A New Plate Buckling Design Formula (3rd Report)
——On the Real Edge Condition——

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Summary

The previous plate buckling design formula proposed by the authors is advanced by taking into account the real rotational restraint effect of supporting members at edges. For this purpose, a series of the theoretical plate buckling analysis is performed with variation in the aspect ratio and the torsional rigidity of the supporting members at edges. Based on the computed results, an approximate plate buckling design formula is formulated. Verification examples of 288 cases varying the aspect ratio, the torsional rigidity and the loading ratio between longitudinal and transverse compression are demonstrated by comparing the approximate solutions with the exact and FE results.

1. Introduction

Ship structures are essentially an assembly of plates, and evaluation of the plate buckling strength is one of special tasks which we must perform in the structural design of ships. For practical design, a simplified plate buckling design formula is necessary. Classification societies have proposed their own design guides, but we are still confronted with a great number of questions due to loading condition, boundary condition, etc.

Almost all current plate buckling design formulas have been derived under the assumption that the edges are simply supported or clamped, which corresponds to zero or infinite rotational restraints at plate edges, respectively. In real ship platings, however, the edges are supported by supporting members of which the rotational restraints are neither zero nor infinite, and thus such ideal edge conditions never occur.

Lundquist and Stowell[1] investigated the effect of the edge condition on the buckling strength of rectangular plate under uniaxial compression, in which the support along the unloaded edges is intermediate between simply supported and clamped. Also Bleich[2], Timoshenko and Gere[3] and Hughes[4] introduced the cases in which one of both edges is elastically restrained and the other simply supported or clamped. Design tables or charts are provided. However, works for evaluating the buckling strength of plates in which both edges are elastically restrained and/or combined loading is applied are not found. Also it would be highly desirable for the designer to have an explicit formula, instead of design charts or design tables.

In the previous studies[5,6], a new and accurate buckling design formula for ship platings subjected to in-plane combined loading and lateral pressure is proposed. The effect of welding residual stress is also involved. For a plate in which the buckling occurs in the elasto-plastic range, a new expression for the plasticity correction is derived based on the numerical FE solutions. In this formula, however, the plate edges are still assumed to be simply supported or clamped.

In this paper, the previous design formula is advanced by taking into account the effect of the real edge condition. For this purpose, an elastic buckling condition for a rectangular plate elastically restrained at edges is derived as a characteristic value problem. A series of the buckling analysis is performed with variation in the aspect ratio and the torsional rigidity of the supporting members at edges. Based on the computed results, an approximate plate buckling design formula is formulated in terms of aspect ratio and torsional rigidity. The accuracy of the formula is verified by comparing with the exact and FE solutions.

2. Buckling Problem of a Plate Elastically Restrained at Edges

2.1 Basic Assumptions

In ship platings, supporting members are attached both at shorter and longer edges. As mentioned, the real condition of support at plate edges is neither simply supported nor clamped. It is very difficult to obtain the theoretical buckling strength of a plate under the above-mentioned support condition. In this regard, some

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assumptions are made:

Assumption 1: Geometric and material properties of supporting members attached in the same direction of the plate are the same, and thus the bending/torsional rigidity at the longer or shorter edges is also the same at the corresponding direction.

Assumption 2: The bending rigidity of supporting members attached in ship platings is large enough so that the relative lateral deformation at plate edges is protected.

Assumption 3: Plate edges and supporting members keep straight until the inception of plate buckling.

2.2 Plate Buckling Problem As a Characteristic Value Problem

In this study, the longer edge of the plate is always taken as x direction such that the aspect ratio of the plate is always equal to or greater than 1.0. The fundamental differential equation for the deflection w of a flat plate subjected to in-plane combined loading is derived under the assumption that the deflectoin w is small compared with the thickness t of the plate[3].

$$D(w_{,xxxx}+2w_{,xxyy}+w_{,yyyy}) + t(\sigma_x w_{,xx}+\sigma_y w_{,yy}+2\tau_{xy}w_{,xy})=0 \quad (1)$$

where

σ_x, σ_y = axial stresses in x, y direction

τ_{xy} = shear stress

$D=Et^3/12(1-\nu^2)$

E = Young's modulus

ν = Poisson's ratio

Solution w of Eq. (1) indicates the deflected form of the plate under the corresponding applied load, which represents equilibrium but unstable position. The buckling strength is defined by the load at a bifurcation point where beside the plane equilibrium form $w=0$, a deflected but unstable form of equilibrium occurs.

To solve Eq. (1), the edge condition which is obtained from the conditions of support of the plate should be prescribed. Under the assumptions indicated in the previous section, the following edge conditions are considered:

$w=0$ at all edges (2.a)

$M_x=m_x$ at shorter edges (2.b)

$M_y=m_y$ at longer edges (2.c)

where

M_x, M_y = bending moment of plate part at shorter, longer edges

m_x, m_y = torsional moment of supporting member at shorter, longer edges

The bending moments of plate part are expressed by[2)]

$$M_x=-D(w_{,xx}+\nu w_{,yy}) \quad (3.a)$$

$$M_y=-D(w_{,yy}+\nu w_{,xx}) \quad (3.b)$$

where since the edges are assumed to remain straight, $w_{,yy}$ at shorter edge and $w_{,xx}$ at longer edge must be zero.

Also the torsional moments of supporting member are expressed by[2)]

$$m_x=E\Gamma_S w_{,yyyyx}-GJ_S w_{,yyx} \quad (4.a)$$

$$m_y=E\Gamma_L w_{,xxxxy}-GJ_L w_{,xxy} \quad (4.b)$$

where

J_S, J_L = torsion constants of supporting member at shorter, longer edges

$=1/3\Sigma(b_i t_i^3)$ for one plate

$=1/6\Sigma(b_i t_i^3)$ for a continuously stiffened plate

b_i, t_i = breadth, thickness of web or flange of the corresponding supporting member

Γ_S, Γ_L = flexural-torsional constants of supporting members at shorter, longer edges

When a continuously stiffened plate is considered, the supporting member will contribute to resist the rotational deformation of two adjacent plates along the plate edge. In this regard, J_S and J_L should take a half amount of the whole torsion constant. Also the values Γ_S and Γ_L are usually very small, and for practical applications $\Gamma_S=\Gamma_L=0$ may be used[2)].

