3. Non-Linear Hydrodynamic Forces Acting on Two-Dimensional Bodies

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Summary

The second-order forces acting on a cylindrical body which oscillates with an arbitrary frequency at a free surface of infinitely deep water are calculated on the basis of the perturbation thoery.

The first and second-order boundary value problems are solved by the Boundary Element Method (BEM) which includes both boundaries of the body and the free surfaces. The pressure distribution including the quadratic terms of the Bernoulli equation is evaluated from the solution. The hydrodynamic forces acting on the body are obtained by the integration along the instantaneous wetted contour of the body. Finally, motions of the body in waves are determined by the solution of the equation of the motion up to the second order.

Experiments are carried out for the radiation problems of the heaving and swaying oscillations, and the diffraction problems for a fixed body and a free floating body in steep regular waves.

Those results are discussed in comparison with the numerical calculations.

1. Introduction

In the field of seakeeping quality of ships in waves, many remarkable achievements have been made to predict the hydrodynamic forces acting on ships on the basis of the linearized wave theory. On the other side, a few researchers have attempted to the nonlinear problems. The nonlinear forces have been thought to be very small compared with the linear ones in this field, and are usually neglected. However, they sometimes play a primary role in the problems such as the drift forces, the slowly drifting oscillations or unstable swaying oscillations of a mored vessel. Motions of the recent ocean-platform are fairly different from those of a traditional ship that the nonlinear problems seem to be more important.

Kochin[1] derived the formulas of the steady forces on the two- and three-dimensional body in waves, and Maruo[2] showed the well-known formula of the drift force. Ogilvie[3] obtained the second-order steady forces on a submerged circular cylinder by the perturbation method.

Lee[4] and Parisis[5] presented the complete solution of the second-order forces on a cylindrical body heaving at the free surface. Their formulation seems to have provided the fundamentals of the investigations thereafter, and the theory was extended to the further problems by Potash[6], Söding[7], Masumoto[8] and Papanikolaou-Nowacki[9]. Kim[10] and Yamashita[11] derived an approximate solution for the heaving oscillation of a two-dimensional body.

On the other side, there are few reports on the experiments of the full second-order forces except Tasai-Koterayama[12] and Yamashita[11] in the heaving oscillations of the two-dimensional problem.

In this paper, the author would summarize the theoretical and experimental results of

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four papers on the second-order forces acting upon a cylindrical body of the radiations of swaying and heaving oscillations, and of the diffractions of a fixed and free-floating cylinder in waves.[13,14,15,16]

2. Mathematical Formulation

2.1 *Boundary Conditions*

Let us suppose the motions of a floating body in waves as shown in Fig. 1. We employ two coordinate systems, o-xy be a right-handed coordinate system fixed in space with o-y vertical downward and o-x lying in the undisturbed free surface, $\bar{o}$-$\bar{x}\bar{y}$ be a system fixed in the body and coincides with o-xy when the body lies in a position of equilibrium.

Let the displacement of the motion denote $x_j(t)$, where subscript $j=(1,2,3)$ refers to sway, heave and roll motions respectively, then the relation between the two systems is as follows:

$$\left.\begin{aligned} x(t)&=\bar{x}\cos x_3(t)-\bar{y}\sin x_3(t)+x_1(t)\\ y(t)&=\bar{y}\cos x_3(t)+\bar{x}\sin x_3(t)+x_2(t)\ . \end{aligned}\right\} \tag{1}$$

Here, we assume the floating body oscillates about its equilibrium position and the drift motion is restrained, that is, the external force cancels out the drifting force acting on it.

We will now assume that the fluid is ideal (invicid, imcompressible) and its motion is irrotational. Hence there exists a velocity potential $\Phi(x,y,t)$ satisfying Laplace's equation

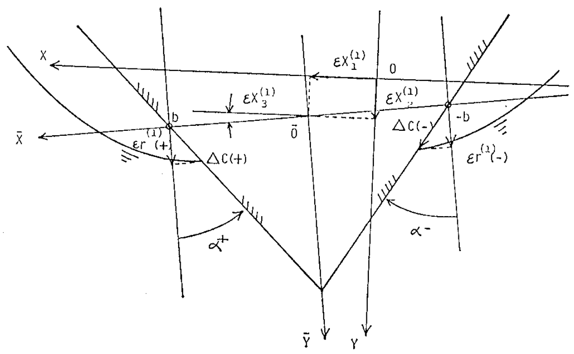

Fig. 1 Coordinate systems

$$[L]\quad \nabla^2\Phi=\Phi_{xx}+\Phi_{yy}=0\ . \tag{2}$$

The fluid pressure $P\ (x,y,t)$ is determined by Bernoulli's equation

$$P=-\rho\Phi_t-\frac{1}{2}\rho(\nabla\Phi)^2+\rho gy+P_0\ , \tag{3}$$

where ρ is the fluid density, g the gravitational acceleration constant and P_0 a constant of integration.

On the free surface two boundary conditions must be imposed. If the free surface is described by $y=\eta(x,t)$, the kinematic boundary condition is

$$0=\frac{D}{Dt}(y-\eta(x,t))=\Phi_y-\Phi_x\eta_x-\eta_t \quad \text{on } y=\eta(x,t)\ , \tag{4}$$

and the dynamic condition is obtained from the Bernoulli's equation on the free surface where the pressure should be atmospheric constant, we choose as $P=P_0$

$$\eta=\frac{1}{g}\left(\Phi_t+\frac{1}{2}\nabla\Phi\nabla\Phi\right) \quad \text{on } y=\eta(x,t)\ . \tag{5}$$

From these conditions, we obtain the non linear free surface boundary condition

$$\begin{aligned}[F]\quad 0&=\frac{D}{Dt}\left(\Phi_t+\frac{1}{2}\nabla\Phi\nabla\Phi-gy\right)\\ &=\Phi_{tt}-g\Phi_y+2\nabla\Phi\nabla\Phi_t\\ &\quad+\frac{1}{2}\nabla\Phi\nabla(\nabla\Phi\nabla\Phi)\\ &\quad \text{on } y=\eta(x,t)\ . \end{aligned} \tag{6}$$

If the body surface is described by $C\ (x,y,t)=C_0(\bar{x},\bar{y})=0$, the kinematic boundary condition on it states that the fluid at a point on the body must have the same velocity component in the direction of the normal to the body.

$$\begin{aligned}[H]\quad \Phi_n(x,y,t)&=V_n(x,y,t)\\ &=\frac{\partial x}{\partial n}\frac{\partial x}{\partial t}+\frac{\partial y}{\partial n}\frac{\partial y}{\partial t}\\ &\quad \text{on } C(x,y,t)=0\ , \end{aligned} \tag{7}$$

where the subscript n denotes the unit normal on body into the fluid.

If the fluid has a horizontal bottom at $y=h$, the kinematic boundary condition is

$$[B] \quad \Phi_y(x, h, t)=0 . \tag{8}$$

If it is infinitely deep, then

$$[B] \quad \lim_{y\to\infty} \Phi_y=0 . \tag{8$'$}$$

The remaining boundary condition is a radiation condition at infinity, i.e., the waves must be propagating outward at a large distance from the body.

2.2 *Linearized Problems*

The problem formulated above is nonlinear and the boundaries in which the velocity potential is defined change with time. In order to reduce the nonlinear boundary conditions, we assume the potential Φ can be expanded in a perturbation series in terms of ε:

$$\Phi(x, y, t)=\varepsilon\Phi^{(1)}(x, y, t) +\varepsilon^2\Phi^{(2)}(x, y, t)+0(\varepsilon^3) , \tag{9}$$

where ε denotes a small perturbation parameter and may be defined such as the ratio of the incident wave amplitude to the half-beam of the body.

Similarly, the free surface elevation is assumed to be expanded:

$$\eta(x, t)=\varepsilon\eta^{(1)}(x, t) +\varepsilon^2\eta^{(2)}(x, t)+0(\varepsilon^3) . \tag{10}$$

The boundary condition on the free surface can be reduced by expanding $\Phi(x, y, t)$ in Taylor series about $y=0$ like as:

$$\Phi(x, \eta, t)=\varepsilon\Phi^{(1)}(x, 0, t) +\varepsilon^2\{\eta^{(1)}\Phi_y^{(1)}(x, 0, t) +\Phi^{(2)}(x, 0, t)\}+0(\varepsilon^3) . \tag{11}$$

Then we obtain the first and second order boundary conditions on the free surface as:

$$\left.\begin{aligned}[F] \quad \varepsilon: &\quad \Phi_{tt}^{(1)}-g\Phi_y^{(1)}=0 \\ \varepsilon^2: &\quad \Phi_{tt}^{(2)}-g\Phi_y^{(2)} \\ &\quad =-2(\Phi_x^{(1)}\Phi_{xt}^{(1)}+\Phi_y^{(1)}\Phi_{yt}^{(1)}) \\ &\quad +\Phi_t^{(1)}\left(\Phi_{yy}^{(1)}-\frac{1}{g}\Phi_{tty}^{(1)}\right) .\end{aligned}\right\} \tag{12}$$

Further, we will assume the body motions are also small as the amplitude of the incident waves and expand as:

$$x_j(t)=\varepsilon x_j^{(1)}(t)+\varepsilon^2 x_j^{(2)}(t)+0(\varepsilon^3) \quad (j=1, 2, 3) . \tag{13}$$

Therefore Eq. (1) can be reduced as

$$\left.\begin{aligned} x-\bar{x}&=\varepsilon(x_1^{(1)}-\bar{y}x_3^{(1)}) \\ &\quad +\varepsilon^2\left(x_1^{(2)}-\bar{y}x_3^{(2)}-\frac{1}{2}\bar{x}x_3^{(1)2}\right)+0(\varepsilon^3) \\ y-\bar{y}&=\varepsilon(x_2^{(1)}+\bar{x}x_3^{(1)}) \\ &\quad +\varepsilon^2\left(x_2^{(2)}+\bar{x}x_3^{(2)}-\frac{1}{2}\bar{y}x_3^{(1)2}\right)+0(\varepsilon^3) .\end{aligned}\right\} \tag{14}$$

