# Electrical Resistivity Due to Electron-Electron Umklapp Scattering in Ferromagnetic Metals

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(Received July 17, 1991)

We consider electrical resistivity of ferromagnetic metals in which magnetism and electrical currents are carried by the same electrons of single bands. Relaxation times of electrons with up (or down) spin are expressed in terms of collision integrals by solving a coupled Boltzmann equation. We analyze Umklapp —— as well as normal —— processes of electron-electron scattering in Born approximation for ferromagnetic state. The coefficient of the quadratic term in temperature dependence of resistivity is found to decrease when magnetization develops at low temperatures.

## § 1. Introduction

Electrical resistivity of most of the ferromagnetic metals, having subtracted the residual one, is nearly proportional to square of temperature considerably below the Curie point, i.e., $\rho \fallingdotseq BT^2$ (for $T \ll T_c$). Usual explanation for this characteristic assumes two kinds of electrons, say, $d$-electrons and $s$-electrons. The former have heavy masses and are nearly localized at lattice sites. They have magnetic moments responsible for ferromagnetism. The latter have light masses and are itinerant from site to site carrying electric currents in metals. Interaction between them, i.e., $s$-$d$ interaction causes scattering: Then, $s$-electrons loose their energy by exciting magnetic fluctuations or spin waves in $d$-electron system.[1] As a result, electrical resistivity is proportional to square of temperature, that reflects the dispersion relation of spin waves, $\omega \propto q^2$. General discussions on $T^2$-term of resistivity in paramagnetic case have been given by Ziman using Boltzmann equation[2] and recently by Yamada et al. on a basis of Kubo formula.[3]

However, if the $s$-band is unoccupied we must deal with magnetism and electrical conduction based on the same ground taking only the $d$-band electrons into account. Such a situation is found in ferromagnetic transition metal chalcogenides, for example, in $CoS_2$, of which temperature dependence of resistivity changes at $T_c$ and nearly proportional to $T^2$ at low temperatures.[4] The characteristic suggests that electron-electron scattering in $d$-band restricted by Pauli's exclusion principle would be the main origin of resistivity. As known, Umklapp processes are indispensable to give rise to finite resistivity in such a case.

When conduction electrons are ferromagnetic, the sizes of the Fermi surfaces of up-spin electrons and of down-spin electrons are different. Since the electron scatter-

ing takes place near the Fermi surfaces at low temperatures, a direct influence of ferromagnetism on electrical conduction is expected.

In this paper we study the relation between magnetism and resistivity focusing on the change in electron-electron scattering process in ferromagnetic state within a simplest model and methods. For an electron system with single band we solve a coupled Boltzmann equation in relaxation time approximation to obtain a general expression for resistivity. Normal and Umklapp scattering processes for given magnetization are examined, first in Born approximation. We find that the coefficient of quadratic term in temperature-dependent resistivity, $B(\alpha)=R(T)/T^2$ decreases when the magnetization, $\alpha$, develops. Enhancement of resistivity due to higher-order processes of scattering is briefly discussed.

## § 2. A coupled Boltzmann equation

In the ferromagnetic state the current carried by up-spin electrons is different from the current carried by down-spin electrons. We set up a coupled Boltzmann equation for distribution functions of electron of wave vector $\boldsymbol{k}$ with up-spin, $f_{\boldsymbol{k}}{}^{+}$, and of that with down-one, $f_{\boldsymbol{k}}{}^{-}$ as follows,

$$-\frac{e\boldsymbol{E}}{\hbar}\cdot\frac{\partial f_{\boldsymbol{k}}{}^{\sigma}}{\partial \boldsymbol{k}}=\left.\frac{\partial f_{\boldsymbol{k}}{}^{\sigma}}{\partial t}\right|_{\text{coll}}, \tag{2·1}$$

$\sigma=+(-)$ for up- (down-) spin,

that is,

$$\begin{aligned}-\frac{\hbar e\boldsymbol{E}\cdot\boldsymbol{k}_1}{m}\frac{\partial f_{k_1}^{+}}{\partial \varepsilon_{k_1}^{+}}=\sum_{k_2k_3k_4}\{&W(\boldsymbol{k}_1+,\ \boldsymbol{k}_2-;\ \boldsymbol{k}_3+,\ \boldsymbol{k}_4-)\\ &\times[(1-f_{k_1}^{+})(1-f_{k_2}^{-})f_{k_3}^{+}f_{k_4}^{-}-f_{k_1}^{+}f_{k_2}^{-}(1-f_{k_3}^{+})(1-f_{k_4}^{-})]\\ &+W(\boldsymbol{k}_1+,\ \boldsymbol{k}_2+;\ \boldsymbol{k}_3+,\ \boldsymbol{k}_4+)[(1-f_{k_1}^{+})(1-f_{k_2}^{+})f_{k_3}^{+}f_{k_4}^{+}\\ &-f_{k_1}^{+}f_{k_2}^{+}(1-f_{k_3}^{+})(1-f_{k_4}^{+})]\}\end{aligned} \tag{2·2}$$

