Fermi Liquid Theory on Heavy Electron Systems

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Theory of heavy electron is reviewed on the basis of the Fermi liquid theory. The importance of the orbital degeneracy in realizing the heavy electrons is stressed by using the scaling theory on the Kondo temperature. With use of the periodic Anderson Hamiltonian, the expressions for the physical quantities, such as electronic specific heat, magnetic susceptibility, conductivity, relaxation time of nuclear spin and anomalous Hall coefficient are derived. These results explain the essential properties of heavy electrons. For further development the extension to the orbital degenerate case is essential and discussed mainly on the specific heat and the susceptibility. At the end the relation between the Fermi liquid state and spin fluctuation is discussed with use of the orthogonality theorem.

§ 1. Introduction

In this paper we discuss the normal state properties of the heavy fermion systems. At the beginning we discuss the reason why the heavy electron system is realized at low temperatures.[1] In usual rare earth metals the magnetic long range order, such as helical structure, appears at a temperature around 50 K~300 K. The magnetic long range order is due to the RKKY (Ruderman-Kittel-Kasuya-Yosida) interaction, $J_{\mathrm{RKKY}}\boldsymbol{S}_i\cdot\boldsymbol{S}_j$, which is proportional to S^2, S being the magnitude of localized spin. For example, Gd metal with localized spin $S=7/2$ orders in the ferromagnetic state below 300 K. This RKKY interaction between Ce (Yb) ions with a single f-electron (hole) is weak because of $S_z^2=1/4$ compared with $S_z^2=49/4$ for Gd. If the magnetic ordering temperature is scaled by S^2 from that of Gd, the critical temperature for Ce (Yb) is given by 6 K. If we scale the paramagnetic Curie temperature by the de Gennes factor $(g_J-1)^2j(j+1)$, we obtain 3 K for that of Ce^{3+}. These values for the magnetic ordering temperature of Ce system are reasonable compared with observed ones. Moreover, if we take into account the Kondo effect, it lowers further the ordering temperature estimated above.

In addition to the low ordering temperature due to small localized spins, the Kondo temperature becomes high owing to the orbital degeneracy, even for the weak coupling of the exchange interaction.[2)~4)] This is the important fact in realizing the heavy electron in the Ce and Yb systems. We shall discuss it in the next section. The other rare earth metals than Ce and Yb have the multiply occupied f-shell. For the f-shell with plural electrons or holes, the Hund's coupling is strong enough to reduce the degree of freedom of angular momentum exchanged by the s-f exchange.

This effect reduces remarkably the Kondo temperature.

The heavy electrons are nothing but quasi-particles in the Fermi liquid. The Fermi liquid state in the correlated system can be obtained continuously by starting with the non-interacting state and increasing the mutual interactions. In the heavy electron systems these heavy electrons themselves become superconducting states and/or magnetically ordered states, as shown by the large jumps of the specific heat at the transition. Therefore, in order to describe the physics in the heavy electron systems, it is important to describe them in the Fermi liquid states at the first stage.

In this article we explain the analysis of heavy electron systems on the basis of the Fermi liquid theory. We derive the Kondo temperature in the system with the orbital degeneracy by using the scaling theory in § 2. In § 3, we derive the expressions for the physical quantities in the case with the non-degenerate f-orbital. In § 4, the T^2-term of resistivity due to the electron-electron scattering is discussed. In § 5, the extension to the case with f-orbital degeneracy is discussed. In § 6, the relaxation time of nuclear spin and the Hall coefficient are given. It is shown in § 7 that the Fermi liquid state behaves as the local singlet state and conserves the local spin by accompanying the neighboring spin with transfer of the f-electron. In the last section concluding remarks are given shortly.

§ 2. Scaling theory in the presence of crystal field

In this section, we consider the Kondo temperature by using the scaling theory for the Hamiltonian,[1),4),5)]

$$\begin{aligned}\mathcal{H}=&\sum_{k,M,m}\varepsilon_k(c_{kM}^{\dagger}c_{kM}+c_{km}^{\dagger}c_{km})+\sum_{M}E_M a_M^{\dagger}a_M+\sum_{m}E_m a_m^{\dagger}a_m\\&-\sum_{\substack{M,M'\\k,k'}}\frac{J_1}{2N}c_{kM}^{\dagger}c_{k'M'}a_{M'}^{\dagger}a_M-\sum_{\substack{m,m'\\k,k'}}\frac{J_0}{2N}c_{km}^{\dagger}c_{k'm'}a_{m'}^{\dagger}a_m\\&-\sum_{\substack{m,M\\k,k'}}\frac{J_2}{2N}(c_{kM}^{\dagger}c_{k'm}a_m^{\dagger}a_M+c_{km}^{\dagger}c_{k'M}a_M^{\dagger}a_m)\,.\end{aligned}\tag{2·1}$$

Here, capital M represents a higher level state in a cubic crystal field, and m stands for a lower level state. The energy levels, E_M and E_m, are defined so as to satisfy the condition,

$$\sum_{M}E_M+\sum_{m}E_m=0\,.\tag{2·2}$$

Since we consider the case with one f-electron, we confine ourselves in the subspace,

$$\sum_{M}a_M^{\dagger}a_M+\sum_{m}a_m^{\dagger}a_m=1\,.\tag{2·3}$$

The exchange interactions J_0 and J_1 work in the subspace of lower and higher levels, respectively and J_2 is that between the two subspaces.

Following Poorman's derivation by Anderson,[4)] we obtain the scaling equations:

$$\frac{d\tilde{J}_0}{dD}=\sum_{m}\frac{\tilde{J}_0^{\,2}}{D+E_m-z}+\sum_{M}\frac{\tilde{J}_2^{\,2}}{D+E_M-z},\tag{2·4}$$

$$\frac{d\tilde{J}_1}{dD}=\sum_M \frac{\tilde{J}_1^2}{D+E_M-z}+\sum_m \frac{\tilde{J}_2^2}{D+E_m-z}, \tag{2·5}$$

$$\frac{d\tilde{J}_2}{dD}=\sum_M \frac{\tilde{J}_1\tilde{J}_2}{D+E_M-z}+\sum_m \frac{\tilde{J}_0\tilde{J}_2}{D+E_m-z}, \tag{2·6}$$

where z is the total energy of the electron system with the impurity in the crystalline field.

Here, we assume a cubic crystal field and define crystal potential $E_D=-2\Delta/3$ for Γ_7 doublet and $E_Q=\Delta/3$ for Γ_8 quartet. Hereafter, we discuss two cases depending on the sign of $\Delta=E_Q-E_D$.

Case 1. $E_Q-E_D=\Delta>0$. The doublet Γ_7 is the ground state and $z\simeq-2\Delta/3$.

$$\frac{d\tilde{J}_0}{dD}=\frac{2\tilde{J}_0^2}{D}+\frac{4\tilde{J}_2^2}{D+\Delta}, \tag{2·7}$$

$$\frac{d\tilde{J}_1}{dD}=\frac{4\tilde{J}_1^2}{D+\Delta}+\frac{2\tilde{J}_2^2}{D}, \tag{2·8}$$

$$\frac{d\tilde{J}_2}{dD}=\frac{4\tilde{J}_1\tilde{J}_2}{D+\Delta}+\frac{2\tilde{J}_0\tilde{J}_2}{D}. \tag{2·9}$$

If we assume $J_0=J_1=J_2=J$ and $\tilde{J}_0=\tilde{J}_1=\tilde{J}_2=\tilde{J}$, the above equations reduce to

$$\frac{d\tilde{J}}{dD}=\frac{2\tilde{J}^2}{D}+\frac{4\tilde{J}^2}{D+\Delta}. \tag{2·10}$$

The isotropic case is realized, when the localized f-level, ε_f, is low enough from the Fermi level, E_F, and satisfies $U\gg|E_F-\varepsilon_f|\gg|\Delta|$, U being the intra-Coulomb repulsion between f-electrons. The solution of (2·10) is given by

$$-\frac{1}{\tilde{J}}+\frac{2N}{\rho J}=2\log\frac{D}{D_0}+4\log\left(\frac{D+\Delta}{D_0+\Delta}\right), \tag{2·11}$$

where D_0 and $\rho J/2N$ are given by an initial condition.

For the case with $D_0\gg\Delta$, $\tilde{J}$ is given by

$$\tilde{J}=\frac{\rho J}{2N}\left[1+\frac{\rho|J|}{N}\log(D/D_0)+\frac{2\rho|J|}{N}\log((D+\Delta)/D_0)\right]^{-1}. \tag{2·12}$$

Since the Kondo temperature is defined as D giving rise to infinite coupling constant, T_K is determined by the equation ($k_B=1$),

$$1+\frac{\rho|J|}{N}\log\frac{T_K}{D_0}+\frac{2\rho|J|}{N}\log\frac{T_K+\Delta}{D_0}=0, \tag{2·13}$$

$$T_K=\left(\frac{D_0}{T_K+\Delta}\right)^2 D_0 e^{-N/\rho|J|}. \tag{2·14}$$

If $T_K\ll\Delta$, we obtain

$$T_K=\left(\frac{D_0}{\Delta}\right)^2 D_0 e^{-N/\rho|J|}=\left(\frac{D_0}{\Delta}\right)^2 T_K^0, \tag{2·15}$$

where T_K^0 is the Kondo temperature for the case without higher quartet level Γ_8. Thus, we can see that higher states in a crystal field cannot be neglected as far as $\Delta \ll D_0$, even if $\Delta \gg T_K^0$. If we assume $D_0=10^4$ K and $\Delta=10^2$ K, we obtain $(D_0/\Delta)^2=10^4$ as the prefactor of T_K^0 in (2·15). This result shows the importance of the higher levels in determining the Kondo temperature.

Case 2. $E_Q-E_D=\Delta<0$. The quartet Γ_8 is the ground state and $z\simeq-|\Delta|/3$. For this case the scaling equation is given by

$$\frac{d\tilde{J}}{dD}=\frac{4\tilde{J}^2}{D}+\frac{2\tilde{J}^2}{D+|\Delta|}. \tag{2·16}$$

With use of the initial values D_0 and $\rho J/2N$,

$$\tilde{J}=\frac{\rho J}{2N}\left[1+\frac{\rho|J|}{N}\log\frac{D+|\Delta|}{D_0+|\Delta|}+\frac{2\rho|J|}{N}\log\frac{D}{D_0}\right]^{-1}. \tag{2·17}$$

The Kondo temperature T_K giving $\tilde{J}=\infty$ is determined as

$$\begin{aligned} T_K&=(D_0/(T_K+|\Delta|))^{1/2}D_0\exp[-N/2\rho|J|] \\ &\simeq(D_0/|\Delta|)^{1/2}D_0\exp[-N/2\rho|J|] \quad \text{for} \quad T_K\ll|\Delta|. \end{aligned} \tag{2·18}$$

The Kondo temperature takes a large value by the prefactor 2 of $\rho|J|$ due to the degeneracy of crystal level.

In the case with general crystal field splittings, T_K is given by

$$T_K=(D_0/\Delta_1)^{N_1/N_0}(D_0/\Delta_2)^{N_2/N_0}\cdots(D_0/\Delta_m)^{N_m/N_0}D_0\exp\left[-\frac{2N}{\rho|J|}\frac{1}{N_0}\right], \tag{2·19}$$

where Δ_i and N_i are the level splitting between i and 0 levels and the degeneracy of i level. $i=0$ is the ground level. If we include up to the next divergent terms, we obtain T_K by multiplying T_K in (2·19) by the factor $\sqrt{\rho|J|/N}$.

§ 3. Fermi liquid theory on the periodic Anderson Hamiltonian

Heavy electrons realized at low temperatures have been investigated by several approaches such as slave boson method, Gutzwiller approximation.[6] Among them the Fermi liquid theory seems to be the most general approach to describe the essential properties of heavy electrons. The other methods also describe the heavy electrons as the correlated narrow band and give the expressions for the physical quantities similar to those derived by the Fermi liquid theory. In the present paper, we describe the Fermi liquid theory on the basis of the periodic Anderson Hamiltonian.[7] Here we discuss for simplicity the periodic Anderson Hamiltonian without orbital degeneracy. The orbital degeneracy is important to discuss the real heavy electron systems and is taken into account in § 5.

$$\mathcal{H}=\mathcal{H}_0+\mathcal{H}', \tag{3·1}$$

$$\mathcal{H}_0=\sum_{k,\sigma}\varepsilon_k c_{k\sigma}^\dagger c_{k\sigma}+\sum_{k,\sigma}E_k a_{k\sigma}^\dagger a_{k\sigma}+\sum_{k,\sigma}(V_k a_{k\sigma}^\dagger c_{k\sigma}+V_k^* c_{k\sigma}^\dagger a_{k\sigma})+\frac{N}{4}\langle n_0^f\rangle^2, \tag{3·2}$$

$$\mathcal{H}'=\sum_{k,k',q}{}'\frac{U}{N}a^{\dagger}_{k+q\uparrow}a^{\dagger}_{k'-q\downarrow}a_{k'\downarrow}a_{k\uparrow}\,. \tag{3·3}$$

Creation operator $a^{\dagger}_{k\sigma}(c^{\dagger}_{k\sigma})$ is that for f-electron (conduction electron) with energy E_k (ε_k) and spin σ. Heavy f-electrons in the same atom interact with each other via Coulomb repulsion U. The f and conduction electrons hybridize through matrix element V_k. Our results derived in the following can be applied also to the transition and actinide metals as well as rare earth metals, since our Hamiltonian includes the dispersion of f-electrons.

