Supergauge Field Theory of Covariant Heterotic Strings*)

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We present the gauge covariant second quantized field theory for free heterotic strings, which is leading candidate for a unified theory of all known particles. Our action is invariant under the semi-direct product of the super Virasoro and the Kac-Moody $E_8 \times E_8$ or Spin(32)/Z_2 group. We derive the covariant action by path integrals in the same way that Feynman originally derived the Schrödinger equation. By adding an infinite number of auxiliary fields, we can also make the action explicitly local.

We stress that our path integral methods can be generalized to the interacting case of splitting strings. We expect that the complete interacting theory will be a non-linear realization of the Virasoro and Kac-Moody algebras. Understanding the geometry behind such theories may eventually help in a *non-perturbative* formulation of the theory, in which 10 dimensional space-time is dynamically broken down to our four dimensional universe.

§ 1. Introduction

The heterotic string of Gross, Harvey, Martinec and Rohm[1] is a promising candidate for a finite theory which can unite gravity with the rest of the known particle interactions. The heterotic string uses the observation that the 26 dimensional string model of Nambu-Goto[2] can be reduced down to a 10 dimensional theory (which in turn can be combined to form the superstring model of Green and Schwarz[3]) by compactifying the remaining 16 dimensions on the root lattice space of $E_8 \times E_8$ or Spin(32)/Z_2. The group $E_8 \times E_8$, in turn, is sufficient to eliminate anomalies[4] and is large enough to accomodate symmetry breakings which can yield a low energy theory compatible with all known particle interactions.[5]

Furthermore, the theory may be finite as well. Previously, supergravity[6] was the most promising theory for uniting gravity with the known particle interactions. Unfortunately, the $O(8)$ supergravity action is too small to accomodate $SU(3) \times SU(2) \times U(1)$ and the theory also possesses a potential counterterm at the 7th loop level, so the theory is unlikely to be finite.

By contrast, the closed superstring model is probably finite for purely topological reasons. The ultraviolet divergences found in quantum gravity arise when we pinch the propagators of the Feynman graphs and distort the topology. However, the closed string

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has an entirely new topological structure. Loop graphs defined for the closed string are spheres with handles, which cannot be pinched in the same way to yield the usual infinities. (Although the divergence structure of the bosonic string is known to all loop orders,[7] one must still check these results for the superstring model.)

The theory, however, is only a first quantized one, i.e., we treat the location of the string X_μ as a dynamical variable, rather than using a field functional $\phi(X)$ which is defined at the location of the string. There are several advantages to formulating the gauge covariant second quantized field theory of strings:

(a) The theory can formally be shown to be unitary. In the first quantized theory, the counting of diagrams is ill-defined. (In fact, the first quantized action has no interaction terms at all; the interactions are introduced by summing over different topologies, which must be added in by hand.) By contrast, all interaction terms are explicitly present in the second quantized theory, so all weights are uniquely specified in a unitary theory.

(b) The theory can ultimately be used for non-perturbative calculations. For example, geometries like Calabi-Yau are at present beyond the scope of string theories (these geometries are obtained as solutions of the Einstein action, rather than as solutions of the string action). Non-perturbative approaches may eventually solve the question of how a 10 dimensional universe breaks down to a four dimensional one, which, we feel, can only be understood dynamically.

(c) The theory may explain the underlying geometry behind the string theory. (One feels that, if Einstein had never written down general relativity, a field theorist would have discovered the Einstein action perturbatively as an infinite power expansion of an interacting spin two field. As a power series, however, the field theorist would completely miss the black-hole solution, the Robertson-Walker universe, etc., which would only be discovered by a geometer working with curvatures.) Presently, we are constructing the covariant interactions for the theory, proceeding as field theorists, not as geometers, by power expanding around free strings. Hopefully, we will be able to discover the underlying geometry behind such a power expansion.

Years ago, we wrote down the field theory of interacting strings.[8] Unfortunately, the theory was formulated in the light cone gauge[9] because we did not know how to eliminate the ghosts of the theory covariantly. Since then, however, our understanding of the gauges has been increased by the work of mathematicians, who have written down the representations of Kac-Moody algebras.[10] Using their results, we have been able to formulate the gauge covariant field theory of bosonic strings[11] and superstrings.[12]

We will employ the Kac-Moody construction in order to write down the field theory of heterotic strings. There are a few new features of this second quantized theory which are not manifest in the first quantized, light cone theory. First, we now have a non-trivial coupling between the Virasoro algebra and the Kac-Moody algebra (in fact, the semi-direct product of these two algebras). The union of the Virasoro algebra with the Kac-Moody $E_8 \times E_8$ algebra is crucial to the construction of the gauge covariant field theory. Second, we find that the 10 dimensional supersymmetry of the Neveu-Schwarz-Ramond model (which is obscure in the first quantized theory) is manifest in the second quantized field theory (if certain matrices exist).

We want to stress the fact that this theory is *gauge* invariant. (If we explicitly break this gauge invariance, then we would obtain a BRST[13]-type theory.) This gauge invariance may prove decisive in reformulating the model geometrically.

Also, we note that we can always make this model local by adding an infinite number of auxiliary fields. These fields simply soak up the non-local terms arising from the zeros of the determinant of the Shapovalov matrix.

Lastly, we stress that we are using the language of path integrals because we wish ultimately to write down the theory of interacting strings, which should be invariant under a non-linear realization of the Virasoro and Kac-Moody algebra. This is currently under investigation.

§ 2. Path integral derivation of covariant field theories

In our previous work, we showed how to use path integrals to write down the gauge covariant second quantized field theory of Nambu-Goto and Green-Schwarz strings.

We will now quickly review this approach, which is based on the method used by Feynman to extract the Schrödinger equation from the first quantized action $mv^2/2$.[14] We will use the path integral formalism[15] which has been generalized for strings.

First, we begin our discussion with the identity which establishes the link between the first quantized and second quantized theories, which works both for propagators as well as the interacting string:

$$G_{ij}=\int_{X_i}^{X_j}\mathcal{D}X\exp\left[i\int_{\tau_i}^{\tau_j}d\tau\int_0^\pi d\sigma\mathcal{L}(X)\right]$$

$$=\int\mathcal{D}\phi\mathcal{D}\phi^\dagger\phi(X_i,\,\tau_i)\phi^\dagger(X_j,\,\tau_j)\exp\left[i\int\mathcal{L}(\phi)\mathcal{D}X\right], \tag{2·1}$$

where $\mathcal{L}(\phi)$ is the second quantized action that we want to extract from the heterotic string. This path integral expression represents the Green's function for the propagation of a string at "time" τ_i and location X_i to "time" τ_j. (When we generalize this expression for gauge theories, we must be careful because we are manipulating quantities which cannot be inverted.)