It is very difficult to solve Eq. (1) directly for a plate elastically restrained at both shorter and longer edges. In the following, therefore, a rectangular plate, one edge simply supported and the other elastically restrained is at first considered.

The general solution of the differential equation (1) may be expressed by

$$w=C_1w_1(\alpha_1, x,y)+C_2w_2(\alpha_1, x, y) + C_3w_3(\alpha_2, x, y)+C_4w_4(\alpha_2, x, y) \quad (5)$$

where

$C_1 \sim C_4$ = constants to be determined

α_1, α_2 = transcendental functions of the buckling strength

Substitution of the solution (5) into the boundary conditions (2) yields a set of linear homogeneous equations, namely

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = 0 \quad (6)$$

where λ = transcendental function of the parameter α_i

For arbitrary values of λ, Eq. (6) has solutions different from zero only if the determinant Δ vanishes, that is,

$$\Delta=\begin{vmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} & \lambda_{14} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} & \lambda_{24} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} \\ \lambda_{41} & \lambda_{42} & \lambda_{43} & \lambda_{44} \end{vmatrix}=0 \quad (7)$$

Expanding the condition (7) furnishes an equation for the parameter α_i, and solving this equation the buckling strength will be obtained.

3. Derivation of a New Plate Buckling Design Formula in One Load Component

3.1 Bckling Strength of a Rectangular Plate in Longitudinal Compression σ_x

(1) Elastically Restrained at the Longer Edges and Simply Supported at the Shorter Edges

The buckling of a plate subjected to only longitudinal compression along the shorter edges, that is, σ_y and τ_{xy} in Eq. (1) vanish, is considered. The rotation at the loaded, shorter edges occurs freely but the unloaded, longer edges are elastically restrained by supporting members. As a result, the bending moment M_x and the torsional moment m_x at the shorter edges must be zero.

At first, when the coordinate is taken as in Fig. 1, the deflection w of the plate which satisfies the differential equation of Eq. (1) and the condition of simple support at the shorter edges (i. e. $w=0$, $M_x=m_x=0$ at $x=0$ and a) is assumed:

$$w=Y(y)\sin(m\pi x/a) \tag{8}$$

where $Y(y)$ indicates a function of y and m represents the number of half-waves in x direction.

Substituting Eq. (8) into Eq. (1) and considering $\sigma_y=\tau_{xy}=0$, we obtain the ordinary differential equation of the fourth order, replacing σ_x by σ_{xcr1}:

$$Y_{,yyyy}-2(m\pi/a)^2Y_{,yy}+(m\pi/a)^4(1-\mu_{x1}^2)Y=0 \tag{9}$$

where

$\mu_{x1}=\beta/m\cdot\sqrt{k_{x1}}$

$\beta=a/b$

$k_{x1}=\sigma_{xcr1}/\sigma_E$

$\sigma_E=\pi^2D/b^2t$

σ_{xcr1}=elastic buckling stress under the present loading and edge conditions

The general solution of the above differential equation is

$$Y(y)=C_1\cosh\alpha_1y+C_2\sinh\alpha_1y+C_3\cos\alpha_2y+C_4\sin\alpha_2y \tag{10}$$

where

$\alpha_{1,2}=m\pi/a\cdot(\mu_{x1}\pm1)^{1/2}$

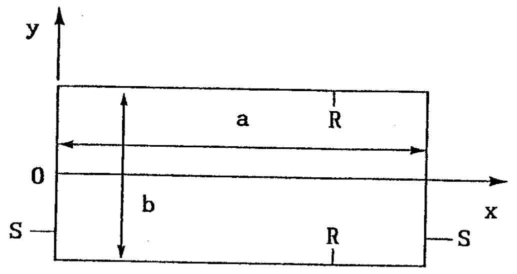
$C_1\sim C_4$=constants

Considering the deflection w at the buckling load is a symmetric function of y, only the first and third term in the general solution (10) remain. Thus

$$w=(C_1\cosh\alpha_1y+C_3\cos\alpha_2y)\sin(m\pi x/a) \tag{11}$$

Unknown constants C_1 and C_3 are determined from the conditions of support and moment equilibrium at the longer edges, as indicated in Eq. (2.c), that is,

$$w=0 \text{ and } M_y=m_y \text{ at } y=\pm b/2 \tag{12}$$



S : simply supported
R : rotationally restrained

Fig. 1 Coordinate of a plate, elastically restrained at the longer edges and simply suppored at the shorter edges

Substitution of Eq. (11) into Eq. (12), considering $w_{,xx}=0$ and $\Gamma_L=0$ at $y=\pm b/2$, yields

$$\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{Bmatrix}C_1\\ C_3\end{Bmatrix}=0 \tag{13}$$

where

$A_{11}=\cosh(\alpha_1b/2)$

$A_{12}=\cos(\alpha_2b/2)$

$A_{21}=\alpha_1^2\cosh(\alpha_1b/2)+\zeta_Lb(m\pi/a)^2\alpha_1\sinh(\alpha_1b/2)$

$A_{22}=-\alpha_2^2\cos(\alpha_2b/2)-\zeta_Lb(m\pi/a)^2\alpha_2\sin(\alpha_2b/2)$

$\zeta_L=GJ_L/bD$

The condition of determinant $\Delta=0$ yields finally the exact buckling condition.

$$\Delta=A_{11}A_{22}-A_{12}A_{21}=0 \tag{14}$$

By solving the above equation, the buckling coefficient k_{x1} will be found. Fig. 2 indicates the computed results with variation in the aspect ratio and the torsional rigidity of the supporting member at the longer edges, in which the shorter edges are simply supported. It is observed that the buckling coefficient is affected by the torsional rigidity ζ_L, but the effect of the aspect ratio may be neglected for practical use.

It will be very convenient for applications if the buckling coefficient can be estimated by an explicit formula. It is known that the normalized torsional rigidity of most supporting members in the actual ship platings is less than about 5.0[8].

