of encounter ($=2\pi/\omega$), the wave elevations at bow and stern become larger due to the superimpose of steady and unsteady waves components, and at $1/2T$ they become smaller due to the cancel of steady and unsteady waves. Thus by using the present method, not only steady waves but also unsteady waves around the ship can be calculated easily. This is one of the features of the present method, and the valuable information such as pressure distribution on ship hull and the velocity components in addition to the wave configuration can also be obtained.

5. Concluding Remarks

In this paper an investigation was made of panel method for unsteady ship hydrodynamic forces. First, formulations were made for both steady and unsteady free-surface conditions under the assumption of small perturbation over the double body flow. A newly derived free-surface condition for unsteady motion make a theoretical pair with Dawson's steady linearized free-surface condition[3]. By use of a Rankine panel method, calculations were made of unsteady ship hydrodynamic forces such as added mass, damping and wave exciting forces for a two-dimensional submerged cylinder and an ellipsoidal ship hull form. By comparing with other calculations and experiments it is confirmed that the present results show good agreement with them. Further, an improvement of the accuracy is observed in the wave exciting force for surge motion with forward speed. It is noted that the present method can give valuable information such as wave configurations around a ship in unsteady case. Thus it can be said that the present method is useful for better understanding of unsteady free-surface flow problems. Extension of the present method to the calculations of ship motion and second order steady forces such as added resistance and wave drifting forces is a future work together with an improvement of the accuracy of the present method.

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References

1) Baba, E.: Wave Resistance of Ships in Low Speed, Mitsubishi Technical Bulletin, No. 109 (1976).
2) Chang, M. S.: Computations of Three Dimen-