Second, we must rewrite the action in terms of the first order formalism (defined in terms of X_μ and its canonical conjugate P_μ). This will allow us to make "time" slices and then to calculate the propagation of the string from one "time" to the next slice:

$$G_{ij}=\int_{X_i}^{X^j}\mathcal{D}X\mathcal{D}P\exp\left[i\int_{\tau_i}^{\tau^j}d\tau\int_0^\pi d\sigma[\rho\dot{X}+\lambda^m C_m]\right]$$

$$\to\int\prod_{k=i}^{j-1}\mathcal{D}X_k\mathcal{D}P_k\exp\left[i\int_0^\pi d\sigma P_{k\mu}(X_k{}^\mu-X_{k+1}^\mu)\prod_m\delta(C_{m,k})\right]$$

$$=\int\prod_{k=i}^{j-1}\langle X_k|P_k\rangle\mathcal{D}X_k\mathcal{D}P_k\langle P_k|X_{k+1}\rangle\prod_m\delta(C_{m,k}). \tag{2·2}$$

Notice that, because we are dealing with a gauge theory, the Hamiltonian is formally equal to zero (actually, it is equal to a sum of first and second class Dirac constraints C_m).

The transition to quantum mechanics and to harmonic oscillators is now made by noticing that we can convert a c-number expression into an operator expression:

$$|X\rangle=\prod_{n=1}^{\infty}|X_n\rangle=\prod_{n=1}^{\infty}\exp\left(-\frac{1}{2}X_n{}^2-i\sqrt{2}X_na_n{}^\dagger+\frac{1}{2}a_n{}^\dagger a_n{}^\dagger\right)|0\rangle,$$

$$|P\rangle=\prod_{n=1}^{\infty}|P_n\rangle=\prod_{n=1}^{\infty}\exp\left(-\frac{1}{2}P_n{}^2+\sqrt{2}P_na_n{}^\dagger-\frac{1}{2}a_n{}^\dagger a_n{}^\dagger\right)|0\rangle,$$

$$\langle P|X\rangle=\prod_{n=1}^{\infty}\frac{1}{\sqrt{2}}\exp(-iP_nX_n)\ ,$$

$$P\exp(iP\dot{X})=-i\frac{\delta}{\delta X}\exp(iP\dot{X})\ . \tag{2·3}$$

Third, we now change the basic intermediate states of the theory. Instead of inserting complete sets of string eigenstates at every "time" slice, we can equally insert complete sets of string functionals at every slice:

$$\mathbf{1}=\int|\phi\rangle\mathcal{D}\phi\langle\phi|\exp\left[-\int\mathcal{D}X\varphi^\dagger(X)\varphi(X)\right],$$

$$|\phi\rangle\equiv\phi(X_0)|0\rangle+A_\mu(X_0)a_1{}^\dagger|0\rangle+g_{\mu\nu}a_{1\mu}^\dagger a_{1\nu}^\dagger|0\rangle+\cdots,$$

$$\phi(X)\equiv\langle\phi|X\rangle,$$

$$\mathcal{D}\phi=\mathcal{D}\varphi(X_0)\mathcal{D}A_\mu(X_0)\mathcal{D}g_{\mu\nu}(X_0)\cdots\ . \tag{2·4}$$

Dirac now tells us that we should apply the first class constraints directly onto the basic eigenstates of the theory. These first class constraints are preserved as the string evolves in "time", so we can apply them at each slice:

$$\left\{\int_0^\pi e^{in\sigma}d\sigma C_m{}^{\text{1st class}}(P,X)\right\}|\phi\rangle=0,\qquad n\geq 1\ . \tag{2·5}$$

This means, of course, that we must now project out the spurious states from the theory using standard projection operator techniques. In this sense, the propagator of the theory is generated by the delta functions over the first class constraints. The projection operator $\boldsymbol{P}$ can be constructed in a variety of ways.[16] Putting everything together, we now integrate over the restricted Hilbert space given by $\boldsymbol{P}|\phi\rangle$:

$$\mathcal{L}(\phi)=\phi^\dagger\boldsymbol{P}(i\partial_\tau-(L_0-\alpha_0))\boldsymbol{P}\phi\ . \tag{2·6}$$

We have the option of eliminating the "time" derivative in the action by making a Fourier transform. The "energy" corresponding to the "time" can then be absorbed in a redefinition of the intercept, and in this way we simply retrieve the mass shell condition. Although we can always eliminate the "time" derivative, we will find that the "time" evolution of the string will be extremely valuable when we generalize our results to the interacting case, where there are non-trivial complications due to counting problems. (Notice that, properly speaking, our formalism is a "double time" theory. This is a common feature of covariant string theories.)

§ 3. The heterotic string

We will now apply these results directly to the heterotic string. Our notation will be as follows: M will be a 26 dimensional index which contains a 10 dimensional index μ and a 16 dimensional index I. The two dimensional space will be represented by a flat index a, while α will represent the curved 2D index, i.e., $M=(\mu, I)=1$ to 26; $\mu=1$ to 10; $I=1$ to 16; $a=1,2;\alpha=1,2$. All fermions will be 16 component real Majorana-Weyl spinors. We take

$$\gamma^0=-i\sigma_2,\quad \gamma^1=\sigma_1,\quad \eta^{00}=-1,\quad \eta^{11}=+1,\quad \bar{\psi}=\psi^T\gamma^0$$

and

$$e_a{}^\alpha\eta^{ab}e_b{}^\beta=g^{\alpha\beta},\quad e=|e_\alpha{}^a|.$$

We begin by using the Neveu-Schwarz-Ramond model for the right-moving sector. Later, we will generalize our results for the Green-Schwarz action in the right-moving sector.

We begin our discussion by rewriting the first quantized action in the first order form, where we have conjugate variables X_μ and P_μ. (This transition to the first order form must be carefully checked, because in principle it is not at all obvious that the separation into the left- and right-moving sectors will be preserved under a reparametrization. Naively, in fact, we expect that a reparametrization will mix-up the left- and right-moving sectors, so we will check this carefully when we go to the first order formalism.)

