Prediction of Wave Drift Damping by a Higher Order BEM

by B. Teng*, *Member* S. Kato*, *Member*
K. Hoshino*, *Member*

Summary

The paper develops a numerical method for predicting wave drift damping of three dimensional bodies. The first order potentials are calculated by a new integral equation method, in which the second derivatives of the steady potential has been removed from the integration on the free surface. The second order mean potential is calculated by an integral equation method which is firstly proposed and implemented. Comparison is made with analytic solution for a uniform cylinder. Numerical examinations are made on the convergence with radius of the mesh on the free surface, and magnitude of each component of wave drift damping. Timman-Newman relation is also used to check the correction of the first order potentials. Comparison with experimental results is made on an array of four cylinders which are restrained and freely moving respectively. It was found that good agreements exist between the present calculation and experimental results and negative wave damping may occur at some wave frequencies.

1. Introduction

Tension leg platforms (TLPs) are semi-submersible structures moored to seabed with a number of pretensioned vertical cables (tethers). The response motion of upper structure with wave exciting induces tethers vibrating continuously, which will break when their fatigue life has been reached. Damping can decrease amplitudes of response of structures. Thus, accurately predicting damping is important for the prediction of fatigue life of tethers. Usually damping of a compliment structure can be divided into the viscous damping, the radiation damping and the wave drift damping. The wave drift damping, defined by Wichers and Sluijs[1], is due to the increase of drift force with forward moving speed of a floating body. Its calculation needs the nonlinear knowledge on wave diffraction and radiation in a steady flow.

In this respect, significant progress has been made recently. Matsui, Lee and Sano[2] and Emmerhoff and Sclavounos[3] have derived analytic solutions for uniform cylinders in finite and infinite water depth. Bao and Kinoshita[4] expended to truncated cylinders. For 3D arbitrary bodies, integral equation method had been developed by Nossen, Grue and Palm[5], Grue and Palm[6], Zhao et al[7], Huijsmans and Hermans[8] and Eatock Taylor and Teng[9], Newman[10], among others.

* Ship Research Institute

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The present research examines the wave damping of floating bodies by a higher order boundary element method based on perturbation with respect to wave slope and current velocity. The oscillating wave potentials are resolved by a new developed integral equation (Teng and Kato[11]). Comparing with some widely used ones, the present one removes second derivatives of steady potential from the integral on the free surface. Thus, the present integral equation can be dealt with more accurately. Cauchy principal value (CPV) integrals on the body surface and the free surface are dealt with by direct and indirect methods respectively. The present work also derived an integral equation for the calculation of second order steady potential, which will give rise to some contribution to wave drift damping (Grue and Palm[6]). The second order drift forces on forward moving bodies are calculated both by a near field and a far field method. The wave damping is obtained by numerical differentiation of second order mean drift forces in current.

Numerical test is made on the convergence of drift force with radius of the mesh on the free surface, and the examination of the contribution of the second order steady potential on drift force. The comparison between the far field and near field method is made upon horizontal modes at restrained case. The Timman-Newman relationship is also used to certify the correction of the present method. Comparison is made with Kinoshita, Sunahara and Bao's[12] experimental results on an array of four cylinders which are restrained and freely moving respectively. It was found that good agreements exist between the present calculation and experimental results and negative wave drift damping may occur at some wave frequency.

2. Perturbation expansion

2.1 Velocity potential and wave surface

The fluid is assumed to be homogenous and incompressible, and the motion irrotational. Waves are assumed to be periodic. There exists a velocity potential Φ that satisfies the Laplace equation, the nonlinear free surface condition

$$\Phi_{tt}+2\nabla\Phi\cdot\nabla\Phi_t+g\Phi_z+\frac{1}{2}\nabla\Phi\cdot\nabla(\nabla\Phi\cdot\nabla\Phi)=0 \tag{1}$$

on the free surface $z=\zeta(x, y, t)$, defined by

$$\zeta=-\frac{1}{g}\left(\frac{d\Phi}{dt}+\frac{1}{2}\nabla\Phi\cdot\nabla\Phi\right)_{z=\zeta} \tag{2}$$

and the body condition

$$\frac{\partial\Phi}{\partial n}=V_s\cdot n \tag{3}$$

on the instantaneous body surface S_0. V_s is the velocity of body motion and n is the unit normal vector of the body surface pointing out of the fluid.

Under the assumption of small wave slope $\varepsilon=kA$, the velocity potential can be expended into a perturbation series

$$\Phi(x, t)=\Phi^{(0)}(x)+\varepsilon\Phi^{(1)}(x, t)+\varepsilon^2\Phi^{(2)}(x, t)+\cdots \tag{4}$$

The potentials at each order of ε can further be expanded with respect to the current parameter $\tau=\sigma U/g$

$$\begin{aligned}\Phi^{(0)}(x)&=U\chi_s\\ \Phi^{(1)}(x, t)&=\Phi^{(10)}(x, t)+\tau\Phi^{(11)}(x, t)+\cdots\\ \Phi^{(2)}(x, t)&=\Phi^{(20)}(x, t)+\tau\Phi^{(21)}(x, t)+\cdots\end{aligned} \tag{5}$$

where σ is the wave encounter frequency which has a relation with wave frequency ω of

$$\sigma=\omega-kU\cos\beta, \tag{6}$$

β is the incident angle of the waves, and k is the wave number which is the real solution of the dispersion relation. The first index in the superscript corresponds to wave steepness, and the second to current parameter.

