CP Violation in the Standard Model

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In order to study if the standard six-quark model is compatible with *CP* violation in $K\to 2\pi$ decays, the parameter B is calculated. Theoretical errors associated with the evaluation of B based on $SU(3)$, PCAC and $K^+\to\pi^+\pi^0$ data are estimated. It is found that in order to find if the standard six-quark model is compatible with *CP* violation experiments it is most important to obtain a better upper bound on the ratio of semi-leptonic branching ratios of B-mesons $\mathrm{Br}(b\to ul\nu)/\mathrm{Br}(b\to cl\nu)$ experimentally.

§ 1. Introduction

The purpose of this article is to study if the *CP* violation in $K\to 2\pi$ decays is compatible with the standard six-quark model for strong, weak and electromagnetic interactions.[1),2)]

In this model the source of the *CP* violation is the angle δ which appears in the weak charged-current,

$$J_\mu=(g_2/2\sqrt{2})(\bar{u},\ \bar{c},\ \bar{t})\,i\gamma_\mu(1+\gamma_5)\,U\begin{pmatrix} d \\ s \\ b \end{pmatrix}, \tag{1·1}$$

where

$$U=\begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3-s_2s_3e^{i\delta} & c_1c_2s_3+s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3+c_2s_3e^{i\delta} & c_1s_2s_3-c_2c_3e^{i\delta} \end{pmatrix}. \tag{1·2}$$

In the 3×3 unitary matrix U, $c_i=\cos\theta_i$ and $s_i=\sin\theta_i$, and θ_1, θ_2, θ_3 and δ are Kobayashi-Maskawa angles.

As is well known, the *CP* violating $K\to 2\pi$ decay amplitudes are expressed as[3)]

$$\eta_{+-}=A(K_L\to\pi^+\pi^-)/A(K_S\to\pi^+\pi^-)=\varepsilon+\varepsilon'\,,$$
$$\eta_{00}=A(K_L\to\pi^0\pi^0)/A(K_S\to\pi^0\pi^0)=\varepsilon-2\varepsilon'\,. \tag{1·3}$$

The *CP* violation parameters ε and ε' are expressed as

$$\varepsilon\approx\frac{1}{2\sqrt{2}}e^{i\pi/4}\left[\frac{\mathrm{Im}M_{12}}{\mathrm{Re}M_{12}}+2\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right],$$

$$\varepsilon'\approx-\frac{1}{\sqrt{2}}e^{i\pi/4}\left(\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\right)\left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}-\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}\right), \tag{1·4}$$

where

$$A_I \exp(i\delta_I) = \langle (\pi\pi)_I | H_w | K^0 \rangle,$$

$$A_I^* \exp(i\delta_I) = \langle (\pi\pi)_I | H_w | \bar{K}^0 \rangle, \tag{1·5}$$

$$M_{12} = \sum_{n \neq K^0, \bar{K}^0} P \frac{\langle K^0 | H_w | n \rangle \langle n | H_w | \bar{K}^0 \rangle}{m_K - m_n}, \tag{1·6}$$

$$-2\mathrm{Re}M_{12} \approx \Delta m = m_L - m_S\,. \tag{1·7}$$

Since all phase conventions are chosen so that the predominant quark amplitude contributing to $K \to 2\pi$ decays $s \to u + \bar{u} + d$ is real, the CP violation is associated with the imaginary part of the quark amplitudes $s \to c + \bar{c} + d$ and $s \to t + \bar{t} + d$. These amplitudes automatically obey the $\Delta I = 1/2$ rule and, therefore, $\mathrm{Im}A_2 = 0$ in (1·4). Thus, we obtain

$$\varepsilon' \approx -\frac{1}{\sqrt{2}} e^{i\pi/4} \left(\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\right) \left(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right). \tag{1·8}$$

According to recent experimental results,[4),5)]

$$\varepsilon'/\varepsilon = 0.0017 \pm 0.0082 \text{ (BNL-Yale)}$$

$$= -0.0046 \pm 0.0058 \text{ (FNAL-Saclay)}. \tag{1·9}$$

From (1·8), (1·9) and experimental results $\mathrm{Re}A_2/\mathrm{Re}A_0 \approx 1/20$, we find $|2\,\mathrm{Im}A_0/\mathrm{Re}A_0| \ll |\mathrm{Im}M_{12}/\mathrm{Re}M_{12}|$ and, therefore,

$$\varepsilon \approx -e^{i\pi/4} (\mathrm{Im}M_{12}) / (\sqrt{2}\Delta m). \tag{1·10}$$

