Hadron-Nucleon Scattering Lengths from QCD Sum Rules

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Recent works of applying the QCD sum rule to the calculation of hadron-nucleon scattering lengths are reported. After a brief introduction to the QCD sum rule, how to obtain scattering lengths in the QCD sum rule is explained. Then it is applied first to the nucleon-nucleon and hyperon-nucleon channels and secondly to the pion-nucleon and kaon-nucleon channels.

§ 1. Introduction

Let us start by two examples of the QCD sum rule,[1),2)] which are related with our following discussion:

$$M_N = -\frac{8\pi^2}{M_B{}^2}\langle \bar{u}u\rangle_0\,, \tag{1}$$

$$2m_\pi{}^2 f_\pi{}^2 = -(m_u + m_d)(\langle \bar{u}u\rangle_0 + \langle \bar{d}d\rangle_0)\,, \tag{2}$$

where M_B is the Borel mass. The former is derived by Ioffe[3)] and shows that the condensation of quarks is responsible for the nucleon mass. The latter is the well-known Gell-Mann-Oakes-Renner relation,[4)] which was rederived in the context of the QCD sum rule by Shifman, Vainshtein and Zakharov.[1)] Both of these are relations between observables of hadrons, such as masses and decay constants, and vacuum expectation values of quark-gluon operators.

The QCD sum rule is a nonperturbative method which provides us with relations between physical quantities. However, it does not directly solve the QCD. Therefore, it cannot answer questions concerning such as confinement and/or spontaneous chiral symmetry breaking.

The structure of the QCD sum rules, Eqs. (1) and (2), can be illustrated as follows:

$$\underset{\text{(mass, decay constant)}}{\text{Observables of 1-Hadron States}} \Leftarrow \underset{\text{(vacuum)}}{\text{Information on the 0-Hadron State}}\,.$$

We proposed in Refs. 5) and 6) to extend the QCD sum rule by increasing the number of hadrons by one on both sides of the above relation:

$$\underset{(S\text{-matrix})}{\text{Observables of 2-Hadron States}} \Leftarrow \text{Information on the 1-Hadron State}\,.$$

Namely, the scattering quantities of hadrons are related with the expectation values of quark-gluon operators with respect to the one-hadron state. To be specific, we show here two examples of new QCD sum rules:[5),6)]

$$a_{NN}^{1}=\frac{4\pi}{M_B^2}M_N(\langle\bar{d}d\rangle_p+3\langle u^\dagger u\rangle_p+\langle d^\dagger d\rangle_p)\,, \tag{3}$$

$$\frac{1}{3}(a_{\pi N}^{I=1/2}-a_{\pi N}^{I=3/2})=\frac{1}{4\pi}\frac{m_\pi M_N}{m_\pi+M_N}(\langle u^\dagger u\rangle_p-\langle d^\dagger d\rangle_p)\,. \tag{4}$$

The former shows that the nucleon-nucleon scattering length is determined by the quark densities in the nucleon. The latter is a similar relation for the pion-nucleon scattering length, which reproduces the well-known Tomozawa-Weinberg relation.[7),8)]

In the next section we will show how to obtain scattering lengths in the QCD sum rule.

§ 2. Formulation

Let us consider the correlation function,

$$\Pi^H(q)=-i\int d^4x e^{iqx}\langle\psi|T(\eta_H(x)\eta_H^\dagger(0))|\psi\rangle\,. \tag{5}$$

η_H is the interpolating field for the hadron, H, i.e. a quark-gluon composite operator which creates H. The spectral function is then defined in terms of the correlation function as

$$\rho^H(\omega,\boldsymbol{q})=\frac{i}{2\pi}\{\Pi^H(\omega+i\eta,\boldsymbol{q})-\Pi^H(\omega-i\eta,\boldsymbol{q})\}\,. \tag{6}$$