As the same the wave profile can be expanded into

$$\zeta(x, t)=\varepsilon\zeta^{(1)}(x, t)+\varepsilon^2\zeta^{(2)}(x, t)+\cdots \tag{7}$$

and

$$\begin{aligned}\zeta^{(1)}(x, t)&=\zeta^{(10)}(x, t)+\tau\zeta^{(11)}(x, t)+\cdots\\ \zeta^{(2)}(x, t)&=\zeta^{(20)}(x, t)+\tau\zeta^{(21)}(x, t)+\cdots\end{aligned} \tag{8}$$

where

$$\begin{aligned}\zeta^{(10)}&=-\Phi_t^{(10)}/g\\ \zeta^{(11)}&=-U\nabla\chi_s\cdot\nabla\Phi_t^{(10)}/g\end{aligned} \tag{9}$$

2.2 Hydrodynamic forces

After getting the diffraction potential, the hydrodynamic pressure in the fluid domain can be obtained from Bernoulli's equation. By perturbation expansion, the pressure may be written in the form

$$p(x, t)=p^{(0)}(x)+\varepsilon p^{(1)}(x, t)+\varepsilon^2 p^{(2)}(x, t)+\cdots \tag{10}$$

and

$$\begin{aligned}p^{(0)}(x, t)&=p^{(00)}(x)+0(\tau^2)\\ p^{(1)}(x, t)&=p^{(10)}(x, t)+\tau p^{(11)}(x, t)+0(\tau^2)\\ p^{(2)}(x, t)&=p^{(20)}(x, t)+\tau p^{(21)}(x, t)+0(\tau^2)\end{aligned} \tag{11}$$

$p^{(00)}$ is the hydrostatic pressure and $p^{(10)}$ the linear oscillating pressure. The remaining components are defined by the relations

$$\begin{aligned}p^{(11)}&=-\rho\left[\frac{\partial\Phi^{(11)}}{\partial t}+\nabla\Phi^{(10)}\cdot\nabla\Phi^{(01)}\right]\\ p^{(20)}&=-\rho\left[\frac{\partial\Phi^{(20)}}{\partial t}+\frac{1}{2}\nabla\Phi^{(10)}\cdot\nabla\Phi^{(10)}\right]\\ p^{(21)}&=-\rho\left[\frac{\partial\Phi^{(21)}}{\partial t}+\nabla\Phi^{(01)}\cdot\nabla\Phi^{(20)}+\nabla\Phi^{(11)}\cdot\nabla\Phi^{(10)}\right]\end{aligned} \tag{12}$$

The mean values of the second order pressure in terms of ε are

$$\begin{aligned}p^{(20)m}&=-\frac{\rho}{4}\overline{\nabla\Phi^{(10)}\cdot\nabla\Phi^{(10)}}\\ p^{(21)m}&=-\rho\left[U\nabla\chi_s\cdot\nabla\Phi^{(20)m}+\frac{1}{2}\overline{\nabla\Phi^{(11)}\cdot\nabla\Phi^{(10)}}\right]\end{aligned} \tag{13}$$

where $\Phi^{(20)m}$ is the second order steady velocity potential in terms of wave steepness.

3. Integral equations

3.1 Zero order steady potential

The steady velocity potential χ_s can be expressed as the sum of a steady incident potential and the disturbance from a body

$$\chi_s(x)=\chi-x. \tag{14}$$

Under the assumption of small forward speed, χ satisfies the 'rigid wall' condition

$$\frac{\partial\chi}{\partial n}=0 \tag{15}$$

on the free surface,

$$\frac{\partial\chi}{\partial n}=n_1 \tag{16}$$

on the mean body surface S_B, and

$$\nabla\chi=0 \quad |x|\to\infty \tag{17}$$

in the field far away from the body. The calculation of the zero order steady potential is straightforward by using the Green's function

$$G_0(x, x_0)=-\frac{1}{4\pi}\left[\frac{1}{r}+\frac{1}{r_1}+\sum_{n=1}^{\infty}\left(\frac{1}{r_{2n}}+\frac{1}{r_{4n}}+\frac{1}{r_{3n}}+\frac{1}{r_{5n}}\right)\right] \tag{18}$$

where

$$\begin{aligned}r&=[(R^2+(z-z_0)^2]^{1/2},\ r_1=[R^2+(z+z_0)^2]^{1/2},\\ r_{2n}&=[R^2+(z-z_0-2nh)^2]^{1/2},\\ r_{3n}&=[R^2+(z+z_0+2nh)^2]^{1/2},\\ r_{4n}&=[R^2+(z-z_0+2nh)^2]^{1/2},\\ r_{5n}&=[R^2+(z+z_0-2nh)^2]^{1/2},\\ R^2&=(x-x_0)^2+(y-y_0)^2,\end{aligned} \tag{19}$$

and h is the water depth. The above Green's function satisfies the rigid wall free surface condition on the mean water surface and the impermeable condition at sea bed.

3.2 The first order oscillating potential

The first order oscillating potential in wave slope ε can be expressed as

$$\Phi^{(1)}(x, t)=A\,Re[\Phi^{(1)}(x)e^{i\sigma t}]=A\,Re\left[\left(\phi_0^{(1)}(x)+\phi_7^{(1)}(x)+i\sigma\sum_{j=1}^{6}\xi_j\phi_j^{(1)}(x)\right)e^{i\sigma t}\right], \tag{20}$$

where A is the amplitude of the incident waves, ϕ_0 the incident potential, ϕ_7 diffraction potential and $\phi_j (j=1, \cdots, 6)$ the radiation potentials corresponding to six generalized body motions. (ξ_1, ξ_2, ξ_3) are the amplitudes of translational motion, and $(\xi_4, \xi_5, \xi_6)=(\alpha_1, \alpha_2, \alpha_3)$ the amplitudes of rotation.

Approximating to the leading order in current factor τ, the free surface condition for the first order potentials in wave slope ε can be written as

$$-\nu_0\phi_j^{(1)}+2i\tau\nabla_2\phi_j^{(1)}\cdot\nabla_2\chi_s+i\tau\phi_j^{(1)}\nabla_2^2\chi + \frac{\partial\phi_j^{(1)}}{\partial z}=0 \quad j=1, \cdots, 6 \text{ and } 0+7 \tag{21}$$

on the still water surface, where $\nu_0=\sigma^2/g$, and ∇_2 is a two dimensional gradient operator on a horizontal plane. In the field far away from the body, the above equation can be simplified as

$$-\nu_0\phi_j^{(1)}-2i\tau\frac{\partial\phi_j^{(1)}}{\partial x}+\frac{\partial\phi_j^{(1)}}{\partial z}=0. \tag{22}$$

The body condition can be written as

$$\frac{\partial\phi_B^{(1)}}{\partial n}=0, \quad \phi_B^{(1)}=\phi_0^{(1)}+\phi_7^{(1)}$$
$$\frac{\partial\phi_j^{(1)}}{\partial n}=n_j+\frac{U}{i\sigma}m_j, \; j=1, \cdots, 6 \tag{23}$$

on the mean body surface S_B, where

$$(m_1, m_2, m_3)=-(n\cdot\nabla)\nabla\chi_s,$$
$$(m_4, m_5, m_6)=-(n\cdot\nabla)(x\times\nabla\chi_s). \tag{24}$$