We begin with the action (with the NSR model[16] in the right-moving sector):

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_r+\mathcal{L}_l\,,$$

$$\mathcal{L}_0=-\frac{1}{2}e\partial_\alpha X^\mu\partial_\beta X_\mu g^{\alpha\beta}\,,$$

$$\mathcal{L}_l=-\frac{1}{2}e\partial_\alpha X^I\partial_\beta X^I g^{\alpha\beta}+\lambda^{--}(e_-{}^\alpha\partial_\alpha X^I)^2\,,$$

$$\mathcal{L}_r=-\frac{i}{2}e\bar{\psi}_\mu\gamma^a e_a{}^\alpha\partial_\alpha\psi^\mu-ie\bar{\xi}_\alpha\gamma^\beta\gamma^\alpha\psi_\mu\partial_\beta X^\mu+\frac{e}{2}\bar{\xi}_\alpha\gamma^\beta\gamma^\alpha\psi_\mu\bar{\xi}_\beta\psi^\mu+\bar{\omega}_\mu\gamma^+e_+{}^\alpha\partial_\alpha\psi^\mu,\quad(3\cdot1)$$

where λ^{--} and $\bar{\omega}$ are Lagrange multipliers which eliminate the left- and right-moving sectors.

Notice that the action is invariant under local 2D SUSY, also local Lorentz invariance, local Weyl invariance, and local 2D reparametrization, but not under global Lorentz transformations (because of the left- and right-moving separation).

In order to extract out the first and second class constraints, we will find it useful to break local Weyl invariance by setting $e=1$ and local 2D Lorentz invariance by setting $e_1{}^0=0$.

With this particular parametrization of the metric (which still preserves 2D SUSY and reparametrization invariance), we can write the metric as

$$e_a{}^\alpha=\begin{pmatrix}\frac{1}{\sqrt{\lambda}} & \frac{\rho}{\sqrt{\lambda}} \\ 0 & \sqrt{\lambda}\end{pmatrix};\quad g^{\alpha\beta}=\begin{pmatrix}-\frac{1}{\lambda} & -\frac{\rho}{\lambda} \\ -\frac{\rho}{\lambda} & \lambda-\frac{\rho}{\lambda}\end{pmatrix}. \tag{3·2}$$

We will choose this particular parametrization because ρ and λ will conveniently become the Lagrange multipliers for the first class constraints.

Because the original action was also invariant under $\delta\zeta_\alpha=\gamma_\alpha\theta$, we can choose $\gamma^\alpha\zeta_\alpha=0$.

With this choice of parametrization, the calculation is now entirely straightforward. However, since the details are rather tedious, we will present the calculation in the Appendix.

We complete the transition to the first order formalism by introducing the conjugate variables P^M and Π_μ. Putting everything together, we find that our new action is equal to

$$\begin{aligned}\mathcal{L}&=P^M\dot{X}_M+\Pi^\mu\dot{\psi}_\mu+\rho(P^MX'_M-\Pi\cdot\psi')\\&-\frac{\lambda}{2}(P^{M^2}+X'^{M^2}+2\Pi\sigma_3\psi')+2i\lambda\bar{\xi}_1(P\sigma_3+X')_\mu\psi^\mu\\&+\left[\frac{\lambda^{--}e_2{}^{0^2}\lambda}{1+2\lambda\lambda^{--}e_-{}^{0^2}}\right](P^I\pm X'^I)^2+\left(\Pi+\frac{i}{2}\frac{\bar{\psi}\gamma^0}{\sqrt{\lambda}}-\bar{\omega}\begin{pmatrix}0&0\\1&0\end{pmatrix}\sqrt{\lambda}\right)\delta\,.\end{aligned} \tag{3·3}$$

Notice that the Hamiltonian is formally equal to zero and that λ, ρ and $\bar{\zeta}$ are the Lagrange multipliers for the first class constraints. The second class constraints merely tell us that we can set the left-(right-) moving parts to zero, i.e., $P^I+X^I=0$ and one of the spinor components of the fermion is set to zero. These second class constraints can be incorporated into our theory by making the transition from Poisson brackets to Dirac brackets (which simply set equal to zero the left- or right-moving commutators that we wish to eliminate).

We can now apply these first class constraints directly onto the bosonic field functional $|\phi\rangle$ and the fermionic field functional $|\psi\rangle$:

$$\begin{aligned}&\langle P^{M^2}+X'^{M^2}+2\Pi\sigma_3\psi'+2P^MX'^M-2\Pi\cdot\psi'\rangle_n\left|\begin{matrix}\phi\\\psi\end{matrix}\right\rangle\cong L_n\left|\begin{matrix}\phi\\\psi\end{matrix}\right\rangle=0\,,\\&\langle(P\sigma_3+X')_\mu\psi^\mu\rangle_{n-1/2}|\phi\rangle\cong G_{n-1/2}|\phi\rangle=0\,,\\&\langle(P\sigma_3+X')_\mu\psi^\mu\rangle_n|\psi\rangle\cong F_n|\psi\rangle=0\,,\qquad n\geq1\,.\end{aligned} \tag{3·4}$$

§ 4. Green-Schwarz action in the right-moving sector

In the previous section, we wrote down the heterotic string action (with the NSR model in the right-moving sector) in the first order formalism. This allowed us to calculate the first class constraints, which are essential in calculating the "time" evolution of the string from one slice to the next slice.

We will now quantize the Green-Schwarz action in the right-moving sector, where 10D SUSY is manifest. This has the advantage that instead of two field functionals $|\phi\rangle$ and

$|\psi\rangle$, we only have one field functional in the theory.

We start with the usual Green-Schwarz action, where θ^1 and θ^2 are Majorana-Weyl spinors in 10D with 16 real components. Notice that these two spinors are 2D scalars and do not form 2D spinors in the two dimensional space spanned by the string. The action has a term which will set the left-moving sector to zero:

$$\begin{aligned}\mathcal{L}=&-\sqrt{-g}-i\partial_\tau X_\mu\bar{\theta}\gamma^\mu\partial_\sigma\rho\theta+i\partial_\sigma X_\mu\bar{\theta}\gamma^\mu\partial_\tau\rho\theta\\&+\bar{\theta}^1\gamma^\mu\partial_\tau\theta^1\bar{\theta}^2\gamma^\mu\partial_\sigma\theta^2-\bar{\theta}^1\gamma_\mu\partial_\sigma\theta^1\bar{\theta}^2\gamma^\mu\partial_\tau\theta^2+\bar{\zeta}e_+{}^a\partial_a\theta^1\,,\\\Pi_\tau{}^\mu&\equiv\partial_\tau X^\mu-i\bar{\theta}\gamma^\mu\partial_\tau\theta\,;\qquad\Pi_\sigma{}^\mu=\partial_\sigma X^\mu-i\bar{\theta}\gamma^\mu\partial_\sigma\theta\,,\\g&\equiv-(\Pi_\tau\cdot\Pi_\sigma)^2+(\Pi_i\cdot\Pi_\tau)(\Pi_\sigma\cdot\Pi_\sigma),\\\rho&=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.\end{aligned}\tag{4·1}$$