Now, we expand both sides of Eq. (7) in Taylor series about its mean position

$$\begin{aligned}\Phi_n(x, y, t) &=\Phi_n^{(1)}(\bar{x}, \bar{y}, t)+(x-\bar{x})\Phi_{xn}^{(1)}+(y-\bar{y})\Phi_{yn}^{(1)} \\ &\quad +\Phi_n^{(2)}(\bar{x}, \bar{y}, t)+0(\varepsilon^3) \\ &=\left(\frac{\partial\bar{x}}{\partial n}-\varepsilon\frac{\partial}{\partial n}\bar{y}x_3^{(1)}\right)\left\{\varepsilon(\dot{x}_1^{(1)}-\bar{y}\dot{x}_3^{(1)}) +\varepsilon^2\left(\dot{x}_1^{(2)}-\bar{y}\dot{x}_3^{(1)}-\frac{1}{2}\bar{x}\dot{x}_3^{(1)2}\right)\right\} \\ &\quad +\left(\frac{\partial\bar{y}}{\partial n}+\varepsilon\frac{\partial}{\partial n}\bar{x}x_3^{(1)}\right)\left\{\varepsilon(\dot{x}_2^{(1)}+\bar{x}\dot{x}_3^{(1)}) +\varepsilon^2\left(\dot{x}_2^{(2)}+\bar{x}\dot{x}_3^{(1)}-\frac{1}{2}\bar{y}\dot{x}_3^{(1)2}\right)\right\}+0(\varepsilon^3) ,\end{aligned} \tag{15}$$

where $\dot{x}_j=\dfrac{\partial}{\partial t}x_j \quad (j=1, 2, 3)$.

And using the following relations

$$\left.\begin{aligned}\frac{\partial}{\partial n}\bar{x}&=\frac{\partial}{\partial s}\bar{y}\equiv\bar{y}' \\ \frac{\partial}{\partial n}\bar{y}&=-\frac{\partial}{\partial s}\bar{x}\equiv-\bar{x}' ,\end{aligned}\right\} \tag{16}$$

we transform Eq. (15) to tangential and normal components of the body surface and obtain the following results[7].

$$\left.\begin{aligned}[H] \quad \varepsilon: &\quad \Phi_n^{(1)}=f_t^{(1)} \\ \varepsilon^2: &\quad \Phi_n^{(2)}=f_t^{(2)}+x_3^{(1)}C_t^{(1)}-x_3^{(1)}\Phi_s^{(1)} \\ &\quad -f^{(1)}\Phi_{nn}^{(1)}-d^{(1)}\Phi_{sn}^{(1)}\end{aligned}\right\} \quad \text{on } C_0(x, y)=0 , \tag{17}$$

where

$$a=\bar{x}\bar{x}'+\bar{y}\bar{y}', \quad b=\bar{x}\bar{y}'-\bar{y}\bar{x}',$$
$$c^{(n)}=\bar{x}'x_1^{(n)}+\bar{y}'x_2^{(n)}, \quad d^{(n)}=c^{(n)}+bx_3^{(n)},$$
$$h^{(n)}=\bar{y}'x_1^{(n)}-\bar{x}'x_2^{(n)}, \quad f^{(n)}=h^{(n)}-ax_3^{(n)}$$
$$(n=1,2)$$

And we can use the following relations

$$\left.\begin{aligned} \Phi_{nn}^{(1)}&=-\Phi_{ss}^{(1)}-\frac{1}{\rho}\Phi_n^{(1)} \\ \Phi_{sn}^{(1)}&=\Phi_{ns}^{(1)}-\frac{1}{\rho}\Phi_s^{(1)} \end{aligned}\right\} \tag{18}$$

$1/\rho=\bar{x}'\bar{y}''-\bar{y}'\bar{x}''$;

ρ: radius of the curvature.

Neglecting the transient response, we may expand the velocity potential associated with the each harmonic components using the fundamental angular frequency of the incident wave

$$\Phi(x,y,t)=Re\left\{\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}\varepsilon^n{}_k\varphi^{(n)}(x,y)e^{ik\omega t}\right\}. \tag{19}$$

Further we can reduce the combination of n and k in Eq. (19) as is already proved by Lee[4]

$$\Phi(x,y,t)=Re\{\varepsilon\varphi^{(1)}e^{i\omega t}+\varepsilon^2({}_0\varphi^{(2)}+{}_2\varphi^{(2)}e^{2i\omega t})\}+0(\varepsilon^3), \tag{20}$$

and ${}_0\varphi^{(2)}$ is also proved to contribute only to the mass transport of the fluid and not contribute to the pressure or the force of the body, then we will omit hereafter.

The second order incident wave potential is expressed as follows, if it is infinitely deep,

$$\left.\begin{aligned} \varepsilon\varphi_0^{(1)}&=\frac{iga_w}{\omega}e^{-Ky+iKx}, \\ \varepsilon^2\varphi_0^{(2)}&=0, \\ \frac{\omega^2}{g}&=K=2\pi/\lambda \end{aligned}\right\} \tag{21}$$

The free surface elevation in this case is given as

$$\left.\begin{aligned} \eta_0(x,t)&=Re\{\eta_0^{(1)}e^{i\omega t}+\varepsilon^2\eta_0^{(2)}e^{i2\omega t}\}+0(\varepsilon^3), \\ \varepsilon\eta_0^{(1)}&=-a_we^{iKx}, \\ \varepsilon^2\eta_0^{(2)}&=-\frac{Ka_w^2}{2}e^{i2Kx}. \end{aligned}\right\} \tag{22}$$

The first and second order boundary value problems in infinitely deep water are summerized as follows:

First order problem

$$\left.\begin{aligned} &[L] \quad \nabla^2\varphi^{(1)}(x,y)=0, \\ &[F] \quad \left\{K+\frac{\partial}{\partial y}\right\}\varphi^{(1)}(x,0)=0, \\ &[H] \quad \varphi_n^{(1)}=i\omega f^{(1)} \qquad \text{on } C_0, \\ &[B] \quad \varphi_y^{(1)}(x,\infty)=0, \\ &[R] \quad \left\{\frac{\partial}{\partial x}\pm iK\right\}\varphi^{(1)}(\pm\infty,y)=0, \end{aligned}\right\} \tag{23}$$

where $K=\dfrac{\omega^2}{g}=2\pi/\lambda$, λ: wave length,

$[R]$ is well-known as Sommerfeld's radiation condition.

Second order problem $({}_2\varphi^{(2)}\to\varphi^{(2)})$

$$\left.\begin{aligned} &[L] \quad \nabla^2\varphi^{(2)}(x,y)=0, \\ &[F] \quad \left\{4K+\frac{\partial}{\partial y}\right\}\varphi^{(2)}(x,0)=Q(x), \\ &[H] \quad \varphi_n^{(2)}=f_t^{(2)}+\frac{1}{2}(x_3^{(1)}C_t^{(1)}-x_3^{(1)}\varphi_s^{(1)} \\ &\qquad\qquad -f^{(1)}\varphi_{nn}^{(1)}-d^{(1)}\varphi_{sn}^{(1)}) \qquad \text{on } C_0, \\ &[B] \quad \varphi_y^{(2)}(x,\infty)=0, \\ &[R] \quad \left\{\frac{\partial}{\partial x}\pm i4K\right\}\varphi^{(2)}(\pm\infty,y)=0, \end{aligned}\right\} \tag{24}$$

where $Q(x)=\dfrac{i\omega}{2g}\{2(\nabla\varphi^{(1)})^2-\varphi^{(1)}(\varphi_{yy}^{(1)}+K\varphi_y^{(1)})\}$.

2.3 *Pressure, Forces and Moment*

Solving the boundary value problem of Eqs. (23) and (24), we obtain the distributions of the velocity potential and the pressure.

Let expand the pressure and the hydrodynamic forces in accordance with Eq. (20)

$$\left.\begin{aligned} P(x,y,t)&=Re\{p^{(0)}+\varepsilon p^{(1)}p^{i\omega t}+\varepsilon^2({}_0p^{(2)}+{}_2p^{(2)}e^{i2\omega t})\}+0(\varepsilon^3), \\ F_j(t)&=Re\{F_j^{(0)}+\varepsilon F_j^{(1)}e^{i\omega t}+\varepsilon^2({}_0F_j^{(2)}+{}_2F_j^{(2)}e^{i2\omega t})\}+0(\varepsilon^3), \end{aligned}\right\} \tag{25}$$

where subscript k of ${}_kp^{(2)}$ and ${}_kF_j^{(2)}$ denotes the frequency parameter of $e^{ik\omega t}$.

Pressure on the body surface are determined by Eq. (3) as follows:

$$\left.\begin{aligned}
p^{(0)}&=\rho g\bar{y}\ ,\\
p^{(1)}&=-i\rho\omega\varphi^{(1)}+\rho g(x_2^{(1)}+\bar{x}x_3^{(1)})\ ,\\
{}_0p^{(2)}&=\rho g\left({}_0x_2^{(2)}+\bar{x}_0x_3^{(2)}\right)\\
&\quad-\frac{1}{4}\rho g y|x_3^{(1)}|^2-\frac{1}{4}\rho|\nabla\varphi^{(1)}|^2\\
&\quad-\frac{i\rho\omega}{2}\{(x-\bar{x})\varphi_x^{(1)*}+\varphi_y^{(1)*}\}\ ,\\
{}_2p^{(2)}&=\rho g({}_2x_2^2+\bar{x}_2x_3^2)-i2\rho\omega\varphi^{(2)}\\
&\quad-\frac{1}{4}\rho g\bar{y}x_3^{(1)2}-\frac{1}{4}\rho(\nabla\varphi^{(1)})^2\\
&\quad-\frac{i\rho\omega}{2}\{(x-\bar{x})\varphi_x^{(1)}+(y-\bar{y})\varphi_y^{(1)}\}\ ,
\end{aligned}\right\}\tag{26}$$

where $\varphi^{(1)*}$ denotes the complex conjugate of $\varphi^{(1)}$.