and

$$\begin{aligned}-\frac{\hbar e\boldsymbol{E}\cdot\boldsymbol{k}_2}{m}\frac{\partial f_{k_2}^{-}}{\partial \varepsilon_{k_2}^{-}}=\sum_{k_1k_3k_4}\{&W(\boldsymbol{k}_1+,\ \boldsymbol{k}_2-;\ \boldsymbol{k}_3+,\ \boldsymbol{k}_4-)\\ &\times[(1-f_{k_1}^{+})(1-f_{k_2}^{-})f_{k_3}^{+}f_{k_4}^{-}-f_{k_1}^{+}f_{k_2}^{-}(1-f_{k_3}^{+})(1-f_{k_4}^{-})]\\ &+W(\boldsymbol{k}_1-,\ \boldsymbol{k}_2-;\ \boldsymbol{k}_3-,\ \boldsymbol{k}_4-)[(1-f_{k_1}^{-})(1-f_{k_2}^{-})f_{k_3}^{-}f_{k_4}^{-}\\ &-f_{k_1}^{-}f_{k_2}^{-}(1-f_{k_3}^{-})(1-f_{k_4}^{-})]\},\end{aligned} \tag{2·3}$$

where $-e$, $m$, $\varepsilon_{\boldsymbol{k}}{}^{\sigma}$ are electron charge, effective mass and energy with wave-vector $\boldsymbol{k}$, spin $\sigma$, respectively. We have approximated $\varepsilon_{\boldsymbol{k}}{}^{\sigma}$ by $\hbar^2k^2/(2m)-\sigma\Delta/2$($\Delta$=exchange splitting) for simplicity. $\boldsymbol{E}$ is the electric field, which is chosen as directed to $x$- axis, $\boldsymbol{E}=(E,0,0)$, in the following. $W(a;\ b)$ represents the transition rate from state $b$ to state $a$ or from $a$ to $b$. To linearize the coupled equation we take the devia-

tion from the equilibrium distribution function $f^0(\varepsilon_k{}^\sigma)=\{\exp[(\varepsilon_k{}^\sigma-\mu)/(k_B T)]+1\}^{-1}$, ($\mu$=chemical potential, $k_B$=Boltzmann constant):

$$f_k{}^\sigma=f^0(\varepsilon_k{}^\sigma)+\tau^\sigma\frac{\hbar eEk_x}{m}\frac{\partial f^0(\varepsilon_k{}^\sigma)}{\partial\varepsilon_k{}^\sigma}. \tag{2·4}$$

Here we have employed the relaxation time approximation by introducing $\tau^\sigma$, a constant relaxation time of spin-$\sigma$ electron on the Fermi surface. After substituting Eq. (2·4) into Eqs. (2·2) and (2·3), we multiply $k_{1x}$ and $k_{2x}$ on both sides of the equations and sum over $k_{1x}$ and $k_{2x}$, respectively (this is a variational method[2]). For an elastic scattering $\varepsilon_1+\varepsilon_2=\varepsilon_3+\varepsilon_4$, a useful relation between products of Fermi distribution function,

$$f^0(\varepsilon_1)f^0(\varepsilon_2)[1-f^0(\varepsilon_3)][1-f^0(\varepsilon_4)]=[1-f^0(\varepsilon_1)][1-f^0(\varepsilon_2)]f^0(\varepsilon_3)f^0(\varepsilon_4), \tag{2·5}$$

helps us to reduce the Boltzmann equation to a coupled equation for $\tau^+$ and $\tau^-$. For fixed numbers of total electrons with spin $\sigma$, $N^\sigma$, we obtain

$$\begin{pmatrix} A_{U+N}^{+}+E_U{}^+ & A_{U+N} \\ A_{U+N} & A_{U+N}^{-}+E_U{}^- \end{pmatrix}\begin{pmatrix}\tau^+\\ \tau^-\end{pmatrix}=\frac{2m}{\hbar^2}\begin{pmatrix}N^+\\ N^-\end{pmatrix}, \tag{2·6}$$

$$A_{U+N}^{+}\equiv\langle W(\boldsymbol{k}_1+,\boldsymbol{k}_2-;\boldsymbol{k}_3+,\boldsymbol{k}_4-)\cdot(k_{1x}-k_{3x})^2\rangle_{U+N}, \tag{2·7}$$