From the free surface boundary condition (Eq. 22) and the out going condition of oscillating waves at infinity, we can derive a Green's function as

$$4\pi G(x, x_0)=-\frac{1}{r}-\frac{1}{r_{31}}-\int_0^\infty\int_0^{2\pi} e^{\lambda w} \frac{(\lambda f(\tau)+\nu)\cosh\lambda(h+z)\cosh\lambda(h+z_0)}{\pi(\lambda F(\lambda, \tau)-\nu\cosh\lambda h)}d\lambda\, d\theta \tag{25}$$

where

$$W=-h+i[(x-x_0)\cos\theta+(y-y_0)\sin\theta)],$$
$$f(\tau)=1-2\tau\cos\theta, \tag{26}$$
$$F(\tau, \lambda)=\sinh\lambda h+2\tau\cos\theta\cosh\lambda h.$$

Applying Green's second identity to the unsteady potentials and an oscillating source with a reverse speed, as shown by Nossen et al[5] for infinite water depth, we can obtain the integral equation

$$\alpha\phi_j^{(1)}(x_0)-\iint_{S_B}\phi_j^{(1)}(x)\frac{\partial G}{\partial n}ds +2i\tau\iint_{S_F}\phi_j^{(1)}(x)\left(\nabla_2 G\cdot\nabla_2\chi+\frac{1}{2}G\nabla_2^2\chi\right)ds = \begin{cases} \phi_0^{(1)}(x_0) & \text{for } \phi_j^{(1)}=\phi_B^{(1)} \\ \iint_{S_B}\left(G+\frac{i\tau}{\nu_0}\nabla G\cdot\nabla\chi_s\right)n_j ds & \text{for } j=1, \cdots, 6 \end{cases} \tag{27}$$

after using the Tuck's theorem[13] to remove the second order derivative of the steady potential for smooth body. Here S_F is the outer free surface. Examination on floating cylinders by Eatock Taylor and Teng[9] has suggested that the local geometry of 'corners' could have an important effect on the flow when the body has forward speed. Wave drift damping, however, is very little influenced by this effect. The second order derivative of the steady potential on the free surface can be removed by applying the transform (Teng and Kato[11])

$$\iint_{S_F} G(x, x_0)\nabla\phi_j^{(1)}\cdot\nabla\chi ds = -\oint_{C_B} G(x, x_0)\phi_j^{(1)}n_1 dl - \iint_{S_F}\phi_j^{(1)}(\nabla_2 G\cdot\nabla_2\chi+G\nabla_2^2\chi)ds \tag{28}$$

where C_B is the water line, the intersecting line of the body and the still water surface, and the line integral is taken clockwise. This yields a new integral equation of

$$\alpha\phi_j^{(1)}(x_0)-\iint_{S_B}\phi_j^{(1)}(x)\frac{\partial G}{\partial n}ds = i\tau\oint_{C_B} G\phi_j^{(1)}(x)n_1 dl - i\tau\iint_{S_F}[\phi_j^{(1)}(x)\nabla_2 G\cdot\nabla_2\chi(x) - G\phi_j^{(1)}(x)\cdot\nabla_2\chi(x)]ds + \begin{cases} \phi_0^{(1)}(x_0) & \text{for } \phi_B^{(1)} \\ \iint_{S_B}\left(G+\frac{i\tau}{\nu_0}\nabla G\cdot\nabla\chi_s\right)n_j ds & \text{for } j=1, \cdots, 6 \end{cases} \tag{29}$$

For benefiting the discretization by higher order elements, we combine the above equation with a corresponding integral equation obtained inside the body, as Eatock Taylor and Chau[14] did for the wave diffraction in still water, and obtain a new integral equation

$$\left[1-\iint_{S_W}(\nu_0 G-2i\tau G_x)dx\,dy\right]\phi_j^{(1)}(x_0) + \iint_{S_B}[\phi_j^{(1)}(x_0)-\phi_j^{(1)}(x)]\frac{\partial G}{\partial n}ds = i\tau\oint_{C_B} G\phi_j^{(1)}n_1 dl - i\tau\iint_{S_F}[\phi_j^{(1)}\nabla_2 G\cdot\nabla_2\chi - G\nabla_2\phi_j^{(1)}\cdot\nabla_2\chi]ds + \begin{cases} \phi_0^{(1)}(x_0) & \text{for } \phi_B^{(1)} \\ \iint_{S_B}\left(G+\frac{i\tau}{\nu_0}\nabla G\cdot\nabla\chi_s\right)n_j ds & \text{for } j=1, \cdots, 6. \end{cases} \tag{30}$$

Since the derivative of the steady disturbance χ on the free surface decays rapidly with increasing distance from the body, the integration on the free surface is needed only in a small area around the body.

Because the calculation of the Green's function is very expensive and the unknowns are both on the body surface and the free surface, it is not economic to use Eq. (30) directly for practical application. Here the perturbation method is introduced to expand the Green's function into Taylor series in terms of the current parameter τ

$$G(-\tau)=G^{(0)}+\tau G^{(1)}+0(\tau^2), \tag{31}$$

where $G^{(0)}$ is the same as the Green function for the wave problem without current, and

$$G^{(1)}=-2i\partial^2 G^{(0)}/\partial\nu_0\partial x. \tag{32}$$

Substituting equations (5) and (31) into equation (30) and collecting the same order terms in τ, we can derive two sets of integral equations as follows

$$\left[1-\iint_{S_W}\nu G^{(0)}(x, x_0)dxdy\right]\phi_j^{(10)}(x_0)$$

$$+\iint_{S_B}[\phi_j^{(10)}(x_0)-\phi_j^{(10)}(x)]\frac{\partial G^{(0)}}{\partial n}ds \quad (33)$$
$$=\begin{cases}\phi_0^{(1)} & \text{for } \phi_D^{(10)}\\ \iint_{S_B}G^{(0)}n_jds & \text{for } j=1,\cdots,6\end{cases}$$

