Mechanisms for Infinity Cancellation in $SO(32)$ Superstring Theory

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We examine the mechanisms for infinity cancellation in the type I superstring theory with an $SO(32)$ internal symmetry.

Recently,[1] we provided a proof that the parity conserving one loop N point functions with arbitrary numbers of external gauge bosons are finite in type I superstring theory with an $SO(32)$ internal symmetry. This extends a result[2] of Green and Schwarz for the 4 point function. The proof utilized the old Ramond-Neveu-Schwarz[3] manifestly Lorentz invariant formulation of the theory, projecting onto the supersymmetric subspace[4] with even G parity bosons and Majorana-Weyl fermions. This approach has several advantages over the newer light cone gauge formulation. First of all, the light cone gauge becomes highly nonlinear unless the external momenta and polarization vectors can be chosen transverse to some quantization axis. This can be done without loss of generality if there are four or fewer external particles. The full treatment of N point functions above $N=5$ seems prohibitively difficult in light cone gauge although some results have been recently obtained.[5]

Secondly, it is not clear whether the light cone gauge can be used without ambiguity in the calculation of parity violating amplitudes.

Finally, the use of the light cone gauge may obscure the role of the G parity and Majorana-Weyl projections in cancelling potential ultraviolet divergences.

In this article, we provide some further refinements of the result of Ref 1). String quantization on the annulus and the role of the modular group in loop finiteness are discussed.

In a string theory, the N point planar loop with bosons circulating takes the form

$$\mathcal{L}_B = g^N d\Omega\, \mathrm{Tr}\, P \prod_{i=1}^{N} [V(p_i, 1)\Delta]. \tag{1}$$

Here P is the projection operator onto the physical subspace. The vertices V for particle emission are constructed out of conformal fields $\phi_{Jk}(\rho, a, a^\dagger)$ with J indicating the $SU(1,1)$ spin of the representation generated by some L_0, $L_\pm$ and k being the non integer part of the L_0 eigenvalues. These fields are composed of linear combinations of harmonic basis functions on the unit disk [6] with quantized coefficients. The string coordinate $Q_\mu(z)$ and the Neveu Schwarz[3] field $H_\mu(z)$ for example are obtained as direct sums of bounded below ($k=-J$) and bounded above ($k=J$) representations.

$$Q_\mu(z, a, a^\dagger) = \sum_{n=0}^{\infty}\left[\frac{\Gamma(n+2\epsilon)}{n!}\right]^{1/2}(a_\mu^{n\dagger}z^{n+\epsilon}+a_\mu^n z^{-n-\epsilon}),$$

$$[a_\mu^n, a_\nu^{m\dagger}] = g_{\mu\nu}\delta_{mn}, \quad J=-\epsilon, \quad k=\pm\epsilon\,, \tag{2}$$

$$H_\mu(z, b, b^\dagger) = \sum_{n=0}^{\infty}(b_\mu^{n\dagger}z^{n+1/2}+b_\mu^n z^{-n-1/2}),$$

$$\{b_\mu^n, b_\nu^{m\dagger}\} = g_{\mu\nu}\delta_{mn}, \quad J=-\frac{1}{2}, \quad k=\pm\frac{1}{2}\,. \tag{3}$$

In addition one has

$$P_\mu(z) = iz\frac{d}{dz}Q_\mu(z), \tag{4}$$

which transforms as $J=-1$, $k=\pm\epsilon$ without requiring the introduction of new oscillators. Vertices for the emission of excited states involve further derivatives of H_μ and P_μ with correspondingly more negative values of J.