We will calculate the potential distribution and its derivatives on the body surface from the solution of the first order problem as follows:

$$\left.\begin{aligned}
(\nabla\varphi^{(1)})^2&=(\varphi_n^{(1)})^2+(\varphi_s^{(1)})^2\\
&=(f_t^{(1)})+(\varphi_s^{(1)})^2\ ,\\
(x-\bar{x})\varphi_x^{(1)}+(y-\bar{y})\varphi_y^{(1)}&=f^{(1)}\varphi_n^{(1)}+d^{(1)}\varphi_s^{(1)}\\
&=f^{(1)}f_t^{(1)}+d^{(1)}\varphi_s^{(1)}\ .
\end{aligned}\right\}\tag{27}$$

The forces and moment are obtained in the following form by integrating the pressure along the body surface at instantaneous position and wetted contour.

$$F_j(t)=-\int_{C(t)}(P-P_0)\frac{\partial}{\partial n}x_j(t)\,ds\ ,\tag{28}$$

where $C(t)$ denotes the wetted contour; $C(t)=C_0-\varDelta C(t)$.

$\varDelta C(t)$ may be expanded with the same perturbation series as Eq. (10) and we take into account this effect in the hydrodynamic force to the accuracy of the second order and obtain

$$\begin{aligned}
\begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix}&=\int_{C(t)}(P-P_0)\\
&\quad\cdot\begin{Bmatrix}-\bar{y}'-\varepsilon x_3^{(1)}\bar{x}'-\varepsilon^2({}_2x_3^{(2)}\bar{x}'-(1/2)x_3^{(1)2}\bar{y}')\\ \bar{x}'-\varepsilon x_3^{(1)}\bar{y}'-\varepsilon^2({}_2x_3^{(2)}\bar{y}'+(1/2)x_3^{(1)2}\underline{x}\ )\\ a+\varepsilon c^{(1)}+\varepsilon^2(c^{(2)}-h^{(1)}x_3^{(1)})\end{Bmatrix}ds\\
&=\int_{C_0}(P-P_0)\\
&\quad\cdot\begin{Bmatrix}-\bar{y}'-\varepsilon x_3^{(1)}\bar{x}'-\varepsilon^2({}_2x_3^{(2)}\bar{x}'-(1/2)x_3^{(1)2}\bar{y}')\\ \bar{x}'-\varepsilon x_3^{(1)}\bar{y}'-\varepsilon^2({}_2x_3^{(2)}\bar{y}'+(1/2)x_3^{(1)2}\bar{x}')\\ a+\varepsilon c^{(1)}+\varepsilon^2(c^{(2)}-h^{(1)}x_3^{(1)})\end{Bmatrix}ds\\
&\quad-\int_{\varDelta C(t)}(P-P_0)\begin{Bmatrix}-\bar{y}'\\ \bar{x}'\\ a\end{Bmatrix}ds+0(\varepsilon^3)
\end{aligned}\tag{29}$$

Second term of the right hand side of Eq. (29) can be calculated by applying Leibniz's rule as

$$\begin{aligned}
&-\int_{\varDelta C(t)}(P-P_0)\frac{\partial x_j}{\partial n}ds\\
&=\int_0^{r^+(t)}(P-P_0)\,dy\left[\frac{1}{\bar{y}'}\frac{\partial\bar{x}_j}{\partial n}\right]_{(b,0)}\\
&\quad-\int_0^{r^-(t)}(P-P_0)\,dy\left[\frac{1}{\bar{y}'}\frac{\partial\bar{x}_j}{\partial n}\right]_{(-b,0)},\\
&=-\frac{\rho g\varepsilon^2}{2}\left[r^{+2}(t)\begin{Bmatrix}1\\ \tan\alpha^+\\ b\cdot\tan\alpha^+\end{Bmatrix}\right.\\
&\quad\left.-r^{-2}(t)\begin{Bmatrix}1\\ -\tan\alpha^-\\ b\cdot\tan\alpha^-\end{Bmatrix}\right]+0(\varepsilon^3)\ ,
\end{aligned}\tag{30}$$

where

$$r^{\pm}(t)=\varepsilon\left\{x_2^{(1)}\pm bx_3^{(1)}-\frac{i\omega}{g}\varphi^{(1)}(\pm b,0)\right\}e^{i\omega t}$$

=relative wave elevation from $\bar{y}=0$.

$\alpha^{\pm}$ denotes the angles of the intersection between hull-side and the free-surface as shown in Fig. 1.

In Eq. (30), we find that the angles of the intersections at free-surface do not effect to the horizontal forces in the second-order theory.

The aforementioned formulation can be simply applied for the radiation problem of j-mode, if we substitute the following equations.

$$\left.\begin{aligned}
a_w&=x_j^{(1)}=a_j\\
\varphi_0&=0
\end{aligned}\right\}\tag{31}$$

2.4 *The Equation of the Motion*

The inertia forces with respect to the origin o fixed on the body are determined with the consideration of displacements of the motion in Eq. (1). Equating them to the pressure forces of Eq. (28), we obtain the following results.

First Order

$$\left.\begin{aligned} &M(\ddot{x}_1^{(1)}-y_G\ddot{x}_3^{(1)})=F_1^{(1)}\\ &M\ddot{x}_2^{(1)}=F_2^{(1)}\\ &I\ddot{x}_3^{(1)}+Mgy_Gx_3^{(1)}-M(y_G\ddot{x}_1^{(1)}+gx_1^{(1)})=F_3^{(1)} \end{aligned}\right\} \tag{32}$$

where

$M=\rho A$; mass of the body per unit length,

$I=\iint\rho(x^2+y^2)\,dx\,dy=Mr_G{}^2$;

mass moment of inertia of the body with respect to the coordinate origin,

$(x_G,y_G)=(0,y_G)$; center of gravity of the body.

Quasi-Hydrostatics of the second order

We have assumed the external force which cancels out the drifting force. Let the D_f denote the external force as follows.

$$\left.\begin{aligned} &D_f=\sum_{n=1}^{4}{}_0F_1^{(2)}(n)\\ &0=\sum_{n=1}^{4}{}_0F_2^{(2)}(n)\\ &Mgy_{G\,0}x_3^{(2)}=\sum_{n=1}^{4}{}_0F_3^{(2)}(n) \end{aligned}\right\} \tag{33}$$

If the external force are provided by the restoring force of a mooring system, it may be expressed in the form.

$$D_f=k_0x_1^{(2)}\,, \tag{34}$$

where

k: spring constant of the mooring system,

${}_0x_1^{(2)}$: drifting displacement in sway.

Further, let us define the sinkage force (S_f) and the steady heeling moment (H_m) as follows:

$$\left.\begin{aligned} S_f&=2\rho gb_0x_2^{(2)}\\ &=\sum_{n=1}^{4}{}_0F_2^{(2)}(n)+2\rho gb_0x_2^{(2)}\\ H_m&=Mg\,\overline{GM}_0x_3^{(2)}\\ &=\sum_{n=1}^{4}{}_0F_3^{(2)}(n)-Mg\,y_{G\,0}x_3^{(2)} \end{aligned}\right\} \tag{35}$$

These steady forces may be obtained from the drift displacement of the motion in the free floating problem in waves.

Hydrodynamics of the second order

Although the heaving equation is influenced by the first order rolling motion, the swaying and rolling equations are the same as the first order case.

$$\left.\begin{aligned} &M({}_2\ddot{x}_1^{(2)}-y_{G\,2}\ddot{x}_3^{(2)})=\sum_{n=1}^{5}{}_2F_1^{(2)}(n)\\ &M\left[{}_2\ddot{x}_2^{(2)}-\frac{1}{2}y_G\{x_3^{(1)}\ddot{x}_3^{(1)}+(\dot{x}_3^{(1)})^2\}\right]\\ &\qquad=\sum_{n=1}^{5}{}_2F_2^{(2)}(n)\\ &I_2\ddot{x}_3^{(2)}+Mgy_{G\,2}x_3^{(2)}-M(y_{G\,2}\ddot{x}_1^{(2)}+g_2x_1^{(2)})\\ &\qquad=\sum_{n=1}^{5}{}_2F_3^{(2)}(n) \end{aligned}\right\} \tag{36}$$

3. Solution of the Problem

3.1 *First Order Problem*

The boundary value problems formulated in the preceding chaper can be reduced to the first order theories which have been solved by making use of the methods of multi-pole expansions, Green functions and variational method.

However, in the second order problem there appears inhomogenous boundary condition on the free surface. This means that we must evaluate the potential and its derivatives to calculate the pressure distribution on it and also evaluate their contribution to the body by integrating over the free surface. These processes may be simplified by applying the Boundary Element Method (BEM) which enable us to deal with the free surface as same as the body surface.

As for the problem treated here, Yeung[17] showed the numerical examples and found its satisfactory accuracy in the two and three dimensional radiation problems. He obtained the solution by integrating all the boundaries around the domain in which the potential satisfies Laplace equation. We will formulate the same problem by a slightly modified approach.

The potential can be expressed in the following form by applying Green's theorem.

$$\varphi(P)=\frac{1}{2\pi}\int_{C+F+B+R^{\pm}}\left(\frac{\partial}{\partial n}\varphi(Q)-\varphi(Q)\frac{\partial}{\partial n}\right)\cdot\log r(P,Q)\,ds(Q)\ , \tag{37}$$

where

$P=(x,y)$, $Q=(x',y')$ and $r^2=(x-x')^2+(y-y')^2$, $C,F,R^{\pm}$ and B denote the each boundaries on the body, free surface, right and left radiation boundaries and bottom of the fluid.