for the zero order terms in τ; and

$$\left[1-\iint_{S_W}\nu G^{(0)}dxdy\right]\phi^{(11)}(x_0)$$
$$+\iint_{S_B}[\phi^{(11)}(x_0)-\phi^{(11)}(x)]\frac{\partial G^{(0)}}{\partial n}ds$$
$$=\iint_{S_B}\frac{\partial G^{(1)}}{\partial n}[\phi^{(10)}(x)-\phi^{(10)}(x_0)]ds$$
$$+\iint_{S_W}(\nu G^{(1)}-2iG_x^{(0)})\phi^{(10)}ds$$
$$+i\oint_{C_B}G\phi^{(10)}n_1dl$$
$$-i\iint_{S_F}[\phi^{(10)}\nabla_2G^{(0)}\cdot\nabla_2\chi+G^{(0)}\nabla_2\phi^{(10)}\cdot\nabla_2\chi]ds$$
$$+\begin{cases}0 & \text{for } \phi_D^{(11)}\\ \iint_{S_B}\left(G^{(1)}+\frac{i}{\nu_0}\nabla G^{(0)}\cdot\nabla\chi_s\right)n_jds & \text{for } j=1,\cdots,6\end{cases} \quad (34)$$

for the first order terms in τ. The calculation of the remaining Cauchy principal value (CPV) integrations is conducted directly by a numerical method (Teng and Eatock Taylor[15]), in which a technique is applied to separate out a singularity whose CPV integration vanishes, while assuring that the integration of the remaining term is straightforward.

3.3 Second order steady potential

The second order steady potential satisfies the boundary conditions

$$\frac{\partial\Phi^{(20)m}}{\partial n}=-\frac{\sigma}{2g}\mathrm{Im}[\phi^{(10)}\partial^2\phi^{(10)*}/\partial z^2] \quad (35)$$

on the still water surface, and

$$\frac{\partial\Phi^{(20)m}}{\partial n}=\frac{1}{2}Re[-n\cdot[(\xi^{(10)}+\alpha^{(10)}\times x)\cdot\nabla]\nabla\phi^{(10)*}$$
$$+(\alpha^{(10)}\times n)\cdot[i\sigma(\xi^{(10)}+\alpha^{(10)}\times x)-\nabla\phi^{(10)}]^*] \quad (36)$$

on the body surface, where * denotes the complex conjugate. The second order steady potential comes from the evanescent modes of first order potentials. It always exists in current, even in the case where it vanishes in still water, for example, a fixed uniform cylinder.

Applying G_0 as Green's function, the integral equation for the second order steady potential can be written as

$$\Phi^{(20)m}(x_0)+\iint_{S_B}\frac{\partial G_0}{\partial n}[\Phi^{(20)m}(x_0)-\Phi^{(20)m}(x)]ds$$
$$=-Re\left[\iint_{S_B}\frac{1}{2}\{-n\cdot[(\xi^{(10)}+\alpha^{(10)}\times x)\cdot\nabla]\nabla\phi^{(10)*}\right.$$
$$\left.+(\alpha^{(10)}\times n)\cdot[i\sigma(\xi^{(10)}+\alpha^{(10)}\times x)-\nabla\phi^{(10)}]^*\}G_0ds\right]$$
$$-\frac{\sigma}{2g}Im\left[\iint_{S_F}\phi^{(10)}(x)\nabla_2^2\phi^{(10)*}(x)G_0ds\right] \quad (37)$$

The integral equation also includes second derivatives in both the integral on the free surface and the body surface. To get rid of second derivatives, following transforms are used

$$Im\left[\iint_{S_F}G_0\phi^{(10)}(x)\nabla_2^2\phi^{(10)*}(x)ds\right]$$
$$=Im\left[-\iint_{S_F}\phi^{(10)}(x)\nabla_2G_0\cdot\nabla_2\phi^{(10)*}(x)ds\right.$$
$$\left.+\oint_{C_I-C_B}G_0\phi^{(10)}(x)\frac{\partial\phi^{(10)*}(x)}{\partial n}dl\right] \quad (38)$$

for the integral on the free surface, where C_I is a contour at outer boundary of the mesh on the free surface, and

$$\iint_{S_B}G_0[(\delta^{(10)}\cdot\nabla)\nabla\phi^{(10)*}]\cdot nds$$
$$=\iint_{S_B}[(n\cdot\delta^{(10)})(\nabla G_0\cdot\nabla\phi^{(10)*})-(\nabla G_0\cdot\delta^{(10)})$$
$$(\nabla\phi^{(10)*}\cdot n)-G_0(\alpha^{(10)}\times n)\cdot\nabla\phi^{(10)*}]ds$$
$$-\oint_{C_B}G_0(\nabla\phi^{(10)*}\times\delta^{(10)})\cdot dl \quad (39)$$

for the integral on the body surface, where $\delta=\xi+\alpha\times x$. Furthermore, to remove CPV integrals, which appear in the discretization by high order elements, the following relation

$$\iint_{S_B}[(n\cdot\delta^{(10)})(\nabla G_0\cdot\nabla\phi^{(10)*}(x_0))$$
$$-(\nabla G_0\cdot\delta^{(10)})(\nabla\phi^{(10)*}(x_0)\cdot n)$$
$$-G_0(\alpha^{(10)}\times n)\cdot\nabla\phi^{(10)*}(x_0)]ds$$
$$-\oint_{C_B}G_0(\nabla\phi^{(10)*}(x_0)\times\delta^{(10)})\cdot dl=0 \quad (40)$$

is added to the integral on the body surface, and it yields

$$\iint_{S_B}[(\delta^{(10)}\cdot\nabla)\nabla\phi^{(10)*}]\cdot nds$$
$$=\iint_{S_B}[(n\cdot\delta^{(10)})(\nabla G_0\cdot(\nabla\phi^{(10)*}(x)-\nabla\phi^{(10)*}(x_0))$$
$$-(\nabla G_0\cdot\delta^{(10)})(\nabla\phi^{(10)*}(x)-\nabla\phi^{(10)*}(x_0))\cdot n)$$
$$-G_0(\alpha^{(10)}\times n)\cdot(\nabla\phi^{(10)*}(x)-\nabla\phi^{(10)*}(x_0))]ds$$
$$-\oint_{C_B}G_0[(\nabla\phi^{(10)*}(x)-\nabla\phi^{(10)*}(x_0))\times\delta^{(10)}]\cdot dl \quad (41)$$

Then, integration can be done in a straightforward manner.