The self-energy part of the f-electron due to the Coulomb repulsion, $\Sigma_k(z)$, is introduced to describe the Fermi liquid theory in the general picture. With use of the self-energy, the Green's function of f-electron and conduction electron are given by

$$(z\hat{1}-\hat{H})\hat{G}=\hat{1}\,, \tag{3·4}$$

where

$$(z\hat{1}-\hat{H})=\begin{pmatrix} z-E_k-\Sigma_k(z) & -V_k \\ -V_k^* & z-\varepsilon_k \end{pmatrix}, \tag{3·5}$$

$$\hat{G}=\begin{pmatrix} G^f_{k\sigma}(z) & G^{fc}_{k\sigma}(z) \\ G^{cf}_{k\sigma}(z) & G^c_{k\sigma}(z) \end{pmatrix}. \tag{3·6}$$

The matrix $\hat{1}$ represents the two-dimensional unit matrix. The diagonal parts of Green's functions, $G^f_{k\sigma}$ and $G^c_{k\sigma}$ are written, respectively, as

$$G^f_{k\sigma}(z)=[z-E_{k\sigma}-\Sigma_{k\sigma}(z)-|V_k|^2/(z-\varepsilon_{k\sigma})]^{-1}\,, \tag{3·7}$$

$$G^c_{k\sigma}(z)=[z-\varepsilon_{k\sigma}-|V_k|^2/(z-E_{k\sigma}-\Sigma_{k\sigma}(z))]^{-1}\,. \tag{3·8}$$

The energy eigenvalue $z=E^*_{k\sigma}$ is given by the pole of $\hat{G}$ and determined by the equation,

$$(z-E_{k\sigma}-\Sigma_{k\sigma}(z))(z-\varepsilon_{k\sigma})-|V_k|^2=0\,. \tag{3·9}$$

Following Luttinger,[8] we obtain the T-linear term of the specific heat, ($\omega_+=\omega+i\delta$)

$$\begin{aligned}\gamma&=\frac{\pi^2k_B^2}{6\pi i}\sum_{\substack{k\\ \sigma}}\left\{\frac{\partial}{\partial\omega}[\ln(\omega_++\mu-E_k-\Sigma_k{}^R(\omega_+)-|V_k|^2/(\omega_++\mu-\varepsilon_k))-\text{c.c.}]\right\}_{\omega=0}\\
&=\frac{2\pi^2k_B^2}{3}\sum_k-\frac{1}{\pi}\mathrm{Im}(\mu+i\delta-E_k-\Sigma_k{}^R(0)-|V_k|^2/(\mu+i\delta-\varepsilon_k))^{-1}\\
&\quad\times\left(1-\frac{\partial\Sigma_k{}^R(\omega)}{\partial\omega}\Big|_{\omega=0}+\frac{|V_k|^2}{(\mu-\varepsilon_k)^2}\right)\\
&=\frac{2\pi^2k_B^2}{3}\left\{\sum_k\rho_k{}^f(0)\left(1-\frac{\partial\Sigma_k{}^R(\omega)}{\partial\omega}\Big|_{\omega=0}\right)+\sum_k\rho_k{}^c(0)\right\},\end{aligned} \tag{3·10}$$

where density of states for f-electron $\rho_k{}^f(\omega)$ and that for conduction electron $\rho_k{}^c(\omega)$ are defined, respectively, as

$$\rho_k{}^f(\omega)=-\frac{1}{\pi}\mathrm{Im}[\mu+\omega_+-E_k-\Sigma_k{}^R(\omega)-|V_k|^2/(\mu+\omega_+-\varepsilon_k)]^{-1}, \tag{3·11}$$

$$\rho_k{}^c(\omega)=-\frac{1}{\pi}\mathrm{Im}[\mu+\omega_+-\varepsilon_k-|V_k|^2/(\omega_++\mu-E_k-\Sigma_k{}^R(\omega))]^{-1}. \tag{3·12}$$

The coefficient of T-linear term can be written by the density of states of quasiparticles as

$$\gamma=\frac{\pi^2 k_B{}^2}{3}\sum_{k,\sigma}\delta(\mu-E^*_{k\sigma}). \tag{3·13}$$

Here, we introduce the wave-function renormalization factors, $z_k{}^f$ and $z_k{}^c$, which are given by the residues of $G_k{}^f$ and $G_k{}^c$ at $\omega=0$,

$$z_k{}^f=\left(1-\frac{\partial\Sigma_k(\omega)}{\partial\omega}\Big|_{\omega=0}+\frac{|V_k|^2}{(\mu-\varepsilon_k)^2}\right)^{-1}, \tag{3·14}$$

$$z_k{}^c=\frac{|V_k|^2}{(\mu-\varepsilon_k)^2}\Big/\left(1-\frac{\partial\Sigma_k(\omega)}{\partial\omega}\Big|_{\omega=0}+\frac{|V_k|^2}{(\mu-\varepsilon_k)^2}\right). \tag{3·15}$$

We put

$$\tilde{\gamma}_k=(1-\partial\Sigma_k(\omega)/\partial\omega)_{\omega=0}, \tag{3·16}$$

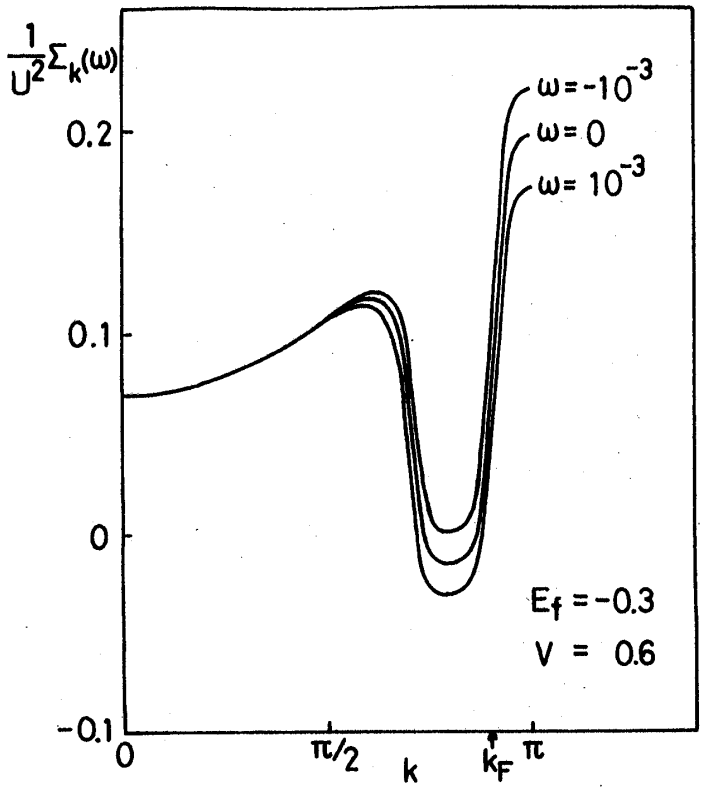

Fig. 1. k-dependence of the f-electron self-energy $\Sigma_k{}^{(2)}(\omega)$ with ω as a parameter. The arrow shows the Fermi wavevector obtained under the condition that the f-electron number per site is unity. For conduction electrons $\varepsilon_k=-\cos k$ is assumed.

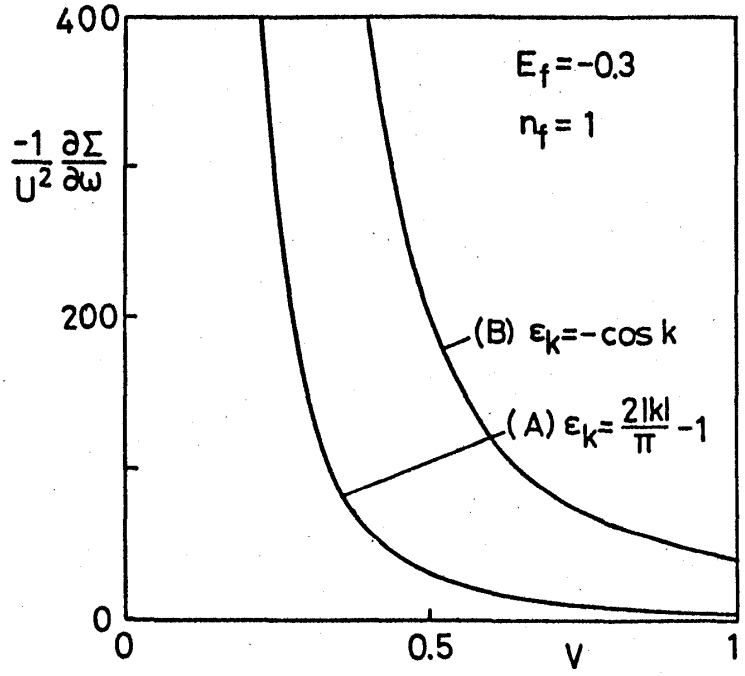

Fig. 2. The mass enhancement factor divided by U^2 as a function of hybridization V.

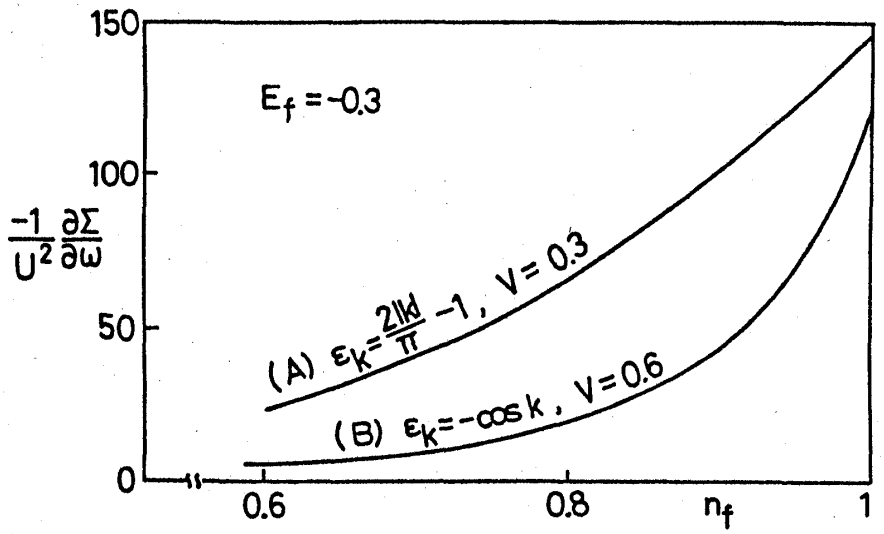

Fig. 3. The mass enhancement factor divided by U^2 as a function of f-electron number n_f.

and obtain the relation

$$\tilde{\gamma}_k z_k{}^f + z_k{}^c = 1 . \tag{3·17}$$

From (3·13), we derive Eq. (3·10),

$$\gamma = \frac{\pi^2 k_B{}^2}{3}\sum_{k,\sigma}(\tilde{\gamma}_{k\sigma} z^f_{k\sigma} + z^c_{k\sigma})\delta(\mu - E^*_{k\sigma})$$

$$= \frac{\pi^2 k_B{}^2}{3}\sum_{k,\sigma}[\tilde{\gamma}_{k\sigma}\rho^f_{k\sigma}(0) + \rho^c_{k\sigma}(0)] . \tag{3·18}$$

This expression means that γ is given by the sum of $\rho^c_{k\sigma}$ and $\rho^f_{k\sigma}$ enhanced by the factor of $\tilde{\gamma}_k$. The large enhancement of γ in heavy electron systems originates from the first term of (3·18), because this term is enhanced by $\tilde{\gamma}_k$ due to the electron

interaction between f-electrons.

The U^2-term of $\tilde{\gamma}_k$ has been calculated by Zlatic et al. and Okada et al. for the one-dimensional case.[9),10)] In Figs. 1~3, we show the numerical results for the electron mass, namely $-\partial\Sigma/\partial\omega|_{\omega=0}$. These results show that the heavy electron is realized in the cases with a large density of states for f-electron at the Fermi energy. Okada et al. confirmed that the U^2-term of $\tilde{\gamma}_k$ is well scaled by the square of the bare f-electron density at Fermi energy, $\rho^f(0)^2$. This is because the coupling constant is given by $\rho^f(0)U$ and the second order term is proportional to $[\rho^f(0)]^2$. For the cases when the total number of f-electrons per site approaches to unity a large electron mass is obtained. In the nearly half-filled case of f-electrons the Coulomb repulsion among f-electrons reduces strongly the hybridization and the transfer term and realizes the extremely heavy electrons. The similar calculation for the d-p model for high T_c superconducting oxides has been done by Kanki.[11)] For this case the mass enhancement is weak owing to a large band width of bare d-electrons.

From the above discussion, we can expect $\tilde{\gamma}_k$ in the heavy electron system is much larger than unity, $\tilde{\gamma}_k \gg 1$. For this case we can derive the eigenvalue equation as follows.

At first, we expand $\Sigma_{k\sigma}(\omega_+)$ as

$$\Sigma_{k\sigma}(\omega_+) \simeq \Sigma_{k\sigma}(0)+\partial\Sigma_{k\sigma}(\omega)/\partial\omega|_{\omega=0}\omega - i\Delta_k\,, \tag{3·19}$$

where

$$\Delta_k = -\mathrm{Im}\Sigma_{k\sigma}(\omega)\,. \tag{3·20}$$

By substituting (3·19) into (3·9), we obtain

$$\begin{aligned} E_k^* &= \frac{1}{\tilde{\gamma}_k}(E_{k\sigma}+\Sigma_{k\sigma}(0)+|V_k|^2/(\mu-\varepsilon_{k\sigma})) \\ &= \tilde{E}_k+|\tilde{V}_k|^2/(\mu-\varepsilon_{k\sigma})\,, \end{aligned} \tag{3·21}$$

$$\tilde{E}_k = (E_{k\sigma}+\Sigma_{k\sigma}(0))/\tilde{\gamma}_k\,, \tag{3·22}$$

$$|\tilde{V}_k|^2 = |V_k|^2/\tilde{\gamma}_k\,. \tag{3·23}$$

Thus, (3·21) shows that the energy of E_k^* is renormalized by $\tilde{\gamma}_k^{-1}$ and constructs a very narrow band. Inversely, this narrow band gives the large density of states of quasi-particles at the Fermi energy. This may be the simplest explanation of heavy electrons.

At this point we have to explain the reason why the heavy electron systems with nearly half-filled f-electrons can stay still in the metallic state even for the large value of U. The reason can be explained by the existence of conduction band, with which f-electrons hybridize and can be delocalized through the conduction bands, as far as the hybridization remains a finite value in spite of the strong reduction.