The spectral function contains the information on the physical spectrum of the system:

$$\rho^H(q)=(2\pi)^3\sum_n\{\delta^4(q+p_\psi-p_n)|\langle n|\eta_H^\dagger(0)|\psi\rangle|^2\pm\delta^4(q-p_\psi+p_n)|\langle n|\eta_H(0)|\psi\rangle|^2\}\,. \tag{7}$$

In the physical region, $q^2\sim m_H^2$, Π^H is nonperturbative and difficult to calculate. However, in the deep-Euclid region, $q^2\to-\infty$, Π^H is perturbative so that the operator product expansion (OPE),

$$-i\int d^4x e^{iqx}T(\eta_H(x)\eta_H^\dagger(0))=C_I(q)I+C_{\bar{q}q}(q)\,\bar{q}q+C_{G^2}(q)G_{\mu\nu}G^{\mu\nu}+\cdots \tag{8}$$

makes sense. Namely, in the deep-Euclid region the correlation function can be expressed in terms of the expectation values of quark-gluon operators.

Then the deep-Euclid region is related to the physical region by the Lehmann representation for the correlation function as

$$\Pi^H(\omega,\boldsymbol{q})=\int_{-\infty}^{\infty}d\omega'\frac{\rho^H(\omega',\boldsymbol{q})}{\omega-\omega'}\,. \tag{9}$$

ω' in the integral runs over the physical region while ω can be taken to be any value in the complex plane, therefore in the deep-Euclid region. Thus, by evaluating the left-hand side by the OPE and expressing the right-hand side in terms of physical quantities of hadrons with ω in the deep-Euclid region, one obtains relations between observables of hadrons and the expectation values of quark-gluon operators. These relations are the QCD sum rules.

As we have seen, there are two essential ideas employed in order to derive the QCD sum rule, i.e. the OPE and the Lehmann representation. These two ideas are valid independent of the choice of $|\psi\rangle$ with which the correlation function is defined. In the original sum rules[1),2)] $|\psi\rangle$ is taken to be the vacuum. In Refs. 5) and 6) it was proposed to take $|\psi\rangle$ to be the one-nucleon state, $|N(\boldsymbol{p}=0)\rangle$. Then, the reduction formula tells us that the correlation function has second-order poles at $q^2=m_H{}^2$ with their coefficients proportional to the T-matrices for the forward elastic HN and $\bar{H}N$ scatterings:

$$T_{\binom{HN}{\bar{H}N}}(qp\to qp)\propto \lim_{q^2\to m_H{}^2}(q^2-m_H{}^2)^2\Pi_N{}^H(q)\,. \tag{10}$$

Thus, the spectral function has the following structure,

$$\begin{aligned}\rho_N{}^H(\omega,\boldsymbol{q})\propto\,&\delta'(\omega-\sqrt{m_H{}^2+\boldsymbol{q}^2})T_{HN}-\delta(\omega-\sqrt{m_H{}^2+\boldsymbol{q}^2})\delta T_{HN}\\&\pm\delta'(\omega+\sqrt{m_H{}^2+\boldsymbol{q}^2})T_{\bar{H}N}\pm\delta(\omega+\sqrt{m_H{}^2+\boldsymbol{q}^2})\delta T_{\bar{H}N}\\&+(\text{continuum part})\,,\end{aligned} \tag{11}$$

where δT_{HN} and $\delta T_{\bar{H}N}$ are given by the T-matrix and its derivative. If there is a bound state in the hadron-nucleon channel the spectral function has another first-order pole term, which will be incorporated if it is necessary. By substituting the above expression to the r.h.s. of Eq. (9) and evaluating the l.h.s by the OPE, we obtain the relation between the T-matrix and the matrix elements of quark-gluon operators with respect to the one-nucleon state.