The Ramond Γ_μ transforms as an unbounded representation with $J=-1/2$, $k=0$. As a consequence, the zeroth mode is self-adjoint and does not annihilate the vacuum.

$$\Gamma_\mu(z, c, c^\dagger) = \gamma_\mu + i\sqrt{2}\gamma_{11}\sum_{n=1}^{\infty}(c_\mu^{n\dagger}z^n + c_\mu^n z^{-n}),$$

$$\{c_\mu^n, c_\nu^{m\dagger}\} = g_{\mu\nu}\delta_{mn}\,, \quad \{\gamma_\mu, \gamma_\nu\} = -2g_{\mu\nu}\,,$$

$$J=-\frac{1}{2}, \quad k=0\,. \tag{5}$$

The propagator Δ in Eq. (1) takes the form

$$\Delta = \frac{1}{L_0-1}(1+\Omega)\left(\frac{1+G}{2}\right), \tag{6}$$

where Ω is the twist operator and G the G parity operator. For an orthogonal group internal symmetry

$$\Omega = -e^{i\pi R}\,, \tag{7}$$

$$G = e^{2\pi i R} \tag{8}$$

with

$$R = L_0 - p_0^2/2 = \sum_{m=1}^{\infty} m a^{m\dagger}a^m + L_{0b,c}\,. \tag{9}$$

Thus we can formally write (modulo internal symmetry factors)

$$\Delta = \frac{1}{2}\int_0^1 \frac{dx}{x}x^{L_0-1}\sum_{n=1}^{4}e^{i\pi n(R+1)}\,. \tag{10}$$

The Z_4 symmetry of the propagator,

$$e^{im\pi(R+1)}\Delta = \Delta\,, \quad m=1, 2, 3, 4 \tag{11}$$

is broken for an arbitrary internal symmetry group by the Chan-Paton factors but is restored as we shall see in the case of $SO(32)$.

The vertex for gauge boson emission from a boson line is

$$V_B(p_i, \rho_i, a, a^\dagger)=e^{ip_iQ(\rho_i,a,a^\dagger)}[\zeta_i\cdot P(\rho_i)+p_i\cdot H(\rho_i)\zeta_i\cdot H(\rho_i)]. \tag{12}$$

We pull the propagators to the right in Eq. (1) and supply the internal symmetry factor, $\mathrm{Tr}\lambda^0$, for the planar loop inner boundary of the circulating string to which no external lines are attached. The Chan-Paton factor, $\mathrm{Tr}\prod_{i=1}^{N}\lambda_i$, for the external legs is a common factor in all amplitudes which we suppress. We then have

$$\mathcal{L}_B=\frac{g^N}{2}\int d\Omega\,\mathrm{Tr}\,w^{L_0-1}P(\mathrm{Tr}\lambda^0-e^{i\pi R})(1+e^{2\pi iR})\prod_{i=1}^{N}V_B(p_i,\rho_i) \tag{13}$$

with

$$d\Omega=\frac{dw}{w}\prod_{i=2}^{N}\frac{d\rho_i}{\rho_i}. \tag{14}$$

The integration range is defined by

$$0<w<|\rho_N|<|\rho_{N-1}|<\cdots<|\rho_2|<\rho_1=1. \tag{15}$$

The effect of multiple twists is to extend the integration range of the ρ_i over negative as well as positive values consistent with Eq. (15). In order to put the loop in this form, we must understand the zeroth modes in $V_B(k_i, \rho_i)$ to depend only on the absolute value of ρ_i.

Using the trace technique developed in the treatment of Pomeron factorization in the general dual model[7] one can write[1]

$$\mathcal{L}_B=\frac{g^N}{2}d\Omega\sum_{n=1}^{4}\left[-1+\left(\cos^2\frac{n\pi}{2}\right)(1+\mathrm{Tr}\lambda^0)\right]F_{NS}(we^{in\pi})\,T_{NS}(\rho_i,we^{in\pi}). \tag{16}$$

In writing (16), we defined in Ref. 1)

$$F_{NS}(w)=w^{-1/2}\prod_{n=1}^{\infty}\left(\frac{1+w^{n-1/2}}{1-w^n}\right)^{D-2}. \tag{17}$$

In terms of the Jacobi theta functions

$$F_{NS}(w)=w^{(D-10)/16}\left[2\pi\frac{\theta_3(0|\tau)}{\theta_1'(0|\tau)}\right]^{(D-2)/2}. \tag{18}$$