The transition to the first order form is made by introducing the canonical fields P and $\bar{P}_\theta$. After a fair amount of algebra,[12] we find

$$\begin{aligned}\mathcal{L}&=P_\mu\dot{X}^\mu-\bar{P}_\theta\dot{\theta}+\frac{\lambda}{2}(\Pi_0{}^2+\Pi_1{}^2)+\rho(\Pi_0\Pi_1)+\bar{Q}^a\epsilon^a+\bar{\zeta}e_+{}^1\partial_1\theta^1\,,\\\Pi_{0\mu}&\equiv P_\mu+i\bar{\theta}\gamma_\mu\partial_\sigma\rho\theta\,;\ \Pi_{1\mu}\equiv\partial_\sigma X_\mu-i\bar{\theta}\gamma_\mu\partial_\sigma\theta\,,\\\bar{Q}^1&=\bar{P}_\theta{}^1-i\bar{\theta}^1\gamma_\mu\Pi_0{}^\mu+i\bar{\theta}^1\gamma_\mu\Pi_1{}^\mu-\bar{\theta}^1\gamma_\mu\partial_\sigma\theta^1\bar{\theta}^1\gamma^\mu-\bar{\zeta}e_+{}^0\,,\\\bar{Q}^2&=\bar{P}_\theta{}^2-i\bar{\theta}^2\gamma_\mu\Pi_0{}^\mu-i\bar{\theta}^2\gamma_\mu\Pi_1{}^\mu+\bar{\theta}^2\gamma_\mu\partial_\sigma\theta^2\bar{\theta}^2\gamma^\mu\,.\end{aligned}\tag{4·2}$$

Once again, notice that the Hamiltonian of the system is equal to zero and that the second class constraints simply remove the unwanted left-moving spinor from the theory.

The new feature of this first order action, however, is that the Q field contains *both* first and second class constraints. Previously, when Green and Schwarz began the covariant quantization of their action, it was found that all the commutators were formally infinite.[17] For example, if one starts with the term $\bar{\theta}\gamma_\mu\dot{\theta}\Pi^\mu$ in the action and then forms the canonical conjugate: $\bar{P}_\theta=\delta\mathcal{L}/\delta\dot{\theta}=\bar{\theta}\gamma_\mu\Pi^\mu$, then we find the absurd result that

$$[\bar{\theta},\theta]\sim\frac{1}{\gamma_\mu\Pi^\mu}\sim\infty\,,\tag{4·3}$$

where $\Pi_\mu{}^2$ is the Virasoro generator, which is set equal to zero.

Thus, a naive covariant quantization of the model leads to meaningless, infinite results. The reason for this infinite result lies in the difficulty of separating the 16+16 components of Q into 8+8 components without destroying Lorentz invariance in ten dimensions. Because a 16 component spinor is the smallest representation of the Lorentz group, then one must necessarily break Lorentz invariance when reducing out the 16+16 components in Q down to 8+8 components. Indeed, this is precisely the way in which the light cone formalism for the superstring was first introduced.[18]

Recently, however, we found a way in which we can separate out the 8+8 first class constraints contained within Q without violating Lorentz invariance.[12] The trick is to notice that although $(\Pi_0 \pm \Pi_1)^2 = 0$, we can show that $\Pi_+ \cdot \Pi_- \neq 0$. In this way, we can now write down the correct separation of Q into 8+8 first class and 8+8 second class constraints:

$$\bar{S}^1 = \bar{Q}^1 \Pi_{\mu -} \gamma^\mu ,$$

$$\bar{S}^2 = \bar{Q}^2 \Pi_{\mu +} \gamma^\mu ,$$

$$\bar{T}^1 = \bar{Q}^1 \Pi_{\mu +} \gamma^\mu ,$$

$$\bar{T}^2 = \bar{Q}^2 \Pi_{\mu -} \gamma^\mu , \tag{4·4}$$

where S are first class and T are second class.

Notice that we have eliminated all infinities from the model while retaining 10D Lorentz invariance and 10D SUSY. The price we pay for this, however, is that we must employ rather involved Dirac brackets to incorporate the effect of the second class constraint generated by T.

§ 5. Construction of the algebra

At this point, we have now reformulated the heterotic string in first order formalism (for the NSR and the GS model in the right-moving sector). We have explicitly displayed the first and second class constraints, and can show that the effect of the second class constraints is simply to set equal to zero the left- (right-) moving sectors that we wish to eliminate. Thus, reparametrization invariance is maintained although we have the separation into left- and right- moving sectors.

We will now proceed to construct the algebra of the system. (We will use the NSR formalism, although our results can be generalized to the GS model in the right-moving sector.)

First, we wish to construct the field functionals $|\phi\rangle$ and $|\psi\rangle$, which are simply eigenstates of the operator L_0. If the right-moving states are formed by the bosonic vacuum $|0\rangle_{\rm BR}$ and the fermionic vacuum $|0\rangle_{\rm FR}$, while the left-moving states are labeled by $|k\rangle_{\rm L}$, then the field functionals can be written as

$$|\phi\rangle = \sum \varphi^{ik}(X_0) |i\rangle_{\rm BR} \otimes |k\rangle_{\rm L} ,$$

$$|\psi\rangle = \sum \psi^{jk}(X_0) |j\rangle_{\rm FR} \otimes |k\rangle_{\rm L} . \tag{5·1}$$

In order to explicitly calculate the eigenstates of the string, let us first parametrize the string as

$$X_\mu(\sigma, \tau) = X_\mu + P_\mu \tau + \frac{i}{2} \sum_{n \neq 0} \frac{\alpha_{n\mu}}{n} e^{-2in(\tau - \sigma)} + \frac{i}{2} \sum_{n \neq 0} \frac{\tilde{\alpha}_{n\mu}}{n} e^{-2in(\tau + \sigma)} ,$$

$$X^I(\tau + \sigma) = X^I + P^I(\tau + \sigma) + \frac{i}{2} \sum \frac{\tilde{\alpha}_n{}^I}{n} e^{-2in(\tau + \sigma)} . \tag{5·2}$$

(We will follow the notation of Ref. 1), where the indices are generalized to the covariant case in an obvious way.) We find

$$N=\sum_{n=1}^{\infty}(\alpha_{-n}^{\mu}\alpha_{n\mu}+b_{-n+1/2}^{\mu}b_{n-1/2\mu}+d_{-n}^{\mu}d_{n\mu}),$$

$$\tilde{N}=\sum_{n=1}^{\infty}(\tilde{\alpha}_{-n}^{\mu}\tilde{\alpha}_{n\mu}+\tilde{\alpha}_{-n}^{I}\tilde{\alpha}_{n}^{I}),\tag{5·3}$$

where we have introduced the NSR creation/annihilation operators b and d.