Before closing this section we summarize here the parameters used in the following calculations: the quark masses are

$$m_u=m_d=7\text{ MeV}\,,\quad m_s=170\text{ MeV}\,,$$

the condensates of quark-gluon operators are[2)]

$$\langle\bar{u}u\rangle_0=\langle\bar{d}d\rangle_0=-(225\text{ MeV})^3\,,\quad\langle\bar{s}s\rangle_0=-(217\text{ MeV})^3\,,$$

$$\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0=(340\text{ MeV})^4\,,$$

and the expectation values of quark-gluon operators with the nucleon are[9)~11)]

$$\langle u^\dagger u\rangle_p=\langle d^\dagger d\rangle_n=2\,,\quad\langle u^\dagger u\rangle_n=\langle d^\dagger d\rangle_p=1\,,\quad\langle s^\dagger s\rangle_n=\langle s^\dagger s\rangle_p=0\,,$$

$$\langle\bar{u}u\rangle_p=\langle\bar{d}d\rangle_n=3.46\,,\quad\langle\bar{u}u\rangle_n=\langle\bar{d}d\rangle_p=2.96\,,\quad\langle\bar{s}s\rangle_p=\langle\bar{s}s\rangle_n=0.77\,,$$

$$i\langle\mathcal{S}[\bar{u}\gamma_\mu D_\nu u]\rangle_p=i\langle\mathcal{S}[\bar{d}\gamma_\mu D_\nu d]\rangle_n=222\text{ MeV}\,,$$

$$i\langle\mathcal{S}[\bar{d}\gamma_\mu D_\nu d]\rangle_p=i\langle\mathcal{S}[\bar{u}\gamma_\mu D_\nu u]\rangle_n=95\text{ MeV}\,,$$

$$i\langle\mathcal{S}[\bar{s}\gamma_\mu D_\nu s]\rangle_p=i\langle\mathcal{S}[\bar{s}\gamma_\mu D_\nu s]\rangle_n=18\text{ MeV}\,,$$

$$\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \right\rangle_N = -738\ \mathrm{MeV}\,, \quad \left\langle \frac{\alpha_s}{\pi} S\,[G_{\mu 0} G^{\mu 0}] \right\rangle_N = -50\ \mathrm{MeV}\,.$$

Having understood how to obtain hadron-nucleon scattering lengths in the QCD sum rule, we will discuss its applications in the following two sections.

§ 3. Nucleon-nucleon and hyperon-nucleon scattering lengths

The interpolating fields of the octet baryons are taken as[2)]

$$\eta_p(x) = \epsilon_{abc}(u^{Ta}(x) C\gamma_\mu u^b(x))\gamma_5 \gamma^\mu d^c(x)\,,$$

$$\eta_\Lambda(x) = \sqrt{\frac{2}{3}}\epsilon_{abc}\{(u^{Ta}(x) C\gamma_\mu s^b(x))\gamma_5\gamma^\mu d^c(x) - (d^{Ta}(x) C\gamma_\mu s^b(x))\gamma_5 \gamma^\mu u^c(x)\}\,,$$

$$\eta_{\Sigma^+}(x) = \epsilon_{abc}(u^{Ta}(x) C\gamma_\mu u^b(x))\gamma_5\gamma^\mu s^c(x)\,,$$

$$\eta_{\Xi^0}(x) = -\epsilon_{abc}(s^{Ta}(x) C\gamma_\mu s^b(x))\gamma_5\gamma^\mu u^c(x)\,. \tag{12}$$

The spectral functions in the vacuum and in the nucleon are assumed to be saturated by the nucleon (hyperon) pole terms as

$$\rho_0{}^B(\omega) = \lambda^2\delta(\omega - M_B)P_+ - \lambda^2\delta(\omega + M_B)P_-\,,$$

$$\rho_N{}^B(\omega) = \lambda^2\{-T^+\delta'(\omega - M_B) + \delta\lambda^+\delta(\omega - M_B)\}P_+ + \lambda^2\{-T^-\delta'(\omega + M_B) - \delta\lambda^-\delta(\omega + M_B)\}P_-\,, \tag{13}$$

where λ is the coupling strength of the interpolating field to the octet baryon and $P_\pm = (1\pm\gamma^0)/2$. The continuum part is neglected for simplicity. In the spin-triplet nucleon-nucleon channel, the contribution of the deuteron to the spectral function has to be taken into account. In nonrelativistic approximation it amounts to replacing the scattering length as, $a^3_{NN} \to \tilde{a}^3_{NN} = a^3_{NN} + 2\pi^2 M_N B_D |\tilde{f}_D(0)|^2$, where B_D is the binding energy and $\tilde{f}_D(\boldsymbol{p})$ is the relative wave function of the deuteron in momentum space.