$$A_{U+N}^{-}\equiv\langle W(\boldsymbol{k}_1+,\boldsymbol{k}_2-;\boldsymbol{k}_3+,\boldsymbol{k}_4-)\cdot(k_{2x}-k_{4x})^2\rangle_{U+N}, \tag{2·8}$$

$$A_{U+N}\equiv\langle W(\boldsymbol{k}_1+,\boldsymbol{k}_2-;\boldsymbol{k}_3+,\boldsymbol{k}_4-)\cdot(k_{1x}-k_{3x})(k_{2x}-k_{4x})\rangle_{U+N}, \tag{2·9}$$

$$E_U{}^\sigma\equiv\frac{1}{2}\langle W(\boldsymbol{k}_1\sigma,\boldsymbol{k}_2\sigma;\boldsymbol{k}_3\sigma,\boldsymbol{k}_4\sigma)\cdot G_x{}^2\rangle_U, \tag{2·10}$$

where collision integral

$$\langle W(\boldsymbol{k}_1\sigma,\boldsymbol{k}_2\sigma';\boldsymbol{k}_3\sigma,\boldsymbol{k}_4\sigma')\cdot***\rangle_{U+N}\equiv\frac{1}{k_B T}\sum_{\boldsymbol{k}_1\sim\boldsymbol{k}_4,\boldsymbol{G}}\delta_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{G},\boldsymbol{k}_3+\boldsymbol{k}_4}$$
$$\times W(\boldsymbol{k}_1\sigma,\boldsymbol{k}_2\sigma';\boldsymbol{k}_3\sigma,\boldsymbol{k}_4\sigma')\cdot***f^0(\varepsilon_{\boldsymbol{k}_1}^{\sigma})f^0(\varepsilon_{\boldsymbol{k}_2}^{\sigma'})[1-f^0(\varepsilon_{\boldsymbol{k}_3}^{\sigma})][1-f^0(\varepsilon_{\boldsymbol{k}_4}^{\sigma'})] \tag{2·11}$$

includes Umklapp process, $\boldsymbol{G}\neq0$, as well as normal process $\boldsymbol{G}=0$, $\boldsymbol{G}$ being reciprocal lattice vector. Similar notation $\langle\cdots\rangle_U$ and $\langle\cdots\rangle_N$ refers to the Umklapp process and the normal one, respectively. These equations lead us to a general expression for resistivity in the relaxation time approximation for given relative magnetization, $\alpha=(N^+-N^-)/N_{el}$ and $N_{el}=N^++N^-$,

$$\rho=\frac{Vm}{e^2}(N^+\tau^++N^-\tau^-)^{-1}$$
$$=\frac{V\hbar^2}{2e^2N_{el}^2}\langle W_{tot}G_x{}^2\rangle_U\frac{1-C^2(D\cdot\langle W_{tot}G_x{}^2\rangle_U)^{-1}}{1-\alpha(2C-\alpha\langle W_{tot}G_x{}^2\rangle_U)/D}, \tag{2·12}$$

where

$$W_{tot}\equiv\frac{1}{2}\sum_{\sigma\sigma'}W(\boldsymbol{k}_1\sigma,\boldsymbol{k}_2\sigma';\boldsymbol{k}_3\sigma,\boldsymbol{k}_4\sigma'), \tag{2·13}$$

$$C \equiv A_U^+ + E_U^+ - A_U^- - E_U^-$$
$$= \langle W(\boldsymbol{k}_1+, \boldsymbol{k}_2-; \boldsymbol{k}_3+, \boldsymbol{k}_4-)\cdot(k_{1x}-k_{2x}-k_{3x}+k_{4x})G_x\rangle_U + E_U^+ - E_U^- , \quad (2\cdot14)$$
$$D \equiv A_{U+N}^{+} + A_{U+N}^{-} - 2A_{U+N} + E_U^+ + E_U^-$$
$$= A_U^+ + A_U^- - 2A_U - 4A_N + E_U^+ + E_U^-$$
$$= \langle W(\boldsymbol{k}_1+, \boldsymbol{k}_2-; \boldsymbol{k}_3+, \boldsymbol{k}_4-)\cdot(k_{1x}-k_{2x}-k_{3x}+k_{4x})^2\rangle_{U+N} + E_U^+ + E_U^- . \quad (2\cdot15)$$

We note $A_N^+ = A_N^- = -A_N$. In paramagnetic state, $\alpha=0$, then $C=0$, and the expression (2·12) reduces to a simple one; only the Umklapp process $\langle W_{tot}G_x^2\rangle_U$ contributes to resistivity, as expected. On the other hand in ferromagnetic state we notice that the normal process coupled to Umklapp one also contributes to resistivity through a term $A_N$ in $D$ given by Eq. (2·15).