This modified partition function contains a factor of $1/w$ from the volume element. The effect of the projection operator is to provide the -2 in the exponent. Although the projection operator in Möbius loops has not been studied, we assume in (16) that (17) remains correct with the replacement $w\to we^{in}$. The T's are defined in Ref. 1) to be

$$T_{NS}(\rho_i, w)=(-\epsilon\ln|w|)^{-D}\langle 0|\prod_{i=1}^{N}V_B(\rho_i, d(w),\bar{d}(w))|0\rangle, \tag{20}$$

where

$$d^n(w)=\frac{a^n}{1\pm w^{N+k}}+a'^{n\dagger}, \tag{21}$$

$$\bar{d}^n(w)=a^{n\dagger}+\frac{a'^n w^{n+k}}{1\pm w^{n+k}}. \tag{22}$$

The upper signs are taken in the case of anti-commuting fields and the lower signs in the case of commuting fields. The k's appearing in (21) and (22) are the absolute values of the non-integer parts of the L_0 eigenvalues defined in Eqs. (2) to (5); they should not be confused with particle momenta.

The notation may be significantly improved by replacing the d and $\bar{d}$ of Eq. (22) by, respectively

$$d^n=\frac{a^n}{\sqrt{1\pm w^{n+k}}}+\frac{a'^{n\dagger}w^{(n+k)/2}}{\sqrt{1\pm w^{n+k}}}, \tag{23}$$

$$\bar{d}^n=\frac{a^{n\dagger}}{\sqrt{1\pm w^{n+k}}}+\frac{a'^{n}w^{(n+k)/2}}{\sqrt{1\pm w^{n+k}}}. \tag{24}$$

It is clear that (20) is invariant under this replacement since the vacuum expectation value of any pair of conformal fields

$$\langle 0|\Phi_{Jk}(\rho_i, d, \bar{d})\Phi_{Jk}(\rho_j, d, \bar{d})|0\rangle$$

is independent of which definition one uses for the d and $\bar{d}$. The two definitions are related by a similarity transformation. In the following, we shall use the more symmetric definitions (23) and (24). In the string coordinate $Q_\mu(\rho_i, d, \bar{d})$, we can now recognize the doubly infinite set of harmonic basis functions[8] for the wave equation quantized on a manifold conformally equivalent to an annulus.

Similarly, the parity conserving loops with external gauge bosons and internal fermion lines are given by

$$\mathcal{L}_F=\frac{g^N}{2}\int d\Omega\sum_{n=1}^{2}\left[1-\left(\cos^2\frac{n\pi}{2}\right)(1+\mathrm{Tr}\,\lambda^0)\right]F_R(we^{in\pi})\,T_R(\rho_i, we^{in\pi}), \tag{25}$$

where

$$F_R(w)=2^{(D-2)/2}\prod_{n=1}^{\infty}\left(\frac{1+w^n}{1-w^n}\right)^{D-2}=\left[2\pi\frac{\theta_2(0|\tau)}{\theta_1'(0|\tau)}\right]^{(D-2)/2}, \tag{26}$$

$$T_R(\rho_i, w)=(-\epsilon\ln|w|)^{-D}\langle 0|\prod_{i=1}^{N}V_F(\rho_i, d(w), \bar{d}(w))|0\rangle, \tag{27}$$

$$V_F(\rho_i, a, a^\dagger)=e^{ip_i\cdot Q(\rho_i,a,a^\dagger)}\left[\zeta_i\cdot P(\rho_i)+\frac{p_i\cdot\Gamma(\rho_i)\,\zeta_i\cdot\Gamma(\rho_i)}{(-2)}\right]. \tag{28}$$

The vacuum expectation value in (27) is defined to include a normalized trace over Dirac matrices as in Ref. 1).