In the nucleon-nucleon channel the OPE for the correlation function is given up to dimension six as follows:

$$\Pi_0{}^p(q) = \frac{1}{4\pi^4}\gamma_\mu q^\mu\left[q^4 \ln(-q^2)\frac{1}{16} + \ln(-q^2)\pi^2\left\{m_d\langle\bar{d}d\rangle_0 + \frac{1}{8}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle_0\right\} + \frac{1}{q^2}\frac{8}{3}(\pi^2\langle\bar{u}u\rangle_0)^2\right] + \frac{1}{4\pi^4}\left[q^4\ln(-q^2)\frac{1}{8}m_d + q^2\ln(-q^2)(-\pi^2\langle\bar{d}d\rangle_0)\right],$$

$$\Pi_N{}^p(q) = \frac{1}{4\pi^4}\gamma^\mu\left[q^2\ln(-q^2)\pi^2\left\{-\frac{7}{3}\langle\bar{u}\gamma_\mu u\rangle_N - \frac{1}{3}\langle\bar{d}\gamma_\mu d\rangle_N\right\} + q_\mu q^\nu \ln(-q^2)\pi^2\left\{-\frac{2}{3}\langle\bar{u}\gamma_\nu u\rangle_N - \frac{2}{3}\langle\bar{d}\gamma_\nu d\rangle_N\right\}\right] + \frac{1}{4\pi^4}\left[q^2\ln(-q^2)(-\pi^2\langle\bar{d}d\rangle_N)\right]$$

$$+\frac{1}{4\pi^4}\gamma_\mu q^\mu\Big[\ln(-q^2)\pi^2\Big\{m_d\langle\bar{d}d\rangle_N+\frac{1}{8}\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_N\Big\}$$

$$+\frac{1}{q^2}\frac{16}{3}\pi^4\langle\bar{u}u\rangle_0\langle\bar{u}u\rangle_N\Big]$$

$$+\frac{1}{4\pi^4}\gamma^\mu q^\nu\Big[\ln(-q^2)\pi^2\Big\{\frac{16}{3}i\langle\mathcal{S}[\bar{u}\gamma_\mu D_\nu u]\rangle_N+\frac{4}{3}i\langle\mathcal{S}[\bar{d}\gamma_\mu D_\nu d]\rangle_N\Big\}\Big]$$

$$+\frac{1}{4\pi^4}q^\mu\Big[\ln(-q^2)\pi^2\{-2m_d\langle\bar{u}\gamma_\mu u\rangle_N+2i\langle\bar{d}D_\mu d\rangle_N\}$$

$$+\frac{1}{q^2}\frac{16}{3}\pi^4\langle\bar{d}d\rangle_0\langle\bar{u}\gamma_\mu u\rangle_N\Big]. \qquad (14)$$

By substituting Eqs. (13) and (14) into Eq. (9) and Borel transforming both sides we obtain

$$a^1_{pp}=\frac{4}{\pi}M_p\frac{A_pM_B{}^4-M_p(B_pM_B{}^2+C_p)}{M_B{}^6+\pi^2\Big(8m_d\langle\bar{d}d\rangle_0+\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_0\Big)M_B{}^2+\frac{64}{3}(\pi^2\langle\bar{u}u\rangle_0)^2},$$

$$\frac{1}{4}a^1_{pn}+\frac{3}{4}\tilde{a}^3_{pn}=\frac{4}{\pi}\frac{M_pM_n}{M_p+M_n}$$