In this regard, we aim to express the buckling coefficient by the algebraic formula in the practical range of the torsional rigidity ζ_L. Based on the theoretical results obtained by the series analysis using Eq. (14), the buckling coefficient k_{x1} can be formulated as a function of the torsional rigidity ζ_L, namely

$$\begin{aligned}k_{x1}&=0.396\zeta_L^3-1.974\zeta_L^2+3.565\zeta_L+4.0 && \text{for } 0.0\le\zeta_L\le2.0\\ &=6.951-0.881/(\zeta_L-0.4) && \text{for } 2.0<\zeta_L\le20.0\\ &=7.025 && \text{for } \zeta_L=\infty\end{aligned} \tag{15}$$

The accuracy of this approximate formulation is checked by the dotted lines in Fig. 2 and it is observed

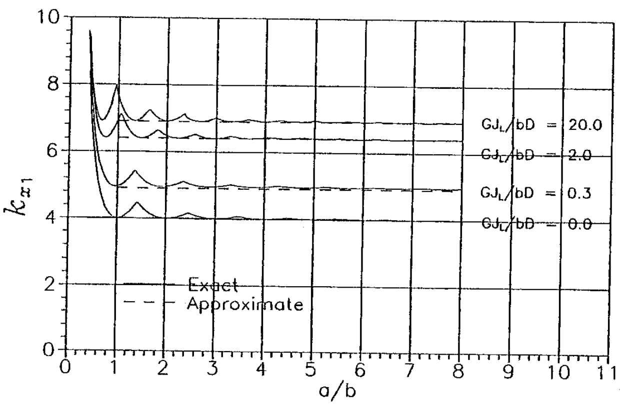


Fig. 2 Buckling coefficient k_{x1} of a plate in longitudinal compression, elastically restrained at the longer edges and simply supported at the shorter edges

that the simplified formula provides an accurate solution. Accordingly, the elastic buckling stress in this case can be predicted by

$$\sigma_{xcr1} = k_{x1} \cdot \sigma_E \qquad (16)$$

(2) Elastically Restrained at the Shorter Edges and Simply Supported at the Longer Edges

In this case, the rotation at the longer edges occurs freely. As a result, the bending moment M_y and the torsional moment m_y at the longer edges must be zero. The coordinate of the plate is taken as in Fig 3. The deflection w of the plate which satisfies the differential equation of Eq. (1) and the condition of simple support at the longer edges (i. e. $w=0$, $M_y=m_y=0$ at $y=0$ and b) is assumed:

$$w = X(x) \sin(n\pi y/b) \qquad (17)$$

where $X(x)$ indicates a function of x and n represents the number of half-waves in y direction.

Substituting Eq. (17) into Eq. (1) and considering $\sigma_y = \tau_{xy} = 0$, we obtain the ordinary differential equation of the fourth order, replacing σ_x by σ_{xcr2}:

$$X_{,xxxx} - 2(n\pi/b)^2(1-\mu_{x2}^2)X_{,xx} + (n\pi/b)^4 X = 0 \qquad (18)$$

where

$\mu_{x2} = \sqrt{k_{x2}}/\sqrt{2}\, n$

$k_{x2} = \sigma_{xcr2}/\sigma_E$

σ_{xcr2}=elastic buckling stress under the present loading and edge conditions

The general solution of the above differential equation is

$$X(x) = C_1 \cos \alpha_1 x + C_2 \sin \alpha_1 x + C_3 \cos \alpha_2 x + C_4 \sin \alpha_2 x \qquad (19)$$

where

$\alpha_{1,2} = n\pi/\sqrt{2}\, b \cdot [\mu_{x2} \mp (\mu_{x2}^2 - 2)^{1/2}]$

$C_1 \sim C_4$=constants

The deflected shape in x direction after buckling will be dependent on the aspect ratio, and two patterns are expected: one symmetric and the other antisymmetric. In this regard, the general solution will become as follows:

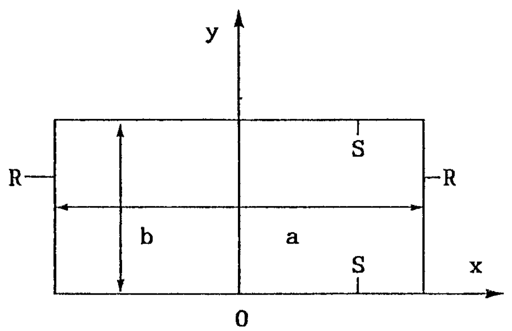


Fig. 3 Coordinate of a plate, elastically restrained at the shorter edges and simply supported at the longer edges

(i) Symmetric Pattern

In this case, the first and third term in the general solution remain, namely

$$X(x) = C_1 \cos \alpha_1 x + C_3 \cos \alpha_2 x \qquad (20)$$

The constants C_1 and C_3 will be determined from the bounday condition:

$$w=0 \text{ and } M_x = m_x \text{ at } x = \pm a/2 \qquad (21)$$

Substitution of Eq. (20) into Eq. (21), considering $w_{,xy}=0$ and $\Gamma_S=0$ at $x=\pm a/2$, yields

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} C_1 \\ C_3 \end{Bmatrix} = 0 \qquad (22)$$

where

$A_{11} = \cos(\alpha_1 a/2)$

$A_{12} = \cos(\alpha_2 a/2)$

$A_{21} = \alpha_1^2 \cos(\alpha_1 a/2) + \zeta_S a(n\pi/b)^2 \alpha_1 \sin(\alpha_1 a/2)$

$A_{22} = \alpha_2^2 \cos(\alpha_2 a/2) + \zeta_S a(n\pi/b)^2 \alpha_2 \sin(\alpha_2 a/2)$

$\zeta_S = GJ_S/aD$

The condition of determinant $\Delta=0$ yields finally the exact buckling condition.

$$\Delta = A_{11}A_{22} - A_{12}A_{21} = 0 \qquad (23)$$

(ii) Antisymmetric Pattern

In this case, the second and fourth term in the general solution remain, namely

$$X(x) = C_2 \sin \alpha_1 x + C_4 \sin \alpha_2 x \qquad (24)$$

The constants C_2 and C_4 will be determined from the boundary conditions (21). Substitution of Eq. (24) into Eq. (21), considering $w_{,yy}=0$ and $\Gamma_S=0$ at $x=\pm a/2$, yields

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix} = 0 \qquad (25)$$

where

$A_{11} = \sin(\alpha_1 a/2)$

$A_{12} = \sin(\alpha_2 a/2)$

$A_{21} = \alpha_1^2 \sin(\alpha_1 a/2) - \zeta_S a(n\pi/b)^2 \alpha_1 \cos(\alpha_1 a/2)$