Then the on-shell condition $L_0-1=0$ is given by

$$\frac{1}{4}P_{\mu}^{2}=N+\tilde{N}-1+\frac{1}{2}\sum_{I=1}^{16}(P^{I})^{2}.\tag{5·4}$$

The Hilbert space is also constrained by the condition

$$N=\tilde{N}-1+\frac{1}{2}\sum_{I=1}^{16}(P^{I})^{2},\tag{5·5}$$

which is simply a reflection of the fact that closed strings must be independent of where we choose the origin of our σ parametrization.

Let us now begin a simple description of the Hilbert space of the covariant theory, which is the product space of the right- and left-moving sectors. Before the projection operator eliminates the spurious states of the theory, we find the right-moving states are given by

$$b_{-1/2}^{\mu}|0\rangle_{\mathrm{BR}}\,;\quad \psi|0\rangle_{\mathrm{FR}}\tag{5·6}$$

and the left-moving states are given by

$$\tilde{\alpha}_{-1}{}^{\mu}|0\rangle_{\mathrm{L}},\quad \tilde{\alpha}_{-1}^{I}|0\rangle,\quad |P^{I};(P^{I})^{2}=2\rangle\tag{5·7}$$

for $p_{\mu}^{2}=0$.

Notice that if the P^I are defined on the root lattice of a 16 dimensional self-dual lattice, then there are 480 states with $(P^I)^2=2$. When combined with the 16 components of α^I, we find that we have 496 states in the left-moving sector generated by compactification on the self-dual lattice, which form the adjoint representation of $E_8\times E_8$.

At the next level $P^2=8$, the states become more complicated. In the right-moving sector, we have

$$b_{-3/2}^{\mu}|0\rangle_{\mathrm{BR}},\quad b_{-1/2}^{\mu}b_{-1/2}^{\nu}b_{-1/2}^{\rho}|0\rangle_{\mathrm{BR}},\quad \alpha_{-1}^{\mu}b_{-1/2}^{\nu}|0\rangle_{\mathrm{BR}}$$

$$d_{-1}^{\mu}|0\rangle_{\mathrm{FR}}\psi,\quad \alpha_{-1}^{\mu}|0\rangle_{\mathrm{FR}}\psi\,,\tag{5·8}$$

while in the left-moving sector we have

$$\tilde{\alpha}_{-2}^{\mu}|0\rangle,\quad \tilde{\alpha}_{-1}^{\mu}\tilde{\alpha}_{-1}^{\nu}|0\rangle,\quad \tilde{\alpha}_{-2}^{I}|0\rangle,\quad \tilde{\alpha}_{-1}^{I}\tilde{\alpha}_{-1}^{J}|0\rangle,$$

$$\tilde{\alpha}_{-1}^{\mu}|P^{I},(P^{I})^{2}=2\rangle\,,\quad |P^{I},(P^{I})^{2}=4\rangle\,.\tag{5·9}$$

For the states generated by compactification, we have $16+16\times(17/2)+16\times480+480\times(1+2^7)=69{,}752$ states, which in turn can be decomponed into $E_8\times E_8$ representations:[19)]

$(3875,1)+(1,3875)+(248,248)+(248,1)+(1,248)+2(1,1)$.

The states generated by compactification rapidly proliferate. For example, at the next level, just the compactified left-moving states can be written down as

$$\tilde{\alpha}^{I}_{-3}|0\rangle, \quad \tilde{\alpha}^{I}_{-1}\tilde{\alpha}^{J}_{-2}|0\rangle, \quad \tilde{\alpha}^{I}_{-1}\tilde{\alpha}^{J}_{-1}\tilde{\alpha}^{K}_{-1}|0\rangle,$$
$$\tilde{\alpha}^{I}_{-2}|P^I, (P^I)^2=2\rangle, \quad \tilde{\alpha}^{I}_{-1}\tilde{\alpha}^{J}_{-1}|P^I, (P^I)^2=2\rangle$$
$$\tilde{\alpha}^{I}_{-1}|P^I, (P^I)^2=4\rangle, \quad |P^I, (P^I)^2=6\rangle, \tag{5·10}$$

which in turn form $16+16\times16+816+480\times16+480\times16\times(17/2)+16\times480\times(1+2^7)+480\times(1+3^7)=2115008$ states. These states, in turn, can be reduced down to:[19)]

$(30380,1)+(1,30380)+(3875,248)+(248,3875)+2(248,248)$
$+(3875,1)+(1,3875)+3(248,1)+3(1,248)+2(1,1)$.

Obviously, we wish to find another way in which to show that these states can be grouped into $E_8\times E_8$ representations. We will use the Kac-Moody construction to show that we can regroup all these states into representations of the algebra to all levels in the theory.

To begin a discussion of the Kac-Moody construction, let us first parametrize the open string (for the sake of argument) as

$$X^I(\theta)=X^I+P^I\theta+i\sum_{n\neq0}\frac{\alpha_n e^{in\theta}}{n},$$
$$q^I(\theta)=C^I{:}\exp(iX^I(\theta)){:}\,, \tag{5·11}$$

where C_I defines a "twist".[20)]

From these operators, we can now define three types of operators which generate a closed algebra, the semi-direct product of the Virasoro algebra Diff(S_1) with the Kac-Moody algebra g. (We will follow the notation of Ref. 20). See also Ref. 19).)

Let $\boldsymbol{e}_I$ be an orthonormal vector in the 16 dimensional root lattice space. For $SU(n)$, for example, we know that the roots can be represented by $\boldsymbol{e}_J-\boldsymbol{e}_I$. This allows us to construct the following operators:

$$J(I,\theta)={:}q_I^*(\theta)q_I(\theta){:}=-\frac{d}{d\theta}X^I(\theta)\,,$$
$$J(\boldsymbol{\alpha},\theta)={:}q_J^*(\theta)q_I(\theta){:},\quad \boldsymbol{\alpha}=\boldsymbol{e}_J-\boldsymbol{e}_I\,,$$
$$L(\theta)=\frac{N}{2}[\Sigma{:}J(I,\theta)^2{:}+\sum_{\alpha}{:}J(\boldsymbol{\alpha},\theta)J(-\boldsymbol{\alpha},\theta){:}], \tag{5·12}$$

where $N=(1+C_2(G)/2)^{-1}$ and $C_2(G)$ is the value of the quadratic Casimir operator for the adjoint representation.