4. Hydrodynamic force

The hydrodynamic forces and moments on bodies can be obtained by direct integration of the hydrodynamic pressure on body surface. This method is called as the near field method. When approximating to the first order in terms of current parameter τ, the leading order exciting force in terms of wave slope ε can be written as

$$F^{(1)}=-\rho\iint_{S_B}Re[(i\sigma\phi^{(1)}$$
$$+U\nabla\chi_s\cdot\nabla\partial^{(01)})e^{i\sigma t}]nds \quad (42)$$

The first order force in wave slope is usually divided into exciting force and hydrodynamic coefficients, which correspond to diffraction and radiation potentials, respectively. The hydrodynamic coefficients satisfy the Timman-Newman relation[5)16)]

$$f_{ij}(\tau)=\omega^2a_{ij}+i\omega_{bij}$$
$$=\iint_{S_B}\phi_in_jds=f_{ji}(-\tau)\quad(j,i=1,\cdots,6) \quad (43)$$

which can be used to check the correction and the accuracy of obtained potentials at first order of wave slope.

The equation of near field method for the second order mean drift force is

$$F_m^{(2)}=F_{\mathrm{I}}^{(2)}+F_{\mathrm{II}}^{(2)}+F_{\mathrm{III}}^{(2)}+F_{\mathrm{IV}}^{(2)}+F_W^{(2)} \qquad (44)$$

where $F_{\mathrm{I}}^{(2)}$, $F_{\mathrm{II}}^{(2)}$, $F_{\mathrm{III}}^{(2)}$, $F_{\mathrm{IV}}^{(2)}$ and $F_W^{(2)}$ are defined by

$$F_{\mathrm{I}}^{(2)}=-\rho\iint_{S_B} Re[\frac{1}{4}\nabla\phi^{(10)*}\cdot(\nabla\phi^{(10)}+2\tau\nabla\phi^{(11)})]nds$$

$$F_{\mathrm{II}}^{(2)}=-\rho\iint_{S_B} U\nabla\chi_s\cdot\nabla\phi^{(20)m}nds$$

$$F_{\mathrm{III}}^{(2)}=-\frac{\rho}{2}\iint_{S_B} Re[i\sigma\nabla\phi^{(1)*}\cdot(\xi^{(1)}+\alpha^{(1)}\times(x-x_c))n + i\sigma\alpha^{(1)*}\times n\phi^{(1)}]ds$$

$$F_{\mathrm{IV}}^{(2)}=-\frac{\rho g}{2}A_{WP}Re[(x_f-x_c)\xi_4^{(1)*}\xi_6^{(1)} + (y_f-y_c)\xi_5^{(1)*}\xi_6^{(1)} - \frac{1}{2}(\xi_4^{(1)}\xi_4^{(1)*}+\xi_5^{(1)}\xi_5^{(1)*})z_c]k$$

$$F_W^{(2)}=\frac{\rho g}{2}\int_{C_B} Re[\frac{1}{2}\zeta^{(10)}\zeta^{(10)*}+\tau\zeta^{(10)}\zeta^{(11)*} + \zeta^{(1)*}(\xi_3^{(1)}+(y-y_c)\xi_4^{(1)}-(x-x_c)\xi_5^{(1)})]ndl \qquad (45)$$

(x_c, y_c, z_c) and (x_f, y_f, z_f) are the coordinates of centres of gravity and floatation, A_{WP} is the area of waterplane.

For fixed bodies, the application of the above equation is not a difficult job. However, when bodies are free to move, the multiply of the first derivatives of first order potentials will introduce some difficulty. The multiply in $F_{\mathrm{I}}^{(2)}$ can be represented as

$$\iint_{S_B}\nabla\phi^{(10)*}\cdot\nabla\phi^{(11)}nds=\iint_{S_B}\Big[\frac{\partial\phi^{(10)*}}{\partial t_1}\frac{\partial\phi^{(11)}}{\partial t_1} + \frac{\partial\phi^{(10)*}}{\partial t_2}\frac{\partial\phi^{(11)*}}{\partial t_2}+\frac{\partial\phi^{(10)*}}{\partial n}\frac{\partial\phi^{(11)}}{\partial n}\Big]ds \qquad (46)$$

by two independent unit vectors t_1 and t_2 in the plane tangent to the body. The normal derivative of velocity potential $\phi^{(11)}$ is a combination of m_j terms, which includes second derivatives of steady potential on the body surface. Thus, to apply the near field method to compute the second order mean drift forces, a direct evaluating method for the second derivatives of potentials on body surface has to be developed.

As in still water, a far field method can also be developed for the horizontal modes of mean drift force upon using the principle of conservation of momentum. This method can avoid using the second order derivatives of the steady potential on body surface and is believed to give more accurate results. Nossen et al[5] have obtained the far field equations for the case of infinity water depth, and Grue and Biberg[17] got the following equations

$$\frac{F_x}{\rho g A^2}=-\frac{g\nu}{4\omega^2}\Big\{\int_0^{2\pi}(C_g(\nu_1 h)\cos\theta + 2\tau\sin^2\theta)|H(\theta)|^2 d\theta + 2C_g(kh)\cos\beta\, Re[S]\Big\}$$

$$\frac{F_y}{\rho g A^2}=-\frac{g\nu}{4\omega^2}\Big\{\int_0^{2\pi}(C_g(\nu_1 h)\sin\theta - 2\tau\sin\theta\cos\theta)|H(\theta)|^2 d\theta + 2C_g(kh)\sin\beta\, Re[S]\Big\}$$

$$\frac{M_z}{\rho g A^2}=-\frac{g}{4\omega^2}\Big\{\int_0^{2\pi}(C_g(\nu_1 h) - 2\tau\cos\theta)Im\Big[H(\theta)\frac{\partial H(\theta)^*}{\partial\theta}\Big]d\theta - 2\Big[\Big(1-\frac{k}{C_g}\frac{dC_g}{dk}\Big)\tau\sin\beta\, Im[S] + (C_g(kh)-2\tau\cos\beta)Im[S']\Big]\Big\} \qquad (47)$$

for finite water depth. The parameters in the above equation are defined by

$$S=\sqrt{\frac{2\pi}{\nu}}e^{i\pi/4}H^*(\beta+2\tau^h\sin\beta),$$
$$\tau^h=\tau/C_g(kh),\ C_g(kh)=\tanh kh+\frac{kh}{\cosh^2 kh}, \qquad (48)$$
$$g\nu\tanh(\nu h)=\nu_0,\ \nu_1=\nu(1+2\tau^h\cos\theta),$$

where H is the distribution of scattering wave amplitude at infinity.