$A_{22} = \alpha_2^2 \sin(\alpha_2 a/2) - \zeta_S a(n\pi/b)^2 \alpha_2 \cos(\alpha_2 a/2)$

$\zeta_s = GJ_s/aD$

The condition of determinant $\Delta=0$ yields finally the exact buckling condition.

$$\Delta = A_{11}A_{22} - A_{12}A_{21} = 0 \qquad (26)$$

By solving Eq. (23) or (26), the buckling coefficient k_{x2} will be found. Fig. 4 indicates the computed results with variation in the aspect ratio and the torsional rigidity ζ_S. Based on the theoretical results, it is also possible to formulate the buckling coefficient by an algebraic expression in the practical range of the torsional rigidity ζ_S:

$$k_{x2} = p_2\zeta_S^4 + q_2\zeta_S^3 + r_2\zeta_S^2 + s_2\zeta_S + t_2 \qquad (27)$$

where

$p_2 = -1.010\beta^4 + 12.827\beta^3 - 52.553\beta^2 + 67.072\beta - 27.585$ for $0.0 \le \zeta_S < 0.4$

$= 0.047\beta^4 - 0.586\beta^3 + 2.576\beta^2 - 4.410\beta + 1.748$ for $0.4 \le \zeta_S < 0.8$

$= -0.017\beta^2 + 0.099\beta - 0.150$ for $0.8 \le \zeta_S < 2.0$

$= 0.0$ for $2.0 \le \zeta_S \le 20.0$

$= 0.0$ for $\zeta_S = \infty$

$q_2 = 0.881\beta^4 - 10.851\beta^3 + 41.688\beta^2 - 43.150\beta + 14.615$ for $0.0 \le \zeta_S < 0.4$

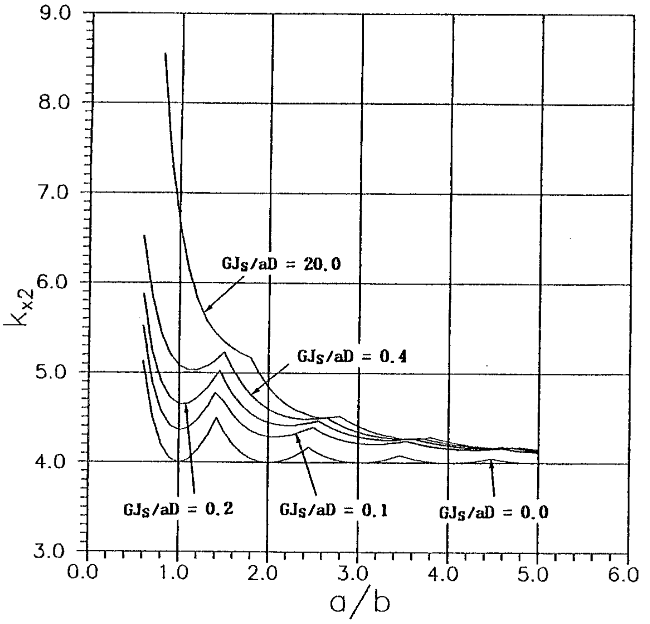


Fig. 4 Buckling coefficient k_{x2} of a plate in longitudinal compression, elastically restrained at the shorter edges and simply supported at the longer edges

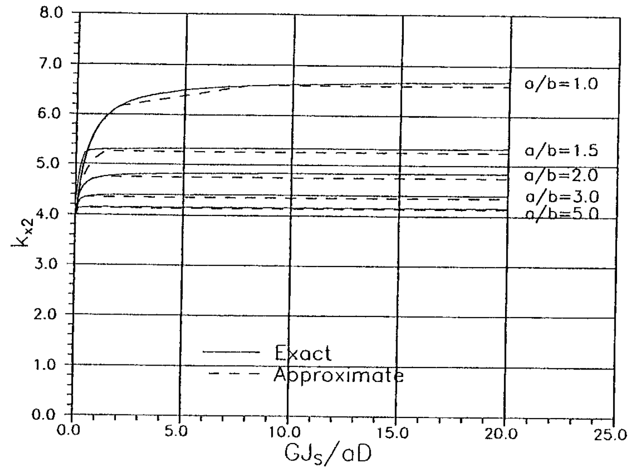


Fig. 5 Accuracy of the approximate formula of buckling coefficient k_{x2}

$=-0.123\beta^4+1.549\beta^3-6.788\beta^2$
$\quad +11.299\beta-3.662$ for $0.4\leq\zeta_s<0.8$
$=0.138\beta^2-0.793\beta+1.171$ for $0.8\leq\zeta_s<2.0$
$=0.0$ for $2.0\leq\zeta_s\leq 20.0$
$=0.0$ for $\zeta_s=\infty$

$r_2=-0.190\beta^4+2.093\beta^3-5.891\beta^2$
$\quad -2.096\beta+1.792$ for $0.0\leq\zeta_s<0.4$
$=0.114\beta^4-1.412\beta^3+5.933\beta^2$
$\quad -8.638\beta+0.224$ for $0.4\leq\zeta_s<0.8$
$=-0.457\beta^2+2.571\beta-3.712$ for $0.8\leq\zeta_s<2.0$
$=0.0$ for $2.0\leq\zeta_s\leq 20.0$
$=0.0$ for $\zeta_s=\infty$

$s_2=0.004\beta^4-0.007\beta^3-0.243\beta^2$
$\quad +0.630\beta+3.617$ for $0.0\leq\zeta_s<0.4$
$=-0.021\beta^4+0.184\beta^3-0.126\beta^2$
$\quad -2.625\beta+6.457$ for $0.4\leq\zeta_s<0.8$
$=0.822\beta^2-4.516\beta+6.304$ for $0.8\leq\zeta_s<2.0$
$=-0.106\beta+0.176$ for $2.0\leq\zeta_s\leq 20.0$
$=0.0$ for $\zeta_s=\infty$

$t_2=4.0$ for $0.0\leq\zeta_s<0.4$
$=-0.001\beta^4+0.033\beta^3-0.241\beta^2$
$\quad +0.684\beta+3.539$ for $0.4\leq\zeta_s<0.8$
$=-0.148\beta^2+0.596\beta+3.847$ for $0.8\leq\zeta_s<2.0$
$=-1.822\beta+7.850$ for $2.0\leq\zeta_s\leq 20.0$
$=0.041\beta^4-0.602\beta^3+3.303\beta^2$
$\quad -8.176\beta+12.144$ for $\zeta_s=\infty$