All of the collision integrals in Eqs. (2·7)~(2·10) are generally represented as

$$\frac{2\pi}{\hbar k_B T}\sum_{k_1\sim k_4}|\langle f|T|i\rangle|^2\delta(\varepsilon_1^\sigma+\varepsilon_2^{\sigma'}-\varepsilon_3^\sigma-\varepsilon_4^{\sigma'})f^0(\varepsilon_1^\sigma)f^0(\varepsilon_2^{\sigma'})[1-f^0(\varepsilon_3^\sigma)][1-f^0(\varepsilon_4^{\sigma'})]$$
$$\times(\text{a quadratic term of wave-vector differences}), \quad (2\cdot16)$$

where $\langle f|T|i\rangle$ refers to a related $t$-matrix element. We now change the sum over $\boldsymbol{k}$ to the integral over energy and to the integral on the constant energy surface in $\boldsymbol{k}$-space:

$$\sum_{\boldsymbol{k}}\cdots = \frac{V}{(2\pi)^3}\int d^3\boldsymbol{k}\cdots = \frac{V}{(2\pi)^3}\int d\varepsilon\int\frac{mdS}{\hbar^2 k}\cdots . \quad (2\cdot17)$$

By taking integral over energy in advance, we can obtain the temperature dependence of the collision integrals imposed by Pauli's principle at low temperatures ($k_B T \ll k_B T_F = E_F$=Fermi energy):

$$\frac{1}{k_B T}\iiint_{-\infty}^{\infty} d\varepsilon_1 d\varepsilon_2 d\varepsilon_3 d\varepsilon_4\,\delta(\varepsilon_1+\varepsilon_2-\varepsilon_3-\varepsilon_4)f^0(\varepsilon_1)f^0(\varepsilon_2)[1-f^0(\varepsilon_3)][1-f^0(\varepsilon_4)]$$
$$= \frac{2\pi^2}{3}(k_B T)^2 . \quad (2\cdot18)$$

Therefore it is natural to express the temperature dependence of resistivity at low temperatures as

$$\rho(T) = B(\alpha)T^2 . \quad (2\cdot19)$$

It is noted that the magnetization $\alpha$ is still temperature dependent in ferromagnetic state, $T < T_c \ll T_F$.

In order to evaluate the coefficient $B(\alpha)=\rho(T)/T^2$, knowledge of collision integrals for each scattering process is necessary. Since the calculation becomes complicated for general $t$-matrices we restrict ourselves at first to Born approximation to discuss semi-quantitative features of the phenomena. Further simplification is achieved when we use a single-band Hubbard model, the Hamiltonian of which is written in conventional notations as follows:

$$H=\sum_{k,\sigma}\varepsilon_k a_{k\sigma}^{+}a_{k\sigma}+\frac{U}{N}\sum_{k_1\sim k_4,G}\delta_{k_1+k_2+G,k_3+k_4}a_{k_1+}^{\dagger}a_{k_2-}^{\dagger}a_{k_4-}a_{k_3+}\,, \tag{2·20}$$

$U$=repulsive interaction on site, $N$=number of lattice sites.

In Born approximation the $t$-matrix element of two-electron scattering with opposite spins is independent of wave-vectors, $\langle f|T|i\rangle=U/N$, while that of same-spin electrons is zero. On the other hand, we describe ferromagnetism of conduction electrons in rigid band picture, that is,

$$\varepsilon_k^{\pm}=\varepsilon_k\mp\frac{\Delta}{2}\,, \tag{2·21}$$

where $\Delta=U(N^+-N^-)/N$ is the exchange splitting. Thus the Fermi wave-vector of up- or down-spin electrons is given by

$$k_F^{\pm}=(1\pm\alpha)^{1/3}k_F^{0}\,, \tag{2·22}$$

where $k_F^{0}$ represents the Fermi wave-vector in paramagnetic state.

## § 3. Analysis of scattering processes

As shown in Eq. (2·12), electron-electron scattering causes resistivity through the Umklapp process, in which sum of the momenta of two electrons with opposite spins in the initial state, $\boldsymbol{K}=\boldsymbol{k}_1+\boldsymbol{k}_2$, changes to $\boldsymbol{K}+\boldsymbol{G}=\boldsymbol{k}_3+\boldsymbol{k}_4$ in the final state. Here $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ ($\boldsymbol{k}_2$ and $\boldsymbol{k}_4$) are on the Fermi surface of up-spin (down-spin) electrons. The momentum difference $\boldsymbol{G}$ is one of the reciprocal lattice vectors. Among them only those parallel to the electric field $\boldsymbol{E}$ is relevant to our discussion.