Now we discuss the magnetic susceptibility. Here we assume that the g-values for f- and conduction electrons are given by g^f and g^c, respectively. In this case $E_{k\sigma}$ and $\varepsilon_{k\sigma}$ are given by

$$E_{k\sigma} = E_k - H\sigma^f\,, \tag{3·24}$$

$$\varepsilon_{k\sigma}=\varepsilon_k-H_\sigma^c\,, \tag{3·25}$$

$$H_\sigma^f=\frac{1}{2}g^f\sigma\mu_B H\,, \tag{3·26}$$

$$H_\sigma^c=\frac{1}{2}g^c\sigma\mu_B H\,, \tag{3·27}$$

μ_B being the Bohr magneton. If $g^f\neq g^c$, the magnetic moment, $M=(1/2)g^f\mu_B(n_\uparrow^f-n_\downarrow^f)+(1/2)g^c\mu_B(n_\uparrow^c-n_\downarrow^c)$, does not commute with the Hamiltonian and does not conserve. For this case we discuss in § 5. Here, for simplicity, we assume $g^f=g^c=2$ and $H_\sigma^f=H_\sigma^c=H_\sigma$. Total electron number N_e and magnetization M are given by

$$N_e=\sum_{k\sigma}\theta(\mu-E_{k\sigma}^*)\,, \tag{3·28}$$

$$M=\mu_B\sum_{k\sigma}\sigma\theta(\mu-E_{k\sigma}^*)\,, \tag{3·29}$$

where $E_{k\sigma}^*$ is an eigenvalue of the quasi-particle in the presence of the magnetic field and is defined well near the Fermi energy. From (3·29), the spin susceptibility is given by

$$\begin{aligned}\chi_s&=\lim_{H\to 0}\mu_B\sum_{k,\sigma}\sigma\delta(\mu-E_{k\sigma}^*)(-\partial E_{k\sigma}^*/\partial H|_{H=0})\\&=2\mu_B\sum_k\sigma\delta(\mu-E_k{}^*)(-\partial E_{k\sigma}^*/\partial H|_{H=0})\,.\end{aligned} \tag{3·30}$$

By using the eigenvalue equation (3·9), we obtain

$$\frac{\partial E_{k\sigma}^*}{\partial H}=\left[1-\frac{\partial\Sigma_k(\omega)}{\partial\omega}+\frac{|V_k|^2}{(E_k{}^*-\varepsilon_k)^2}\right]^{-1}(-\mu_B\sigma)\left[\tilde{\chi}_s(\boldsymbol{k})+\frac{|V_k|^2}{(E_k{}^*-\varepsilon_k)^2}\right], \tag{3·31}$$

where

$$\tilde{\chi}_s(\boldsymbol{k})=\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})+\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})\,, \tag{3·32}$$

$$\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})=1-\partial\Sigma_{k\sigma}(0)/\partial H_\sigma|_{H_\sigma=0}\,, \tag{3·33}$$

$$\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})=\partial\Sigma_{k\sigma}(0)/\partial H_{-\sigma}|_{H_{-\sigma}=0}\,. \tag{3·34}$$

Thus, the spin susceptibility is given by

$$\chi_s=2\mu_B{}^2\{\sum_k\rho_k{}^f(0)\tilde{\chi}_s(\boldsymbol{k})+\rho^c(0)\}\,, \tag{3·35}$$

where $\rho_k{}^f(0)$ and $\rho^c(0)=\sum_k\rho_k{}^c(0)$ are given by (3·11) and (3·12), respectively. The susceptibility χ_s is enhanced by $\tilde{\chi}_s$ due to the electron interaction between f-electrons. Now, we define the four-point vertex $\Gamma_{\sigma\sigma'}(\boldsymbol{k}_1,\boldsymbol{k}_2;\boldsymbol{k}_3,\boldsymbol{k}_4)$ and discuss the relation between $\tilde{\gamma}(\boldsymbol{k})$ and $\tilde{\chi}_{\sigma\sigma'}(\boldsymbol{k})$ by using Ward's identities.[7] The results are the following,

$$\tilde{\gamma}(\boldsymbol{k})=\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})+\sum_{k'}\rho_{k'}{}^f(0)\Gamma_{\sigma\sigma}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}',\boldsymbol{k})\,, \tag{3·36}$$

$$\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})=\sum_{\boldsymbol{k}'}\rho_{\boldsymbol{k}}{}^{f}(0)\Gamma_{\sigma-\sigma}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}',\boldsymbol{k})\,. \tag{3·37}$$

For simplicity we define

$$\gamma^{f}=\sum_{\boldsymbol{k}}\rho_{\boldsymbol{k}}{}^{f}(0)\,\tilde{\gamma}(\boldsymbol{k})\,, \tag{3·38}$$

$$\chi^{f}_{\uparrow\uparrow}=\sum_{\boldsymbol{k}}\rho_{\boldsymbol{k}}{}^{f}(0)\,\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})\,, \tag{3·39}$$

$$\chi^{f}_{\uparrow\downarrow}=\sum_{\boldsymbol{k}}\rho_{\boldsymbol{k}}{}^{f}(0)\,\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})=\sum_{\boldsymbol{k}\boldsymbol{k}'}\rho_{\boldsymbol{k}}{}^{f}(0)\Gamma_{\uparrow\downarrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}',\boldsymbol{k})\rho_{\boldsymbol{k}'}{}^{f}(0)\,, \tag{3·40}$$

$$\delta^{f}_{\uparrow\uparrow}=\sum_{\boldsymbol{k},\boldsymbol{k}'}\rho_{\boldsymbol{k}}{}^{f}(0)\Gamma_{\sigma\sigma}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}',\boldsymbol{k})\rho_{\boldsymbol{k}'}{}^{f}(0)\,. \tag{3·41}$$

With use of these quantities, we obtain the following expressions,

$$\gamma=\frac{\pi^{2}k_{\mathrm{B}}{}^{2}}{3}\left(\sum_{\sigma}\rho_{\sigma}{}^{c}(0)+2\gamma^{f}\right), \tag{3·42}$$

$$\gamma^{f}=\chi^{f}_{\uparrow\uparrow}+\delta^{f}_{\uparrow\uparrow}\,, \tag{3·43}$$

$$\chi_{s}=2\mu_{\mathrm{B}}{}^{2}[\chi_{s}{}^{f}+\rho^{c}(0)]\,, \tag{3·44}$$

$$\chi_{c}{}^{f}=\chi^{f}_{\uparrow\uparrow}-\chi^{f}_{\uparrow\downarrow}=\sum_{\boldsymbol{k}}\rho_{\boldsymbol{k}}{}^{f}(0)(\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})-\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k}))\,, \tag{3·45}$$

where $\chi_{c}{}^{f}$ is the charge susceptibility of f-electron and

$$\chi_{c}=\rho^{c}(0)+\chi_{c}{}^{f}\,. \tag{3·46}$$

§ 4. Resistivity

The T^{2}-term of resistivity in the heavy electron system is very large and its coefficient is nearly proportional to γ^{2}. Now we derive the exact coefficient of the T^{2}-term in the heavy electron system on the basis of the Kubo formula. Current operator $\hat{J}$ in our system is given by

$$\hat{J}=e\sum_{\boldsymbol{k}\sigma}(\boldsymbol{v}_{\boldsymbol{k}}{}^{f}a^{\dagger}_{\boldsymbol{k}\sigma}a_{\boldsymbol{k}\sigma}+\boldsymbol{v}_{\boldsymbol{k}}{}^{c}c^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma})+e\sum_{\boldsymbol{k}\sigma}(\nabla_{\boldsymbol{k}}V_{\boldsymbol{k}}a^{\dagger}_{\boldsymbol{k}\sigma}c_{\boldsymbol{k}\sigma}+\nabla_{\boldsymbol{k}}V_{\boldsymbol{k}}{}^{*}c^{\dagger}_{\boldsymbol{k}\sigma}a_{\boldsymbol{k}\sigma})\,, \tag{4·1}$$

where

$$\boldsymbol{v}_{\boldsymbol{k}}{}^{c}=\nabla_{\boldsymbol{k}}\varepsilon_{\boldsymbol{k}}\,, \tag{4·2}$$

$$\boldsymbol{v}_{\boldsymbol{k}}{}^{f}=\nabla_{\boldsymbol{k}}E_{\boldsymbol{k}}\,. \tag{4·3}$$

In Ref. 7), $\hat{J}$ in (4·1) is used to derive the conductivity. Here, we derive the conductivity with use of the physical quantities expressed by quasi-particles. At finite temperatures, the eigenvalue of the quasi-particle, $z=E_{\boldsymbol{k}}{}^{*}-i\Gamma_{\boldsymbol{k}}{}^{*}(\Gamma_{\boldsymbol{k}}{}^{*}>0)$, is determined by

$$(z-\varepsilon_k)(z-E_k^0-\Sigma_k^R(z))-|V_k|^2=0\,. \tag{4·4}$$

We confine ourselves to the low temperatures and substitute into (4·4) the expansion form of the self-energy part,

$$\Sigma_k^R(z)=\Sigma_k^R(0)+\partial\Sigma_k^R(z)/\partial z|_{z=0}\cdot z-i\Delta_k\,, \tag{4·5}$$

we obtain the eigenvalue for $\Gamma_k^*\leq\Delta_k\ll|E_k^*|$ as

$$E_k^*=\frac{1}{\tilde{\gamma}_k}(E_k+|V_k|^2/(E_k^*-\varepsilon_k))\,,$$

$$E_k=E_k^0+\Sigma_k^R(0)\,, \tag{4·6}$$

$$\Gamma_k^*=\frac{\Delta_k}{\tilde{\gamma}_k+|V_k|^2/(E_k^*-\varepsilon_k)^2}=z_k^f\Delta_k\,, \tag{4·7}$$

where $z_k^f=z_k^f(E_k^*)$ given by (3·14). The velocity of the quasi-particle is derived from the eigenvalue equation as

$$\boldsymbol{v}_k^*=\boldsymbol{\nabla}_k E_k^*=z_k^f\tilde{\boldsymbol{v}}_k^f+z_k^c\boldsymbol{v}_k^c+z_k^f\frac{1}{\mu-\varepsilon_k}\boldsymbol{\nabla}_k|V_k|^2\,, \tag{4·8}$$

where

$$\tilde{\boldsymbol{v}}_k^f=\boldsymbol{\nabla}_k E_k=\boldsymbol{\nabla}_k(E_k^0+\Sigma_k(0))\,. \tag{4·9}$$

At $T=0$ and external frequency $\omega=0$, the vertex correction $\boldsymbol{\Lambda}_k^0$ is given by

$$\boldsymbol{\Lambda}_{k\sigma}^0(0)=\sum_{k',\sigma'}\int\frac{d\omega'}{2\pi i}\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')[G_{k'}^f(\omega')]^2\left[\boldsymbol{v}_{k'}^f+\frac{|V_{k'}|^2}{(\omega'+\mu-\varepsilon_{k'})^2}\boldsymbol{v}_{k'}^c+\frac{\partial|V_{k'}|^2/\partial\boldsymbol{k}'}{(\omega'+\mu-\varepsilon_{k'})}\right], \tag{4·10}$$

where

$$\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')\equiv\Gamma_{\sigma\sigma'}(\boldsymbol{k}\boldsymbol{k}';\boldsymbol{k}'\boldsymbol{k})\,. \tag{4·11}$$

On the other hand, the momentum derivative of the f-electron self-energy is given by

$$\begin{aligned}\boldsymbol{\nabla}_k\Sigma_{k\sigma}(0)&=\sum_{k',\sigma'}\int\frac{d\omega'}{2\pi i}\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')\lim_{q\to0}\frac{1}{\boldsymbol{q}}[G_{k'+q}^f(\omega')-G_{k'}^f(\omega')]\\&=\sum_{k'\sigma'}\int\frac{d\omega'}{2\pi i}\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')[G_{k'}^f(\omega')]^2\left[\boldsymbol{v}_{k'}^f+\frac{|V_{k'}|^2\boldsymbol{v}_{k'}^c}{(\omega'+\mu-\varepsilon_{k'})^2}+\frac{\partial|V_{k'}|^2/\partial\boldsymbol{k}'}{(\omega'+\mu-\varepsilon_{k'})}\right]\\&\quad-\sum_{k',\sigma'}\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')z_{k'}^f\delta(\mu-E_{k'}^*)\boldsymbol{v}_{k'}^*\,,\end{aligned} \tag{4·12}$$

where $\boldsymbol{v}_{k'}^*$ is the velocity of the quasi-particle given by (4·8). The last term of (4·12) is the backflow term and is absent in the expression of $\sigma(\omega)$ for the finite temperatures with $\omega<\Gamma_k^*$. The real velocity $\boldsymbol{J}_k$ giving the conductivity is given by

$$\boldsymbol{J}_k=z_k^f\left(\boldsymbol{v}_k^f+\boldsymbol{\Lambda}_k^0(0)+\frac{1}{\mu-\varepsilon_k}\boldsymbol{\nabla}_k|V_k|^2\right)+z_k^c\boldsymbol{v}_k^c$$

$$=z_k{}^f\left(\boldsymbol{v}_k{}^f+\boldsymbol{\nabla}_k\Sigma_k(0)+\frac{1}{\mu-\varepsilon_k}\boldsymbol{\nabla}_k|V_k|^2\right)+z_k{}^c\boldsymbol{v}_k{}^c$$

$$+z_k{}^f\sum_{k'}\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')z_{k'}{}^f\delta(\mu-E_{k'}{}^*)\boldsymbol{v}_{k'}{}^*$$

$$=\boldsymbol{v}_k{}^*+\sum_{k',\sigma}f_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')\delta(\mu-E_{k'}{}^*)\boldsymbol{v}_{k'}{}^*\,. \tag{4·13}$$

The interaction between quasi-particles is given by

$$f_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}')=z_k{}^f\Gamma_{\sigma\sigma'}(\boldsymbol{k}\boldsymbol{k}';\boldsymbol{k}'\boldsymbol{k})z_{k'}{}^f\,. \tag{4·14}$$