In the above equation, the following condition should be applied.

i) If $4.0<\beta\leq 4.5$ and $\zeta_s\geq 0.2$ then $\zeta_s=0.2$,
ii) If $\beta>4.5$ and $\zeta_s\geq 0.1$ then $\zeta_s=0.1$,
iii) If $\beta\geq 2.2$ and $\zeta_s\geq 0.4$ then $\zeta_s=0.4$,
iv) If $\beta\geq 1.5$ and $\zeta_s\geq 1.4$ then $\zeta_s=1.4$,
v) If $8.0\leq\zeta_s\leq 20.0$ then $\zeta_s=8.0$,
vi) If $\beta\geq 5.0$ then $\beta=5.0$

The accuracy of the approximate formula is checked by the dotted lines in Fig. 5. Accordingly, the elastic buckling stress in this case can be predicted by

$$\sigma_{xcr2}=k_{x2}\cdot\sigma_E \tag{28}$$

(3) Both Edges Elastically Restrained

It is very difficult to obtain analytical solution of buckling strength of a plate elastically restrained at both edges. It is considered that the edge condition of this case is the combined one of the previous two conditions, that is , one edge simply supported and the other elastically restrained. In this regard, we assume the following combination for estimating the buckling strength of both edges elastically restrained plates, namely

$$\sigma_{xE}=k_x\cdot\sigma_E \tag{29}$$

where

σ_{xE}=elastic buckling stress in the longitudinal compression

$k_x=k_{x1}+k_{x2}-k_{x0}$

k_{x0}=buckling coefficient of a plate simply supported at all edges in longitudinal compression($\cong 4.0$)

3.2 Buckling Strength of a Rectangular Plate in Transverse Compression σ_y

(1) Elastically Restrained at the Longer Edges and Simply Supported at the Shorter Edges

The buckling of a plate under transverse compression is considered, in which the rotation at the unloaded edges (the shorter edges) occurs freely but the loaded edges (the longer edges) are elastically restrained by supporting members. Since only transverse compression σ_y is applied along the longer edges, σ_x and τ_{xy} in Eq. (1) vanish. Also since the conditions of simple support at the unloaded edges (or the shorter edges) are considered, the deflection w and the bending moment M_x at the unloaded edges must be zero.

At first, when the coordinate is taken as in Fig. 1, the deflection w of the plate which satisfies the differential equation of Eq. (1) and the condition of simple support at the shorter edges (i. e. $w=0$, $M_x=m_x=0$ at $x=0$ and a) is assumed, namely

$$w=Y(y)\sin(m\pi x/a) \quad (30)$$

where $Y(y)$ indicates a function of y and m represents the number of half-waves in x direction.

Substituting Eq. (30) into Eq. (1) and considering $\sigma_x=\tau_{xy}=0$, we obtain the ordinary differential equation of the fourth order, replacing σ_y by σ_{ycr1}:

$$Y_{,yyyy}-2(m\pi/a)^2(1-\mu_{y1}^2)Y_{,yy}+(m\pi/a)^4Y=0 \quad (31)$$

where

$\mu_{y1}=\beta/m\cdot\sqrt{k_{y1}/2}$

$k_{y1}=\sigma_{ycr1}/\sigma_E$

σ_{ycr1}=elastic buckling stress under the present loading and edge conditions

The general solution of Eq. (31) is

$$Y(y)=C_1\cos\alpha_1 y+C_2\sin\alpha_1 y+C_3\cos\alpha_2 y+C_4\sin\alpha_2 y \quad (32)$$

where

$\alpha_{1,2}=m\pi/\sqrt{2}a\cdot(\mu_{y1}\mp\sqrt{\mu_{y1}^2-2})$

$C_1\sim C_4$=constants to be determinded

Considering the deflection w at the bucklilng load is a symmetric function of y, only the first and third term in Eq. (32) remain, namely

$$Y(y)=C_1\cos\alpha_1 y+C_3\cos\alpha_2 y \quad (33)$$

Unknown constants C_1 and C_3 are determined from the conditions of support and moment equilibrium at the longer edges, as given in Eq. (2), that is,

$$w=0 \text{ and } M_y=m_y \text{ at } y=\pm b/2 \quad (34)$$

Substitution of Eqs. (3.b) and (4.b) into Eq. (34), considering $w_{,xx}=0$ and $\Gamma_L=0$ at $y=\pm b/2$, yields

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{Bmatrix} C_1 \\ C_3 \end{Bmatrix}=0 \quad (35)$$

where

$A_{11}=\cos(\alpha_1 b/2)$

$A_{12}=\cos(\alpha_2 b/2)$

$A_{21}=\alpha_1^2\cos(\alpha_1 b/2)+\zeta_L m^2\pi^2 b/a^2\alpha_1\sin(\alpha_1 b/2)$

$A_{22}=\alpha_2^2\cos(\alpha_2 b/2)+\zeta_L m^2\pi^2 b/a^2\alpha_2\sin(\alpha_2 b/2)$

$\zeta_L=GJ_L/bD$

The condition of determinant $\Delta=0$ yields finally the exact buckling condition.

$$A_{11}A_{22}-A_{12}A_{21}=0 \quad (36)$$

By solving the above equation, the buckling coefficient k_{y1} will be found. Fig. 6 indicates the computed results with variation in the aspect ratio and the torsional rigidity. It is observed that the buckling coefficient is affected by both aspect ratio and torsional rigidity.