At first we consider the region of the total momentum $\boldsymbol{K}$ available for the Umklapp process. Since the absolute values $|\boldsymbol{K}|$ and $|\boldsymbol{K}+\boldsymbol{G}|$ must satisfy $k_F^{+}-k_F^{-}\leq|\boldsymbol{K}|$, $|\boldsymbol{K}+\boldsymbol{G}|\leq k_F^{+}+k_F^{-}$ simultaneously, the region of $\boldsymbol{K}$ is inside the two spheres of radius $k_F^{+}+k_F^{-}$ separated by $G=|\boldsymbol{G}|$. It is shown as the hatched region in Fig. 1. The other conditions $k_F^{+}-k_F^{-}\leq|\boldsymbol{K}|$, $|\boldsymbol{K}+\boldsymbol{G}|$ are automatically satisfied since we consider the Fermi sphere always being inside the first Brillouin zone, i.e., $k_F^{\pm}<G/2$,

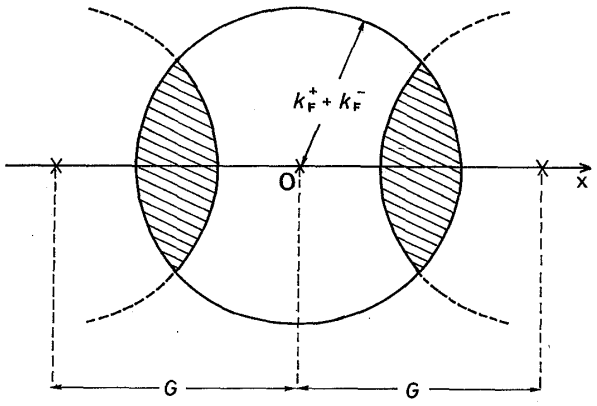


Fig. 1. Region of total wave-vector $\boldsymbol{K}$ (hatched) available for electron-electron Umklapp scattering.

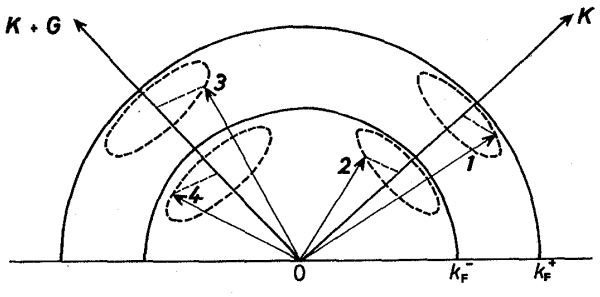


Fig. 2. Umklapp scattering process in wave-vector space. Initial wave-vectors *1* and *2* jump to *3* and *4* by Umklapp scattering. Total wave-vector changes by $\boldsymbol{G}$.

so as to be consistent with the approximation $\varepsilon_k=\hbar^2k^2/(2m)$. The volume of the region, $V_K(\alpha)$, decreases with $k_F^+ + k_F^-$ when the magnetization increases:

$$V_K(\alpha)=\frac{8\pi}{3}\left\{(k_F^+ + k_F^-)^3-\frac{3}{4}(k_F^+ + k_F^-)^2G+\frac{G^3}{16}\right\}. \tag{3·1}$$

The tendency manifests a general feature of the magnetization dependence of resistivity caused by Umklapp process.

Let us look at some details of each scattering process. For given total momentum $\boldsymbol{K}(=\boldsymbol{k}_1+\boldsymbol{k}_2)$, two wave-vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ end on the two circles perpendicular to $\boldsymbol{K}$-vector and on the spin-up and spin-down Fermi spheres, respectively, as illustrated in Fig. 2. The radii of the circles are the same and given by

$$r(K)=\frac{|\boldsymbol{k}_1\times\boldsymbol{k}_2|}{K}=\frac{1}{2K}\sqrt{[(k_F^+ + k_F^-)^2-K^2][K^2-(k_F^+ - k_F^-)^2]}. \tag{3·2}$$

After the Umklapp scattering $\boldsymbol{K}$ has changed to $\boldsymbol{K}+\boldsymbol{G}$; then, similarly, two wave-vectors $\boldsymbol{k}_3$ and $\boldsymbol{k}_4$ are on two circles with the same radius, $r(|\boldsymbol{K}+\boldsymbol{G}|)$, where

$$|\boldsymbol{K}+\boldsymbol{G}|=\sqrt{K^2+G^2-2KG\cos\Theta_K}, \tag{3·3}$$

and $\Theta_K$ is the angle between $\boldsymbol{K}$ and $-\boldsymbol{G}$. These geometrical observations lead us to express the first collision integral in Eq. (2·12) as