Now, we will decompose the first order potential into the following terms.

$$\left.\begin{aligned}&\varepsilon\varphi^{(1)}=\frac{iga_w}{\omega}\left\{\phi_0^{(1)}+\phi_4^{(1)}+K\sum_{j=1}^{3}\bar{x}_j^{(1)}\phi_j^{(1)}\right\}\ ,\\&\bar{x}_j^{(1)}=x_j^{(1)}/a_w\ ,\end{aligned}\right\} \tag{38}$$

where subscript $j=(0,1,2,3,4)$ refers to the incident wave, sway, heave, roll and diffraction respectively,

$\phi_0^{(1)}=e^{-Ky+iKx}$,
a_w=amplitude of the incident wave,
ω=angular frequency of the incident wave,
g=gravitational acceleration,
$K=\omega^2/g$=wave number.

The boundary conditions for each subproblems are rewritten as:

$$\left.\begin{aligned}&[L]\quad \nabla^2\phi_j^{(1)}(x,y)=0\\&[F]\quad \left\{K+\frac{\partial}{\partial y}\right\}\phi_j^{(1)}(x,0)=0\\&[H]\quad \frac{\partial}{\partial n}\phi_j^{(1)}=\frac{\partial}{\partial n}\bar{x}_j \qquad \text{for } j=1,2,3\\&\qquad\ \ \frac{\partial}{\partial n}\phi_j^{(1)}=-\frac{\partial}{\partial n}\phi_0^{(1)} \qquad \text{for } j=4\\&[B]\quad \frac{\partial}{\partial n}\phi_j^{(1)}(x,\infty)=0\\&[R]\quad \left\{\frac{\partial}{\partial x}\pm iK\right\}\phi_j^{(1)}(\pm\infty,y)=0\end{aligned}\right| \tag{39}$$

For a problem symmetrical with respect to $\bar{y}$-axis, let $\phi_S(x,y)$ be a source potential placed at the origin. Then $\phi_S(x,y)$ can be expressed by

$$\begin{aligned}\phi_S(P)&=\frac{1}{2\pi}\int_{C+F+R^{\pm}+B}\left(\frac{\partial}{\partial n}\phi_S(Q)-\phi_S(Q)\frac{\partial}{\partial n}\right)\log r(P,Q)\,ds(Q)\\&=-\frac{1}{\pi}\oint\frac{e^{-ky}\cos kx}{k-K}dk+ie^{-ky}\cos Kx\ ,\end{aligned} \tag{40}$$

where $\oint$ denotes the Cauchy's principal value. Asymptotic expansions for ϕ_S and $\phi_j^{(1)}$ are well-known as[18]:

$$\left.\begin{aligned}&\phi_S(P)\to ie^{-Ky\mp iKx} && as\ x\to\pm\infty\\&\phi_j^{(1)}(P)\to iH_j^{\pm}(K)e^{-Ky\mp iKx} && \text{as } x\to\pm\infty\end{aligned}\right\} \tag{41}$$

where

$$H_j^{\pm}(K)=\int_{C_0}\left(\frac{\partial}{\partial n}\phi_j^{(1)}-\phi_j^{(1)}\frac{\partial}{\partial n}\right)e^{-Ky\pm iKx}\,ds\ ;$$

Kochin function.

Let $\phi_N(P)$ be a new potential defined as follows:

$$\phi_N(P)=\phi_j^{(1)}(P)-H_j^{\pm}\phi_S(P)\ , \tag{42}$$

then, at infinity

$$\phi_N(P)\to 0 \qquad \text{as } x\to\pm\infty\ .$$

If we take the radiation boundaries $R^{\pm}$ far from the body and the problems are restricted in deep water case, the integration on $R^{\pm}$ and B will vanish and we obtain the following equation.

$$\phi_N(P)=\frac{1}{2\pi}\int_{C+F}\left(\frac{\partial}{\partial n}\phi_N(Q)-\phi_N(Q)\frac{\partial}{\partial n}\right)\cdot\log r(P,Q)\,ds(Q)$$

$$=\frac{1}{2\pi}\int_{C+F}\left(\frac{\partial}{\partial n}\phi_j^{(1)}-\phi_j^{(1)}\frac{\partial}{\partial n}\right)$$
$$\cdot\log r\,ds-\frac{1}{2\pi}H_j^{\pm}(K)$$
$$\cdot\int_{C+F}\left(\frac{\partial}{\partial n}\phi_S-\phi_S\frac{\partial}{\partial n}\right)\log r\,ds \tag{43}$$

Thus we obtain the following integral equation making use of the free surface condition and taking into account $\log r$ becomes singular when P approaches the boundaries.

$$\pi\phi_j^{(1)}(P)+\int_{C+F}\phi_j^{(1)}\frac{\partial}{\partial n}\log r\,ds$$
$$+K\int_F\phi_j^{(1)}\log r\,ds-H_j^{\pm}(K)$$
$$\cdot\left\{\pi\phi_S+\int_{C+F}\phi_S\frac{\partial}{\partial n}\log r\,ds\right.$$
$$\left.+K\int_F\phi_S\log r\,ds-\int_C\frac{\partial}{\partial n}\phi_S\log r\,ds\right\}$$
$$=\int_C\frac{\partial}{\partial n}\phi_j^{(1)}\log r\,ds\,. \tag{44}$$

Similary, for anti-symmetry problems let $\phi_D(x,y)$ be a horizontal doublet potential placed at the origin

$$\phi_D(x,y)$$
$$=\frac{1}{2\pi}\int_{C+F+R^{\pm}+B}\left(\frac{\partial}{\partial n}\phi_D-\phi_D\frac{\partial}{\partial n}\right)\log r\,ds$$
$$=\frac{1}{\pi}\oint_0^\infty\frac{ke^{-ky}\sin kx}{k-K}dk-iKe^{-kx}\sin Kx\,,$$
$$\rightarrow\ \pm Ke^{-Ky\mp iKx}\qquad\text{as } x\rightarrow\pm\infty\,. \tag{45}$$

Therefore, we introduce the following potential

$$\phi_N(x,y)=\phi_j^{(1)}-\frac{i}{K}H_j^{\pm}(K)\phi_D(x,y)\,, \tag{46}$$

and the same procedure as used in the symmetry problem will be applied.

The similar procedures can be applied to the diffraction problem if the potential is split into two parts, symmetry and anti-symmetry with respect to y-axis.

3.2 *Second Order Problem*

We will decompose the second order potential into three components in the second order boundary value problem of Eq. (24), and normalize as

$$\varphi_j^{(2)}={}_m\varphi_j^{(2)}+{}_b\varphi_j^{(2)}+{}_f\varphi_j^{(2)}$$
$$=\frac{iga_w^{(2)}}{2\omega}{}_m\phi^{(2)}+\frac{iga_w^{(1)2}}{2\omega}({}_b\phi^{(2)}+{}_f\phi^{(2)})\,, \tag{47}$$

where

- ${}_m\phi^{(2)}$; second order potential due to the motion of bi-frequency
- ${}_b\phi^{(2)}$; due to the second order body surface condition
- ${}_f\phi^{(2)}$; due to the second order free surface condition.

Then, the force due to these potentials are also split into each terms as:

$${}_2F_j^{(2)}(5)={}_2F_j^{(2)}(M)+{}_2F_j^{(2)}(B)+{}_2F_j^{(2)}(F)\,, \tag{48}$$

where

- ${}_2F_j^{(2)}(M)$; second order force due to the motion of bi-frequency which produces the added-mass and damping forces of $4K$
- ${}_2F_j^{(2)}(B)$; due to the second order body surface condition
- ${}_2F_j^{(2)}(F)$; due to the second order free surface condition.

Therefore, the boundary value problems for each three potentials are rewritten as:

$$\left.\begin{aligned}
&[L]\quad \nabla\,{}_i\phi^{(2)}=0\qquad (i=m,b,f)\\
&[F]\quad \left\{4K+\frac{\partial}{\partial y}\right\}({}_m\phi^{(2)},{}_b\phi^{(2)},{}_f\phi^{(2)})\\
&\qquad\qquad=(0,0,q(x))\qquad\text{on } y=0\\
&[H]\quad \frac{\partial}{\partial n}({}_m\phi^{(2)},{}_b\phi^{(2)},{}_f\phi^{(2)})=({}_2h^{(2)},h_{ij}^{(2)},0)\\
&\qquad\qquad\qquad\text{on } C_0\\
&[B]\quad \frac{\partial}{\partial y}{}_i\phi^{(2)}(x,\infty)=0\,,\qquad (i=m,b,f)\\
&[R]\quad \left\{\frac{\partial}{\partial x}\pm i4K\right\}{}_i\phi^{(2)}(\pm\infty,y)=0\,,\\
&\qquad\qquad\qquad (i=m,b,f)
\end{aligned}\right\} \tag{49}$$

where

$$q(x)=-2(\nabla\phi^{(1)})^2+\phi^{(1)}(\phi_{yy}^{(1)}+K\phi_y^{(1)})$$
$$=q_c(x)+iq_s(x)\,,$$
$${}_2h^{(2)}=4Kf^{(2)}/a_w^{(2)}\,,$$
$$h_{ij}^{(2)}=K\bar{x}_3^{(1)}\bar{c}^{(1)}-\bar{x}_3^{(1)}\phi_s^{(1)}-\bar{f}^{(1)}\phi_{nn}^{(1)}-\bar{d}^{(1)}\phi_{sn}^{(1)}\,.$$

The problems for ${}_m\phi^{(2)}$ and ${}_b\phi^{(2)}$ are the same as the first order problems in Eq. (23), by replacing K to $4K$. The essential difference between the first and second order problems appears in the problems of ${}_f\phi^{(2)}$ which must include the boundary condition on the free surface. However, it may be solved easily by applying the BEM described in the preceeding section in the same way as used in the first order problems.