Hence, the purpose of the phenomenological study of the CP violation in $K \to 2\pi$ decays is to calculate A_0, A_2 and M_{12} theoretically. Though theoretical study of CP violation is incomplete without reproducing $\mathrm{Re}A_0$, $\mathrm{Re}A_2$ and $\mathrm{Re}M_{12}$ theoretically, we study only $\mathrm{Im}M_{12}$ and $\mathrm{Im}A_0$ in this article. The reason is the fact that the CP violation angle δ appears only in $\mathrm{Im}M_{12}$ and $\mathrm{Im}A_0$ and the fact that $\mathrm{Re}M_{12}$ and $\mathrm{Re}A_0$ have significant long-distance contributions[6)] which we cannot calculate reliably at present.[7)] In this article we use experimental results as the magnitudes of $\mathrm{Re}M_{12}$, $\mathrm{Re}A_0$ and $\mathrm{Re}A_2$.

Long-distance contributions to $\mathrm{Im}M_{12}$ and $\mathrm{Im}A_0$ are considered to be negligible since the quark amplitude $s \to u + \bar{u} + d$ is real. Short-distance contributions to M_{12} (contributions from intermediate states with mass$\gtrsim 2m_c$) are considered to be well approximated by the box-diagrams,[8)] M_{12}^{box},

$$\mathrm{Im}M_{12}^{\mathrm{box}} = -(G_F^2/6\pi^2) f_K^2 m_K m_c^2 s_1^2 s_2 s_3 \sin\delta B$$

$$\times [-\eta_1 + \eta_2 (s_2^2 + s_2 s_3 \cos\delta)(m_t/m_c)^2 + \eta_3 \ln(m_t/m_c)^2],$$

$$\mathrm{Re}M_{12}^{\mathrm{box}} = -(G_F^2/12\pi^2) f_K^2 m_K m_c^2 s_1^2 B \eta_1, \tag{1·11}$$

where $G_F = 1.166 \times 10^{-5}\mathrm{GeV}^{-2}$, $f_K = 157\mathrm{MeV}$, and m_K and m_c are the mass of kaons and that of c-quark, respectively. Here

$$\eta_1 \approx 0.7, \quad \eta_2 \approx 0.6 \quad \text{and} \quad \eta_3 \approx 0.4 \tag{1·12}$$

are QCD enhancement factors,[9)] and the parameter B defined by

$$B=-\frac{3}{4m_K f_K{}^2}\langle\bar{K}^0|O^{\Delta S=2}|K^0\rangle \tag{1·13}$$

is the measure of the matrix-element of the $\Delta S=2$ operator

$$O^{\Delta S=2}=[\bar{s}_i\gamma_\mu(1+\gamma_5)d_i][\bar{s}_j\gamma_\mu(1+\gamma_5)d_j] \tag{1·14}$$

compared to the value calculated by inserting the vacuum in all possible ways. In deriving (1·11) we have assumed $m_t{}^2 \ll m_W{}^2$ and used the experimental results s_2, $s_3 \ll 1$, which are obtained from experimental information on semileptonic B-meson decays.[10]

Therefore, theoretical study of CP violation in the standard six-quark model reduces to theoretical evaluation of the parameter B and the imaginary part of the matrix-element $A_0=\langle(\pi\pi)_{I=0}|H_w|K^0\rangle\exp(-i\delta_0)$, where H_w is the effective Hamiltonian for strangeness changing nonleptonic interaction, in which the effects of virtual hard gluons are included through the renormalization group equation,[9),11)]

$$H_w=\sqrt{2}G_F s_1 c_1 c_3 \sum_{i=1}^{6} C_i O_i + \text{h.c.}, \tag{1·15}$$

where

$$O_1=\bar{d}_L s_L \bar{u}_L u_L - \bar{d}_L u_L \bar{u}_L s_L\,,$$

$$O_2=\bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L + 2\bar{d}_L s_L \bar{d}_L d_L + 2\bar{d}_L s_L \bar{s}_L s_L\,,$$

$$O_3=\bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L + 2\bar{d}_L s_L \bar{d}_L d_L - 3\bar{d}_L s_L \bar{s}_L s_L\,,$$