$$\times\frac{A_nM_B{}^4-M_p(B_nM_B{}^2+C_n)}{M_B{}^6+\pi^2\Big(8m_d\langle\bar{d}d\rangle_0+\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_0\Big)M_B{}^2+\frac{64}{3}(\pi^2\langle\bar{u}u\rangle_0)^2}, \qquad (15)$$

where M_B is the Borel mass,

$$A_N=\pi^2\langle\bar{d}d\rangle_N+3\pi^2\langle u^\dagger u\rangle_N+\pi^2\langle d^\dagger d\rangle_N,$$

$$B_N=m_d\pi^2(2\langle u^\dagger u\rangle_N-\langle\bar{d}d\rangle_N)-2\pi^2 i\langle\bar{d}D_0d\rangle_N-\frac{1}{8}\pi^2\Big\langle\frac{\alpha_s}{\pi}G^2\Big\rangle_N$$

$$-\frac{4}{3}\pi^2(4i\langle\mathcal{S}[\bar{u}\gamma_0D_0u]\rangle_N+i\langle\mathcal{S}[\bar{d}\gamma_0D_0d]\rangle_N),$$

$$C_N=-\frac{16}{3}\pi^4(\langle\bar{d}d\rangle_0\langle u^\dagger u\rangle_N+\langle\bar{u}u\rangle_0\langle\bar{u}u\rangle_N), \qquad (16)$$

and $\mathcal{S}[A_\mu B_\nu]=(A_\mu B_\nu+A_\nu B_\mu)/2-g_{\mu\nu}A_\lambda B^\lambda/4$.

Let us first concentrate on the leading order results. The results are

$$a^1_{NN}=\frac{4\pi}{M_B{}^2}M_N(\langle\bar{d}d\rangle_p+3\langle u^\dagger u\rangle_p+\langle d^\dagger d\rangle_p)=23.2\text{ fm},$$

$$\tilde{a}^3_{NN}=\frac{4\pi}{3M_B{}^2}M_N(2\langle\bar{d}d\rangle_n+6\langle u^\dagger u\rangle_n+2\langle d^\dagger d\rangle_n-\langle\bar{d}d\rangle_p-3\langle u^\dagger u\rangle_p-\langle d^\dagger d\rangle_n)$$

$$=5.4\text{ fm}. \qquad (17)$$

Experimental scattering lengths are $a^1_{NN}(\exp)=23.7$ fm and $a^3_{NN}(\exp)=-5.4$ fm. We evaluated the deuteron pole contribution by employing the Paris potential[19] and found that $2\pi^2M_NB_D|\tilde{f}(0)|^2=9.4$ fm. By adding this contribution to $a^3_{NN}(\exp)$, we obtain

$\tilde{a}^3_{NN}$('exp')=4.0 fm. The calculated scattering lengths are surprisingly close to these values. It is also interesting that the scalar and vector densities of quarks in the nucleon induce attraction between two nucleons.

By taking into account higher order terms and determining the Borel mass by stability condition, we obtain $a^1_{NN}=11.6$ fm and $\tilde{a}^3_{NN}=2.8$ fm. These values are in qualitative agreement with the experimental ones. Even though the leading-order results are closer to the experimental values than the full results, we do not take it seriously because of crude approximations used in the present calculation. It should be also noted that a^1_{NN} is sensitive to the change of the interaction strength since there is almost a bound state in the spin-singlet nucleon-nucleon channel.

In the hyperon-nucleon channels the leading order results are as follows:

$$\bar{a}_{\Lambda N}=\frac{4\pi}{M_B{}^2-4m_s{}^2}\frac{M_\Lambda M_N}{M_\Lambda+M_N}\left\{\frac{1}{3}(2\langle\bar{u}u\rangle_p+2\langle\bar{d}d\rangle_p-\langle\bar{s}s\rangle_p)\right.$$
$$\left.+\frac{1}{6}(7\langle u^\dagger u\rangle_p+7\langle d^\dagger d\rangle_p+10\langle s^\dagger s\rangle_p)\right\}=10.9\text{ fm}\,,$$