$$\langle W_{\rm tot}G_x^2\rangle_U=\langle W(\boldsymbol{k}_1+,\boldsymbol{k}_2-;\ \boldsymbol{k}_3+,\boldsymbol{k}_4-)\cdot G_x^2\rangle_U\equiv G_U$$

$$=\frac{m^4V^3}{3\cdot2^7\pi^6\hbar^9}(k_BT)^2\left(\frac{U}{N}\right)^2\tilde{G}_U(\alpha), \tag{3·4}$$

$$\tilde{G}_U(\alpha)=2\left(\frac{G_x}{k_F^+k_F^-}\right)^2\int dS_1dS_2dS_3\delta_{\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{G},\boldsymbol{k}_3+\boldsymbol{k}_4}\delta(|\boldsymbol{k}_4|-k_F^-)$$

$$=\frac{16\pi^3G_x^2}{(k_F^+k_F^-)^2}\int_0^{\max\Theta_K}d\Theta_K\sin\Theta_K\int_{\min K}^{\max K}dKK^2r(K)r(|\boldsymbol{K}+\boldsymbol{G}|), \tag{3·5}$$

where the region of the integrals is the right hatched region in Fig. 1. If $k_F^+ + k_F^- \le G/\sqrt{2}$, the limits of the integrals can be expressed by

$$\min K=G\cos\Theta_K-\sqrt{(k_F^+ + k_F^-)^2-G^2\sin^2\Theta_K}, \tag{3·6}$$

$$\max K=k_F^+ + k_F^-, \tag{3·7}$$

$$\max\Theta_K=\cos^{-1}\left(\frac{G}{2(k_F^+ + k_F^-)}\right), \tag{3·8}$$

since $\max\Theta_K$ is the angle between the vector to a vertex of the hatched region and $x$-axis in Fig. 1. Even for $k_F^+ + k_F^- > G/\sqrt{2}$, (3·6)~(3·8) remain as good

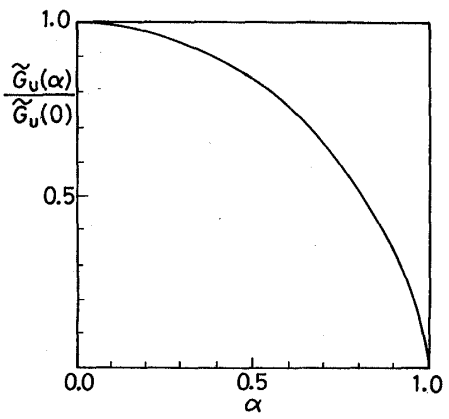


Fig. 3. Normalized collision integral $\tilde{G}_U(\alpha)/\tilde{G}_U(0)$ in Born approximation versus relative magnetization.

approximation for the integration. The result of numerical calculation of $\tilde{G}_U(\alpha)/\tilde{G}_U(0)$ is shown in Fig. 3 for $k_F^0=(1/16)^{1/3}G$. It is a monotonically decreasing function of magnetization.

Other collision integrals are evaluated in a similar manner. An explicit calculation of $A_U^+(A_U^-)$ is described in the Appendix. We note here useful relations between the collision integrals, that are deduced from their definitions:

$$A_U=\frac{1}{2}(G_U-A_U^+-A_U^-) \tag{3·9}$$

and

$$A_N=-A_N^+=-A_N^- . \tag{3·10}$$

As for the collision integrals of the normal process, where, $\boldsymbol{K}$, the total wave-vector of two electrons is conserved, we first integrate over an initial wave-vector $\boldsymbol{k}_1$ and a final vector $\boldsymbol{k}_3$ along the circumference of the circle on the up-spin Fermi surface for given $\boldsymbol{K}$. Then we integrate over $\boldsymbol{K}$ and obtain the analytical expression for $A_N$.

$$A_N=\frac{m^4V^3}{3\cdot 2^7\pi^6\hbar^9}(k_BT)^2\left(\frac{U}{N}\right)^2\tilde{A}_N(\alpha), \tag{3·11}$$

$$\tilde{A}_N(\alpha)=-\frac{2\pi}{(k_F^+k_F^-)^2}\int_{-1}^{1}d(\cos\Theta_K)\int_{k_F^+-k_F^-}^{k_F^++k_F^-}dKK^2r(K)\iint_0^{2\pi}d\varphi_1d\varphi_3(k_{1x}-k_{3x})^2$$

$$=-\frac{512\pi^3}{315}(1-\alpha)\left[7-3\left(\frac{1-\alpha}{1+\alpha}\right)^{2/3}\right](k_F^0)^3 . \tag{3·12}$$

Here we have used

$$(k_{1x}-k_{3x})^2=r^2(K)\cdot(\cos\varphi_1-\cos\varphi_3)^2\cdot\sin^2\Theta_K , \tag{3·13}$$

where $\varphi_1$ and $\varphi_3$ are angles of $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ projected in a plane perpendicular to $\boldsymbol{K}$.