We aim to express the buckling coefficient by the algebraic formula in the practical range of the torsional rigidity. Based on the theoretical results, the buckling coefficient k_{y1} can be formulated in terms of aspect ratio and torsional rigidity, namely

$$k_{y1}=p_1\zeta_L^2+q_1\zeta_L+r_1 \quad (37)$$

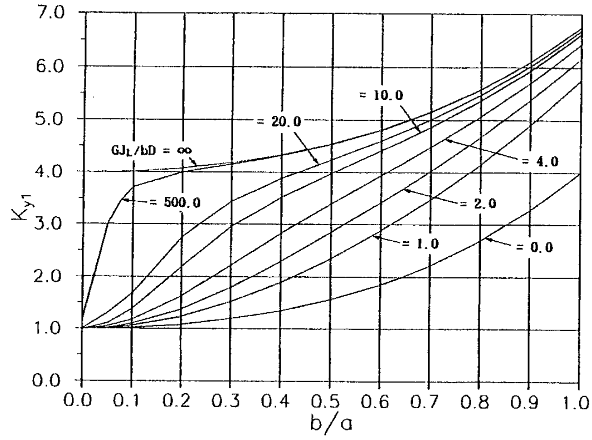


Fig. 6 Buckling coefficient k_{y1} of a plate in transverse compression, elastically restrained at the longer edges and simply supported at the shorter edges

where

$p_1=1.322\alpha^4-1.919\alpha^3+0.021\alpha^2+0.032\alpha$ for $0.0\leq\zeta_L\leq2.0$

$=-0.463\alpha^4+1.023\alpha^3-0.649\alpha^2+0.073\alpha$ for $2.0<\zeta_L\leq8.0$

$=0.0$ for $8.0<\zeta_L\leq20.0$

$=0.0$ for $\zeta_L=\infty$

$q_1=-0.179\alpha^4-3.098\alpha^3+5.648\alpha^2-0.199\alpha$ for $0.0\leq\zeta_L\leq2.0$

$=5.432\alpha^4-11.324\alpha^3+6.189\alpha^2-0.068\alpha$ for $2.0<\zeta_L\leq8.0$

$=-1.047\alpha^4+2.624\alpha^3-2.215\alpha^2+0.646\alpha$ for $8.0<\zeta_L\leq20.0$

$=0.0$ for $\zeta_L=\infty$

$r_1=0.994\alpha^4+0.011\alpha^3+1.991\alpha^2+0.003\alpha+1.0$ for $0.0\leq\zeta_L\leq2.0$

$=-3.131\alpha^4+4.753\alpha^3+3.587\alpha^2-0.433\alpha+1.0$ for $2.0<\zeta_L\leq8.0$

$=20.111\alpha^4-43.697\alpha^3+30.941\alpha^2-1.836\alpha+1.0$ for $8.0<\zeta_L\leq20.0$

$=0.751\alpha^4-0.047\alpha^3+2.053\alpha^2-0.015\alpha+4.0$ for $\zeta_L=\infty$

$\alpha=1/\beta=b/a$

The accuracy of the approximate formula is checked by the dotted lines in Fig. 7. Accordingly, the elastic buckling stress in this case can be predicted by

$$\sigma_{ycr1}=k_{y1}\cdot\sigma_E \quad (38)$$

(2) Elastically Restrained at the Shorter Edges and Simply Supported at the Longer Edges

In this case, the rotation at the loaded edges (the longer edges) occurs freely but the unloaded edges (the shorter edges) are elastically restrained by supporting members. The deflection w and the bending moment M_y at the loaded edges must be zero. The coordinate of the plate is taken as in Fig. 3.

The deflection w of the plate which satisfies the differential equation of Eq. (1) and the condition of simple support at the longer edges (i. e. $w=0$, $M_y=m_y$

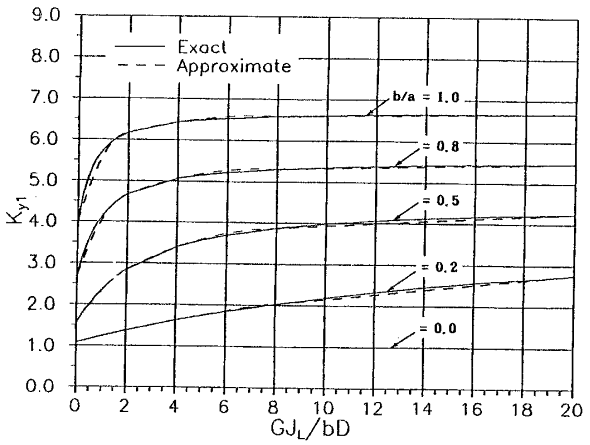


Fig. 7 Accuracy of the approximate formula of buckling coefficient k_{y1}

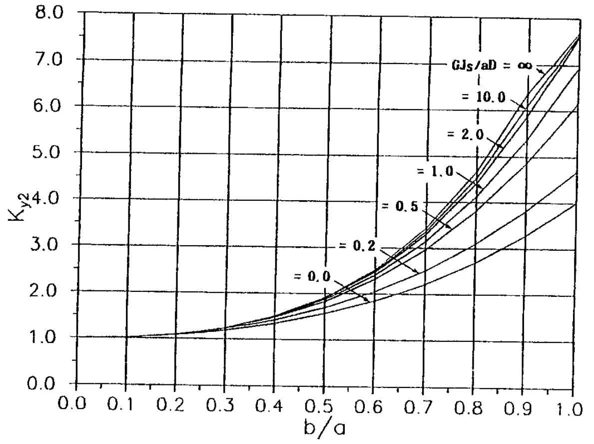


Fig. 8 Buckling coefficient k_{y2} of a plate in transverse compresion, elastically restrained at the shorter edges and simply supported at the longer edges

$=0$ at $y=0$ and b) is assumed, namely

$$w=X(x)\sin(n\pi y/b) \qquad (39)$$

where $X(x)$ indicates a funtion of x and n represents the number of half-waves in y direction.

Substituting Eq. (39) into Eq. (1) and considering $\sigma_x=\tau_{xy}=0$, we obtain the ordinary differential equation of the fourth order, replacing σ_y by σ_{ycr2}:

$$X_{,xxxx}-2(n\pi/b)^2X_{,xx}+(n\pi/b)^4(1-\mu_{y2}^2)X=0 \qquad (40)$$

where

$k_{y2}=\sigma_{ycr2}/\sigma_E$

$\mu_{y2}=\sqrt{k_{y2}}/n$

σ_{ycr2}=elastic buckling stress under the present loading and edge conditions

The general solution of the above differential equation (40) is

$$X(x)=C_1\cosh\alpha_1 x+C_2\sinh\alpha_1 x +C_3\cos\alpha_2 x+C_4\sin\alpha_2 x \qquad (41)$$

where

$\alpha_{1,2}=n\pi/b\cdot\sqrt{\mu_{y2}\pm 1}$

$C_1\sim C_4$=constants

The deflected shape in y direction after buckling will be symmetric and thus by removing the terms representing antisymmetric deflections, the following two terms remain.