$$\bar{a}_{\Sigma N}^{T=3/2}=\frac{4\pi}{M_B{}^2-4m_s{}^2}\frac{M_\Sigma M_N}{M_\Sigma+M_N}\{\langle\bar{s}s\rangle_p+3\langle u^\dagger u\rangle_p+\langle s^\dagger s\rangle_p\}=9.4\text{ fm}\,,$$

$$\bar{a}_{\Sigma N}^{T=1/2}=\frac{4\pi}{M_B{}^2-4m_s{}^2}\frac{M_\Sigma M_N}{M_\Sigma+M_N}\left\{\langle\bar{s}s\rangle_p+\frac{9}{2}\langle d^\dagger d\rangle_p-\frac{3}{2}\langle u^\dagger u\rangle_p+\langle s^\dagger s\rangle_p\right\}=5.0\text{ fm}\,,$$

$$\bar{a}_{\Xi N}^{T=1}=\frac{4\pi}{M_B{}^2-6m_s{}^2}\frac{M_\Xi M_N}{M_\Xi+M_N}\{\langle\bar{u}u\rangle_p+\langle u^\dagger u\rangle_p+3\langle s^\dagger s\rangle_p\}=9.0\text{ fm}\,,$$

$$\bar{a}_{\Xi N}^{T=0}=\frac{4\pi}{M_B{}^2-6m_s{}^2}\frac{M_\Xi M_n}{M_\Xi+M_n}\{2\langle\bar{d}d\rangle_p-\langle\bar{u}u\rangle_p+2\langle d^\dagger d\rangle_p-\langle u^\dagger u\rangle_p+3\langle s^\dagger s\rangle_p\}$$
$$=6.5\text{ fm}\,,$$

$$a_{pp}=\frac{8\pi}{M_B{}^2}\frac{M_N M_N}{M_N+M_N}\{\langle\bar{d}d\rangle_p+3\langle u^\dagger u\rangle_p+\langle d^\dagger d\rangle_p\}=23.2\text{ fm}\,,$$

$$\bar{a}_{pn}=\frac{4\pi}{M_B{}^2}\frac{M_N M_N}{M_N+M_N}\{\langle\bar{d}d\rangle_n+3\langle u^\dagger u\rangle_n+\langle d^\dagger d\rangle_n\}=9.8\text{ fm}\,, \tag{18}$$

where $\bar{a}_{B_1B_2}$ is the spin-averaged scattering length, $\bar{a}_{B_1B_2}\equiv(3a^3_{B_1B_2}+a^1_{B_1B_2})/4$ ($\bar{a}_{pn}\equiv(3\tilde{a}^3_{pn}+a^1_{pn})/4$), and the nucleon-nucleon scattering lengths are also shown for comparison.

From the above results, we obtain the following inequalities:

$$a_{NN}>\bar{a}_{\Lambda N},\ \bar{a}_{\Sigma N},\ \bar{a}_{\Xi N}\,,$$

$$\bar{a}_{\Sigma N}^{T=3/2}>\bar{a}_{\Sigma N}^{T=1/2}\,,$$

$$\bar{a}_{\Xi N}^{T=1}>\bar{a}_{\Xi N}^{T=0}\,.$$

These inequalities can be understood by the following relations between the matrix elements of the quark densities (and a factor 2 in the expression for a^1_{pp}),

$$\langle u^\dagger u\rangle_N, \langle d^\dagger d\rangle_N > \langle s^\dagger s\rangle_N\,, \quad \langle u^\dagger u\rangle_p > \langle u^\dagger u\rangle_n\,, \quad \langle d^\dagger d\rangle_n > \langle d^\dagger d\rangle_p\,,$$

$$\langle \bar{u}u\rangle_N, \langle \bar{d}d\rangle_N > \langle \bar{s}s\rangle_N\,, \quad \langle \bar{u}u\rangle_p > \langle \bar{u}u\rangle_n\,, \quad \langle \bar{d}d\rangle_n > \langle \bar{d}d\rangle_p\,.$$

Namely, the scattering length is larger or roughly the interaction between baryons at low energies is stronger, if the overlap of two baryons in the flavor space is larger. It is extremely interesting to see if above inequalities among scattering lengths really hold in nature.