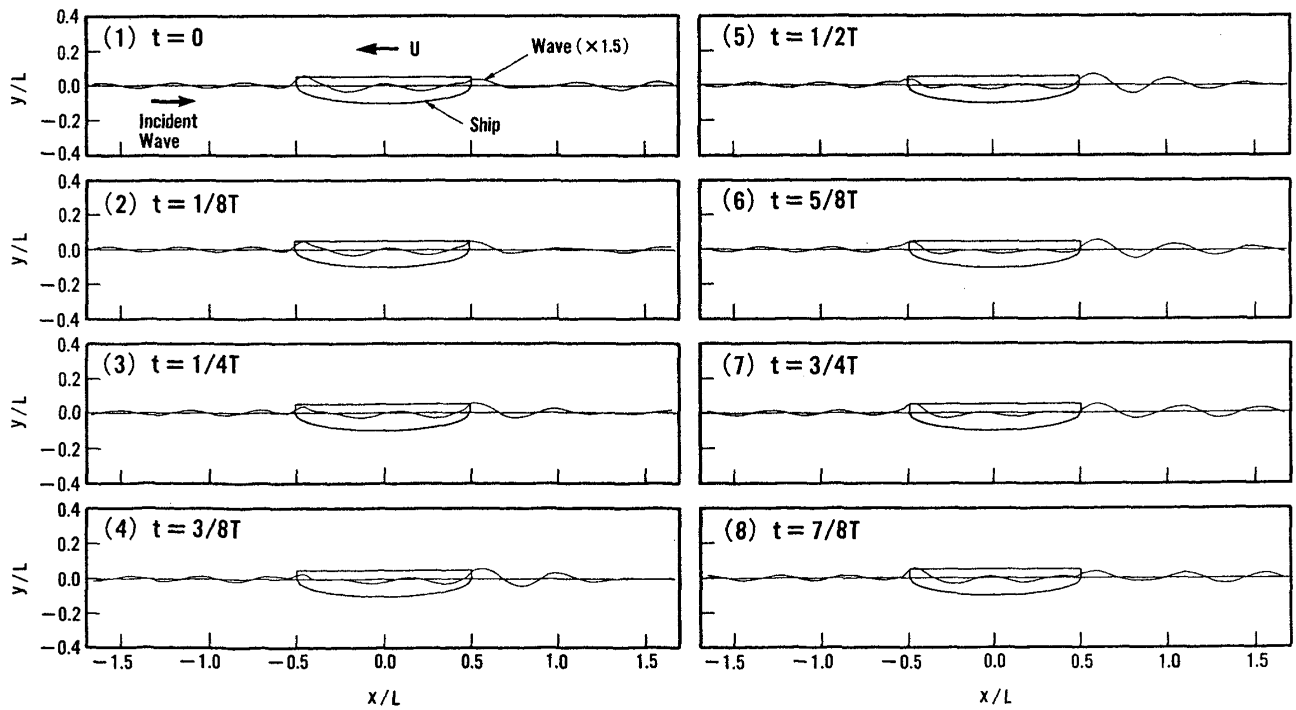

Fig. 15 Change of wave profile in a period of encounter ($Fn=0.3$, $\lambda/L=0.4$, $\zeta_0/L=0.01$)

sional Ship Motions with Forward Speed, Proc. 2nd Int. Conf. on Numerical Ship Hydrodynamics, Univ. Califormia, Berkeley (1977), pp. 124-135.
3) Dawson, C. W.: A Practical Computer Method for Solving Ship-Wave Problems, Proc. 2nd Int. Conf. on Numerical Ship Hydrodynamics, Univ. California, Berkeley (1977), pp. 30-38.
4) Gadd, G. E.: A Method of Computing the Flow and Surface Wave Pattern Around Full Forms, The Royal Institution of Naval Architects, Vol. 18 (1976), pp. 207-219.
5) Hess, J. L. and Smith, A. M. O.: Calculation of Non-Lifting Potential Flow About Arbitrary Three-Dimensional Bodies, Report No. E. S. 40622, Dauglas Aircraft Co. Ltd. (1962).
6) Huijismans, R. H. M. and Hermans, A. J.: The Effect of the Steady Perturbation Potential on the Motions of a Ship Sailing in Random Seas, 5th Int. Conf. on Numerical Ship Hydrodynamics, Hiroshima (1989).
7) Inglis, R. B. and Price, W. G.: Comparison of Calculated Responses for Arbitrary Shaped Bodies using Two and Three-Dimensional Theories, International Shipbuilding Progress, Vol. 27, No. 308 (1980), pp. 86-95.
8) Inoue, Y. and Makino, Y.: The Influence of Forward Speed upon Three Dimensional Hydrodynamic Forces, Journal of the Society of Naval Architects of Japan, Vol. 166 (1989). pp. 207-216.
9) Iwashita, H. and Ohkusu, M.: Hydrodynamic Forces on Ship Moving with Forward Speed in Waves, Journal of the Society of Naval Architects of Japan, Vol. 166 (1989), pp. 187-205.
10) Jensen, G., Söding, H. and Mi, Z.-X.: Rankine Source Methods for Numerical Solutions of the Steady Wave Resistance Problem, Proc. 16th Symp. on Naval Hydrodynamics (1986), pp. 575-581.
11) Jensen, P. S.: On the Numerical Radiation Condition in the Steady-State Ship Wave Problem, Journal of Ship Research, Vol. 31, No. 1 (1987), pp. 14-22.
12) Kashiwagi, M., Varyani, K. and Ohkusu, M.: Forward-Speed Effects on Hydrodynamic Forces Acting on a Submerged Cylinder in waves, Reports of Research Institute for Applied Mechanics, Kyushu Univ., Vol. 34, No. 102 (1987), pp. 1-26.
13) Kashiwagi, M. and Ohkusu, M.: The Effects of Forward Speed in the Radiation Problem of a Surface-Piercing Body, Journal of the Society of Naval Architects of Japan, Vol. 164 (1988), pp. 92-104.
14) Kobayashi, M.: On the Hydrodynamic Forces and Moments Acting on a Three Dimensional Body with a Constant Forward Speed, Journal of the Society of Navel Architects of Japan, Vol. 150 (1981), pp. 175-189.
15) Lee, K.-Y.: Widerstandserhöhung in Kurzen Wellen, Schiffstechnik, Bd. 30, Heft 3 (1983), pp. 133-170.
16) Naito, S., Yamamoto, O. and Takahashi, T.: Ship Hull Forms and Seakeeping Qualities, 5th Marine Dynamics Symposium, The Society of Naval Architects of Japan (1988), pp. 45-65.
17) Nakos, D. E.: Free Surface Panel Methods for Unsteady Forward Speed Flows, Proc. 4th Int. Workshop on Water Waves and Floating Bodies, Oslo (1989), pp. 175-178.
18) Ogiwara, S.: Numerical Solution for Steady Ship Waves by Means of Rankine Sources, Open Forum on Numerical Ship Hydrodynamics, Tokyo (1987), pp. 66-80.
19) Sakamoto, T. and Baba, E.: Minimization of Resistance of Slowly Moving Full Hull Forms in Short Wave, Proc. 16th Symp. on Naval Hydrodynamics (1988), pp. 173-193.
20) Sclavounos, P. D. and Nakos, D. E.: Stability Analysis of Panel Methods for Free Surface Flows with Forward Speed, Proc. 17th Symp. on Naval Hydrodynamics (1988), pp. 173-193.
21) Takagi, K.: An Application of Rankine Source Method for Unsteady Free Surface Flows, Journal of the Kansai Society of Naval Architects, Japan, Vol. 213 (1990), pp. 21-29.
22) Takagi, K.: Calculation of Added Resistance of a Ship in Waves by Rankine Source Method, Journal of the Kansai Society of Naval Architects, Japan (1990) (to be published).
23) Timman, R. and Newman, J. N.: The Coupled Damping Coefficients of a Symmetric Ship, Journal of Ship Research, Vol. 5, No. 4 (1962), pp. 1-7.
24) Zhao, R., Faltinsen, O. M., Krokstad, J. R. and Aanesland, V.: Wave-Current Interaction Effects on Large-Volume Structures, Proc. the Int. Conf. on Behavior of Offshore Structures (1988), pp. 623-638.