The Jacobi relation between the bosonic and fermionic partition functions is

$$F_R(w)=F_{NS}(w)+F_{NS}(we^{2\pi i}). \tag{29}$$

Thus the total parity conserving amplitude in one loop order is

$$\mathcal{L}=\mathcal{L}_B+\mathcal{L}_F=\frac{g^N}{2}\int d\Omega\sum_{n=1}^{4}\left\{\left(\sin^2\frac{n\pi}{2}\right)F_{NS}(we^{in\pi})\left[-T_{NS}(\rho_i, we^{in\pi})+T_R(\rho_i, we^{in\pi})\right]\right.$$
$$\left.+(\mathrm{Tr}\,\lambda^0)\left(\cos^2\frac{n\pi}{2}\right)F_{NS}(we^{in\pi})\left[T_{NS}(\rho_i, we^{in\pi})-T_R(\rho_i, we^{in\pi})\right]\right\}. \tag{30}$$

The T's in Eq. (30) are completely specified by the elementary two point functions of the conformal fields. For example, T_{NS} and T_R are each proportional to

$$A_0(\rho_i, w)=(-\epsilon \ln|w|)^{-D}\langle 0|\prod_{i=1}^{N} e^{ip_i\cdot Q(\rho_i,d(w),\bar{d}(w))}|0\rangle$$

$$=(-\epsilon \ln|w|)^{-D}\prod_{i<j}\exp[-\langle 0|p_i\cdot Q(\rho_i)p_j\cdot Q(\rho_j)|0\rangle]. \tag{31}$$

The vacuum expectation value of two Q's satisfies

$$\langle 0|Q_\mu(\rho_i, d, \bar{d})\,Q_\nu(\rho_j, d, \bar{d})|0\rangle=-g_{\mu\nu}\left(\frac{1}{\epsilon^2\ln w}-\frac{\ln w}{12}-\ln\psi(\rho_j/\rho_i, w)\right) \tag{32}$$

with

$$\psi(x, w)=-2\pi i e^{i\pi\nu^2/\tau}\frac{\theta_1(\nu|\tau)}{\theta_1'(0|\tau)}, \tag{33}$$

where

$$\nu=\frac{\ln x}{2\pi i}, \tag{34}$$

$$\tau=\frac{\ln w}{2\pi i}. \tag{35}$$

The potential divergence of the loop amplitude occurs at $w=1$ as can be seen from the form of the partition functions in (18) and (26) or from the form of the d operators in Eqs. (23) and (24). Fortunately, the behavior of the theta functions near $w=1$ is related to that near $w=0$ via the modular group transformations.[9]

$$\theta_1(\nu|\tau)=\theta_1(\tilde{\nu}|\tilde{\tau})[\epsilon^{-1}(c\tau+d)^{-1/2}e^{-i\pi c\nu^2/(c\tau+d)}], \tag{36}$$

where $\epsilon^8=1$ and

$$\tilde{\nu}=\frac{\nu}{c\tau+d}, \tag{37}$$

$$\tilde{\tau}=\frac{a\tau+b}{c\tau+d} \tag{38}$$

with a, b, c, d being integers satisfying

$$ad-bc=1. \tag{39}$$

For both the planar graph and the Möbius loop, one can find a modular group element relating the behavior near $|w|=1$ to that near $w=0$. In the planar case, we choose

$$a=d=0, \quad -b=c=1, \tag{40}$$

$$\tilde{\nu}=\nu'=\frac{\nu}{\tau}, \quad \tilde{\tau}=\tau'=-\frac{1}{\tau} \tag{41}$$

so that

$$e^{i\pi\nu^2/\tau}\frac{\theta_1(\nu|\tau)}{\theta_1'(0|\tau)}=\tau\frac{\theta_1(\nu'|\tau')}{\theta_1'(0|\tau')} \tag{42}$$

thus

$$\psi(\rho_j/\rho_i, w) = \tau\hat{\psi}(\rho_j'/\rho_i', w'), \tag{43}$$

where

$$\rho_i' = e^{2\pi i(\ln\rho_i/\ln w)}, \tag{44}$$

$$w' = e^{4\pi^2/\ln w} \tag{45}$$

and

$$\langle 0|\hat{Q}_\mu(\rho_i', d', \bar{d}')\,\hat{Q}_\nu(\rho_j', d', \bar{d})|0\rangle = g_{\mu\nu}\left(\frac{1}{2\epsilon} - \ln\hat{\psi}(\rho_j'/\rho_i', w')\right). \tag{46}$$

The $\hat{Q}_\mu$ are obtained from Eq. (2) by substituting d and $\bar{d}$ for a and $a^\dagger$ and putting $w'=0$ in the zeroth mode terms. The $\hat{}$ therefore signifies that the zeroth modes remain untransformed by Eqs. (23) and (24).