5. NUMERICAL RESULTS

The theory described in the foregoing is applied to develop a general numerical procedure for computing the wave run-up and forces on a three dimensional body in a weak current.

Figure 1 shows the convergence with radius of the mesh on the free surface for each component of second order mean drift force on a restrained truncated cylinder. Index $_T$ denotes the total force. The cylinder has a radius a and draft $T/a=1$, and is in a water depth of $h/a=2$. The calculation is made at $ka=1.5$ and Froude number ($Fr=U/\sqrt{ga}$) 0.10. A positive current velocity is defined such that the waves propagate against the current. It can be seen that truncating errors are not very big when radius R/a of the mesh on the free surface is larger than 2, and very small when R/a is larger than 4.

Figure 2 shows the comparison of each component of the wave damping of the same truncated cylinder, obtained by numerical differentiation of each term of

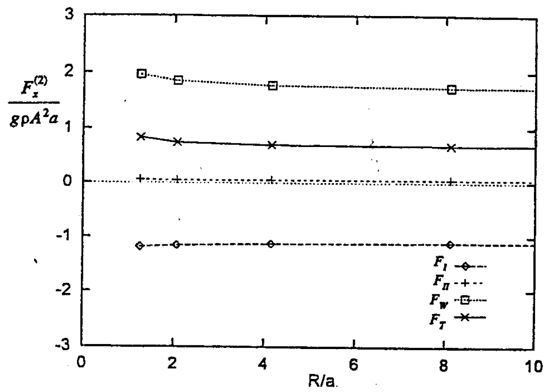

Fig. 1 Examination on the convergence with radius of the mesh on the free surface for second order drift force on a truncated cylinder at $Fr=-0.10$ and $ka=1.5$.

second mean drift force. It can be seen that the dominant contribution comes from the water-line integral and the body integral of the first order potential. At low frequency, the term from the second order mean potential is very small, but it increases with the increase of wave frequency and is not negligible at high frequency.

Figures 3 and 4 are the cross coupling surge and heave added mass and damping coefficients of an hemisphere at Froude numbers of ± 0.1. The reason we chosen these values is that they are zero in the still water problem and only come from the disturbance of the steady flow, so the calculation of those values are very sensible to the methods used, and can show their availability more clearly. From Figs. 3 and 4 it can be seen that the added mass and damping coefficients, $a_{13}(U)$ and $b_{13}(U)$, in the following current is close to the added mass and damping coefficients, $a_{31}(-U)$ and $b_{31}(-U)$, in a corresponding reverse current. Timman and Newman relationship is satisfied very well.

Figure 5 shows the comparison of the second order drift forces on the fixed hemisphere by the near field and the far field method at $Fr=-0.1$. It can be seen from the comparison that the good agreement exists between the two methods.

Figure 6 shows the comparison of the first order exciting force on a uniform circular cylinder of radius a in a water depth of $h/a=1$ with Matsui et al's[2)18)] analytic solution. In the calculation, a mesh of 16 (4 (circumferentially) x 4 (depthwise)) elements on a quadrant of body surface, and 32 (4 (circumferentially) x 8 (radially)) elements on a quadrant of free surface are applied. The comparison shows that the agreement with Matsui's analytic solution is very good. Fig. 7

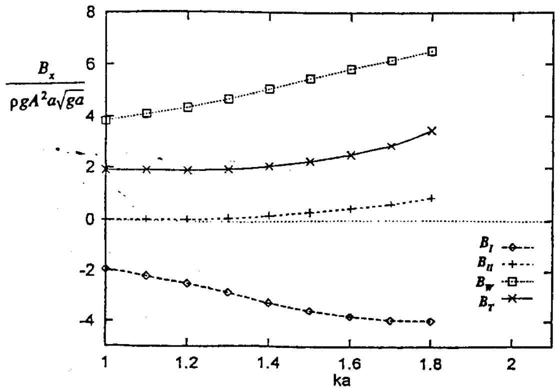

Fig. 2 Examination on the contribution of each term of the wave damping of the truncated cylinder.

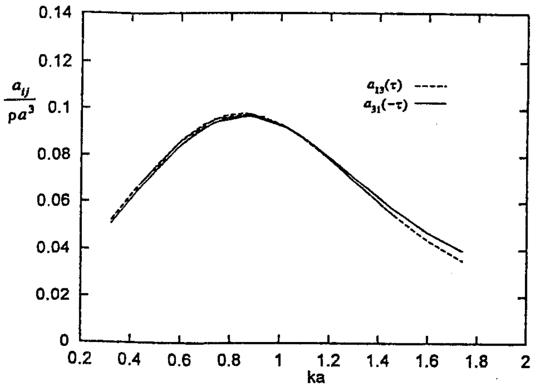

Fig. 3 Cross coupling surge-heave added mass of an hemisphere of radius a in a water depth of $h/a=2$

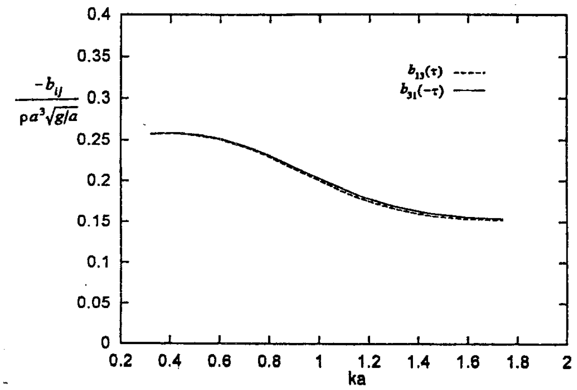

Fig. 4 Cross coupling surge-heave damping coefficients of an hemisphere of radius a in a water depth of $h/a=2$