In Ref. 7), the expression of the conductivity at low temperatures is derived by applying Eliashberg's theory to our case.[7] The result is

$$\sigma_{\mu\nu}(\omega)=\frac{i}{2}e^2\left\{\sum_k J_{k\mu}\frac{\frac{1}{2T}\mathrm{ch}^{-2}(E_k{}^*/2T)}{\omega+2i\Gamma_k{}^*}J_{k\nu}\right.$$

$$\left.+\frac{1}{2}\sum_{kk'}J_{k\mu}z_k{}^f\frac{\frac{1}{2T}\mathrm{ch}^{-2}(E_k{}^*/2T)\,T_{22}(\boldsymbol{k},\boldsymbol{k}';\omega)}{(\omega+2i\Gamma_k{}^*)(\omega+2i\Gamma_{k'}{}^*)}z_{k'}{}^fJ_{k'\nu}\right\}. \tag{4·15}$$

The reciprocal life-time of quasiparticle, $\Gamma_k{}^*$, is given by (4·7). The imaginary part of the self-energy, $-i\Delta_k$, is given at low temperatures as[7]

$$\Delta_k=-\mathrm{Im}\Sigma_k{}^R(\varepsilon)$$

$$=(\varepsilon^2+(\pi T)^2)/2\cdot\sum_{k'q}\pi\rho_{k-q}^f(0)\rho_{k'}{}^f(0)\rho_{k'+q}^f(0)$$

$$\times\left\{\Gamma_{\uparrow\downarrow}^2(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})+\frac{1}{2}\Gamma_{\uparrow\uparrow}^{A2}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\right\}. \tag{4·16}$$

The term with T_{22} is important in order to recover the momentum conservation. This correction is related to the imaginary part of the self-energy and essential to obtain the correct result in the thermodynamic limit, $\omega\to0$. The backflow-term conserving the total current at $T=0$ is replaced by the T_{22}-term at finite temperatures. By treating this vertex correction in a consistent way with the self-energy correction, we can show that the resistivity due to the electron interaction vanishes in a free electron system without any crystal potential. We show the diagrams for the general

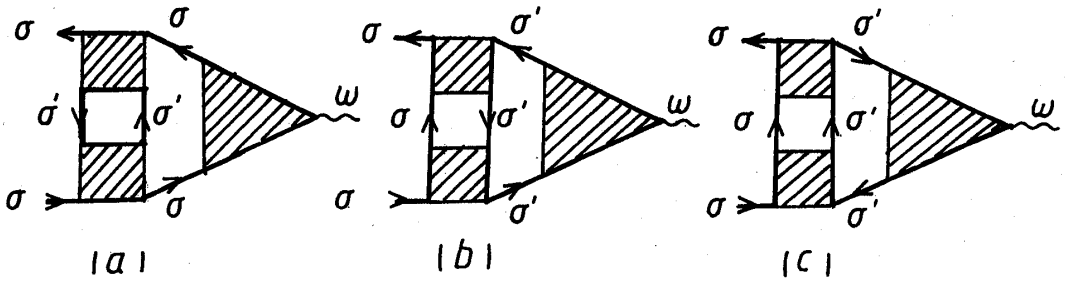

Fig. 4. Three types of vertex corrections giving rise to the T^2-term.

vertex corrections giving rise to the T^2-term in Fig. 4. The three-point vertex part $\boldsymbol{\Lambda}_k(\varepsilon)$ is determined by the equation,

$$\begin{aligned}\boldsymbol{\Lambda}_k(\varepsilon)&=\boldsymbol{J}_k+\boldsymbol{\Lambda}_k^{(a)}(\varepsilon)+\boldsymbol{\Lambda}_k^{(b)}(\varepsilon)+\boldsymbol{\Lambda}_k^{(c)}(\varepsilon)\\&=\boldsymbol{J}_k+\sum_{k'q}\Delta_0(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\left\{\frac{\boldsymbol{\Lambda}_{k-q}(\varepsilon)}{2\Delta_{k-q}(\varepsilon)}+\frac{\boldsymbol{\Lambda}_{k'+q}(\varepsilon)}{2\Delta_{k'+q}(\varepsilon)}-\frac{\boldsymbol{\Lambda}_{k'}(-\varepsilon)}{2\Delta_{k'}(-\varepsilon)}\right\},\end{aligned}\tag{4·17}$$

where

$$\begin{aligned}\Delta_0(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})&=\pi\rho^f_{k-q}(0)\rho^f_{k'+q}(0)\rho_{k'}{}^f(0)\\&\times\left[\Gamma^2_{\uparrow\downarrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})+\frac{1}{2}\Gamma^{A2}_{\uparrow\uparrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\right][(\pi T)^2+\varepsilon^2].\end{aligned}\tag{4·18}$$

The imaginary part of the self-energy, Δ_k, in (4·16) is given by

$$\Delta_k=\frac{1}{2}\sum_{k'q}\Delta_0(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q}).\tag{4·19}$$

We put here

$$\boldsymbol{\Phi}_k(\varepsilon)=\boldsymbol{\Lambda}_k(\varepsilon)/2\Delta_k(\varepsilon)=\boldsymbol{\Phi}_k(-\varepsilon).\tag{4·20}$$

Then we obtain from (4·17) the equation

$$0=\boldsymbol{J}_k+\sum_{k'q}\Delta_0(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})[\boldsymbol{\Phi}_{k-q}+\boldsymbol{\Phi}_{k'+q}-\boldsymbol{\Phi}_{k'}-\boldsymbol{\Phi}_k].\tag{4·21}$$

The conductivity is given by

$$\begin{aligned}\sigma_{\mu\nu}(0)&=e^2\sum_k J_{k\mu}\left(-\frac{\partial f(x)}{\partial x}\right)_{x=E_{k^*}}\frac{\Lambda_{k\nu}}{2\Gamma_k{}^*}\\&=e^2\sum_k J_{k\mu}\left(-\frac{\partial f(x)}{\partial x}\right)_{x=E_{k^*}}\frac{1}{z_k{}^f}\Phi_{k\nu}.\end{aligned}\tag{4·22}$$

Here, if we assume

$$\boldsymbol{\Phi}_k=\boldsymbol{k}F,\tag{4·23}$$

the second term of (4·21) vanishes because of the momentum conservation. To satisfy the equality of (4·21), F tends to infinity. Thus $\sigma_{\mu\nu}$ given by (4·22) tends to infinity. Therefore, we have no resistivity due to the electron interaction in the free electron systems.

On the other hand, in the periodic system there exist Umklapp processes in f-electron scattering and (4·21) can be written as

$$\boldsymbol{J}_k-\sum_{k'q}\Delta_0(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\sum_i\boldsymbol{K}_iF=0.\tag{4·24}$$

Here we have put

$$\boldsymbol{\Phi}_{k-q}+\boldsymbol{\Phi}_{k'+q}-\boldsymbol{\Phi}_{k'}-\boldsymbol{\Phi}_k=-\sum_i\boldsymbol{K}_iF,\tag{4·25}$$

where $\boldsymbol{K}_i$ is a reciprocal lattice vector and we have assumed that Δ_0 with a reciprocal lattice vector in its argument can be replaced by the corresponding value in the reduced zone. For this case

$$\boldsymbol{\Phi}_k=\frac{\boldsymbol{k}}{2\Delta_k}\frac{\boldsymbol{J}_k\cdot\boldsymbol{k}}{\sum_i \boldsymbol{K}_i\cdot\boldsymbol{k}} \qquad \text{and} \qquad \boldsymbol{J}_k\propto\boldsymbol{k} \tag{4·26}$$

and the conductivity is given by

$$\sigma_{\mu\nu}(0)=e^2\sum_k\delta(\mu-E_k^*)J_{k\mu}\frac{1}{2\Gamma_k^*}\frac{k^2}{\sum_i(\boldsymbol{K}_i\cdot\boldsymbol{k})}J_{k\nu}\,. \tag{4·27}$$

In this expression, J_k and Γ_k^* are renormalized by $z_k{}^f$ and density of states of quasiparticles $\rho_k^*(0)=\delta(\mu-E_k^*)$ is enhanced by $1/z_k{}^f$. As the result all factors due to the renormalization cancel out with each other and the resistivity is proportional to Δ_k, which is given by the T^2-term with a strongly enhanced coefficient.

The factor $2\Delta_k$ is given by

$$2\Delta_k\simeq\frac{4}{3}(\pi T)^2\sum_{k'q}\pi\rho^f_{k-q}(0)\rho_{k'}{}^f(0)\rho^f_{k'+q}(0)$$
$$\times\left[\Gamma^2_{\uparrow\downarrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})+\frac{1}{2}\Gamma^{A2}_{\uparrow\uparrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})\right]. \tag{4·28}$$

$\Gamma^A_{\uparrow\uparrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}'+\boldsymbol{q},\boldsymbol{k}-\boldsymbol{q})$ is the antisymmetrized vertex for the electrons with parallel spins and vanishes for $\boldsymbol{q}=0$. If we neglect the momentum dependence in $\Gamma^A_{\uparrow\uparrow}$, $\Gamma^A_{\uparrow\uparrow}=0$. Further, if the large Coulomb repulsion between f-electrons suppresses the charge fluctuation of f-electrons or $\chi_c{}^f=0$, the following relations hold

$$\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})=\tilde{\chi}_{\uparrow\uparrow}(\boldsymbol{k})=\tilde{\gamma}(\boldsymbol{k})\,. \tag{4·29}$$

Here,

$$\tilde{\chi}_{\uparrow\downarrow}(\boldsymbol{k})=\sum_{k'}\rho_{k'}{}^f(0)\Gamma_{\uparrow\downarrow}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{k}',\boldsymbol{k})\,. \tag{4·30}$$

Comparing (4·28) with (3·36) for $\tilde{\gamma}$, we can see that the coefficient of T^2-term of the resistivity, A, is proportional to γ^2 when the momentum dependence in $\Gamma_{\sigma\sigma'}$ is weak. Thus, A can be strongly enhanced as observed in experiments in Ce and U systems.

Here we stress that the large T^2-dependence of the resistivity at low temperatures is an important common feature in heavy electrons. We think that the logarithmic dependence on T at high temperatures is not indispensable for heavy fermion systems. The large T^2-dependence of the resistivity means that the coherent heavy electron band is broken at a comparatively low temperature, because the T^2-dependence reflects that of the reciprocal life-time of quasi-particles. If we define T^* ($k_B=1$) as the temperature equal to the inverse life-time of quasi-particle, we obtain

$$T^*=\Gamma_k{}^*=z_k{}^f\Delta_k=\frac{1}{\tilde{\gamma}}\tilde{\gamma}^2T^{*2}\rho^f(0)\,, \tag{4·31}$$

$$T^* \simeq \frac{1}{\tilde{\gamma}} \frac{1}{\rho^f(0)} = \frac{1}{\rho^*(0)} . \tag{4·32}$$

Thus, T^* is the order of band width of heavy electrons. The rapid increase of the resistivity ceases at the temperature around T^*. The behavior of the resistivity at the temperatures higher than T^* cannot be confined to the logarithmic dependence, because there exist various intersite couplings.

For the actinide system we need another explanation than the Kondo effect for the reason why the heavy electron is realized. For example in the uranium system there exist two or three f-electrons at each U atom. These f-electrons construct narrow coherent bands at low temperatures. The actinide system can be described on the basis of the periodic Anderson Hamiltonian with the orbital degeneracy. The uranium system with two f-electrons has larger degree of freedoms than Ce (Yb) system with one f-electron (hole). The weak Hund's coupling and orbital degeneracy give rise to large entropy and large specific heat at low temperatures. Though the $5f$-band in U systems is not so narrow as $4f$-band in Ce systems, higher density of f-electrons makes more effective the Coulomb repulsion among f-electrons, which is determined by coupling constant $\rho^f(0)U$.

If the static pressure is applied, the heavy electron system becomes light in the general case, because the bare f-electron density at the Fermi energy decreases through the increased hybridization. Thus, the pressure reduces the T-linear term of the specific heat and the T^2-term of the resistivity.

In the low frequency limit, (4·22) or (4·27) can be generalized as

$$\sigma_{\mu\nu}(\omega) = e^2 \sum_k J_{k\mu} \left(-\frac{\partial f(x)}{\partial x} \right)_{x=E_k^*} \frac{2\Gamma^*_{k\omega}}{(2\Gamma^*_{k,\omega})^2 + \omega^2} \Lambda_{k\nu} , \tag{4·33}$$

where

$$\Gamma^*_{k,\omega} = z_k{}^f [\Delta_k(E_k^* + \omega/2) + \Delta_k(E_k^* - \omega/2)] . \tag{4·34}$$

The result (4·33) explains the temperature and frequency dependence of the observed optical conductivity. At low temperatures $\sigma(\omega)$ is given by

$$\sigma(\omega) \propto [A\{(\pi T)^2 + \omega^2\}]^{-1} ,$$

A being proportional to γ^2.

§ 5. Extension to the periodic Anderson model with degenerate f-orbitals

We have developed so far the Fermi liquid theory on the basis of the periodic Anderson model in which localized orbitals are assumed to have no orbital degeneracy. In this section, we remove this assumption and introduce the spin-orbit coupling and the crystalline field splittings.