Thus

$$A_0(\rho_i, w) = \left(\frac{\tau}{i}\right)^{-D/2}(4\pi\epsilon)^{-D/2}\langle 0|\prod_{i=1}^{N} e^{ip_i\cdot\hat{Q}(\rho_i', d(w'), \bar{d}(w'))}|0\rangle. \tag{47}$$

The vacuum expectation value in (47) is well behaved as $w'\to 0$, i.e., as $w\to 1$.

In the Möbius loop, we have τ replaced by $\tau+1/2$. We seek, therefore, a modular transformation in which

$$\tau + \frac{1}{2} \to \frac{a(\tau+1/2)+b}{c(\tau+1/2)+d} = \tau'' + \frac{1}{2} \tag{48}$$

with

$$\tau'' = \frac{\ln w''}{2\pi i} \tag{49}$$

such that $w''\to 0$ as $w\to 1$.

The solution is

$$a = -b = -d = 1\,, \quad c = 2\,, \tag{50}$$

whereby

$$\tau'' = -\frac{1}{4\tau}, \tag{51}$$

$$\nu'' \equiv \frac{\ln\rho_j''/\rho_i''}{2\pi i} = \frac{\nu}{2\tau}. \tag{52}$$

Then from (36) we have

$$e^{i\pi\nu^2/\tau}\frac{\theta_1(\nu|\tau+1/2)}{\theta_1'(0|\tau+1/2)} = 2\tau\frac{\theta_1(\nu''|\tau''+1/2)}{\theta_1'(0|\tau''+1/2)}, \tag{53}$$

$$\psi(\rho_j/\rho_i, we^{i\pi}) = 2\tau\hat{\psi}(\rho_j''/\rho_i'', w''e^{i\pi}). \tag{54}$$

Modular group factors of two such as the one that distinguishes (54) from (43) similarly distinguish all the elementary correlations on an annulus from those on a Möbius strip.

Apart from the correlation of two Q's given above, one has in general

$$\langle 0|\Phi_{J_1k_1}(\rho_i,d(w),\bar{d}(w))\,\Phi_{J_2k_2}(\rho_j,d(w),\bar{d}(w))|0\rangle=\tau^{J_1+J_2}O(\rho_j'/\rho_i',w'), \tag{55}$$

$$\langle 0|\Phi_{J_1k_1}(\rho_i,d(-w),\bar{d}(-w))\,\Phi_{J_2k_2}(\rho_j,d(-w),\bar{d}(-w))|0\rangle$$
$$=(2\tau)^{J_1+J_2}O'(\rho_j''/\rho_i'',-w''). \tag{56}$$

The O and O' are vacuum expectation values of (sometimes different) conformal fields with the same J_1 and J_2. Some of these relations are given in Ref. 1).

Since all superstring vertices have a net $SU(1,1)$ spin $J=-1$, one finds for the terms in (30)

$$\sum_{n=1}^{4}\left(\cos^2\frac{n\pi}{2}\right)F_{NS}(we^{in\pi})[T_{NS}(\rho_i,\,we^{in\pi})-T_R(\rho_i,we^{in\pi})]$$
$$=i\tau^{-N-1}\sum_{n=1}^{4}(\cos^2\frac{n\pi}{2})\,F_{NS}(w'e^{in\pi})[\hat{T}_{NS}(\rho_i',w'e^{in\pi})-\hat{T}_R(\rho_i',w'e^{in\pi})]\,, \tag{57}$$

$$\sum_{n=1}^{4}\left(\sin^2\frac{n\pi}{2}\right)F_{NS}(we^{in\pi})[T_{NS}(\rho_i,we^{in\pi})-T_R(\rho_i,we^{in\pi})]$$
$$=2^{D/2}i(2\tau)^{-N-1}\sum_{n=1}^{4}(\sin^2\frac{n\pi}{2})\,F_{NS}(w''e^{in\pi})$$
$$\times[\hat{T}_{NS}(\rho_i'',w''e^{in\pi})-\hat{T}_R(\rho_i'',w''e^{in\pi})]. \tag{58}$$