Appendix Comparison of Free-Surface Conditions

The free-surface condition presented by Sakamoto and Baba[19] is expressed as:

$$\phi_{Utt}+2\nabla\phi_0\cdot\nabla\phi_{Ut}+\nabla\phi_0\cdot\nabla(\nabla\phi_0\cdot\nabla\phi_U)+g\phi_{Uz}=0 \quad \text{on } z=0. \qquad \text{(A-1)}$$

An index s which denotes a derivative in the streamline direction of the double body flow is introduced. Then eq. (A-1) can be expressed as:

$$\phi_{Utt}+2\phi_{0s}\phi_{Ust}+\phi_{0s}^2\phi_{Uss}+\phi_{0s}\phi_{0ss}\phi_{Us}+g\phi_{Uz}=0 \quad \text{on } z=0. \qquad \text{(A-2)}$$

According to the assumptions on order of magnitude by Sakamoto and Baba, the fourth term of left-side in eq. (A-2) can be neglected because of higher order term. Then eq. (A-2) is expressed as:

$$\phi_{Utt}+2\phi_{0s}\phi_{Ust}+\phi_{0s}^2\phi_{Uss}+g\phi_{Uz}=0 \quad \text{on } z=0. \qquad \text{(A-3)}$$

By comparing eq. (A-3) with the present condition (11), it is found that the present condition has an added term of $2\phi_{0s}\phi_{0ss}\phi_{Us}$ to Sakamoto and Baba's one.

Further, if the lateral disturbance on $z=0$ is small, we can approximate as follows:

$$\frac{\partial}{\partial s}\simeq\frac{\partial}{\partial x},$$

$$\phi_{0s}\simeq U.$$

Then, the present free-surface condition coincides with the classical unsteady linearized free-surface condition as:

$$\phi_{Utt}+2U\phi_{Uxt}+U^2\phi_{Uxx}+g\phi_{Uz}=0 \quad \text{on } z=0. \qquad \text{(A-4)}$$