## § 4. Results in Born approximation

Having numerically calculated the collision integrals $G_U$, $A_U^+$and $A_U^-$ and by using (3·11) for $A_N$ we can express the coefficient of $T^2$-term in resistivity, $B(\alpha)$, as a function of the magnetization. We have chosen the total number of electrons by $k_F^0=(1/16)^{1/3}G$. The ratio, $B(\alpha)/B(0)$, $(=\rho(T,\alpha)/\rho(T,0)$ at low temperatures) is a monotonically decreasing function as shown in Fig. 4 and vanishes at $\alpha=1$. We notice a close similarity in the magnetization dependence of $B(\alpha)/B(0)$ and of $\tilde{G}_U(\alpha)/\tilde{G}_U(0)$, (Fig. 3). That means $\langle W_{tot}G_x^2\rangle_U$ in Eq. (2·12) dominates the resistivity. The result for the relaxation times $\tau^+$ and $\tau^-$ determined from Eq. (2·6) are shown in Fig. 5; $\tau^+(\alpha)$ diverges to infinity, while $\tau^-(\alpha)$ goes to zero when magnetization saturates.

In order to trace the explicit temperature dependence of resistivity, one needs to know the temperature dependence of magnetization. Here we assume a simple form,

$$\alpha(T)=\alpha(0)\sqrt{1-(T/T_c)^2}$$

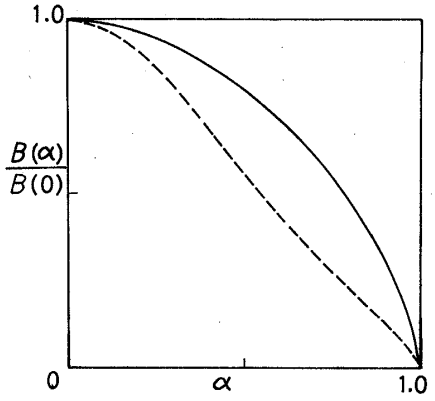


Fig. 4. Coefficient of $T^2$ -term of resistivity versus relative magnetization. (dotted curve: normal process omitted)

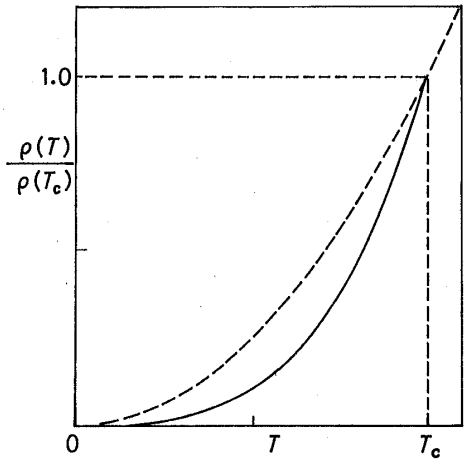


Fig. 6. Temperature dependence of resistivity in ferromagnetic state for $\alpha(0)=1$ (solid curve) and in paramagnetic state (dotted one).

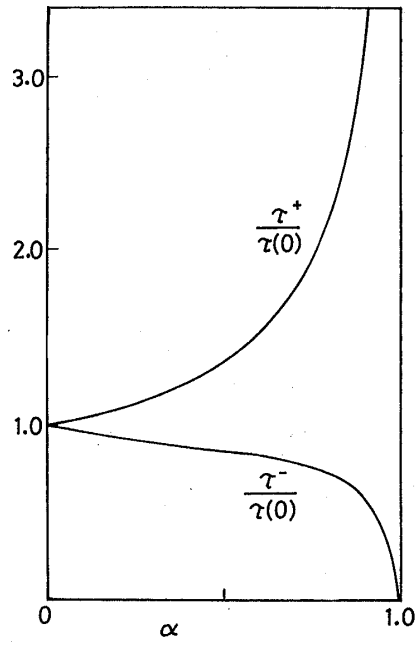


Fig. 5. Relaxation times of electrons, $\tau^+$ and $\tau^-$ (normalized), versus relative magnetization.

just to find a qualitative feature. A result for normalized resistivity $\rho(T)/\rho(T_c)$ is shown in Fig. 6 for $\alpha(0)=1$, saturated magnetization at $T=0$. A reduction of resistivity in ferromagnetic state is evident.