Now, we consider the pressure distribution on the free surface, $q(x)$. In the radiation problems of a single mode, we obtain the potential and the pressure distribution from Eq. (38) and (49):

$$\phi^{(1)}=K\bar{x}_j^{(1)}\phi_j^{(1)} \qquad (j=1,2,3)\ ,$$

$$q_j(x)=K^2\{-2(\nabla\phi_j^{(1)})^2+\phi_j^{(1)}(\phi_{jyy}^{(1)}+K\phi_{jy}^{(1)})\}\ .$$

At a large distance from the body, these terms can be expressed in the asymptotic expansions.

$$\left.\begin{aligned}&\phi_j^{(1)}\rightarrow iH_j^{\pm}(K)e^{-Ky\mp iKx}\\&q_j(x)\rightarrow 0\end{aligned}\right\}\ \text{as}\ x\rightarrow\pm\infty \tag{50}$$

In the case of a symmetric body with respect to y-axis it can be simply shown by the symmetry relation

$$q_{ij}(x)=q_j(-x)\ . \tag{51}$$

Moreover, we find the following relation for radiations of a single mode oscillation of a symmetric body

$$h_{ij}^{(2)}(x,y)=h_{ij}^{(2)}(-x,y)\ . \tag{52}$$

Therefore, the hydrodynamic forces caused by ${}_b\phi^{(2)}$ and ${}_f\phi^{(2)}$ always act as vertical forces even in the swaying or rolling oscillations.

Nextly, we consider the incident wave problems, in which the potential far from the body may be expressed as:

$$\phi^{(1)}=e^{-Ky+iKx}+i\left(H_4^{\pm}+K\sum_{j=1}^{3}\bar{x}_j^{(1)}H_j^{\pm}\right)\cdot e^{-Ky\mp iKx} \qquad \text{as}\ x\rightarrow\pm\infty\ .$$

Therefore, we obtain

$$\left.\begin{aligned}&q(x)=-i8K^2\left(H_4^{+}+K\sum_{j=1}^{3}\bar{x}_j^{(1)}H_j^{+}\right) && \text{as}\ x\rightarrow+\infty\ ,\\&q(x)=0 && \text{as}\ x\rightarrow-\infty\ .\end{aligned}\right\} \tag{53}$$

The complex constant of the pressure on the free surface of the weather side appears from the standing waves made by the interaction of the incident waves and the reflected waves.

The forces are given in the non-dimensional form.

$$f_j^{(2)}(F)=\frac{F_j^{(2)}}{\rho g a_w^2}=-\int_{C_0}\phi^{(2)}\frac{\partial\bar{x}_j}{\partial n}ds \qquad (j=1,2,3)\ . \tag{54}$$

For the evaluation of this term, we introduce three potentials which satisfy the following boundary conditions.

$$\left.\begin{aligned}&[L]\quad \nabla^2\phi_j^R(x,y)=0\\&[F]\quad \left\{4K+\frac{\partial}{\partial y}\right\}\phi_j^R(x,0)=0\\&[H]\quad \frac{\partial}{\partial n}\phi_j^R=\frac{\partial}{\partial n}\bar{x}_j\\&[B]\quad \frac{\partial}{\partial y}\phi_j^R(x,\infty)=0\\&[R]\quad \left\{\frac{\partial}{\partial x}\pm i4K\right\}\phi_j^R(\pm\infty,y)=0\end{aligned}\right\} \tag{55}$$

These can be identified as the first order radiation potentials of wave number of $4K$. Then, we choose the free surface instead of the body surface for the integral path by applying Green's theorem[7].

$$\begin{aligned}\int_{C_0}{}_f\phi^{(2)}\frac{\partial}{\partial n}\bar{x}_j\,ds&=\int_{C_0}{}_f\phi^{(2)}\phi_{jn}^R\,ds\\&=\int_{C_0}{}_f\phi_n^{(2)}\phi_j^R\,ds+\int_F({}_f\phi^{(2)}\phi_{jy}^R-{}_f\phi_y^{(2)}\phi_j^R)\,dx\\&\quad-\int_{R^\pm}({}_f\phi^{(2)}\phi_{jx}^R-{}_f\phi_x^{(2)}\phi_j^R)\,dy\\&=-\int_F q(x)\phi_j^R\,dx\\&\quad-\int_{R^\pm}({}_f\phi^{(2)}\phi_{jx}^R-{}_f\phi_x^{(2)}\phi_j^R)\,dy\ .\end{aligned} \tag{56}$$

It would be little difficult to understand of the radiation condition imposed on R^+ for ${}_f\phi^{(2)}$, because the pressure distribution on the free-surface, $q(x)$, nondecays and lasts at

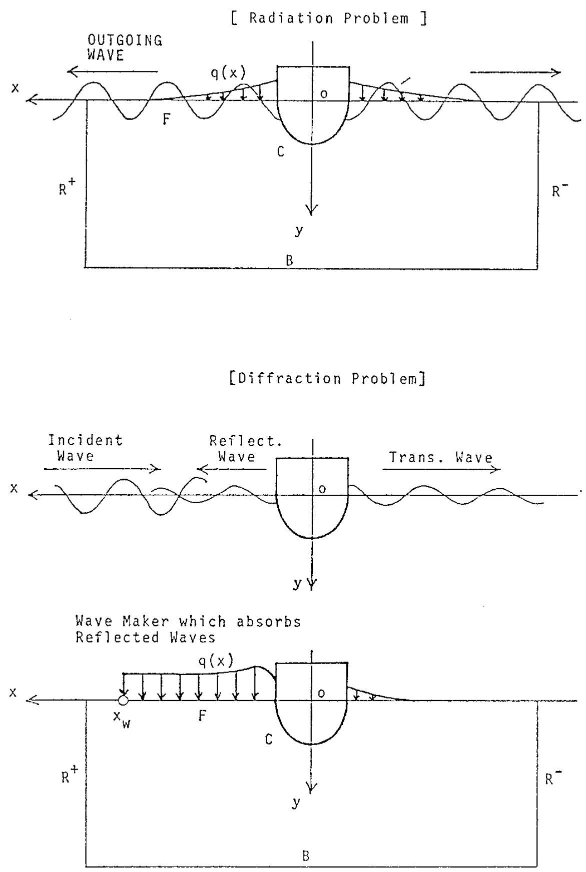

Fig. 2 Second-order boundary value problems

infinity. However, let us introduce an ideal wave-maker which could absorb the reflected waves as shown in Fig. 2, then the radiation condition for ${}_{J}\phi^{(2)}$ could be understood on R^{+} far from the wave-maker. Therefore, we could drop the integrals on $R^{\pm}$ because of the radiation condition for ${}_{J}\phi^{(2)}$ and ϕ_j^R. The first integral on the free-surface in Eq. (56) will oscillate depending on the location of the wave-maker by the asymptotic behavior of ϕ_j^R. However, we could estimate practically it as the mean value of the integral as:

$$\int_{C_0} {}_{J}\phi^{(2)} \frac{\partial}{\partial n} \bar{x}_j \, ds$$
$$= (\text{mean value of}) \left\{ -\int_F q(x) \phi_j^R \, dx \right\} . \tag{57}$$

In the three dimensional problems, Molin[19] derived the same procedures and obtained the second order wave forces upon fixed axisymmetric bodies without any difficulties.

In this study, the second order forces due to the nonlinear free surface condition are evaluated by Eq. (57) making use of the wave-free potential aforementioned. Consequently, they can be obtained by the solutions of the first order problems without solving the second order boundary value prolems.

In the radiation problems, the second order forces are obtained by two methods, the one by the direct solution of the second order boundary value problem, and the other by Eq. (57). They show good agreement each other.

All the quantities are non-dimensionalized as follows.

Radiation

$$\left.\begin{array}{l}
x_j(t) = a_j \cos \omega t, \quad \varepsilon = a_j/b \qquad (i=1,2,3) \\
f_{ij}^{(1)} = |F_{ij}^{(1)}| (\rho g b_i a_i)^{-1} \cos(\omega t + \delta_{ij}^{(1)}) \\
\hfill (j=1,2,3) \\
{}_0f_{ij}^{(2)} = |{}_0F_{ij}^{(2)}| \left(\frac{1}{2}\rho g a_i^2\right)^{-1} \\
{}_2f_{ij}^{(2)} = |{}_2F_{ij}^{(2)}| (\rho g a_i^2)^{-1} \cos(2\omega t + \delta_{ij}^{(2)})
\end{array}\right\} \tag{58}$$

Diffraction

$$\left.\begin{array}{l}
\eta(t) = -\left(a_w \cos \omega t + \frac{K}{2} a_w^2 \cos 2\omega t\right) \\
\varepsilon = a_w/b \\
\bar{x}_j^{(1)} = |x_j^{(1)}| (a_w)^{-1} \cos(\omega t + \alpha_j^{(1)}) \\
\hfill (j=1,2) \\
\bar{x}_3^{(1)} = |x_3^{(1)}| (K a_w)^{-1} \cos(\omega t + \alpha_3^{(1)}) \\
\hfill (j=3) \\
{}_2\bar{x}_j^{(2)} = |{}_2x_j^{(2)}| b_j (a_w)^{-2} \cos(2\omega t + \alpha_j^{(2)}) \\
\hfill (j=1,2,3) \\
f_j^{(1)} = |F_j^{(1)}| (\rho g b_j a_w)^{-1} \cos(\omega t + \delta_j^{(1)}) \\
\hfill (j=1,2,3) \\
{}_0f_j^{(2)} = {}_0F_j^{(2)} \left(\frac{1}{2}\rho g a_w^2\right)^{-1} \quad (j=1,2,3) \\
{}_2f_j^{(2)} = |{}_2F_j^{(2)}| (\rho g a_w^2)^{-1} \cos(2\omega t + \delta_j^{(2)}) \\
\hfill (j=1,2,3)
\end{array}\right\} \tag{59}$$

where b_j denotes as: $b_1 = b_2 = b$, $b_3 = b^2$ (b= half-beam at waterline)

4. Experiments

All the experiments were carried out in a small tank ($L \times B \times D = 9\ m \times 1.2\ m \times 1.2\ m$) of the Defense Academy as shown in Fig. 3. Five models whose cross-sections are shown in Fig. 4 were used and their principal dimensions are shown in Table 1.