1. *The periodic Anderson Hamiltonian for degenerate f-orbitals*

For this case, our Hamiltonian can be written down as

$$\mathcal{H}=\sum_{k\sigma}\varepsilon_{k\sigma}c^{\dagger}_{k\sigma}c_{k\sigma}+\sum_{iMM'}E_{MM'}f^{\dagger}_{iM}f_{iM'}+\frac{U}{2}\sum_{i,M\neq M'}f^{\dagger}_{iM}f_{iM}f^{\dagger}_{iM'}f_{iM'}$$

$$+\frac{1}{\sqrt{N}}\sum_{iMk\sigma}(V_{kM\sigma}e^{-i\boldsymbol{k}\cdot\boldsymbol{R}_i}c^{\dagger}_{k\sigma}f_{iM}+V^{*}_{kM\sigma}e^{i\boldsymbol{k}\cdot\boldsymbol{R}_i}f^{\dagger}_{iM}c_{k\sigma}). \tag{5·1}$$

The wave function of the conduction electron $|\boldsymbol{k}\sigma\rangle$ is approximated by the plane wave as

$$|\boldsymbol{k}\sigma\rangle=\frac{1}{\sqrt{\Omega}}e^{i\boldsymbol{k}\boldsymbol{r}}\chi_{\sigma}, \tag{5·2}$$

where χ_{σ} represents the spin function for spin $\sigma(\pm1)$ and Ω is the total volume of the crystal. In the presence of the magnetic field H, its energy consists of the kinetic energy and the Zeeman energy as

$$\varepsilon_{k\sigma}=\varepsilon_{k}-\sigma\mu_{\mathrm{B}}H, \tag{5·3}$$

where σ represents the Pauli spin matrix σ_z which takes ±1. The plane wave given by (5·2) can be expanded around site i as

$$|\boldsymbol{k}\sigma\rangle=\frac{4\pi}{\sqrt{\Omega}}e^{i\boldsymbol{k}\cdot\boldsymbol{R}_i}\sum_{l=0}^{\infty}i^{l}j_{l}(k|\boldsymbol{r}-\boldsymbol{R}_i|)\sum_{m=-l}^{l}Y_{l}^{m*}(\theta_{k},\phi_{k})Y_{l}^{m}(\theta_{r-R_i},\phi_{r-R_i})\chi_{\sigma}. \tag{5·4}$$

Here, $j_l(kr)$ is the spherical Bessel function and $Y_l^m(\theta,\phi)$ the spherical harmonic.

$f^{\dagger}_{iM}$ and f_{iM} represent creation and annihilation operators of f-electron at site i in the eigenstate denoted by M under the spin-orbit coupling and the crystalline field. Such eigenstates can be expressed as an appropriate linear combination of the states specified by $j=l\pm s$, $j_z=m$ and $\sigma=\pm1$. Namely, the eigenstate $|iM\rangle$ can be expressed by

$$|iM\rangle=R_{nl}(|\boldsymbol{r}-\boldsymbol{R}_i|)\sum_{m,\sigma}a^{M}_{lm\sigma}Y_{l}^{m}(\theta_{r-R_i},\phi_{r-R_i})\chi_{\sigma}, \tag{5·5}$$

where R_{nl} is the radial part in which $n=4$ and $l=3$ for Ce^{3+} ion and $a^{M}_{lm\sigma}$ is the Clebsch-Gordan coefficient. The f-electron energy with the Zeeman term, which has now off-diagonal elements, is given by

$$E_{MM'}=E_{M}\delta_{MM'}-\langle M|l_z+2s_z|M'\rangle\mu_{\mathrm{B}}H. \tag{5·6}$$

Here it is noted that z-axis differs generally from the principal axes of the crystalline field.

The third term of (5·1) represents the on-site Coulomb repulsion between f-electrons, $U>0$, and the fourth term represents the mixing between f and conduction electrons. The mixing matrix element can be calculated with the use of (5·4) and (5·5) as

$$V_{kM\sigma}=\sqrt{4\pi}\sum_{m}a^{M}_{lm\sigma}Y_{l}^{m}(\theta_{k},\phi_{k})V_{knl}, \tag{5·7}$$

$$V_{knl}=(-i)^{l}\sqrt{\frac{4\pi}{\Omega}}\int_{0}^{\infty}j_{l}(kr)V(r)R_{nl}(r)r^{2}dr, \tag{5·8}$$

where $V(r)$ is the effective potential for electrons, assumed spherically symmetric for simplicity.

The Fermi liquid theory is developed by taking $U=0$ case as the unperturbed state. For that purpose, it is convenient to rewrite the Hamiltonian (5·1) as

$$\mathcal{H}=\sum_{k\sigma}\varepsilon_{k\sigma}c^{\dagger}_{k\sigma}c_{k\sigma}+\sum_{kMM'}E_{MM'}f^{\dagger}_{kM}f_{kM'}+\sum_{kM\sigma}(V_{kM\sigma}c^{\dagger}_{k\sigma}f_{kM}+V^{*}_{kM\sigma}f^{\dagger}_{kM}c_{k\sigma})$$
$$+\frac{U}{2N}\sum_{kk'qMM'}f^{\dagger}_{k-qM}f^{\dagger}_{k'+qM'}f_{k'M'}f_{kM}\,, \tag{5·9}$$

where f_{kM} is

$$f_{kM}=\frac{1}{\sqrt{N}}\sum_{i}e^{-ik\cdot R_i}f_{iM}\,. \tag{5·10}$$

2. *Consideration for the non-interacting system*

The main purpose in this section is to derive the expressions for the T-linear coefficient of the specific heat and the magnetic susceptibility on the basis of this Hamiltonian. For that purpose, we begin with the consideration for $U=0$ case. In the absence of magnetic field ($H=0$), the Green's functions of conduction and f-electrons are given by

$$G^{c}_{k\sigma\sigma}(\omega)=A_{k\bar{\sigma}}(\omega)/[A_{k\sigma}(\omega)A_{k\bar{\sigma}}(\omega)-B_{k\sigma}(\omega)B_{k\bar{\sigma}}(\omega)]\,, \tag{5·11}$$

$$G^{c}_{k\sigma\bar{\sigma}}(\omega)=B_{k\bar{\sigma}}(\omega)/[A_{k\sigma}(\omega)A_{k\bar{\sigma}}(\omega)-B_{k\sigma}(\omega)B_{k\bar{\sigma}}(\omega)]\,, \tag{5·12}$$

$$G^{f}_{kMM'}(\omega)=\frac{\delta_{MM'}}{\omega-E_M}+\sum_{\sigma\sigma'}\frac{V_{kM\sigma}}{\omega-E_M}G^{c}_{k\sigma\sigma'}\frac{V^{*}_{kM'\sigma'}}{\omega-E_{M'}}\,, \tag{5·13}$$

where $\bar{\sigma}=-\sigma$ and

$$A_{k\sigma}(\omega)=\omega-\varepsilon_k-\sum_{M}\frac{|V_{kM\sigma}|^2}{\omega-E_M}\,, \tag{5·14}$$

$$B_{k\sigma}(\omega)=\sum_{M}\frac{V^{*}_{kM\sigma}V_{kM\bar{\sigma}}}{\omega-E_M}\,. \tag{5·15}$$

In this $U=0$ and $H=0$ case, the Hamiltonian (5·9) is diagonal with respect to $\boldsymbol{k}$ and M.

In the case that the localized f-electron states have Kramers degeneracy as for Ce^{3+}, $B_{k\sigma}$ always vanishes. The reason for this is as follows. The time reversal operation denoted by K on the state $|iM\rangle$ gives its counterpart $|i\bar{M}\rangle$ with a phase factor which has no physical significance. Using the relations that $K(\psi\chi_{\uparrow})=\psi^{*}\chi_{\downarrow}$ and $K(\psi\chi_{\downarrow})=-\psi^{*}\chi_{\uparrow}$ and $Y_l^{m*}=(-1)^m Y_l^{-m}$, we obtain

$$a^{\bar{M}}_{l-m\uparrow}=e(-1)^{m+1}a^{M}_{lm\downarrow}\,,$$
$$a^{\bar{M}}_{l-m\downarrow}=e(-1)^{m}a^{M}_{lm\uparrow}\,. \qquad (e=1 \text{ or } -1) \tag{5·16}$$

Note that the Clebsch-Gordan coefficients are taken to be real. From (5·7), (5·16) and $E_M=E_{\bar{M}}$, $B_{k\uparrow}(\omega)$ of (5·15) is calculated as

$$B_{k\uparrow}(\omega)=\frac{1}{2}\sum_{M}\frac{V^{*}_{kM\uparrow}V_{kM\downarrow}+V^{*}_{k\bar{M}\uparrow}V_{k\bar{M}\downarrow}}{\omega-E_M}$$

$$=2\pi|V_{knl}|^2\sum_{M}\frac{1}{\omega-E_M}\sum_{mm'}(a^{M}_{lm\uparrow}a^{M}_{lm'\downarrow}Y_l^{m*}Y_l^{m'}+a^{\bar{M}}_{l-m'\uparrow}a^{\bar{M}}_{l-m\downarrow}Y_l^{-m'*}Y_l^{-m})$$

$$=2\pi|V_{knl}|^2\sum_{M}\frac{1}{\omega-E_M}\sum_{mm'}(a^{M}_{lm\uparrow}a^{M}_{lm'\downarrow}-a^{M}_{lm\uparrow}a^{M}_{lm'\downarrow})Y_l^{m*}Y_l^{m'}=0\,.$$

In the same way we obtain also $B_{k\downarrow}(\omega)=0$. Therefore, in this case

$$G^{c}_{k\sigma\sigma}(\omega)=1/A_{k\sigma}(\omega)\,, \tag{5·17}$$

$$G^{f}_{kMM}(\omega)=\frac{1}{\omega-E_M}+\sum_{\sigma}\frac{|V_{kM\sigma}|^2}{(\omega-E_M)^2}G^{c}_{k\sigma\sigma}(\omega)\,. \tag{5·18}$$

On the other hand, since we can show that

$$|V_{kM\sigma}|^2+|V_{k\bar{M}\bar{\sigma}}|^2=|V_{kM\sigma}|^2+|V_{kM\bar{\sigma}}|^2\equiv 2I_{kM} \tag{5·19}$$

is independent of σ, (5·17) and (5·18) can be written as

$$G^{c}_{k\sigma\sigma}(\omega)=1/A_k(\omega)\,, \tag{5·20}$$

$$G^{f}_{kMM}(\omega)=\frac{1}{\omega-E_M}+\frac{2I_{kM}}{(\omega-E_M)^2A_k(\omega)}\,, \tag{5·21}$$

where

$$A_k(\omega)=\omega-\varepsilon_k-\sum_{M}\frac{I_{kM}}{\omega-E_M}\,, \tag{5·22}$$

which is also independent of σ. The Green's functions of (5·20) and (5·21) have the same pole at $\omega=E^{*}_{kn}$ which is an eigenvalue of the hybridized band and determined by

$$A_k(E^{*}_{kn})=E^{*}_{kn}-\varepsilon_k-\sum_{M}\frac{I_{kM}}{E^{*}_{kn}-E_M}=0\,, \tag{5·23}$$

where the subscript n is assigned to the different eigenvalues. The corresponding eigenstates are given by

$$|\boldsymbol{k}n\sigma^{*}\rangle=\frac{1}{\sqrt{A'_k(E^{*}_{kn})}}\left[c^{\dagger}_{k\sigma}+\sum_{M}\frac{V_{kM\sigma}}{E^{*}_{kn}-E_M}f^{\dagger}_{kM}\right]|0\rangle\,, \tag{5·24}$$

where $|0\rangle$ is the vacuum state and A'_k is given by

$$A'_k(E^{*}_{kn})=\frac{dA_k(\omega)}{d\omega}|_{\omega=E^{*}_{kn}}=1+\sum_{M}\frac{I_{kM}}{(E^{*}_{kn}-E_M)^2}=\frac{d\varepsilon_k}{dE^{*}_{kn}}\,. \tag{5·25}$$

Furthermore, $G^{f}_{kMM}(\omega)$ has another pole at $\omega=E_M$. Residues of $G^{c}_{k\sigma\sigma}(\omega)$ and $G^{f}_{kMM}(\omega)$ at these poles are, respectively, given by

$$z^{c}_{k\sigma}(E^{*}_{kn})=\frac{1}{A'_k(E^{*}_{kn})}=\left[1+\sum_{M}\frac{I_{kM}}{(E^{*}_{kn}-E_M)^2}\right]^{-1}\,, \tag{5·26}$$

$$z^f_{kM}(E^*_{kn})=\frac{2I_{kM}}{(E^*_{kn}-E_M)^2}z^c_{k\sigma}(E^*_{kn})\,, \tag{5·27}$$

$$z^f_{kM}(E_M)=1-\frac{2I_{kM}}{\sum_{M'\in(E_{M'}=E_M)}I_{kM'}} \quad \text{for} \quad I_{kM}\neq 0 \tag{5·28}$$

$$=1 \quad \text{for} \quad I_{kM}=0\,. \tag{5·29}$$

Here, it should be noted that $z^c_{k\sigma}(E^*_{kn})$ is independent of σ and

$$z^c_{k\sigma}(E^*_{kn})+\frac{1}{2}\sum_M z^f_{kM}(E^*_{kn})=1\,. \tag{5·30}$$

Introducing the Green function of the electron in the hybridized band state by

$$G^*_{kn\sigma}(\omega)=1/(\omega-E^*_{kn})\,, \tag{5·31}$$

we can rewrite the Green functions of c- and f-electrons as

$$G^c_{k\sigma\sigma}(\omega)=\sum_n z^c_{k\sigma}(E^*_{kn})G^*_{kn\sigma}(\omega)\,, \tag{5·32}$$

$$G^f_{kMM}(\omega)=\sum_n z^f_{kM}(E^*_{kn})G^*_{kn\sigma}(\omega)+z^f_{kM}(E_M)/(\omega-E_M)\,. \tag{5·33}$$

Now, we discuss how are the electronic band structures. We consider the case $I_{kM}\neq 0$. If there are no degeneracies except Kramers one, the residues at any E_M for $\boldsymbol{k}$ vanish because of (5·28), and (5·23) gives the whole band energies. The number of E^*_{kn} is equal to $N_f/2+1$. Here, N_f denotes the number of the local f-electron states, then $N_f/2$ corresponds to the number of the Kramers doublets. In such a case, the whole bands have dispersion.

If some f-levels including the state M has further degeneracy besides Kramers one, a fraction of the state M mixes with the conduction electron states to construct the hybridized bands but the remaining parts stay at the original energy E_M with the weight of (5·28). The total weight of these unhybridized parts of the f-level (denoted by l) is calculated from (5·28) as $\sum_{M\in l\text{-th level}} z^f_{kM}(E_M)=N_{fl}-2$, where N_{fl} is the degeneracy of that f-level. These remaining parts, of course, form dispersionless bands at the original energy E_M.