Then the algebra generated by this construction closes:

$$[J(I,\theta),J(J,\theta')]=0\,,$$
$$[J(I,\theta),J(\boldsymbol{\alpha},\theta')]=-2\pi\delta(\theta-\theta')\boldsymbol{\alpha}_I J(\boldsymbol{\alpha},\theta),$$

$$[J(\boldsymbol{\alpha},\theta), J(-\boldsymbol{\alpha},\theta')]=2\pi\delta(\theta-\theta')\sum_I \alpha_I J(I,\theta)+2\pi i\delta'(\theta-\theta'),$$

$$[J(\boldsymbol{\alpha},\theta), J(\boldsymbol{\beta},\theta')]=2\pi\delta(\theta-\theta')J(\boldsymbol{\alpha}+\boldsymbol{\beta},\theta) \quad \text{if} \quad \boldsymbol{\alpha}+\boldsymbol{\beta}\in\Lambda;$$

$$=0 \text{ otherwise},$$

$$[L(\theta), J(\boldsymbol{\alpha},\theta')]=J(\boldsymbol{\alpha},\theta)\delta'(\theta-\theta'), \qquad \boldsymbol{\alpha}\in G, \tag{5·13}$$

$$[L(\theta), L(\theta')]=2\pi i\delta(\theta-\theta')(L(\theta)+L(\theta'))$$

$$-\frac{c}{12}2\pi i(\delta'''(\theta+\theta')+\delta'(\theta-\theta')), \tag{5·14}$$

where $c=n-1$ for $SU(n)$; $c=n+1/2$ for $SO(2n+1)$; $c=n(2n+1)/(n+2)$ for $Sp(n)$; $c=n$ for $SO(2n)$ and E_n.

We can easily generalize these commutators for the case when $\boldsymbol{\alpha}$ is not equal to a simple difference of two $\boldsymbol{e}_I$ vectors, e.g., E_8 is represented by the system of roots $\boldsymbol{e}_I-\boldsymbol{e}_J$ $(I\neq J)$, and

$$\pm\left(\frac{1}{3}\sum_{I=1}^{q}\boldsymbol{e}_I-\boldsymbol{e}_L-\boldsymbol{e}_M-\boldsymbol{e}_N\right),$$

$$L\neq M\neq N=1-q\,. \tag{5·15}$$

Each commutator has an important meaning. The first few commutators, for example, simply show that the J's generate representations of $E_8\times E_8$. Because the J's have nice commutation properties with L_0, we can now show that the entire Hilbert space spanned by these compactified states rearrange themselves into representations of $E_8\times E_8$.

The last commutator between the Virasoro generator and the Kac-Moody generator shows the nontrivial relationship between these two systems, which was missing in the light cone formulation of the model.

At first, however, there seems to be a problem with this last commutator. If this commutator were zero, then the Kac-Moody states would commute with the Virasoro algebra (much like the DDF states) and thus we can create new real physical states by simply multiplying real states with an arbitrary number of Kac-Moody operators.

However, the last commutator in (5·13) is *not* zero, which shows that the *Kac-Moody operators create spurious states*, which at first sight seems absurd.

The resolution to this seemingly bizarre result is because the true Virasoro generator extracted from the original first class constraints (3·4) contains the *sum* of Virasoro operators constructed out of the usual oscillators and the Kac-Moody oscillators. Thus, one can easily show that the application of the sum of these two operators onto the states of the theory produces real states which are obtained by *re-shuffling the two sectors*. Thus, the ghost states of the theory contain Kac-Moody operators mixed along with the usual oscillators, such that the *total number* of real states is the same as that given by the light cone gauge theory. Thus, there is no contradiction to having ghost states contain mixtures of Kac-Moody operators.

Notice also that the Kac-Moody algebra is a *global* two dimensional symmetry, not a local one. The Kac-Moody operators do *not* emerge as first class constraints of the theory. (In fact, if the Kac-Moody algebra were a local one, then in principle we could use all of its 496 generators to kill ghosts, which is far too many generators.) We note, however, that a new reformulation of the model may exist in which the Kac-Moody algebra is a local one. (The Kac-Moody algebra, although it is a global 2D symmetry, turns into a local Yang-Mills type symmetry once the interactions are turned on, which is a remarkable feature of the model.)

In summary, we now have a general understanding of the importance of the semi-direct product of the Virasoro algebra with the Kac-Moody algebra. It is now a straightforward task to construct the projection operator for the theory from the Shapovalov matrix.[21] The projection operator, like we found in Ref. 11), is constructed level by level in an iterative process.

Using this projection operator, we can now write down the free field theory Lagrangian for the second quantized heterotic string (with the NSR model in the right-moving sector):

$$\mathcal{L}(\phi,\psi)=\phi^\dagger \boldsymbol{P}_{\rm B}(i\partial_\tau-(L_0-\alpha_{\rm B}))\boldsymbol{P}\phi+\bar{\psi}\boldsymbol{P}_{\rm F}(i\partial_\tau-(F_0-\alpha_{\rm F}))\boldsymbol{P}_{\rm F}\psi\,. \tag{5·16}$$

(Once again, we can reabsorb the "time" derivative if we wish by taking the Fourier transform, which simply shifts the intercept of the trajectories to the values: $\alpha_{\rm B}\to 1$, $\alpha_{\rm F}\to 0$. In this way, we retrieve the mass shell condition if we take the equations of motion for the theory.)

This action is manifestly invariant under (a) local 2D SUSY (generated by the G and F gauges) (b) the Virasoro algebra (c) 2D global Kac-Moody algebra, but *not* 10D SUSY. In order to show 10D SUSY for the even G-parity sector of the theory, we can either use the GS formulation of the right-moving sector (see §4) or we can use the observation that there must exist matrices A and B which mix the fermionic and bosonic modes such that the action is invariant under:

$$\begin{aligned}&\delta\phi=A\psi\,,\\&\delta\psi=B\phi\,.\end{aligned} \tag{5·17}$$

§ 6. 10D SUSY

In principle, it should be possible to construct the matrices A and B to any level in the Hilbert space. For example, take the massless level of the open NSR string. The action has the 10D SUSY:

$$\begin{aligned}&\delta A_\mu\sim\bar{\epsilon}\gamma_\mu\psi\,,\\&\delta\psi\sim\sigma_{\mu\nu}F^{\mu\nu}\epsilon\,.\end{aligned} \tag{6·1}$$

This symmetry, in turn, can be represented explicitly as matrices A and B acting on the field functionals:

$$\delta|\phi\rangle\sim\delta(A_\mu b^\mu_{-1/2}|0\rangle_{\rm B})=A|\psi\rangle=A|0\rangle_{\rm F}\psi$$

$$\Rightarrow A=\bar{\epsilon}b_{-1/2\mu}|0\rangle_{\rm B}\langle 0|_{\rm F}\gamma^\mu\,,$$

$$\delta|\psi\rangle \sim \delta(\psi|0\rangle_F) = B|\phi\rangle = B(A_\mu b^\mu_{-1/2}|0\rangle_B)$$

$$\Rightarrow B = \partial\!\!\!/ \gamma_\mu |0\rangle_F \langle 0|_B b^\mu_{1/2}\epsilon\ . \tag{6·2}$$

It is easy to show that these matrices A and B to lowest order generate the correct 10D SUSY in the presence of the projection operators (which absorb an unwanted term).