The differential $d\Omega$ transforms to the primed and double primed variables as

$$d\Omega=d\Omega'\tau^{N+1}=d\Omega''(2\tau)^{N+1}\,. \tag{59}$$

The effect of the multiple twists is merely to double[10] the integration ranges of the w'',ρ_i'' making them equal to those of w', ρ_i'. Then writing z_i for ρ_i' and ρ_i'' and dropping all primes on the w variables we have

$$\mathcal{L}=i\frac{g^N}{2}\int d\Omega\sum_{n=1}^{4}\left[(\mathrm{Tr}\lambda^0)\cos^2\frac{n\pi}{2}-2^{D/2}\sin^2\frac{n\pi}{2}\right]$$
$$\times F_{NS}(we^{in\pi})[\hat{T}_{NS}(z_i,we^{in\pi})-\hat{T}_R(z_i,we^{in\pi})]. \tag{60}$$

In the case of an $SO(32)$ internal symmetry

$$\mathrm{Tr}\,\lambda^0=2^{D/2}=32\,, \tag{61}$$

$$\mathcal{L}=ig^N2^{D/2-1}\int d\Omega\sum_{n=1}^{4}e^{in\pi}F_{NS}(we^{in\pi})[\hat{T}_{NS}(z_i,we^{in\pi})-\hat{T}_R(z_i,we^{in\pi})]. \tag{62}$$

We see that only in the case of an $SO(32)$ internal symmetry is the four fold symmetry of the integrand maintained. It is now a simple matter to expand (61) about $w=0$ (the original $w=1$)

$$\hat{T}_{NS}(z_i,we^{in\pi})-\hat{T}_R(z_i,we^{in\pi})=\hat{T}_{NS}(z_i,0)-\hat{T}_R(z_i,0)$$
$$+w^{1/2}e^{in\pi/2}\hat{T}_{NS}'(z_i,0)+\mathcal{O}(we^{in\pi}), \tag{63}$$

$$F_{NS}(we^{in\pi})=(we^{in\pi})^{-1/2}+8+\mathcal{O}(w^{1/2}e^{in\pi/2}), \tag{64}$$

$$\sum_{n=1}^{4}(-1)^n \hat{F}_{NS}(we^{in\pi})[\hat{T}_{NS}(z_i,we^{in\pi})-\hat{T}_R(z_i,we^{in\pi})]$$
$$=w^{-1/2}[\hat{T}_{NS}(z_i,0)-\hat{T}_R(z_i,0)](\sum_{n=1}^{4}e^{in\pi/2})$$
$$+[8\hat{T}_{NS}(z_i,0)-8\hat{T}_R(z_i,0)+\hat{T}'_{NS}(z_i,0)](\sum_{n=1}^{4}e^{in\pi})$$
$$+\mathcal{O}(w^{1/2})(\sum_{n=1}^{4}e^{3n\pi i/2})+\mathcal{O}(w). \tag{65}$$

Since $d\Omega$ behaves as dw/w it is clear that the amplitudes are finite independent of the number of external gauge bosons.

The four topologies of planar and Möbius loops each with and without a G parity operator are seen to be intimately connected in the $SO(32)$ theory. Just as any finite integral can be made infinite by separating the integrand into two divergent pieces and making a transformation on one piece alone, the superstring loop amplitudes can be made divergent if one separately transforms individual terms in (60). This, in itself, should not be considered a flaw in the type I theory.

In summary, we have pointed out the role of the modular group and the four fold symmetry between planar and Möbius loops with and without the G parity operator in insuring the finiteness of the parity conserving one loop graphs in $SO(32)$ superstring theory.

The parity violating loops have additional complications and are presently under study in collaboration with B. Harms of the University of Alabama to whom thanks are also due for many discussions of the material of this article.

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