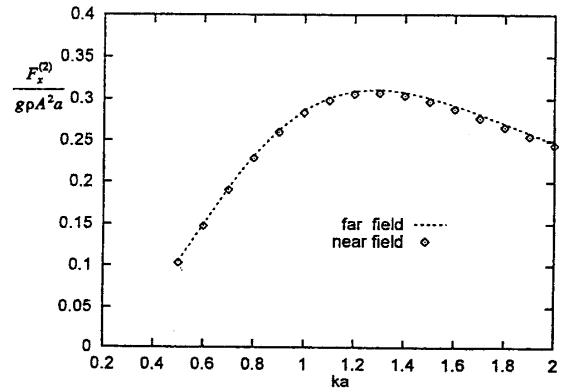

Fig. 5 Comparison of second order drift force on the hemisphere by the near and far field methods

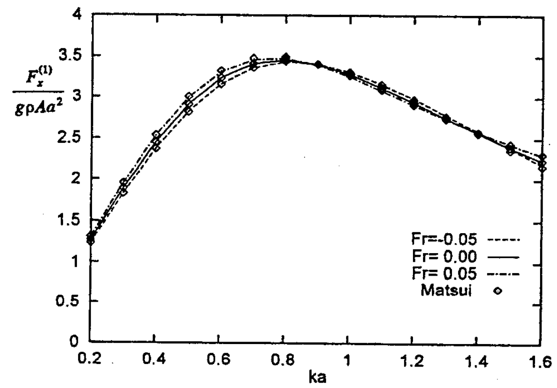

Fig. 6 Surge exciting forces on a uniform cylinder of radius a in a water depth of $h/a=1$.

shows the comparison of the second order mean drift force at different Froude number. The comparison with Matsui's analytic solution shows that at low frequency the two results agree very well, but at high frequency a little difference exists. It seems that the difference comes from the methods used in the calculation of wave forces. Matsui's method is to get them by the Taylor's expansion with the forces and its derivatives at zero current speed, but ours is to compute them directly at a given speed. Due to the nonlinearity of the dispersion equation with current speed and relatively stronger effect of current at high frequency, especially in an opposing current, our results diverge from Matsui et al's at high wave frequency and are not symmetric about the one in still water. From Figs. 6 and 7, it can also be seen that the current effect on the second order drift force is significant, but the effect on the first order exciting forces is relatively weak.

Figure 8 is the wave damping of the cylinder, which is obtained by the numerical differentiation of the mean drift forces at $Fr=\pm 0.05$ with respect to body moving speed. It can be seen that the wave damping reaches its maximum at about $ka=0.7$, and then oscillates with the increase of wave frequency.

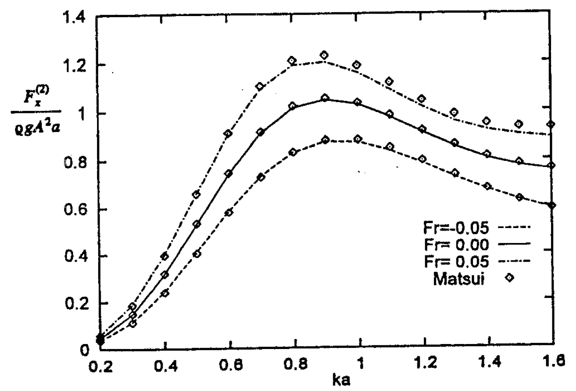

Fig. 7 Second order mean drift forces on a uniform cylinder of radius a in a water depth of $h/a=1$.

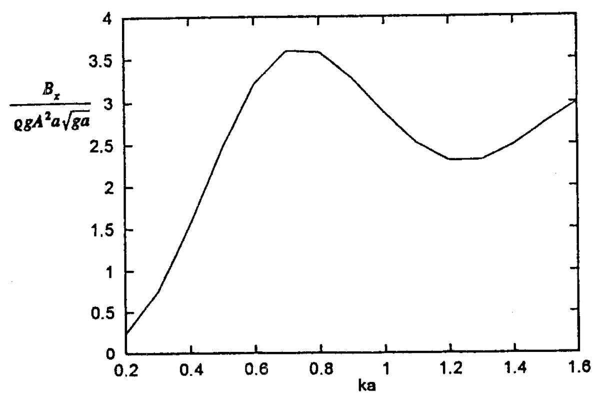

Fig. 8 Wave damping of a uniform cylinder of radius a in a water depth of $h/a=1$, obtained by numerical differentiation of the mean drift forces at $Fr=+/-0.05$.

Figures 9-11 show the comparison of the mean drift forces of present calculation by the far field method with Kinoshita et al's[8] experimental results of an array of four restrained cylinders. The cylinders are with radii of a and draft $T/a=2$, and are located at corners of a square with side length of $5a$. Figs. 12-14 show the comparison with Kinoshita et al's[8] freely moving experiments. The geometric factors of the cylinders are the same as the restrained case, and the inertia factors used in the present calculation are the same as Kinoshita et

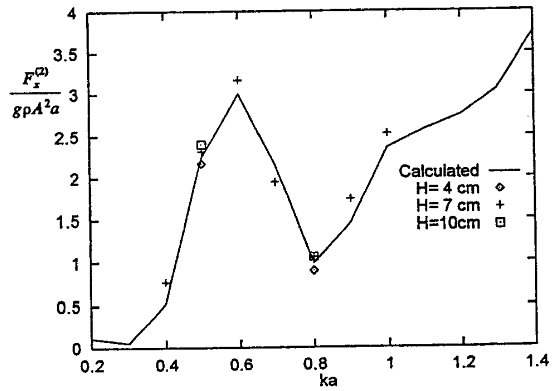

Fig. 9 Comparison of wave drift force on four restrained cylinders at $Fr=0.05$.

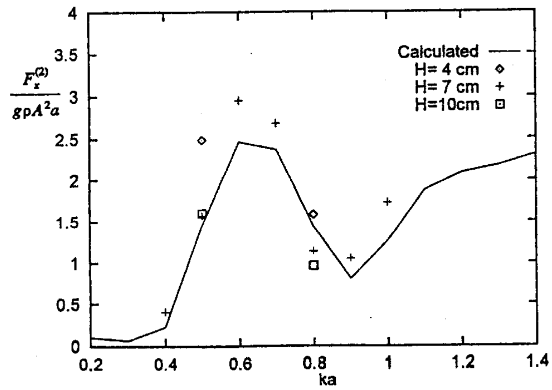

Fig. 10 Comparison of wave drift force on four restrained cylinders at $Fr=0.00$.