Forces were measured by a three-component load-cell in the radiation problems and in the diffractions of fixed cylinders. In case of a free floating cylinder, motions were measured and those records were analized by the Fourier Analysis.

Typical measured records and the corresponding calculations are presented in Figs. 5

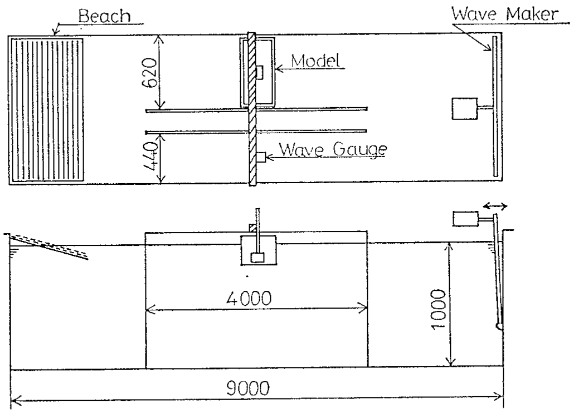

Fig. 3 Experimental set-up for the diffraction problems

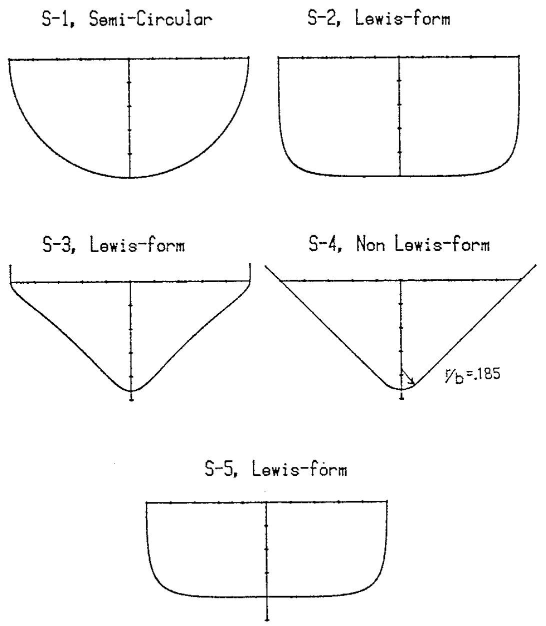

Fig. 4 Model sections used in the experiments

Table 1 Principal dimensions of the models

Item	S-1	S-2	S-3	S-4	S-5
Section	hemi-circle	Lewis form	Lewis form	triang. & R.B.	Lewis form
Half-beam/Draft	1.0	1.0	1.083	1.083	1.25
Sectional area coef.	.785	1.0 (.96)	.537	.537	.95
Length (m)	.6 (.3)	.6 (.3)	.6 (.3)	.6 (.3)	.6
Breadth (m)	.216	.19 (.2)	.216	.216	.2
Draft (m)	.108	.095 (.1)	.1	.1	.08
Displacemt.(Kg)	10.99 (5.45)	10.83 (5.76)	6.98 (3.49)	6.98 (3.49)	9.12

	C-1	C-2
Center of gravity : $\overline{OG}/b$	.031	.163
Metacenter height : $\overline{GM}/b$	.080	.232
Radius of gyration: r_G/b	1.18	.781
Heaving resonance : K_2b	.75	.75
Rolling resonance : K_3b	.056	.340

Note) Figures in parenthesis mean dimensions of the model used in the radiation problem.

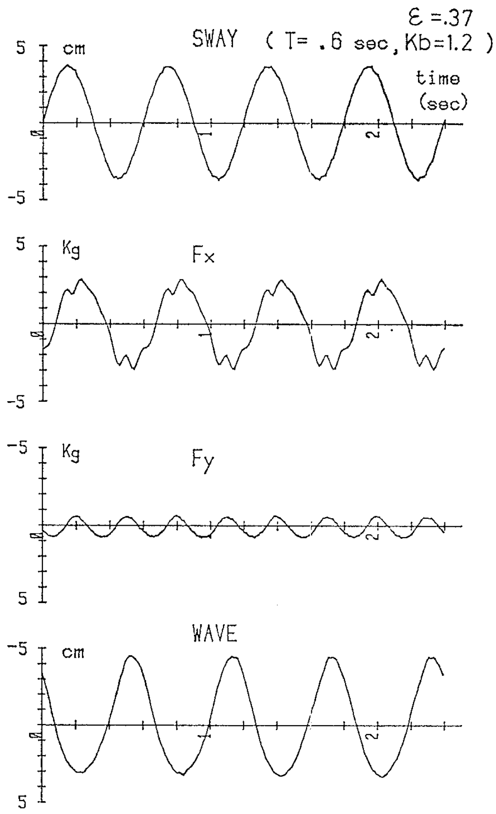

Fig. 5 An example of the experimental records of a swaying circular cylinder ($K_b = 1.2$)

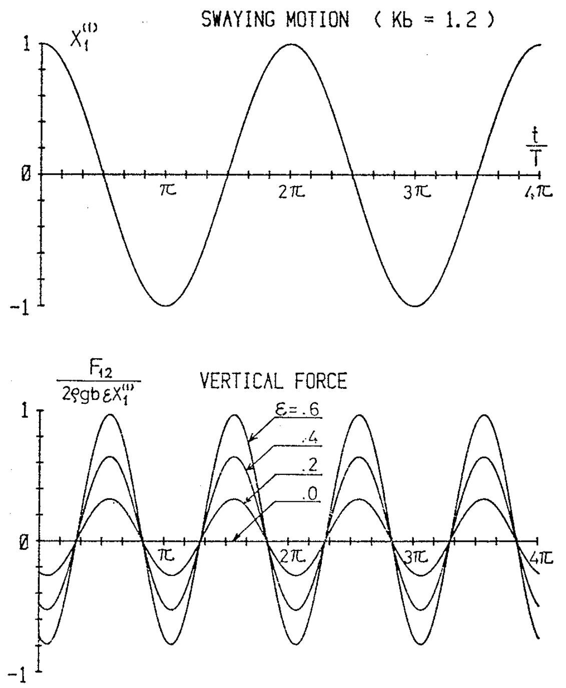

Fig. 6 Calculated second-order forces of a swaying circular cylinder ($K_b=1.2$)

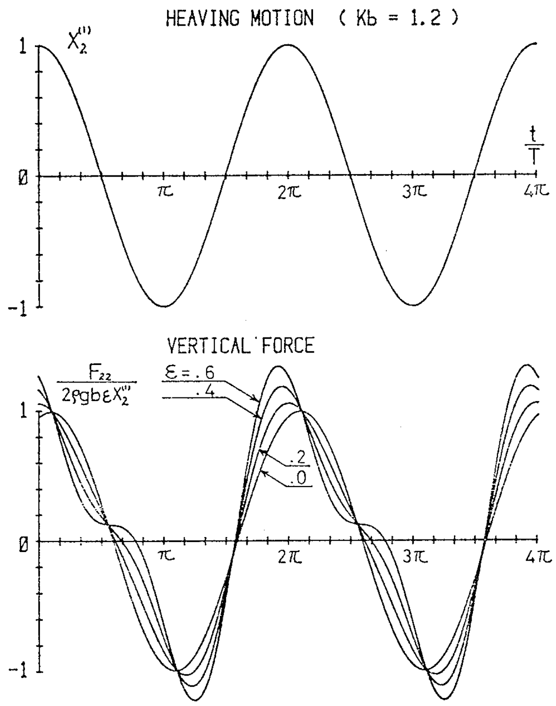

Fig. 8 Calculated total forces of a heaving circular cylinder ($K_b=1.2$)

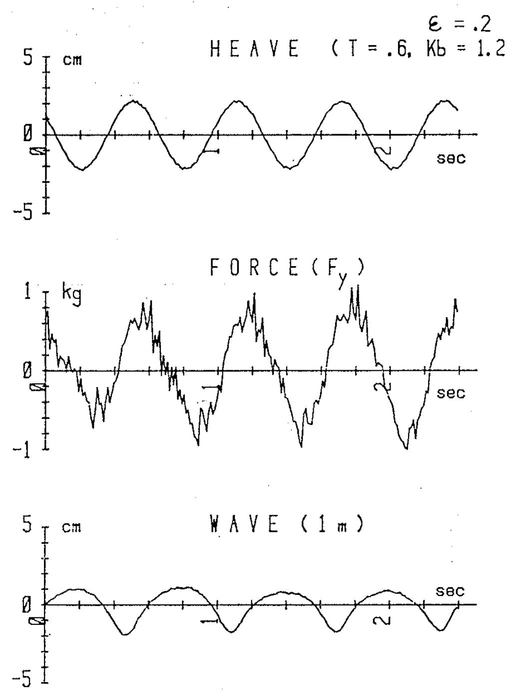

Fig. 7 An example of the experimental records of a heaving circular cylinder ($K_b=1.2$)

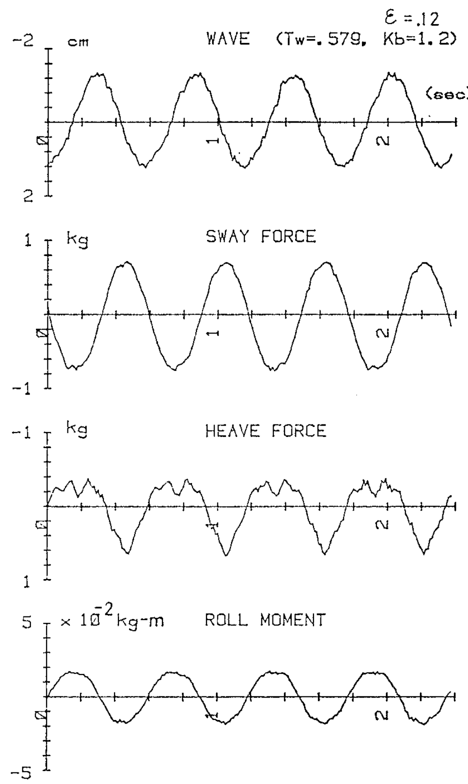

Fig. 9 An example of the experimental records of a fixed Lewis-form cylinder in waves (S-5, $K_b=1.2$)