The generalization of this result to the next massive mode is also straightforward. In fact, we can write the matrix A as follows:

$$A = \bar{\epsilon}\{b^\mu_{-3/2}(a\delta_{\mu\rho} + b\Gamma_{\mu\rho}) + b^\mu_{-1/2}b^\nu_{-1/2}b^\lambda_{-1/2}(c\Gamma_{[\mu\nu}\delta_{\lambda]\rho} + d\Gamma_{\mu\nu\lambda\rho})$$

$$+ a^\mu_{-1}b^\nu_{-1/2}(e\delta_{\mu\nu}\Gamma_\rho + f\delta_{\mu\rho}\Gamma_\nu + g\delta_{\nu\rho}\Gamma_\mu + h\Gamma_{\mu\nu\rho})\}|0\rangle_B\langle 0|_F(md_1{}^\rho + na_1{}^\rho)\psi\ , \tag{6·3}$$

B can always be written in terms of A such that the action is invariant. For higher levels, however, the calculation becomes more tedious. Can we find A and B to all levels, which would make 10D SUSY manifest in the NSR version of the heterotic string?

The answer is probably yes. For example, it has already been shown for the on-shell, light cone formalism that matrices X_{FB}[18] and X_{BF} exist which transform fermion states into bosonic states and which satisfy:

$$X^a_{m,BF}X^b_{n,FB} + (\gamma^0 X_{n,BF})^b(X_{m,FB}\gamma^0)^a = \delta^{ab}\delta_{m+n,0}\ .$$

These operators are constructed out of the fermion-boson vertex function,[22] which is a non-trivial coupling between the bosonic and fermionic Hilbert spaces. If the above relationship can be generalized to the off-shell case, then it should be possible to construct the A and B matrices explicitly.

There is some irony here. Originally, supersymmetry had its origins in the work of Gervais and Sakita,[23] who demonstrated 2D SUSY for the Neveu-Schwarz model. Later, this was generalized to 4D SUSY,[24] which gave rise to the current interest in supersymmetry. However, it was only much later[25] that it was realized that the NSR model itself in the even G-parity sector possessed 10D SUSY as well as 2D SUSY.

Now, with the second quantized action, both 10D and 2D SUSY are manifestly obvious (depending on the existence of the A and B matrices).

One may still object to our action, however, on the grounds that it is not local. However, by introducing an infinite number of auxiliary fields, we can soak up all unwanted non-local terms and obtain an explicitly local action. In particular, we notice that the non-local terms correspond to the zeros of the determinant of the Shapovalov matrix.[21] Kac,[10] however, has explicitly calculated all these zeros, so it is now possible to introduce an infinite number of auxiliary fields which can absorb the non-local terms and convert them into harmless local ones. For details of this construction, for the bosonic case, see Ref. 26), where the result for *all orders* is given in terms of the zeros of the determinant of the Shapovalov matrix. (At least for the lower levels, it can be shown that this series of auxiliary fields reproduces the result of Ref. 27).) Since our results are perfectly general, it should also be possible to construct the fully local version of our heterotic string field theory by simply introducing auxiliary fields which soak up the unwanted non-local terms introduced by the zeros of the (heterotic) Shapovalov determinant.

§ 7. Conclusion

In this paper, we have presented the second quantized field theory of free covariant heterotic strings. We have been careful to preserve the original gauge symmetries of the first quantized theory. The second quantized action is therefore a genuine gauge theory defined for both 2D and 10D SUSY as well as a non-trivial union of the Virasoro and Kac-Moody algebras.

The action is expressed in terms of the projection operator P, which selects out only real states. The novel feature of the real states of this theory is that they are non-trivial mixtures of Kac-Moody and ordinary oscillators, because of the last commutator in (5·13). Thus, Kac-Moody operators can create ghost states, but the total number of real states is equal to the number given by the light-cone theory.

Furthermore, the projection operator P is non-local, but we know all the location of its poles. Thus, by suitably introducing an infinite number of auxiliary fields, it is a simple matter to render the theory explicitly local.[26]

Throughout this paper, we have been careful to express all our results in terms of the path integral method. The reason for this is because the path integral method can be generalized to include interacting strings as well as free strings. The path integral method creates a vertex function of splitting strings which should be part of a non-linear realization of the Virasoro algebra.

This nonlinear realization of the Virasoro algebra, in turn, may give us a clue as to how to reformulate the model geometrically. A geometrical formulation of the model may play a key role in understanding how 10 dimensions can break dynamically down to 4 dimensions. Since the model's amazing properties will be lost if we introduce explicit breaking mechanisms, it is our philosophy that the model's symmetries must break dynamically. This dynamical symmetry breaking may become transparent if the model has a geometric reformulation.

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Appendix

In this appendix, we will fill in the details of how to make the transition from the second to the first order formalism.

We begin by calculating the precise relationship between the metric and the tangent space

$$2e_-{}^0 e_+{}^0 = -1/\lambda\,,$$

$$e_-{}^0 e_+{}^1 + e_+{}^0 e_-{}^1 = -\rho/\lambda\,,$$

$$2e_-{}^1e_+{}^1=\lambda-\rho/\lambda\,. \tag{A·1}$$

Solving, we find

$$\frac{e_-{}^1}{e_-{}^0}=\pm\lambda+\rho\,;\quad \frac{e_+{}^1}{e_+{}^0}=\mp\lambda+\rho\,. \tag{A·2}$$

Now let us introduce yet another Lagrange multiplier into the theory

$$\lambda^{--}(e_-{}^\alpha\partial_\alpha X^I)^2\to-\frac{1}{4}\sigma^{I2}+\sqrt{\lambda^{--}}\,\sigma^Ie_-{}^\alpha\partial_\alpha X^I\,. \tag{A·3}$$

Given this reduction, we can now write the complete action as

$$\mathcal{L}=\frac{1}{2\lambda}[\dot{X}^{M2}+2\dot{X}^M(\rho X'_M+\lambda\sqrt{\lambda^{--}}\,\sigma^Ie_-{}^0\delta_M{}^I-2i\lambda\bar{\zeta}_\alpha g^{\alpha0}\psi_\mu\delta_M{}^\mu)]+\frac{1}{2}X'^{M2}\left(\frac{\rho}{\lambda}-\lambda\right)$$
$$+\left[-\frac{1}{4}\sigma^{I2}+\sigma^I\sqrt{\lambda^{--}}\,e_-{}^0(\pm\lambda+\rho)X'^I\right]+\left[-\frac{i}{2}\bar{\psi}\gamma^0e_0{}^0\dot{\psi}+\bar{\omega}\gamma^+e_+{}^0\dot{\psi}\right]$$
$$+\left[-\frac{i}{2}\bar{\psi}\gamma^ae_a{}^1\partial_1\psi+\bar{\omega}\gamma^+e_+{}^1\partial_1\psi\right]-2i\bar{\zeta}_\alpha g^{\alpha1}\psi_\mu X'^\mu+[\bar{\zeta}_\alpha\psi_\mu\bar{\zeta}_\beta\psi^\mu g^{\alpha\beta}]. \tag{A·4}$$

We have used the fact that $\gamma_\alpha\zeta_\beta=\gamma_\beta\zeta_\alpha$.