$$X(x)=C_1\cosh\alpha_1 x+C_3\cos\alpha_2 x \qquad (42)$$

The constants C_1 and C_3 will be determined from the following boundary condition.

$$w=0 \text{ and } M_x=m_x \text{ at } x=\pm a/2 \qquad (43)$$

Substitution of Eqs. (3.a) and (4.a) into Eq. (43), considering $w_{,yy}=0$ and $\Gamma_S=0$ at $x=\pm a/2$, yields

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{Bmatrix} C_1 \\ C_3 \end{Bmatrix}=0 \qquad (44)$$

where

$A_{11}=\cosh(\alpha_1 a/2)$

$A_{12}=\cos(\alpha_2 a/2)$

$A_{21}=\alpha_1^2\cosh(\alpha_1 a/2)+\zeta_S n^2\pi^2 a/b^2\alpha_1\sinh(\alpha_1 a/2)$

$A_{22}=-\alpha_2^2\cos(\alpha_2 a/2)-\zeta_S n^2\pi^2 a/b^2\alpha_2\sin(\alpha_2 a/2)$

$\zeta_S=GJ_S/aD$

The condition of determinant $\Delta=0$ yields finally the exact buckling condition.

$$A_{11}A_{22}-A_{12}A_{21}=0 \qquad (45)$$

By solving the above equation, the buckling coefficient k_{y2} will be found. Fig. 8 indicates the computed results with variation in the aspect ratio and the torsional rigidity. Based on the theoretical results, it is also possible to formulate the buckling coefficient by an algebraic expression in the practical range of the torsional rigidity, namely

$$k_{y2}=p_2\zeta_S^2+q_2\zeta_S+r_2 \qquad (46)$$

where

$p_2=0.543\alpha^4-1.297\alpha^3$
$\quad+0.192\alpha^2-0.016\alpha$ for $0.0\leq\zeta_S\leq 2.0$
$\quad=-0.347\alpha^4+0.403\alpha^3$
$\quad-0.147\alpha^2+0.016\alpha$ for $2.0<\zeta_S\leq 6.0$
$\quad=0.0$ for $6.0<\zeta_S\leq 20.0$
$\quad=0.0$ for $\zeta_S=\infty$

$q_2=-1.094\alpha^4+4.401\alpha^3$
$\quad-0.751\alpha^2+0.068\alpha$ for $0.0\leq\zeta_S\leq 2.0$
$\quad=2.139\alpha^4-1.761\alpha^3$
$\quad+0.419\alpha^2-0.030\alpha$ for $2.0<\zeta_S\leq 6.0$
$\quad=-0.199\alpha^4+0.308\alpha^3$
$\quad-0.118\alpha^2+0.013\alpha$ for $6.0<\zeta_S\leq 20.0$
$\quad=0.0$ for $\zeta_S=\infty$

$r_2=0.994\alpha^4+0.011\alpha^3$
$\quad+1.991\alpha^2+0.003\alpha+1.0$ for $0.0\leq\zeta_S\leq 2.0$
$\quad=-2.031\alpha^4+5.765\alpha^3$
$\quad+0.870\alpha^2+0.102\alpha+1.0$ for $2.0<\zeta_S\leq 6.0$
$\quad=-0.289\alpha^4+7.507\alpha^3$
$\quad-1.029\alpha^2+0.398\alpha+1.0$ for $6.0<\zeta_S\leq 20.0$
$\quad=-6.278\alpha^4+17.135\alpha^3$
$\quad-5.026\alpha^2+0.860\alpha+1.0$ for $\zeta_S=\infty$

The accuracy of the approximate formula is checked by the dotted lines in Fig. 9. Accordingly, the elastic buckling stress in this case can be predicted by

$$\sigma_{ycr2}=k_{y2}\cdot\sigma_E \qquad (47)$$

(3) Both Edges Elastically Restrained

Using the similar way described before, we assume

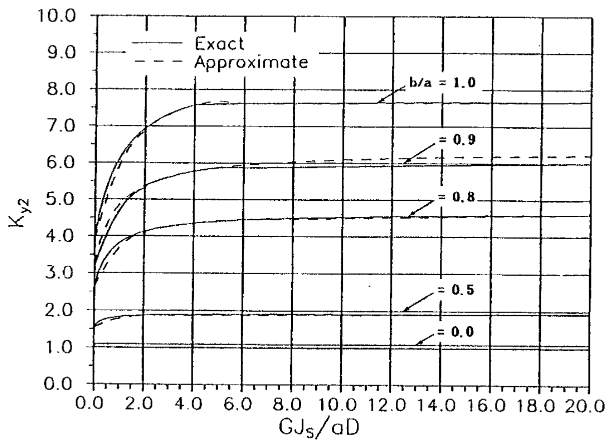


Fig. 9 Accuracy of the approximate formula of buckling coefficient k_{y2}

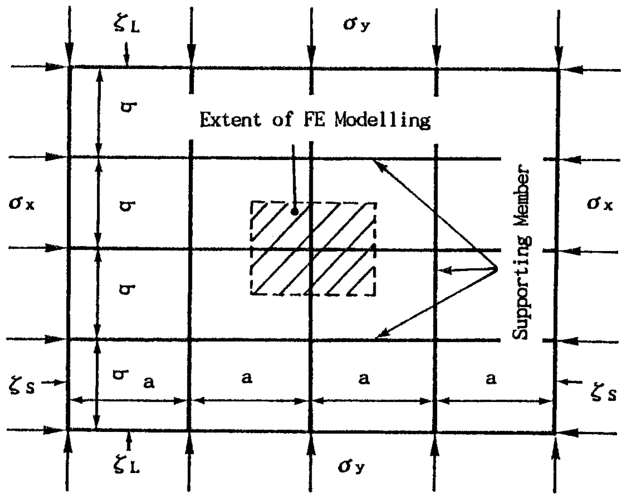


Fig. 10 Object stiffened plate panel

the following combination for estimating the buckling coefficient of a plate elastically restrained at both edges, namely

$$\sigma_{yE}=k_y\cdot\sigma_E \qquad (48)$$

where

σ_{yE}=elastic buckling stress in the transverse compression

$k_y=k_{y1}+k_{y2}-k_{y0}$

k_{y0}=buckling coefficient for a plate simply supported at all edges in transverse compression ($=(b/a)^4+2(b/a)^2+1.0$)