The exceptional case $I_{kM}=0$ occurs, for example, for $\boldsymbol{k}$ parallel to one of the crystal axes for cubic Γ_7 doublet. In such a case, Γ_7 state is outside of the mixing problem solved by (5·23) but joins the band formation by connecting continuously to a solution of (5·23) for $I_{kM}\neq 0$.

Next, we consider a simple case with spherical symmetry in which the crystalline field splitting are not taken into account. Using Clebsch-Gordan coefficients given by

$$a^M_{lm\sigma}=-\sigma\sqrt{\left(l+\frac{1}{2}-j_{1z}\sigma\right)\Big/(2l+1)}\,\delta_{m,j_{1z}-\sigma/2} \quad \text{for} \quad M\colon j_1=l-\frac{1}{2},\ j_{1z}\,, \tag{5·34}$$

$$a^M_{lm\sigma}=\sqrt{\left(l+\frac{1}{2}+j_{2z}\sigma\right)\Big/(2l+1)}\,\delta_{m,j_{2z}-\sigma/2} \quad \text{for} \quad M\colon j_2=l+\frac{1}{2},\ j_{2z}\,, \tag{5·35}$$

and relations $\sum_{m=-l}^{l}|Y_l^m(\theta,\phi)|^2=(2l+1)/4\pi$, $\sum_{m=-l}^{l}m|Y_l^m(\theta,\phi)|^2=0$, we obtain

$$\sum_{M=-j_1}^{j_1} I_{kM}=l|V_{knl}|^2=\frac{2j_1+1}{2}|V_{knl}|^2\,, \tag{5·36}$$

$$\sum_{M=-j_2}^{j_2} I_{kM}=(l+1)|V_{knl}|^2=\frac{2j_2+1}{2}|V_{knl}|^2\,. \tag{5·37}$$

Thus, (5·23) is written as

$$E_{kn}^*-\varepsilon_k-\frac{l|V_{knl}|^2}{E_{kn}^*-E_{j_1}}-\frac{(l+1)|V_{knl}|^2}{E_{kn}^*-E_{j_2}}=0\,, \tag{5·38}$$

which does not depend on the direction of $\boldsymbol{k}$, thereby leading to an expected result that the whole bands are spherical. (We approximated conduction electrons as free.) The degree of (5·38) is three, giving three hybridized bands with two-fold degeneracy. The prefactors of $|V_{knl}|^2$ are equal to a half of the degeneracy of the corresponding spin-orbit levels. Effective mixing matrices are enhanced by the square root of half a degeneracy.

We consider the case that the total number of f- and c-electrons per f-site is less than two, and the lowest band is a hybridized one denoted by E_{k0}^* and that this is partially filled and other bands are vacant. In this case, the linear coefficient of the specific heat is given by

$$\gamma=\frac{2\pi^2}{3}k_B{}^2\sum_k\delta(\mu-E_{k0}^*)\,, \tag{5·39}$$

where μ is the Fermi energy. This is proportional to the density of states at the Fermi energy, which is rewritten with the use of (5·30) as

$$\sum_k\delta(\mu-E_{k0}^*)=\sum_k\left[z_{k\sigma}^c(E_{k0}^*)+\frac{1}{2}\sum_M z_{kM}^f(E_{k0}^*)\right]\delta(\mu-E_{k0}^*)\,. \tag{5·40}$$

By denoting the density of states of the original conduction band per spin by $\rho(\varepsilon_k)$ and replacing the summation over wavevector by integration,

$$\sum_k\delta(\mu-E_{k0}^*)=\int\frac{d\Omega_k}{4\pi}\rho(\varepsilon_k{}^\mu)+\sum_M\int\frac{d\Omega_k}{4\pi}\rho(\varepsilon_k{}^\mu)\frac{I_{kM}}{(\mu-E_M)^2}\,, \tag{5·41}$$

where

$$\varepsilon_k{}^\mu=\mu-\sum_M\frac{I_{kM}}{\mu-E_M}\,. \tag{5·42}$$

The first term of the right-hand side of (5·41) corresponds to the density of states of conduction electrons, which does not change from that of the original conduction band for a constant density of states and the second to that of f-electrons.

Next, we turn to the derivation of the magnetic susceptibility. For this, we must solve the equation for $H\neq 0$,

$$A_{k\sigma}(\omega)A_{k\bar{\sigma}}(\omega)-B_{k\sigma}(\omega)B_{k\bar{\sigma}}(\omega)=0\,, \tag{5·43}$$

to obtain the eigenvalues up to the order of H^2. Here, $A_{k\sigma}(\omega)$ and $B_{k\sigma}(\omega)$ are obtained as

$$A_{k\sigma}(\omega)=\omega-\varepsilon_k-a_k(\omega)+\sigma[1+b_k(\omega)]h-c_k(\omega)h^2+O(h^3)\,, \tag{5·44}$$

$$B_{k\sigma}(\omega)=-d_{k\sigma}(\omega)h+O(h^2)\,, \tag{5·45}$$

where $h=\mu_B H$ and

$$a_k(\omega)=\sum_M \frac{|V_{kM\sigma}|^2}{\omega-E_M}\,, \tag{5·46}$$

$$b_k(\omega)=\sigma\sum_{MM'} \frac{V^*_{kM\sigma}\langle M|l_z+2s_z|M'\rangle V_{kM'\sigma}}{(\omega-E_M)(\omega-E_{M'})}\,, \tag{5·47}$$

$$c_k(\omega)=\sum_{MM'M''} \frac{V^*_{kM\sigma}\langle M|l_z+2s_z|M'\rangle\langle M'|l_z+2s_z|M''\rangle V_{kM''\sigma}}{(\omega-E_M)(\omega-E_{M'})(\omega-E_{M''})}\,, \tag{5·48}$$

$$d_{k\sigma}(\omega)=\sum_{MM'} \frac{V^*_{kM\sigma}\langle M|l_z+2s_z|M'\rangle V_{kM'\bar{\sigma}}}{(\omega-E_M)(\omega-E_{M'})}\,. \tag{5·49}$$

It is easily shown that $a_k(\omega)$, $b_k(\omega)$, $c_k(\omega)$ and $d_{k\sigma}(\omega)d_{k\bar{\sigma}}(\omega)=|d_{k\sigma}(\omega)|^2\equiv e_k(\omega)$ are all independent of σ. Thus, (5·43) is reduced to

$$\omega=\varepsilon_k+a_k(\omega)-\sigma\sqrt{[1+b_k(\omega)]^2+e_k(\omega)}\,h+c_k(\omega)h^2+O(h^3)\,. \tag{5·50}$$

We define the solution of (5·50) for σ by $E^*_{k\sigma n}$ and consider the case that only the lowest band $n=0$ is occupied. From the expressions for the total number of electrons and the energy of the ground state,

$$N_e=\sum_{k\sigma}\theta(\mu-E^*_{k\sigma 0})\,, \tag{5·51}$$

$$E_0=\sum_{k\sigma}E^*_{k\sigma 0}\theta(\mu-E^*_{k\sigma 0})\,, \tag{5·52}$$

the susceptibility is obtained as

$$\chi=\chi_P+\chi_V\,, \tag{5·53a}$$

$$\chi_P=\sum_{k\sigma}\left[\frac{\partial E^*_{k\sigma 0}}{\partial H}\right]^2_{H=0}\delta(\mu-E^*_{k0})\,, \tag{5·53b}$$

$$\chi_V=-\sum_{k\sigma}\frac{\partial^2 E^*_{k\sigma 0}}{\partial H^2}\bigg|_{H=0}\theta(\mu-E^*_{k0})\,. \tag{5·53c}$$

The first term χ_P may be called the Pauli term which gives rise to a usual Pauli susceptibility and the second χ_V the Van Vleck term. The contribution from χ_V is crucial when orbital degeneracy is considered. Differentiating (5·50) by h, we obtain

$$\frac{\partial\omega}{\partial h}\bigg|_{h=0}=-\sigma\frac{\sqrt{[1+b_k(\omega)]^2+e_k(\omega)}}{1-a'_k(\omega)}\,, \tag{5·54}$$

$$\frac{\partial^2\omega}{\partial h^2}\bigg|_{h=0}=\frac{1}{1-a'_k(\omega)}\left[\frac{\partial}{\partial\omega}\left\{\frac{[1+b_k(\omega)]^2+e_k(\omega)}{1-a'_k(\omega)}\right\}+2c_k(\omega)\right]\,, \tag{5·55}$$

where $a'_k(\omega)=\partial a_k(\omega)/\partial\omega$. Noting that $1-a'_k(\omega)$ is equal to $A'_k(E^*_{kn})$ of (5·25) and replacing the summation over $\boldsymbol{k}$ by integration, we obtain

$$\chi_{\mathrm{P}}=2\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\rho(\varepsilon_k{}^{\mu})\frac{[1+b_k(\mu)]^2+e_k(\mu)}{A_k'(\mu)}\,, \tag{5·56}$$

$$\begin{aligned}\chi_{\mathrm{V}}=&-2\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\int_{-D^*}^{\mu}dE_{k0}^*\rho(\varepsilon_k{}^{E_{k0}^*})\left[\frac{d}{dE_{k0}^*}\left\{\frac{[1+b_k(E_{k0}^*)]^2+e_k(E_{k0}^*)}{A_k'(E_{k0}^*)}\right\}+2c_k(E_{k0}^*)\right]\\
=&-2\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\rho(\varepsilon_k{}^{\mu})\frac{[1+b_k(\mu)]^2+e_k(\mu)}{A_k'(\mu)}\\
&+2\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\rho(\varepsilon_k{}^{-D^*})\frac{[1+b_k(-D^*)]^2+e_k(-D^*)}{A_k'(-D^*)}\\
&+2\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\int_{-D^*}^{\mu}dE_{k0}^*\frac{d\rho(\varepsilon)}{d\varepsilon}|_{\varepsilon=\varepsilon_k{}^{E_{k0}^*}}\{[1-b_k(E_{k0}^*)]^2+e_k(E_{k0}^*)\}\\
&-4\mu_{\mathrm{B}}{}^2\int\frac{d\Omega_k}{4\pi}\int_{-D^*}^{\mu}dE_{k0}^*\rho(\varepsilon_k{}^{E_{k0}^*})c_k(E_{k0}^*)\,,\end{aligned} \tag{5·57}$$

where $-D^*$ is the lowest value of E_{k0}^* and

$$\varepsilon_k{}^{E_{k0}^*}=E_{k0}^*-\sum_M\frac{I_{kM}}{E_{k0}^*-E_M}\,. \tag{5·58}$$

The Pauli term is entirely canceled by the first term of (5·57). If we assume that the conduction band width is much larger than the mixing matrix elements, we can safely put $A_k'(-D^*)=1$ and $b_k(-D^*)=e_k(-D^*)=0$. Further, if we assume that the density of states $\rho(\varepsilon)$ is constant we can drop the third term of (5·57). Then, denoting the constant state density by ρ_0 and the mixing integral by V which is assumed to be constant also, we have as the susceptibility

$$\chi=2\mu_{\mathrm{B}}{}^2\rho_0+\mu_{\mathrm{B}}{}^2\rho_0V^2\sum_{MM'}\frac{|\langle M|l_z+2s_z|M'\rangle|^2}{(\mu-E_M)(\mu-E_{M'})}\,, \tag{5·59}$$

where

$$\langle M|l_z+2s_z|M'\rangle=\sum_{m\sigma}(m+\sigma)a_{lm\sigma}^{M}a_{lm\sigma}^{M'}\,. \tag{5·60}$$

The second term of (5·59) comes from the upper limit of the integration of the fourth term of (5·57) which includes $c_k(E_{k0}^*)$. To derive this second term, we have used (5·48), (5·5) and (5·7) and the orthonormalities of $|iM\rangle$ and Y_l^m. The simple expression for the susceptibility given by (5·59) results mainly from the assumption of constant density of states and constant mixing. It is noted here that the anisotropy of the susceptibility is included in (5·60).

Now, we calculate the susceptibility in the spherically symmetric case in which the crystalline field splittings are ignored, retaining only the spin-orbit coupling.

Using (5·34) and (5·35), we have

$$\begin{aligned}\chi=&2\mu_{\mathrm{B}}{}^2\rho_0+\frac{2}{3}\mu_{\mathrm{B}}{}^2\rho_0V^2\left[\frac{g_{j_1}^2j_1(j_1+1)}{(\mu-E_{j_1})^2}\frac{2j_1+1}{2}\right.\\
&\left.+\frac{g_{j_2}^2j_2(j_2+1)}{(\mu-E_{j_2})^2}\frac{2j_2+1}{2}+\frac{2l(l+1)/(2l+1)}{(\mu-E_{j_1})(\mu-E_{j_2})}\right],\end{aligned} \tag{5·61}$$

where $j_1=l-(1/2)$, $g_{j_1}=2l/(2l+1)$; $j_2=l+(1/2)$, $g_{j_2}=(2l+2)/(2l+1)$. If $E_{j_1}=E_{j_2}=E$, we have

$$\chi=2\mu_B{}^2\rho_0+2\mu_B{}^2\rho_0\frac{(2l+1)V^2}{(\mu-E)^2}+\frac{2}{3}\mu_B{}^2\rho_0 l(l+1)\frac{(2l+1)V^2}{(\mu-E)^2}. \tag{5·62}$$

These results for $U=0$ and no crystalline field splittings are derived in Ref. 12).