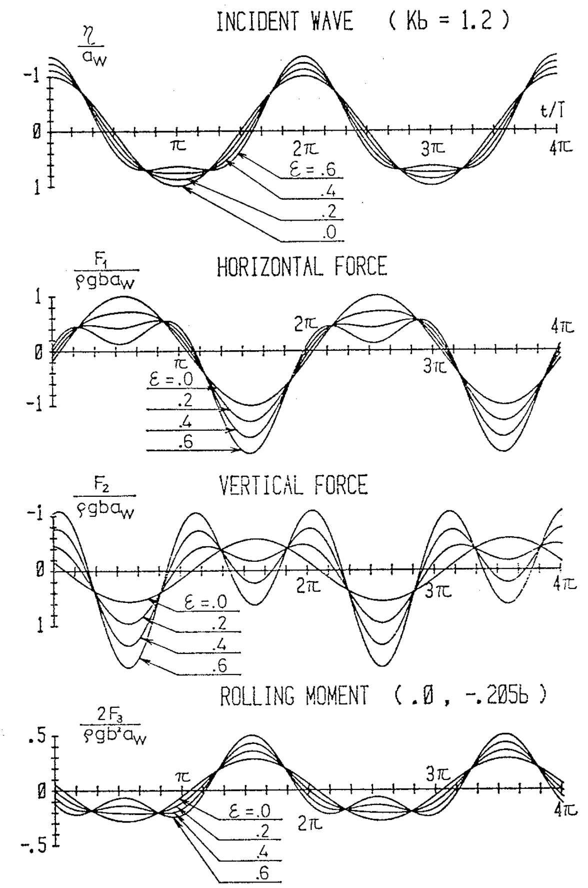

Fig. 10 Calculated wave-exciting forces of a fixed Lewis-form cylinder in waves (S-5, K_b= 1.2)

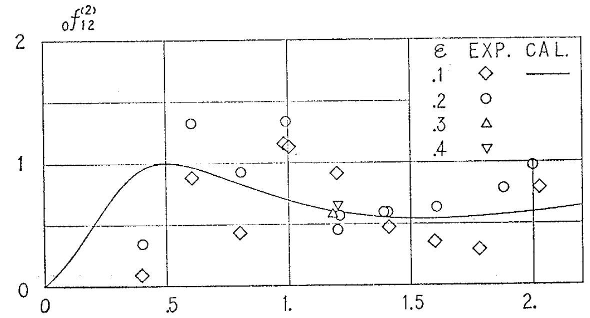

Fig. 11 Second-order vertical steady-forces of a swaying circular cylinder

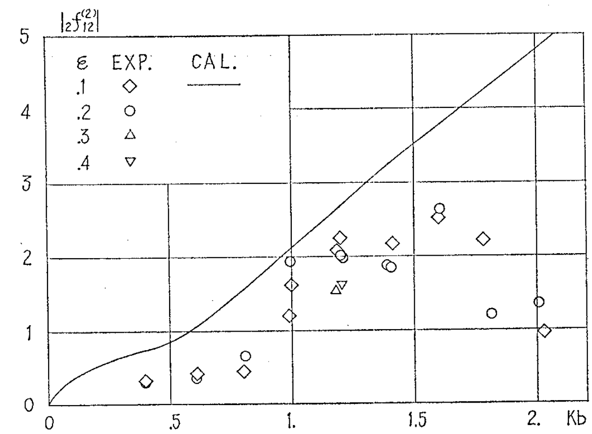

Fig. 12 Second-order vertical oscillating forces of a swaying circular cylinder

through 10. In the radiation problem of the swaying oscillation, the second-order forces are observed directly in the vertical force as shown in Fig. 5. Comparing Fig. 9 with Fig. 6, one would find that the calculation agree with the experiment. In the heaving oscillation, measured force contains all the forces including inertia-one. One would find that both wave-forms of the vertical force show a similarity each other as shown in Figs. 7 and 8, although it would be complicated to compare the second-order forces in those figures. In Fig. 9 and 10, the measured and the calculated time-histories of the diffraction forces of a Lewis-form cylinder are shown.

4.1 *Radiation Problems*[14]

The second-order forces acting on a circular cylinder swaying in the still water are shown in Figs. 11 and 12, the former shows the vertical steady-force and the latter shows the vertical bi-harmonic component. The experiments would verify the validity of the present theory from these results. The second-order forces on a symmetric cylinder in the radiation problems of a single-mode oscillation always act as vertical forces. In Figs. 13 and 14, the second-order forces on a heaving circular cylinder in the still water are shown. Experiments agree well with the calculation except the bi-harmonics in the higher range of wave-number (K_b>1.5), where the viscous effects might appear because the progressing waves reach the limit of the wave-height.

4.2 *Diffraction Forces on a Fixed Cylinder*[13]

The second-order wave-exciting forces of the bi-harmonics in sway and heave on a fixed

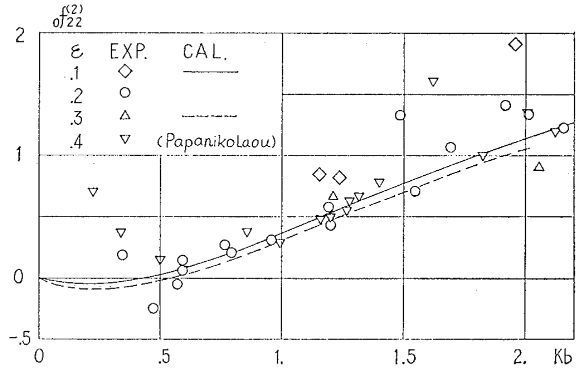

Fig. 13 Second-order vertical steady-forces of a heaving circular cylinder

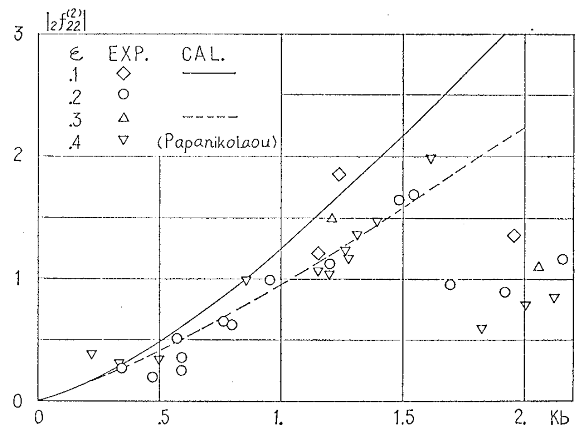

Fig. 14 Second-order vertical oscillating forces of a heaving circular cylinder

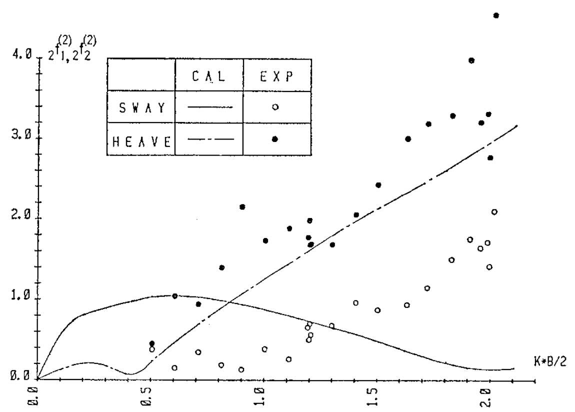

Fig. 15 Second-order wave-exciting forces of a circular cylinder

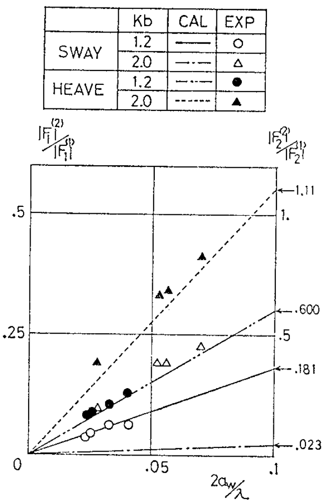

Fig. 16 Ratios of the second-order wave-exciting force to that of the first-order as a function of the amplitude of incident waves (S-1)

circular cylinder is shown in Fig. 15. The calculation for the vertical force would be found to agree well with the experiments but those for the horizontal one show a little different tendency with the experiments in the higher range of the wave-number.

In Fig. 16, the ratios of the second-order force to that of the first-order are given as a function of the amplitude of the incident waves. From these results, one would find that the second-order forces are in proportion to the square of the amplitude of the incident waves and therefore, the second-order forces, particularly in the virtical force, would become important for the large wave problems.

4.3 *Effects of Angles of the Intersection of the Body and Free-surface*[15]

In this section, the hydrodynamic effects of angles of the intersection between hull-side and free-surface, so-called the Wedge-effects[12], are investigated in the radiation and the diffraction problems. Models, S-3 and S-4 have the same beam/draft ratio and the sectional area, S-3 intersects at right-angles at free-surface, on the other side, S-4 intersects at half of right-angles. Therefore, the first-order hydrodynamic characteristics are almost the same between two models.