$$O_4=\bar{d}_L s_L \bar{u}_L u_L + \bar{d}_L u_L \bar{u}_L s_L - \bar{d}_L s_L \bar{d}_L d_L\,,$$

$$O_5=\bar{d}_L \lambda^a s_L(\bar{u}_R\lambda^a u_R + \bar{d}_R\lambda^a d_R + \bar{s}_R\lambda^a s_R),$$

$$O_6=\bar{d}_L s_L(\bar{u}_R u_R + \bar{d}_R d_R + \bar{s}_R s_R) \tag{1·16}$$

and $\bar{d}_L s_L$ is a short hand for $\bar{d}_i\gamma_\mu(1+\gamma_5)s_i/2$ and $\bar{u}_R\lambda^a u_R$ for $\bar{u}_i(\lambda^a)_{ij}\gamma_\mu(1-\gamma_5)u_j/2$. Here, λ^a is a λ-matrix of the color $SU(3)$ group with $\mathrm{Tr}(\lambda^a\lambda^b)=2\delta_{ab}$. The following set of coefficients has been obtained by Gilman and Wise,[11)]

$$C_1=-2.6+0.062\tau, \qquad C_2=0.1-0.006\tau\,,$$

$$C_3=0.08, \qquad C_4=0.4, \qquad C_5=-0.10-0.10\tau\,,$$

$$C_6=-0.030-0.059\tau \tag{1·17}$$

for $\Lambda_{\overline{\mathrm{MS}}}=0.1\,\mathrm{GeV}$, $m_t=30\,\mathrm{GeV}$ and $\alpha_s(\mu^2)=1$, where $\tau=s_2{}^2+s_2c_2s_3\exp(-i\delta)/c_1c_3$. Among six operators O_i, only O_4 is a $\Delta I=3/2$ operator which contributes only to A_2 and all others are $\Delta I=1/2$ operators which contribute only to A_0.

In §2 the magnitude of the parameter B is evaluated. The result has already been derived by Donoghue, Golowich and Holstein,[12)] but there is a controversy on their evaluation[13)] of B. In this article we have checked all the assumptions which are necessary in order to derive their result and have estimated theoretical errors. Discussion and conclusions are given in §3.

§ 2. Evaluation of the parameter B

The magnitude of the parameter B can be extracted from $\Delta I=3/2$ $K\to 2\pi$ decay data by making use of $SU(3)$ symmetry, PCAC and current algebra since both B and $\langle\pi|H_w^{\Delta I=3/2}|K\rangle$, which is derived from $\langle\pi\pi|H_w^{\Delta I=3/2}|K\rangle$ by the soft-pion reduction method, are matrix-elements of the operators which transform as $(27_L, 1_R)$ under the chiral-symmetry group $SU(3)_L\times SU(3)_R$ between one-octet-pseudoscalar-meson states. Though the evaluation of B in this method has been done by Donoghue, Golowich and Holstein,[12] we repeat the derivation in order to estimate the error associated with this estimation of B.

Here, the matrix-element $\langle\pi|H_w^{\Delta I=3/2}|K\rangle$ is derived from $\langle\pi^+(q_+)\pi^0(q_0)|H_w^{\Delta I=3/2}|K^+(q_K)\rangle$ through the soft-pion reduction method by assuming that the $K^+\to\pi^+ +\pi^0$ decay amplitude is a linear function of q_+^2, q_0^2 and q_K^2,

$$(8q_{+0}q_{00}q_{K0})^{1/2}\langle\pi^+(q_+)\pi^0(q_0)|H_w^{\Delta I=3/2}|K^+(q_K)\rangle$$
$$=A+Bq_+^2+Cq_0^2+Dq_K^2\,. \tag{2·1}$$

The error associated with this approximation is not expected to be significant since there are no resonances in $I=2$ $\pi\pi$ state. The error is considered to be less than about 20% since the slopes of the Dalitz plots of $K\to 3\pi$ decays derived by assuming the momentum dependence similar to (2·1) and the soft-pion reduction method agree with the experimental results within an error of about 20%.[14),15)]