3. *Consideration for full Hamiltonian*

For general cases in which the Coulomb repulsion is taken into consideration between f-electrons, we have to introduce the self-energy of f-electrons by $\Sigma_{kMM'}(\omega)$ and determine the Green's function by

$$\hat{P}_k\hat{G}_k=\hat{1}_{N_f+2}, \tag{5·63a}$$

$$\hat{P}_k=\begin{pmatrix}\omega\hat{1}_{N_f}-\hat{E}-\hat{\Sigma}_k & \hat{v}_{k\uparrow} & \hat{v}_{k\downarrow}\\ \hat{v}^{\dagger}_{k\uparrow} & \omega-\varepsilon_{k\uparrow} & 0\\ \hat{v}^{\dagger}_{k\downarrow} & 0 & \omega-\varepsilon_{k\downarrow}\end{pmatrix}, \tag{5·63b}$$

$$\hat{G}_k=\begin{pmatrix}\hat{G}_k{}^f & \hat{G}^{fc}_{k\uparrow} & \hat{G}^{fc}_{k\downarrow}\\ \hat{G}^{cf}_{k\uparrow} & G^c_{k\uparrow\uparrow}(\omega) & G^c_{k\uparrow\downarrow}(\omega)\\ \hat{G}^{cf}_{k\downarrow} & G^c_{k\downarrow\uparrow}(\omega) & G^c_{k\downarrow\downarrow}(\omega)\end{pmatrix}. \tag{5·63c}$$

$\hat{P}_k$ and $\hat{G}_k$ are $(N_f+2)\times(N_f+2)$ matrices. $\hat{1}_{N_f+2}$ and $\hat{1}_{N_f}$ denote the unit matrices of rank N_f+2 and N_f, respectively. These are simply written by $\hat{1}$ in the following. $\hat{E}$, $\hat{\Sigma}_k$ and $\hat{G}_k{}^f$ are $N_f\times N_f$ matrices whose MM'-elements are given by $E_{MM'}$, $\Sigma_{kMM'}(\omega)$ and $G^f_{kMM'}(\omega)$, respectively. $\hat{v}_{k\sigma}$ and $\hat{G}^{fc}_{k\sigma}$ are N_f-dimensional column vectors whose M component is given by $V_{kM\sigma}$ and $G^{fc}_{kM\sigma}(\omega)$, and $\hat{v}^{\dagger}_{k\sigma}$ and $\hat{G}^{cf}_{k\sigma}$ are the row vectors.

The determinant of $\hat{P}_k$ is expressed as

$$\begin{aligned}|\hat{P}_k|&=\begin{vmatrix}\omega\hat{1}-\hat{E}-\hat{\Sigma}_k-\dfrac{\hat{v}_{k\sigma}\hat{v}^{\dagger}_{k\sigma}}{\omega-\varepsilon_{k\sigma}} & \hat{v}_{k\bar{\sigma}}\\ \hat{v}^{\dagger}_{k\bar{\sigma}} & \omega-\varepsilon_{k\bar{\sigma}}\end{vmatrix}(\omega-\varepsilon_{k\sigma})\\ &=\left|\omega\hat{1}-\hat{E}-\hat{\Sigma}_k-\sum_{\sigma}\frac{\hat{v}_{k\sigma}\hat{v}^{\dagger}_{k\sigma}}{\omega-\varepsilon_{k\sigma}}\right|(\omega-\varepsilon_{k\uparrow})(\omega-\varepsilon_{k\downarrow}),\end{aligned} \tag{5·64}$$

where $\hat{v}_{k\sigma}\hat{v}^{\dagger}_{k\sigma}$ is the $N_f\times N_f$ matrix whose MM' element is given by $V^*_{kM\sigma}V_{kM'\sigma}$. Therefore, if we define the $N_f\times N_f$ matrix by

$$\hat{F}_k=\omega\hat{1}-\hat{E}-\hat{\Sigma}_k-\sum_{\sigma}\frac{\hat{v}_{k\sigma}\hat{v}^{\dagger}_{k\sigma}}{\omega-\varepsilon_{k\sigma}}, \tag{5·65}$$

the Green's functions of f- and c-electrons are given by

$$G^f_{kMM'}(\omega)=(\hat{F}_k{}^{-1})_{MM'}, \tag{5·66}$$

$$\begin{aligned}G^c_{k\sigma\sigma}(\omega)&=\left[\omega-\varepsilon_{k\sigma}-\sum_{MM'}V^*_{kM\sigma}\left(\left[\omega\hat{1}-\hat{E}-\hat{\Sigma}_k-\frac{\hat{v}_{k\bar{\sigma}}\hat{v}^{\dagger}_{k\bar{\sigma}}}{\omega-\varepsilon_{k\bar{\sigma}}}\right]^{-1}\right)_{MM'}V_{kM'\sigma}\right]^{-1}\\ &=\frac{1}{\omega-\varepsilon_{k\sigma}}+\frac{1}{\omega-\varepsilon_{k\sigma}}\sum_{MM'}V^*_{kM\sigma}G^f_{kMM'}(\omega)V_{kM'\sigma}\frac{1}{\omega-\varepsilon_{k\sigma}}.\end{aligned} \tag{5·67}$$

By introducing the unitary matrix $\hat{U}_k$, $\hat{P}_k$ is diagonalized as $\hat{U}_k\hat{P}_k\hat{U}_k^{-1}=\hat{Q}_k$ to give eigenvalues E^*_{kn}, where $n=1, 2, \cdots N_f+2$. If we follow Luttinger derivation,[8] the coefficient of the T-linear term of the specific heat is obtained as

$$\gamma=\frac{\pi^2}{3}{k_B}^2\sum_{n=1}^{N_f+2}\delta(\mu-E^*_{kn})\,. \tag{5·68}$$

We calculate this as

$$\begin{aligned}
&\sum_{n=1}^{N_f+2}\delta(\omega-E^*_{kn})\\
&=-\frac{1}{\pi}\mathrm{Im}\frac{\partial}{\partial\omega}\log\prod_{n=1}^{N_f+2}(\omega+i\delta-E^*_{kn})\\
&=-\frac{1}{\pi}\mathrm{Im}\frac{\partial}{\partial\omega}\log|\hat{Q}_k|=-\frac{1}{\pi}\mathrm{Im}\frac{\partial}{\partial\omega}\log|\hat{P}_k|\\
&=-\frac{1}{\pi}\mathrm{Im}\frac{\partial}{\partial\omega}\{\log|\hat{F}_k|+\sum_\sigma\log(\omega+i\delta-\varepsilon_{k\sigma})\}\\
&=-\frac{1}{\pi}\mathrm{Im}\Big\{\sum_{MM'}\Big(\delta_{MM'}-\frac{\partial\Sigma_{kMM'}}{\partial\omega}-\sum_\sigma\frac{V_{kM\sigma}V^*_{kM'\sigma}}{(\omega-\varepsilon_{k\sigma})^2}\Big)\\
&\quad\times G^f_{kM'M}(\omega+i\delta)+\sum_\sigma\frac{1}{\omega+i\delta-\varepsilon_{k\sigma}}\Big\}\\
&=-\frac{1}{\pi}\mathrm{Im}\Big\{\sum_{MM'}\Big(\delta_{MM'}-\frac{\partial\Sigma_{kMM'}}{\partial\omega}\Big)G^f_{kM'M}(\omega+i\delta)+\sum_\sigma G^c_{k\sigma\sigma}(\omega+i\delta)\Big\}\,.
\end{aligned}\tag{5·69}$$

Generally, the imaginary part of the self-energy vanishes at the Fermi surface, so that the T-linear coefficient of the specific heat is obtained as

$$\gamma=\frac{\pi^2}{3}{k_B}^2\sum_k[\mathrm{Tr}\,\hat{\gamma}_k(\mu)\hat{n}_k{}^f(\mu)+\sum_\sigma n^c_{k\sigma}(\mu)]\,, \tag{5·70}$$

where

$$\hat{\gamma}_k(\omega)=\hat{1}-\frac{\partial\hat{\Sigma}_k}{\partial\omega}\,, \tag{5·71}$$

and $\hat{n}_k{}^f(\omega)$ and $n^c_{k\sigma}(\omega)$ are defined by

$$\hat{n}_k{}^f(\omega)=-\frac{1}{\pi}\mathrm{Im}\hat{G}_k{}^f(\omega+i\delta)\,, \tag{5·72}$$

$$n^c_{k\sigma}(\omega)=-\frac{1}{\pi}\mathrm{Im}G^c_{k\sigma\sigma}(\omega+i\delta)\,. \tag{5·73}$$

Thus, the T-linear term of the specific heat is proportional to the density of states of quasi-particles at the Fermi energy in which the f-electron part is enhanced by $\hat{\gamma}_k(\mu)$, while the c-electron part has no such enhancement.

Next, we turn to the susceptibility. As Luttinger[8] has derived, the magnetization is obtained by differentiating the thermodynamic potential with respect to the magnetic field and by keeping in mind its stationary properties to variations of the

self-energy, as

$$M=\mu_B\sum_k\int_{-\infty}^{\infty}d\omega f(\omega)\Big\{\sum_{MM'}\langle M|(l_z+2s_z)|M'\rangle \times\Big(-\frac{1}{\pi}\mathrm{Im}\Big)G^f_{kM'M}(\omega+i\delta)+\sum_\sigma\sigma\Big(-\frac{1}{\pi}\mathrm{Im}\Big)G^c_{k\sigma\sigma}(\omega+i\delta)\Big\}, \tag{5·74}$$

where $f(\omega)$ is the Fermi distribution function. Introducing the $N_f\times N_f$ matrix $\hat{M}$ whose MM' element is given by $\langle M|(l_z+2s_z)|M'\rangle$, making use of (5·66) and (5·67), we can rewrite (5·74) as

$$M=\mu_B\sum_k\int_{-\infty}^{\infty}d\omega f(\omega)\Big(-\frac{1}{\pi}\mathrm{Im}\Big)\Big\{\mathrm{Tr}(\hat{M}\hat{F}_k^{-1}) +\sum_\sigma\frac{\sigma}{\omega+i\delta-\varepsilon_{k\sigma}}+\sum_\sigma\frac{\sigma}{(\omega+i\delta-\varepsilon_{k\sigma})^2}\mathrm{Tr}(\hat{v}_{k\sigma}\hat{v}^\dagger_{k\sigma}\hat{F}_k^{-1})\Big\}. \tag{5·75}$$

If we follow Luttinger's procedure, the next step will be to rewrite (5·75) in the following form,

$$M=\mu_B\sum_k\int_{-\infty}^{\infty}d\omega f(\omega)\Big(-\frac{1}{\pi}\mathrm{Im}\Big)\frac{\partial}{\partial\omega}\mathcal{F}_k(\omega+i\delta)+J. \tag{5·76}$$

If it is possible and J is shown to vanish, then we have the expression for the susceptibility which is described in terms of the quantities at the Fermi surface. However, it is generally impossible to obtain such an expression in the cases including the trace of product of matrices in (5·75), since commutators of $\hat{F}_k$ with $\hat{M}$, $\hat{v}_{k\sigma}\hat{v}^\dagger_{k\sigma}$ do not vanish. In connection with this, in the previous paper[14] in which orbital degeneracy has been taken into consideration for localized f-electrons, non-commutability between these operators was not correctly treated and therefore the final expression given there for the susceptibility is not correct.

Generally, in such cases as here, the expression for the susceptibility is composed of a part expressible in terms of the quantities at the Fermi surface and the other parts given by the integrals over the band. The first part is proper to the Fermi liquid theory and the others can be regarded as correction terms to the first. Here, if we assume that the conduction electron band width is large enough compared with mixing matrix $V_{kM\sigma}$, perturbations which the electronic states undergo by the magnetic field will be limited to the neighbourhood of the Fermi energy. Therefore, in this case, the second correction terms are expected to be small compared with the normal part.

That this is actually the case has been shown by Hanzawa et al.[12] and Yamada and Nakano[15] for a simple special case although we have no space to describe here their detailed calculations. The simple case is that in which the spin-orbit coupling and crystalline field are so strong that we can safely be restricted to the lowest Kramers doublet labeled by M and $\bar{M}$. For this two-dimensional case, it can be shown that $(\hat{M})_{MM}=\mu$, $(\hat{M})_{\bar{M}\bar{M}}=-\mu$ and $(\hat{M})_{M\bar{M}}=0$. The result for the susceptibility is expressed as

$$\chi=2\mu^2\mu_B{}^2\sum_k\left[1-\frac{\partial\Sigma_{kMM}}{\partial(\mu\mu_BH)}\bigg|_{H=0}\right]n^f_{kMM}(\varepsilon_F)$$

$$+2\mu_B{}^2\sum_k n^c_{k\sigma}(\varepsilon_F)+\text{correction term}\,. \tag{5·77}$$

4. *Summary of this section*

In this section, we developed the Fermi liquid theory on the basis of the real periodic Anderson model applicable to Ce metallic compounds, in which the spin-orbit coupling and the crystalline field splittings are introduced. Eigenstates for localized f-electron at site i is denoted by $|iM\rangle$ and mixing between this state and the wavefunction of the conduction electron $|\boldsymbol{k}\sigma\rangle$ is taken into account. The direction of the external field is taken as z-axis and $|iM\rangle$ is described by a linear combination of $Y_l{}^m\chi_\sigma$ referring to z-axis.

First of all, the electronic states for this Hamiltonian have been investigated when $U=0$. Then, Fermi liquid theory has been developed for non-zero U. In this case, the self-energy and Green's function for f-electrons become matrices whose off-diagonal elements are non-zero. This makes the situations complicated. Under these situations, the specific heat has a normal form to the Fermi liquid, but the magnetic susceptibility has, besides the normal part, anomalous correction terms which cannot be expressed by the quantities at the Fermi level. However, this anomalous terms are expected not to play an important role when the width of the conduction band is large compared with the mixing matrix.