In Fig. 17, the Wedge-effects appear in the second-order vertical steady-force in the swaying oscillations, where they act as an upward

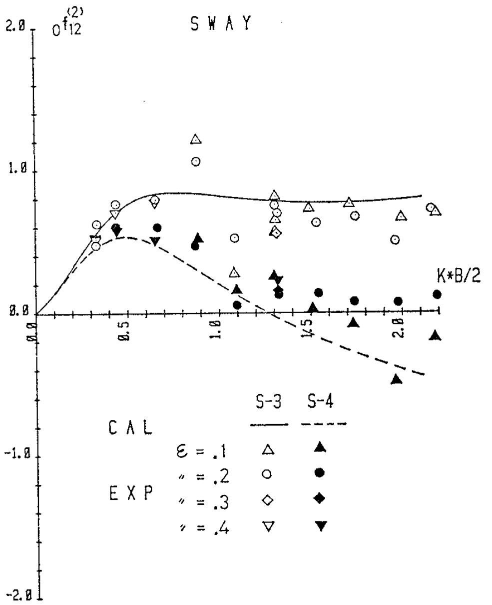

Fig. 17 Second-order vertical steady-forces of swaying cylinders (S-3, S-4)

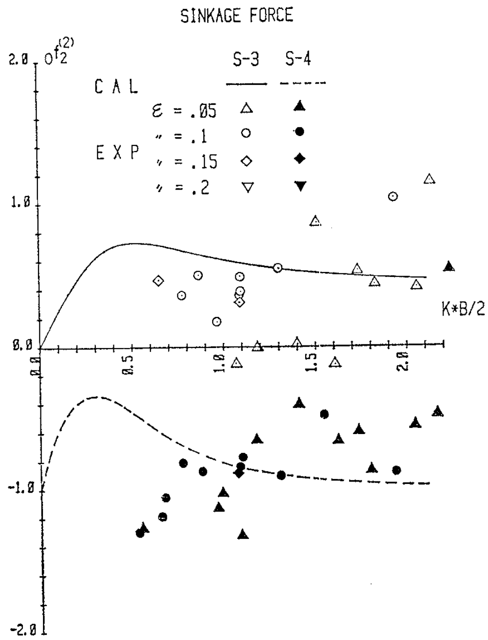

Fig. 19 Second-order vertical steady-forces of fixed cylinder in waves (S-3, S-4)

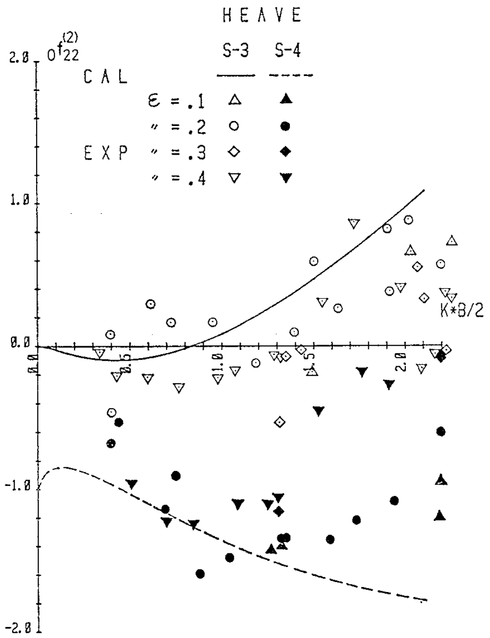

Fig. 18 Second-order vertical steady-forces of heaving cylinders (S-3, S-4)

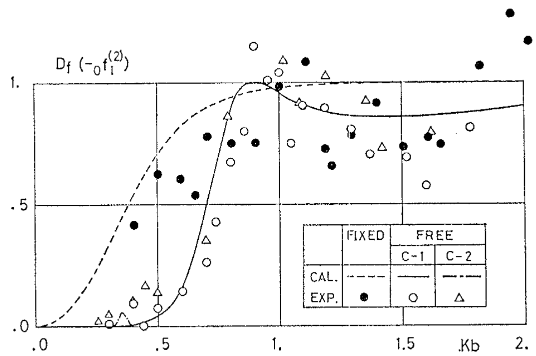

Fig. 20 Drift-forces of a fixed and free-floating Lewis-form cylinder in waves (S-5)

force for S-4. The results for the heaving and diffraction cases are shown in Figs. 18 and 19. The calculations seem to agree with the experiments. The Wedge-effects also appear in the second-order bi-harmonics, but they are not so dominant in the total force that the effects are unclear in the experiments.

4.4 *Second-Order Motions in Waves*[16]

Nextly, the results of the second-order motions of a free-floating Lewis-form cylinder (S-5) in waves are shown in Figs. 20 through 24. Steady-forces of a free-floating cylinder are obtained from the mean drifts in the experimental records multiplied by each restoring-force coefficients as shown in Figs. 20 to 22, together with those of the fixed condition.

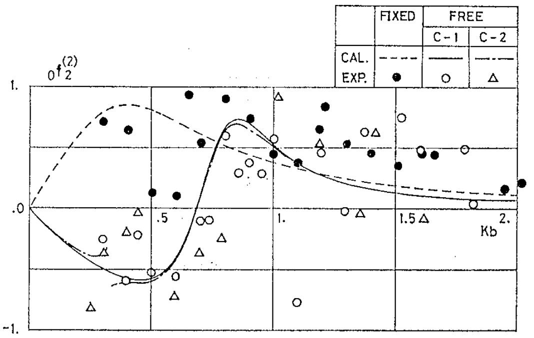

Fig. 21 Second-order vertical steady-forces of a fixed and free-floating Lewis-form cylinder in waves (S-5)

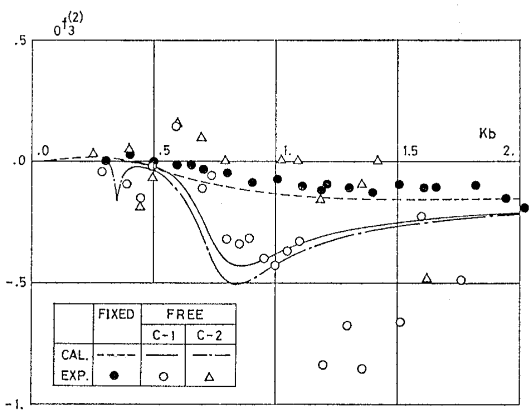

Fig. 22 Second-order heeling-moments of a fixed free-floating Lewis-form cylinder in waves (S-5)

Calculations for those steady-forces are found to agree with the experiments. The steady-heeling angles of a floating-body in waves are observed in the experiments of the smaller $\overline{GM}$ condition (C-1), as are predicted by the calculation.

On the other side, the second-order bi-harmonic motions in sway and heave are shown in Figs. 23 and 24, where the calculations respond largely near the first-order resonances of the heave and roll motions. Although experiments give a small value at the low frequency range, general tendencies seem to be similar to the calculations.

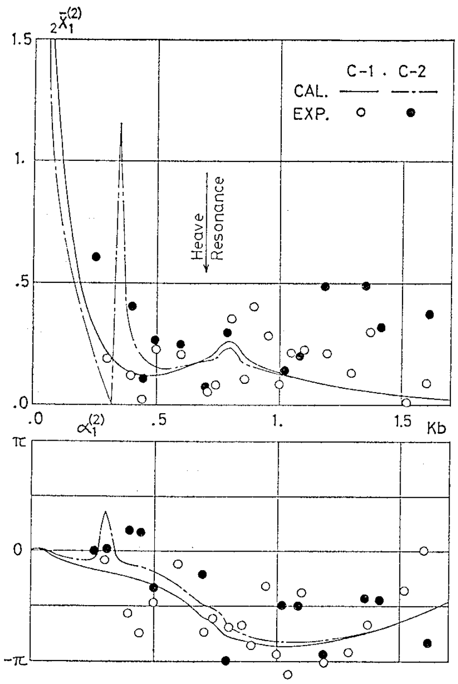

Fig. 23 Second-order swaying-motions of a Lewis-form cylinder in waves (S-5)

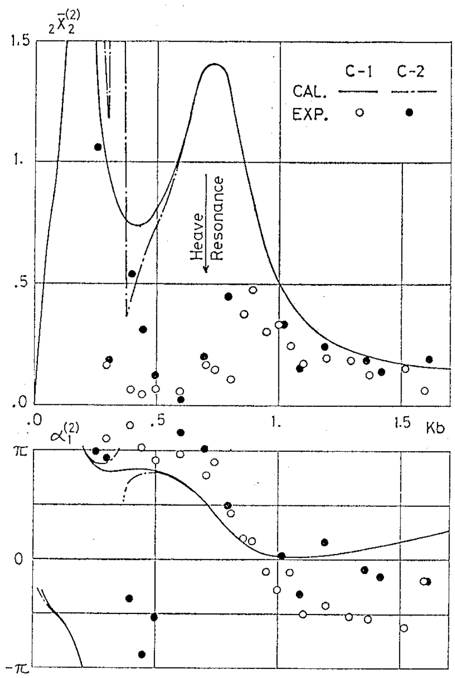

Fig. 24 Second-order heaving-motions of a Lewis-form cylinder in waves (S-5)

5. Conclusion

The first- and second-order forces on a

cylindrical body in waves are calculated on the basis of the regular perturbation theory along with the previous pursuers.

The second-order boundary value problem of the radiations could be solved without any difficulty, while some considerations should be paid in the diffraction problems. The second-order solutions of the diffraction problems strongly depend on the truncation of the free-surface condition. However, the forces acting on the body due to these potentials could be obtained reasonably by the mean value of the integrals on the free-surface applying Green's theorem. Consequently, the second-order forces can be obtained by making use of the first-order solutions without solving the second-order boundary value problems.

The Boundary Element Method which includes both boundaries of the body and the free-surfaces is applied to simplify these procedures, then it enables us to deal with the free-surface as same as the body surface. Those numerical results show good agreement with other theories.

Experiments are carried out for not only the radiations of heaving and swaying oscillations but also the diffractions for a fixed and a free-floating body in waves. Generally speaking, the present theory shows good agreement with experiments of all the problems, although the second-order forces are very small in the extent of the phenomena treated here.

Therefore, we conclude that the present theory can be utilized for the purpose of the predictions of the hydrodynamic forces and the ship motions in wave.

The remaining interests should be turned toward the extremely large amplitude problems and the transient problems. It is hoped that further investigations of such problems will be performed in the future.

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