There are two conditions on the $K^+\to\pi^+ +\pi^0$ decay amplitude. One is the $SU(3)$ condition that the $K\to 2\pi$ decay amplitudes should vanish in the $SU(3)$ symmetry limit,[16)] i.e.,

$$\langle\pi_1(q_1)\pi_2(q_2)|H_w|K(q_K)\rangle=0 \text{ at } q_K^2=q_1^2=q_2^2 \tag{2·2}$$

and another is the $SU(2)$ relation in the soft-pion limit,

$$-4\lim_{q\to 0}\langle\pi^+(p)\pi^0(q)|H_w^{\Delta I=3/2}|K^+(p+q)\rangle$$
$$=\lim_{q\to 0}\langle\pi^+(q)\pi^0(p)|H_w^{\Delta I=3/2}|K^+(p+q)\rangle, \tag{2·3}$$

where $\Delta I=3/2$ interaction $H_w^{\Delta I=3/2}$ is given by

$$H_w^{\Delta I=3/2}=\sqrt{2}G_F s_1 c_1 c_3 C_4 O_4\,. \tag{2·4}$$

The errors associated with the $SU(3)$ and $SU(2)$ conditions (2·2) and (2·3) are considered to be less than about 10% and 5%, respectively.*)

Thus, we find

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*) According to the leading corrections of order $m_K^4\ln m_K$ in chiral perturbation theory,

$$(8q_{K0}q_{+0}q_{00})^{1/2}\langle\pi^+\pi^0|H_w^{\Delta I=3/2}|K^+\rangle|_{q_K^2=q_+^2=q_0^2=-m_K^2/3}$$
$$\times\{(8q_{K0}q_{+0}q_{00})^{1/2}\langle\pi^+\pi^0|H_w^{\Delta I=3/2}|K^+\rangle|_{q_K^2=-m_K^2, q_+^2=q_0^2=-m_\pi^2}\}^{-1}$$
$$=[-(53m_K^2/54)(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2]/[1-(9m_K^2/2)(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2],$$
$$\approx 0.08\,,$$

where $f_\pi=132$MeV and $\mu=1$ GeV.

$$(8q_{K_0}q_{+0}q_{00})^{1/2}\langle\pi^+(q_+)\pi^0(q_0)|H_w^{\Delta I=3/2}|K^+(q_K)\rangle$$
$$=(1/\sqrt{6})A_2(3q_K{}^2-4q_+{}^2+q_0{}^2)/(m_\pi{}^2-m_K{}^2), \qquad (2\cdot5)$$

where the magnitude of the $\Delta I=3/2$ amplitude A_2 is estimated to be

$$A_2=1.30\times10^{-5}\text{MeV} \qquad (2\cdot6)$$

from the experimental value of the $K^+\to\pi^+\pi^0$ decay rate[17] with 13% correction (reduction) due to isospin breaking.[12]

Hence, by making use of the soft-pion reduction method, which is expected to hold with an error less than about 15%, we obtain

$$(2q_0)<\pi^+(q)|H_w^{\Delta I=3/2}|K^+(q)\rangle$$
$$=\sqrt{2}f_\pi\lim_{p\to0}[8p_0q_0(p_0+q_0)]^{1/2}\langle\pi^+(q)\pi^0(p)|H_w^{\Delta I=3/2}|K^+(p+q)\rangle$$
$$=(f_\pi/\sqrt{3})A_2q^2/(m_K{}^2-m_\pi{}^2). \qquad (2\cdot7)$$

By summing up the errors thus estimated, we find that the error associated with this relation is estimated to be less than about 50%

By making use of the $SU(3)$ relation,

$$\langle K^0(q)|O^{\Delta S=2}|\bar{K}^0(q)\rangle=8(f_K/f_\pi)\langle\pi^+(q)|O_4|K^+(q)\rangle, \qquad (2\cdot8)$$

we finally obtain

$$B=(3\sqrt{2}/G_Fm_Kf_\pi f_Ks_1C_4)\langle\pi^+(q)|H_w^{\Delta I=3/2}|K^+(q)\rangle|_{q^2=-m_K{}^2}$$
$$=\sqrt{3/2}(A_2/G_Fm_K{}^2f_Ks_1C_4)$$
$$=0.38\,. \qquad (2\cdot9)$$

The error associated with the $SU(3)$ relation $(2\cdot8)$ is estimated to be less than 50%. Here, we have assigned a generous error since there are ambiguities due to factors like $f_K/f_\pi(\approx1.2)$ which is equal to 1 in the $SU(3)$ symmetry limit.