4. A New Plate Buckling Design Formula in Combined Loads

In this study, the effect of the rotational restraints on the shear buckling which will remain for a further research is not considered. Also it is assumed that the buckling interaction between combined loadings is the same with the one described in the previous papers[5,6]. Therefore, the final expression of the plate buckling design formula accounting for the rotational restraint effects of supporting members at edges becomes

$$\Gamma_B=\left(\frac{\sigma_x+\sigma_{rex}}{\sigma_{xcr}\cdot R_{sx}}\right)^{\alpha_1}+\left(\frac{\sigma_y+\sigma_{rey}}{\sigma_{ycr}\cdot R_{sy}}\right)^{\alpha_2}-\eta_a\leq 0 \qquad (49)$$

where

η_a=safety factor against the buckling, may be taken as 1.0

σ_x, σ_y=applied compressive stress in x, y direction

σ_{xcr}, σ_{ycr}=critical buckling stress for axial compression in x, y direction, Eq. (29) and Eq. (48) will be corrected by the plasticity correction factor

σ_{rex}, σ_{rey}=effective compressive residual stress

$R_{sx}=1.0-(\tau/\tau_{cr})^{\alpha_3}$

$R_{sy}=1.0-(\tau/\tau_{cr})^{\alpha_4}$

τ= applied edge shear stress

τ_{cr}=critical buckling stress for edge shear, corrected by the plasticity correction factor

$\alpha_1\sim\alpha_4$=coefficients[5),6)]

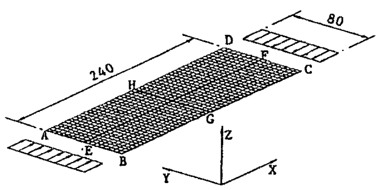
All parameters used in the above equation are described in the previous papers[5,6]. In the application of the above formula, the longer edge is taken as x direction such that the aspect ratio is always equal to or greater than 1.0. Also it should be noted that when the plate buckling of a continuously stiffened plate panel is considered, a half amount of the torsional rigidity for the supporting member should be applied.

5. Verification Examples and Discussions

In order to check the accuracy of the proposed formula, verification examples comparing the approximate solutions with the exact and FE results are demonstrated. The plate buckling of a continuously stiffened plate panel subjected to uniaxial compression or biaxial compression, shown in Fig. 10 is considered. Considering the symmetric pattern of the deflection, the portion along the center line of the adjacent plates is taken as the extent of FE modelling (see Fig. 10).

288 cases with variation in the aspect ratio of plate, the torsional rigidity of supporting member and the loading ratio between longitudinal and transverse compression are analyzed. The exact solution is obtained by solving the exact buckling condition directly for plates, all edges simply supported or one edge simply supported/the other elastically restrained, and the finite-element solution is calculated for all edge conditions through the eigenvalue analysis using the MSC/NASTRAN finite-element code[7].

Fig. 11 shows an example of the finite-element model, in which 16 rectangular plate elements are employed for the plate part in the shorter direction, and for modelling the supporting member the beam elements with only torsional rigidity are used. The boundary condition is also given in Fig. 11. Fig. 12 compares the elastic buckling stress between the exact, numerical and present solution, with variation in the aspect ratio and the torsional rigidity of supporting members. It is observed that the results obtained by the present approximate



Line $\overline{AB}$ and $\overline{CD}$: symmetry or anti-symmetry on YZ plane
Line $\overline{BC}$ and $\overline{AD}$: symmetry on XZ plane
Line $\overline{EF}$ and $\overline{GH}$: beam element with torsional stiffeness only and supported in Z-direction

Fig. 11 An example of the finite-element model of an elastically restrained plate($a/b=3$)

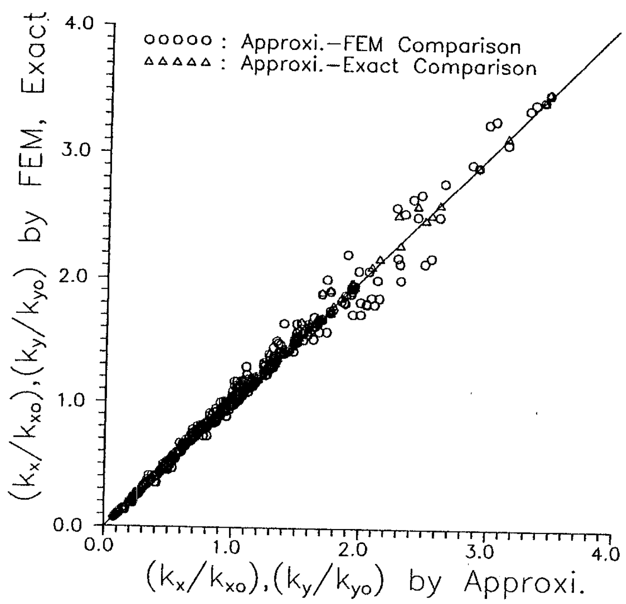


Fig. 12 Comparison of buckling coefficients between the approximate, exact and finite-element solutions

formula agree very well with the exact and FE solutions. In particular, it is clear that the present combination for estimating the buckling strength of a plate elastically restrained at both edges furnishes the accurate solution.

6. Concluding Remarks

In this study, a practical buckling design formula for ship platings providing an accurate solution is derived by taking into account the torsional rigidity effect for supporting members so that we will be able to treat the real plate edge condition in the structural design of ships.

From the present study, the following conclusions can be drawn :

1) The edge condition of the real ship platings is neither simply supported nor clamped, and the condition may be intermediate between two extreme conditions, depending on the rigidity of supporting members at plate edges.

2) When the bending rigidity of supporting members is assumed to be large enough, the real plate edge condition of ship platings can be expressed as a function of the torsional rigidity of supporting members.

3) When the torsional rigidity of supporting members at edges is zero, the edge condition corresponds to simply supported. With increase in the torsional rigidity of supporting members, the plate buckling strength increases. As the torsional rigidity approaches infinite, the edge condition becomes clamped.

4) The present buckling design formula which has been derived based on the theoretical solution provides accurate results with efficiency. In particular, it is clear that the present combination for estimating the buckling strength of a plate elastically restrained at both edges is valid and furnishes the accurate solution.

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