§ 6. T_1 of NMR and anomalous Hall effect

1. *Nuclear spin-lattice relaxation time T_1*

Here, we consider the nuclear spin relaxation due to hyperfine interactions with f-electrons.[16] In this case, relaxation time T_1 is given by[17]

$$\frac{1}{T_1}=k_BT(g_n\mu_n)^2\sum_q|A(\boldsymbol{q})|^2\left[\frac{1}{\omega}\mathrm{Im}\chi^{+-}(\boldsymbol{q},\omega+i0)\right]_{\omega=0}, \tag{6·1}$$

where $A(\boldsymbol{q})$ is a hyperfine coupling constant and $\chi^{+-}(\boldsymbol{q},\omega+i0)$ is the transverse susceptibility of f-electrons. $\chi^{+-}(\boldsymbol{q},\omega)$ is given by

$$\chi^{+-}(\boldsymbol{q},\omega+i0)=i\int_0^\infty dte^{i(\omega+i0)t}\langle[S_q{}^+(t),S_{-q}^-(0)]\rangle\,, \tag{6·2}$$

$$S_q{}^+=\sum_k f^\dagger_{k\uparrow}f_{k+q\downarrow}\,. \tag{6·3}$$

The general discussion was done by Kohno et al.[16] Here, we discuss the case at low temperatures. At zero temperature we can derive the equation,

$$\left[\frac{1}{\omega}\mathrm{Im}\chi^{+-}(\boldsymbol{q},\omega+i0)\right]_{\omega=0}=\pi\sum_k z_k{}^f\delta(\mu-E_k{}^*)z^f_{k+q}\delta(\mu-E^*_{k+q})[\Lambda_{k,k+q}(0)]^2\,. \tag{6·4}$$

The three-point vertex $\Lambda_{k,k+q}(0)$ is given by using the four-point vertex, $\Gamma_{\uparrow\downarrow}$, as

$$\Lambda_{k,k+q}(0)=1-T\sum_{n'}\sum_{k'}\Gamma_{\uparrow\downarrow}(\boldsymbol{k}+\boldsymbol{q},\boldsymbol{k}';\boldsymbol{k},\boldsymbol{k}'+\boldsymbol{q})G^f_{k'+q\uparrow}(i\varepsilon_{n'})G^f_{k'\downarrow}(i\varepsilon_{n'})\,. \tag{6·5}$$

This result can be directly obtained in the imaginary frequency representation, as shown by Shiba for the magnetic impurity.[18] The factor $\Lambda_{k,k+q}$ is expected to be enhanced owing to the electron interaction U, leading to the enhancement of the Korringa constant, i.e., $(T_1T)^{-1}$ at $T=0$.

If we neglect the $\boldsymbol{k}$, ε-dependence of $\Lambda_{k,k+q}(\varepsilon)$,

$$\begin{aligned}\chi(\boldsymbol{q})&=-\sum_{k}\int_{-\infty}^{\infty}\frac{d\varepsilon}{2\pi i}\Lambda_{k,k+q}(i\varepsilon)G_k{}^f(i\varepsilon)G^f_{k+q}(i\varepsilon)\\&\simeq\Lambda(\boldsymbol{q})\bar{\chi}(\boldsymbol{q})\,,\end{aligned} \tag{6·6}$$

where $\chi(\boldsymbol{q})$ is the exact susceptibility and $\bar{\chi}(\boldsymbol{q})$ is defined as that without the vertex correction. Therefore, $\Lambda(\boldsymbol{q})$ can be regarded as an enhancement factor of $\chi(\boldsymbol{q})$ due to vertex corrections. In this case, $(T_1T)^{-1}$ is enhanced by the factor

$$\frac{1}{T_1T}\propto[\rho^f(0)]^2\sum_{q}|A(\boldsymbol{q})|^2\left(\frac{\chi(\boldsymbol{q})}{\bar{\chi}(\boldsymbol{q})}\right)^2. \tag{6·7}$$

If we neglect the momentum dependence of $\Gamma_{\uparrow\downarrow}(\boldsymbol{q})$ in (6·5) and assume a uniform enhancement, we obtain

$$(T_1T)^{-1}\propto\gamma^2\,. \tag{6·8}$$

This is the similar relation to the T^2-term of the resistivity. From (6·4) we can consider also in the picture of the heavy electrons by using the density of states of quasi-particles, $\rho^*(0)$, which is enhanced by $\tilde{\gamma}$. In this picture the vertex correction is given by $z_k{}^f\Lambda_{k,k+q}z^f_{k+q}\Lambda_{k+q,k}$, which is of the order of unity without enhancement. The factor $[\Lambda_{k,k+q}(0)]^2$ is important in understanding of temperature dependence of T_1 in the superconducting state, since it decreases with increasing superconducting gap. Therefore, in order to explain the full temperature dependence of $T_1{}^{-1}$ for the systems such as UPt_3, $CeCu_2Si_2$ and UBe_{13}, we should estimate the temperature dependence of $\Lambda_{k,k+q}$, as well as the life time of heavy electrons.

2. *Anomalous Hall effect in the coherent regime*

The ordinary Hall effect in the system with the electron interaction is developed by Kohno et al.,[19] extending the theory done by Fukuyama[20] to the system with the T^2-terms of resistivity. The normal Hall coefficient R_H has no temperature dependence at low temperatures. On the other hand, typical heavy fermion systems exhibit anomalous Hall effect. The R_H increases rapidly with increasing temperature from a constant value at $T=0$ in the low temperature regime, while it decreases at high temperatures, resulting in a peak structure between the two regimes. The behavior at high temperatures was explained by Coleman et al.[21] and Fert et al.[22] by the mechanism of the single impurity skew scattering.

In Ref. 23) it is shown that the anomalous Hall coefficient in the coherent regime is given by

$$R_H=cR^2\simeq c(R_0+AT^2)^2\,, \tag{6·9}$$

where c is a constant proportional to the susceptibility of f-electrons and R is the true resistivity including the residual resistivity, R_0. The above expression was derived for the case with only the orbital degeneracy.[23] The extension to the case with the spin-orbit coupling is possible to give the same expression (6·9). Equation (6·9) explains the rapid increase of the Hall coefficient at low temperatures. The detail of the derivation for the general case will be given in a future publication.

§ 7. Fermi liquid and spin fluctuations

In this section we would like to stress that the Fermi liquid is a general state including the singlet state as a local structure.[24] The nature of the ground state of a single magnetic impurity is well known as the singlet bound state. The similar nature can be seen also in the periodic Anderson Hamiltonian. For the periodic Anderson Hamiltonian we choose arbitrarily one atomic site of f-electron and call it 0 site. The wavefunction of the ground state can be expanded in the following way, depending on the state of f-electron at 0 site,

$$\Psi_g = A_0\psi_0 + A_\uparrow f_\uparrow{}^\dagger \psi_\uparrow + A_\downarrow f_\downarrow{}^\dagger \psi_\downarrow + A_2 f_\uparrow{}^\dagger f_\downarrow{}^\dagger \psi_2 , \tag{7·1}$$

where $f_\sigma{}^\dagger$, A_a and ψ_a denote f-electron with σ spin, coefficient and neighbouring electron wavefunction associated to each f state, respectively. For Ψ_g to be the ground state, the matrix element of the hybridization term should exist. If it vanishes there is no energy gain, because only the hybridization term reduces the energy in the periodic Anderson Hamiltonian. Thus, the system described by the periodic Anderson Hamiltonian gains the energy by constructing the coherent band of heavy electrons through the hybridization term,

$$\begin{aligned}\mathcal{H}_{\mathrm{mix}} &= \sum_{k,\sigma}(V_k f_{k\sigma}^\dagger c_{k\sigma} + V_k{}^* c_{k\sigma}^\dagger f_{k\sigma}) \\ &= \sum_{\substack{i,\sigma \\ k}}(V_k f_{i\sigma}^\dagger c_{k\sigma} e^{ik\cdot R_i} + V_k{}^* c_{k\sigma}^\dagger f_{i\sigma} e^{-ik\cdot R_i}) \\ &= \sum_{i,\sigma}(V f_{i\sigma}^\dagger c_{i\sigma} + V^* c_{i\sigma}^\dagger f_{i\sigma}) ,\end{aligned} \tag{7·2}$$

where

$$c_{i\sigma} = \sum_k u_k c_{k\sigma} e^{ik\cdot R_i} , \tag{7·3}$$

$$c_{i\sigma}^\dagger = \sum_k u_k c_{k\sigma}^\dagger e^{-ik\cdot R_i} , \tag{7·4}$$

and $V_k = V u_k$ is assumed.

Thus, the necessary condition to be the ground state is the following,

$$\langle \Psi_g | \mathcal{H}_{\mathrm{mix}} | \Psi_g \rangle = \sum_{i,\sigma} V \langle \Psi_g | f_{i\sigma}^\dagger c_{i\sigma} + c_{i\sigma}^\dagger f_{i\sigma} | \Psi_g \rangle \neq 0 . \tag{7·5}$$

For simplicity, we consider the hybridization at site 0 as an example,

$$=\sum_{\sigma} V\{\langle A_{\sigma} f_{\sigma}^{\dagger}\psi_{\sigma}|f_{\sigma}^{\dagger}c_{\sigma}|A_0\psi_0\rangle$$

$$+\langle A_2 f_{\uparrow}^{\dagger} f_{\downarrow}^{\dagger}\psi_2|f_{\sigma}^{\dagger}c_{\sigma}|A_{-\sigma}f_{-\sigma}^{\dagger}\psi_{-\sigma}\rangle+\text{c.c.}\}$$

$$=\sum_{\sigma} V\{A_{\sigma}A_0\langle\psi_{\sigma}|c_{\sigma}|\psi_0\rangle+A_2A_{-\sigma}\langle\psi_2|c_{\sigma}|\psi_{-\sigma}\rangle$$

$$+A_{\sigma}A_0\langle\psi_0|c_{\sigma}^{\dagger}|\psi_{\sigma}\rangle+A_2A_{-\sigma}\langle\psi_{-\sigma}|c_{\sigma}^{\dagger}|\psi_2\rangle\}\,. \tag{7·6}$$

By defining $n_{\sigma\sigma'}$ as the number of local electron with σ' spin associated to f-electron with σ spin, the necessary condition is given on account of the Anderson orthogonality theorem by[25]

$$n_{\sigma\sigma}=n_{0\sigma}-1\,, \tag{7·7}$$

$$n_{\sigma-\sigma}=n_{0-\sigma}\,, \tag{7·8}$$

$$n_{2\sigma}=n_{-\sigma\sigma}-1\,, \tag{7·9}$$

$$n_{2-\sigma}=n_{-\sigma-\sigma}\,. \tag{7·10}$$

Thus, we obtain from (7·7) and (7·8), and the symmetry relation,

$$n_{\sigma\sigma}+1=n_{\sigma-\sigma}=n_{-\sigma\sigma}\,. \tag{7·11}$$

There is no local charge associated to $f_{\sigma}^{\dagger}$ state,

$$n_{\sigma\sigma}+n_{\sigma-\sigma}=0\,. \tag{7·12}$$

From (7·11) and (7·12), we obtain

$$n_{\sigma\sigma}=-\frac{1}{2}\,, \tag{7·13}$$

$$n_{\sigma-\sigma}=\frac{1}{2}\,. \tag{7·14}$$

Thus, we can see that the component of f-electron with σ spin has 1/2 local electron with $-\sigma$ spin and 1/2 local hole with σ spin. This structure is nothing but the singlet ground state for the single impurity. In our case, the number of local electron around a fixed site includes the number of f-electrons at other sites than the fixed site. Thus, in the periodic system, the local singlet state is constructed by both of conduction and f-electrons. The weight of each component, f and conduction electron, depends on the parameters in the Hamiltonian. Therefore, the construction of the singlet in the Fermi liquid is free from the lack of conduction electrons. The same consideration can be applied also to the normal state of the d-p Hamiltonian for the copper-oxide superconductor. In the normal ground state, the d- and p- electrons are combined to construct the quasi-particles in a Fermi liquid state. The local spins due to d-electrons construct a local singlet combined with neighbouring p- and d-electrons.

§ 8. Concluding remarks

Heavy fermion systems are formed at low temperatures in rare earth metallic

compounds, particularly in Ce(Yb) compounds. At high temperatures, these compounds show Kondo like behaviors characterized by the logarithmic increase of the resistivity with lowering temperature although the density of local spins is high. Such a change from the high temperature regime to the low temperature heavy fermion regime resembles the transition observed in dilute alloys including magnetic impurities.

The reason why such a change is possible even for high density of magnetic ions lies in that the characteristic Kondo temperature T_K in magnetically dilute alloys is larger than the characteristic temperature T_N at which the magnetic ordering of the rare earth spins occurs by RKKY interaction. In order that the relation $T_K > T_N$ may hold, T_K must be much larger than that for iron group dilute alloys represented by CuMn. For $4f$-electrons in Ce(Yb) occupying degenerate f-orbitals, T_K can take very large values estimated as $T_K \sim 10$ K. This is the main reason for such a change from the Kondo regime to the heavy fermion regime. In this sense, the degeneracy of f-orbitals has essential importance for formation of the heavy fermion systems.

For the low temperature heavy fermion regime, the electron system including conduction and $4f$-electrons can be described by the Landau Fermi liquid theory. In the main part of this paper the Fermi liquid theory is reviewed which has been developed on the basis of the periodic Anderson model. Here, Anderson model is used for simplicity in which the orbital degeneracy of f-electrons is not taken into account. Considerations on the Anderson model with orbital degeneracy are given in § 5 in which only electronic specific heat and magnetic susceptibility are treated.

We have derived expressions for the electronic specific heat, the susceptibility, the resistivity, the relaxation time of nuclear spins and the normal and anomalous Hall coefficients. We obtain at least qualitative understanding for various experimental results as stated in this paper in terms of the results obtained by the present Fermi liquid theory.

Main results obtained by this theory are as follows. The resistivity due to electron interaction shows T^2-temperature dependence. This resistivity becomes finite only through Umklapp processes and the coefficient A of T^2-resistivity is proportional to the square of the coefficient γ of the T-linear specific heat. The large enhancement of $(T_1T)^{-1}$ for NMR, the temperature dependence of the Hall coefficient at low temperatures and so on are also derived.

For quantitative discussion, however, the various kinds of vertex functions introduced should be calculated explicitly. This is very difficult task and at present low order (2nd) perturbations in one-dimensional system have only been calculated. The other problem is that the periodic Anderson model we have used for the Fermi liquid theory is simple and this should be extended to more realistic ones.

Nevertheless, the Fermi liquid theory has common features for itinerant electrons with strong correlation, irrespective of special models used as the basic Hamiltonian, Anderson model or Hubbard model. Thus, the Fermi liquid theory described here can also be used for high T_c cuprate superconductors and actinide metallic compounds with slight modifications as well.

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