Finally, we have obtained the following prediction for the parameter B,

$$B=0.38\times2 \text{ to } 0.38\times1/2$$
$$=0.8 \text{ to } 0.2\,. \qquad (2\cdot10)$$

However, there is an objection[13] against the result $(2\cdot9)$. In chiral perturbation theory this result is the result of the leading order calculation. The leading corrections of order $m_K{}^4\ln m_K{}^2$ have been calculated in Ref. 13), and the corrections have been found to be large for the $\Delta S=2$ matrix-element. The result $(2\cdot9)$ should be multiplied by a factor*)

*) If we use f_π instead of f_K in $(1\cdot13)$ and 1 instead of f_K/f_π in $(2\cdot8)$, $(2\cdot11)$ becomes[13]

$$1-(55/6)m_K{}^2(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2\approx2.15$$

since

$$f_K/f_\pi=1-(3/2)m_K{}^2(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2\approx1.19.$$

$$1-(23/3)m_K{}^2(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2\approx 1.97 \tag{2·11}$$

(for $\mu=1$ GeV). Thus, it is concluded in Ref. 13) that the large correction (2·11) indicates that chiral perturbation theory for B has broken down.

Our comment on this conclusion is as follows. Even if chiral perturbation theory for B breaks down, our experience in hadron physics teaches us that our prediction for B with estimated error (2·10) is quite reliable as we have shown. We do not have to regard it as the result of chiral perturbation theory.*)

§ 3. Discussion and conclusion

Next let us evaluate ImA_0. We can evaluate it by making use of the soft-pion reduction method as we have done in §2 and by inserting the vacuum in $\langle\pi|O_i|K\rangle$ $(i\neq 4)$ in all possible way. Our experience in the evaluation of B has shown us that the real value can be less than $1/6$ (≈ 0.16) of the predicted value in this method. Situation is worse in this case since $\delta_0\approx\pi/4$ while $\delta_2\approx 0$. Thus, it is very difficult to find if CP violation is compatible with the standard six-quark model by studying the ratio ε'/ε.

Therefore, for our purpose the best way is to study if the theoretical prediction

$$\begin{aligned}\mathrm{Re}\varepsilon &\approx (\mathrm{Im}M_{12})/4(\mathrm{Re}M_{12})\\ &=[(1-D)/4](\mathrm{Im}M_{12}^{\mathrm{box}})/(\mathrm{Re}M_{12}^{\mathrm{box}})\\ &=(B/2)s_2s_3\sin\delta\\ &\quad\times[-\eta_1+\eta_2(s_2{}^2+s_2s_3\cos\delta)(m_t/m_c)^2+\eta_3\ln(m_t/m_c)^2]\end{aligned} \tag{3·1}$$

is compatible with the experimental result, $\mathrm{Re}\varepsilon=(1.621\pm0.088)\times10^{-3}$. In deriving (3·1), we have used the relation,

$$\begin{aligned}-2\,\mathrm{Re}M_{12}^{\mathrm{box}} &\approx \eta_1 B\Delta m\equiv(1-D)\Delta m\\ &=-2(1-D)\mathrm{Re}M_{12}\,.\end{aligned} \tag{3·2}$$

For this purpose we need information on s_2 and s_3, which can be derived from the following experimental results[10] on the B-meson decays,

$$\begin{aligned}&\tau_B=(1.26\pm0.16)\times10^{-12}s\,,\\ &\mathrm{Br}(B\to Xl\nu)=(11.7\pm0.6)\%\,,\\ &\mathrm{Br}(b\to ul\nu)/\mathrm{Br}(b\to cl\nu)<0.08,\ \text{probably } 0.04\,.\end{aligned} \tag{3·3}$$

It has been found[10] that our result $B=(0.8$ to $0.2)$ is compatible with (3·1) and (3·3), but that it is incompatible with (3·1) if $\mathrm{Br}(b\to ul\nu)/\mathrm{Br}(b\to cl\nu)<0.03$. Therefore, in order to find if the standard six-quark model is compatible with the CP violation experiments, it is most important to find a reliable phenomenological model on the semileptonic decay of B-mesons and to obtain better upper bound on $\mathrm{Br}(b\to ul\nu)/\mathrm{Br}(b\to cl\nu)$ exper-

*) It is interesting to notice that the correction factors to both (2·7) and (2·8) are

$$1-(23/6)m_K{}^2(4\pi f_\pi)^{-2}\ln(m_K/\mu)^2\approx 1.48$$

in chiral perturbation theory.

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