There are several troublesome terms in the above expression. Notice that the last term in the brackets is quadratic in the Lagrange multiplier, which is unacceptable. We will find, however, that this unwanted term vanishes if we carefully reduce out the spinor components of the Lagrange multiplier. Given $\gamma^\alpha\zeta_\alpha=0$, we find

$$\begin{pmatrix}\zeta_0{}^1\\ \zeta_0{}^2\end{pmatrix}=\begin{pmatrix}(-\lambda-\rho)\zeta_1{}^1\\ (+\lambda-\rho)\zeta_1{}^2\end{pmatrix}. \tag{A·5}$$

With this decomposition of the 2-component spinor, we can reduce out the term in the bracket,

$$\bar{\zeta}_\alpha\psi_\mu\bar{\zeta}_\beta\psi^\mu g^{\alpha\beta}=\bar{\zeta}_1{}^1\psi^1\bar{\zeta}_1{}^2\psi^2[2g^{11}+2(\rho^2-\lambda^2)g^{00}+2g^{01}(-2\rho)]. \tag{A·6}$$

(This unwanted term will be cancelled later.)

Now let us introduce the canonical conjugates P and Π :

$$\mathcal{L}=-\frac{1}{2}\lambda P^{M2}+P^M[\dot{X}_M+\lambda\sqrt{\lambda^{--}}\,\sigma^Ie_-{}^0\delta_M{}^I+\rho P^MX'_M-2i\lambda\bar{\zeta}_\alpha g^{\alpha0}\psi^M\delta_{\mu M}]-\frac{1}{2}\lambda X'^{M2}$$
$$+\left[-\frac{1}{4}\sigma^{I2}-\rho X'^I\lambda\sigma^Ie_-{}^0\sqrt{\lambda^{--}}+\sigma^I\sqrt{\lambda^{--}}\,e_-{}^0(\pm\lambda+\rho)X'^I-\frac{1}{2}\lambda\lambda^{--}\sigma^{I2}e_-{}^{02}\right]$$
$$+\Pi\dot{\psi}+\left[\Pi+\frac{i}{2}\bar{\psi}\gamma^-e_-{}^0-\bar{\omega}\gamma^+e_+{}^0\right]_\mu\delta^\mu+\left[-\frac{i}{2}\bar{\psi}\gamma^ae_a{}^1\partial_1\psi+\bar{\omega}\gamma^+e_+{}^1\partial_1\psi\right]$$
$$+[2i\bar{\zeta}_\alpha g^{\alpha0}\psi^\mu\rho X_\mu{}^1-2i\bar{\zeta}_\alpha g^{\alpha1}\psi_\mu X'^\mu]+[2\lambda\bar{\zeta}_\alpha g^{\mu0}\psi^\mu\bar{\zeta}_\beta g^{\beta0}\psi_\mu+\bar{\zeta}_\alpha\psi_\mu\bar{\zeta}_\beta\psi^\mu g^{\alpha\beta}]. \tag{A·7}$$

Notice that we have introduced a Lagrange multiplier δ for the Π field.

We wish to cancel the last term in the brackets. We can reduce out the new term that is quadratic in the Lagrange multiplier, which we wish to eliminate

$$2\lambda \bar{\zeta}_\alpha g^{\alpha 0}\psi^\mu \bar{\zeta}_\beta g^{\beta 0}\psi_\mu = \bar{\zeta}_1{}^1\psi^1 \bar{\zeta}_1{}^2\psi^2(4\lambda g^{01^2}+2\lambda g^{00}(\rho^2-\lambda^2)+4\lambda g^{00}g^{01}(-2\rho)). \qquad (\mathrm{A}\cdot 8)$$

Notice that this term exactly cancels the other term which is quadratic in the Lagrange multiplier, which eliminates totally all unwanted terms. This leaves the Lagrange multiplier appearing linearly in the action, which is what we desire.

Now, we can further reduce our action:

$$\begin{aligned}\mathcal{L} = {} & P^M\dot{X}_M+\Pi_\mu\dot{\psi}^\mu-\frac{\lambda}{2}(P^{M^2}+X'^{M^2})+\rho P^\mu X'_M+\Big[\sigma^{I^2}\Big(-\frac{1}{4}-\frac{1}{2}\lambda\lambda^{--}e_-{}^{02}\Big) \\ & +\sigma^I(\lambda P^I\sqrt{\lambda^{--}}\,e_-{}^0+\sqrt{\lambda^{--}}\,e_-{}^0(\pm\lambda)X'^I)\Big]+\Big[\Pi+\frac{i}{2}\frac{\bar{\psi}\gamma^0}{\sqrt{\lambda}}-\bar{\omega}\begin{pmatrix}0&0\\1&0\end{pmatrix}\sqrt{\lambda}\Big]\delta \\ & +[2i\bar{\zeta}_\alpha g^{\alpha 0}\psi_\mu\rho X'^\mu-2i\bar{\zeta}_\alpha g^{\alpha 1}\psi_\mu X'^\mu-2i\lambda P_\mu\bar{\zeta}_\alpha g^{\alpha 0}\psi^\mu]. \end{aligned} \qquad (\mathrm{A}\cdot 9)$$

Except for the last term in brackets, the action is totally in canonical form. We now use the previous identities to show that this last term can be written as

$$\begin{aligned}[\ ] &= -2i\lambda\bar{\zeta}_1\begin{pmatrix}P+X' & 0\\ 0 & -P+X'\end{pmatrix}_\mu \psi^\mu \\ &= -2i\lambda\bar{\zeta}_1(P\sigma_3+X')_\mu\psi^\mu\,. \end{aligned} \qquad (\mathrm{A}\cdot 10)$$

Recombining these terms with earlier terms, we now have the final result which we used earlier in this paper. The action is totally in canonical form, with zero Hamiltonian, with three sets of first class constraints, and with second class constraints which simply remove the unwanted right- (left-) moving sectors.

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