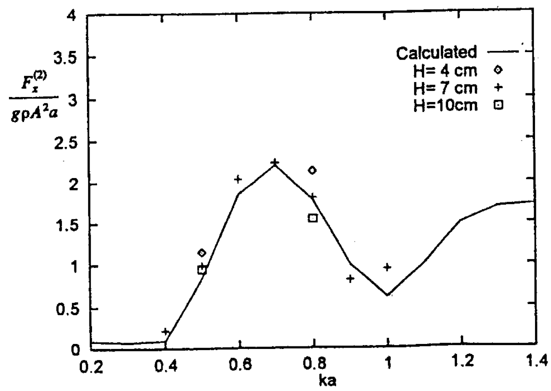

Fig. 11 Comparison of wave drift force on four restrained cylinders at $Fr=-0.05$.

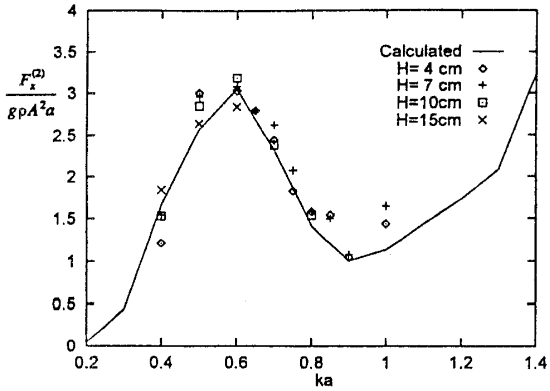

Fig. 12 Comparison of wave drift force on four freely moving cylinders at $Fr=0.05$.

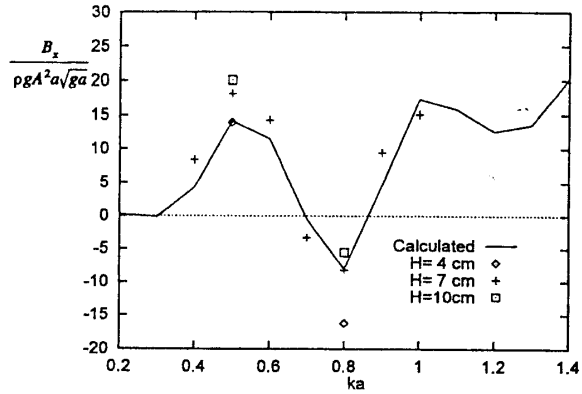

Fig. 15 Comparison of wave drift damping of four restrained cylinders.

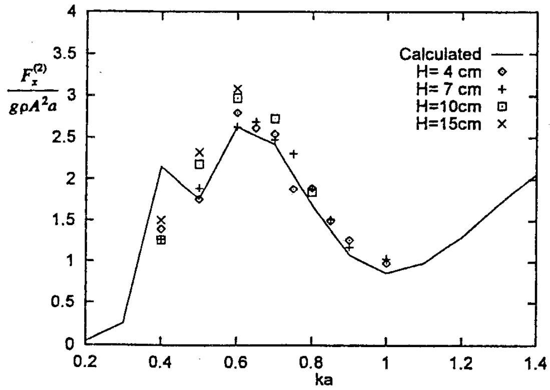

Fig. 13 Comparison of wave drift force on four freely moving cylinders at $Fr=0.00$.

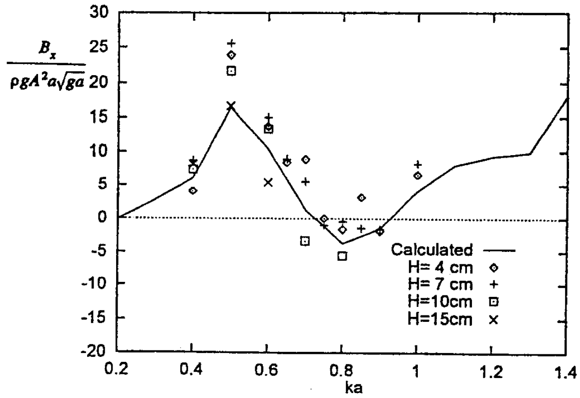

Fig. 16 Comparison of wave drift damping of four freely moving cylinders.

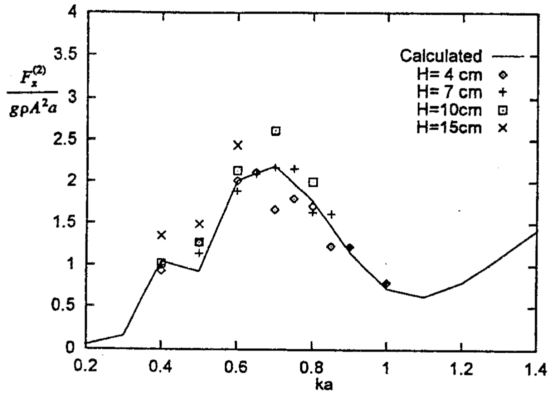

Fig. 14 Comparison of wave drift force on four freely moving cylinders at $Fr=-0.05$.

al's experiment. It can be seen that the present calculation has a good agreement with the experiments no matter in the restrained or freely moving cases.

Figures 15 and 16 show the wave drift damping, obtained by numerical differentiation of the mean drift force at $Fr=\pm 0.05$. It again shows that good agreement exists between the present calculation and the experimental results, and negative wave drift damping appears at about $ka=0.8$ both in experimental and calculated results.

Conclusions

1. All components of second order drift forces converge quickly with the increase of the radius of the mesh on the free surface. Truncating errors can be neglected when R/a is larger than 4.

2. When wave frequency is not very high, the contribution from the second order mean velocity potential is very small for the near field method. However, at high frequency, it is not negligible.

3. Timman-Newman relationship is satisfied very well and good agreement is found from the comparison of the first order exciting force with Matsui's analytic solution. It states that the method for the first order potential is correct and accurate.

4. Good agreements are found between the present calculation and the experimental results on an array of four restrained and freely moving cylinders. It validates the application of the present theory. Negative wave damping is again found at some wave frequency. It may induce big response of the system. Special concern is suggested to be paid for complex structures.

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