## § 5. Discussion

So far we have calculated the resistivity within a simplest approximation, i.e., in Born approximation. Higher order terms in $t$-matrix are expected to be important particularly in ferromagnetic metals. In higher orders, scattering processes between electrons with the same spin as described by $E_U^\sigma$ in Eq. (2·10) also contribute to resistivity. Among others, Umklapp processes accompanied by multi-scattering normal processes with small momentum transfer in intermediate states tend to enhance the resistivity. The enhancement is related to that of dynamical susceptibility in the ferromagnetic system. However, a real excitation of spin fluctuations or spin waves is impossible for a large momentum transfer required in Umklapp processes. The situation is contrasted to that in the two-band (e.g., $s$- and $d$-bands) model of ferromagnetic metals.[1] Further consideration on higher order processes is left to a separate paper.

To summarize: We have shown that the resistivity due to electron-electron Umklapp scattering is suppressed when magnetization develops in ferromagnetic metals in which conduction electrons and magnetic ones are the same. This is mainly because the larger the difference between two Fermi surfaces with opposite spins the smaller the volume of the region available for Umklapp processes in the wave-vector

space. From Boltzmann equation it has been found that not only the Umklapp processes but normal processes coupled to them also contribute to the resistivity. Although we have assumed spherical Fermi surfaces to make our analysis as clear as possible, the conclusion will be useful to discuss real ferromagnetic metals with more general Fermi surfaces.

## Acknowledgements

The authors would like to thank Professor Kei Yosida for calling our attention to the Umklapp processes and for giving us continuous encouragements. They are obliged to Mr. Toshio Funatake for his help in numerical calculations. One of the authors (A.S.) expresses his sincere gratitude to Professor Akio Yoshimori for illuminating discussions in the field of solid state physics for over twenty years.

## Appendix

We calculate the collision integral of the Umklapp process $A_U^+$ ($A_U^-$) in Born approximation. To this end it is convenient to introduce six vectors, $\boldsymbol{s}_{3-1}$, ($\boldsymbol{s}_{4-2}$), $\boldsymbol{t}_1$, ($\boldsymbol{t}_2$) and $\boldsymbol{t}_3$ ($\boldsymbol{t}_4$) for given $\boldsymbol{K}$ and $\boldsymbol{K}+\boldsymbol{G}$. Here, $\boldsymbol{s}_{3-1}$ is a vector from $O_1$, the center of the circle that the end point of the initial wave-vector $\boldsymbol{k}_1$ lies on, to $O_3$ the center of the circle that the final vector $\boldsymbol{k}_3$ ends on. The vectors $\boldsymbol{t}_1$ and $\boldsymbol{t}_3$ are the perpendicular components of $\boldsymbol{k}_1$ and $\boldsymbol{k}_3$ with respect to $\boldsymbol{K}$ and $\boldsymbol{K}+\boldsymbol{G}$, respectively. Other three vectors are defined in the same way. Thus we obtain the following expressions:

$$A_U^+=\frac{m^4V^3}{3\cdot 2^7\pi^6\hbar^9}(k_BT^2)\left(\frac{U}{N}\right)^2\tilde{A}_U^+(a)\,, \tag{A·1}$$

$$\begin{aligned}\tilde{A}_U^+(a)=&\frac{4\pi}{(k_F^+k_F^-)^2}\int d(\cos\Theta_K)\int dKK^2r(K)r(|\boldsymbol{K}+\boldsymbol{G}|)\\&\times\iint_0^{2\pi}d\varphi_1d\varphi_3(s_{3-1,x}-t_{1x}+t_{3x})^2\\=&\frac{4\pi^3}{(k_F^+k_F^-)^2}\int d(\cos\Theta_K)\int dKK^2r(K)r(|\boldsymbol{K}+\boldsymbol{G}|)\\&\times\left\{\frac{1}{2}\left[\frac{1}{K}\left(K^2+(k_F^+)^2-(k_F^-)^2\right)\cos\Theta_K\right.\right.\\&\left.-\frac{|\boldsymbol{K}+\boldsymbol{G}|^2+(k_F^+)^2-(k_F^-)^2}{|\boldsymbol{K}+\boldsymbol{G}|}\cos\Theta_{K+G}\right]^2\\&\left.+\sin^2\Theta_K\left[r^2(K)+\left(\frac{K}{|\boldsymbol{K}+\boldsymbol{G}|}r(|\boldsymbol{K}+\boldsymbol{G}|)\right)^2\right]\right\},\end{aligned} \tag{A·2}$$

where $\Theta_{K+G}$ is the angle between $\boldsymbol{K}+\boldsymbol{G}$ and $-\boldsymbol{G}$. The intervals of the integrals are the same as those in Eq. (3·5). The expression for $A_U^-$ is obtained by interchanging $k_F^+$ and $k_F^-$ in th above formula.

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