§ 4. Pion-nucleon and kaon-nucleon scattering lengths

We take the axial-vector current for the pion and kaon interpolating field,

$$A_\mu(x) = \bar{q}_1(x)\gamma_\mu\gamma_5 q_2(x)\,, \tag{19}$$

which has a property required for the interpolating field,

$$\langle 0|A_\mu(0)|\varphi(k)\rangle = i\sqrt{2}f_\varphi k_\mu \tag{20}$$

with f_φ being the pion (kaon) decay constant. The spectral functions in the vacuum and in the nucleon are assumed to be saturated by the pion (kaon) pole terms:

$$\rho_0{}^\varphi(\omega) = m_\varphi f_\varphi{}^2\{\delta(\omega - m_\varphi) - \delta(\omega + m_\varphi)\}\,,$$

$$\rho_N{}^\varphi(\omega) = -\frac{1}{2}f_\varphi{}^2\Big[\delta'(\omega - m_\varphi)T_{\varphi N} - \delta(\omega - m_\varphi)\Big(T'_{\varphi N} - \frac{3}{m_\varphi}T_{\varphi N}\Big)$$
$$+\delta'(\omega + m_\varphi)T_{\bar{\varphi}N} + \delta(\omega + m_\varphi)\Big(T'_{\bar{\varphi}N} - \frac{3}{m_\varphi}T_{\bar{\varphi}N}\Big)\Big]\,. \tag{21}$$

$T_{\binom{\varphi N}{\bar{\varphi}N}} = T(\pm m_\varphi, 0, m_\varphi{}^2, m_\varphi{}^2)$, $T'_{\binom{\varphi N}{\bar{\varphi}N}} = \pm(\partial/\partial\omega)T(\omega, 0, \omega^2, \omega^2)|_{\omega=\pm m_\varphi}$ with the off-shell T-matrix defined by

$$T(\nu, t, q^2, q'^2) = -i\frac{(q^2 - m_\varphi{}^2)(q'^2 - m_\varphi{}^2)}{2f_\varphi{}^2 m_\varphi{}^4}\int d^4x e^{iqx}$$
$$\times\langle N(p)|T(\partial^\mu A_\mu(x)\partial^\nu A_\nu^\dagger(0))|N(p')\rangle\,, \tag{22}$$

where $\nu = \omega + t/4M_N$, $t = (q - q')^2$ and $q + p = q' + p'$.

The correlation functions in the OPE are given up to dimension six as follows:

$$\Pi_0(\omega) = \frac{3}{8\pi^2}(m_1 + m_2)^2\ln(-\omega^2) - (m_1 + m_2)(\langle\bar{q}_1 q_1\rangle_0 + \langle\bar{q}_2 q_2\rangle_0)\frac{1}{\omega^2}$$
$$-(m_1 + m_2)^2\Big\{-\frac{1}{8}\Big\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\Big\rangle_N\Big\}\frac{1}{\omega^4}\,,$$

$$\Pi_N(\omega) = (\langle q_1^\dagger q_1\rangle_N - \langle q_2^\dagger q_2\rangle_N)\frac{1}{\omega} - (m_1 + m_2)(\langle\bar{q}_1 q_1\rangle_N + \langle\bar{q}_2 q_2\rangle_N)\frac{1}{\omega^2}$$
$$+(m_1 + m_2)^2\Big[(\langle q_1^\dagger q_1\rangle_N - \langle q_2^\dagger q_2\rangle_N)\frac{1}{\omega^3} - \Big\{2(i\langle\mathcal{S}[\bar{q}_1\gamma_0 D_0 q_1]\rangle_N$$

$$+i\langle \mathcal{S}[\bar{q}_2\gamma_0 D_0 q_2]\rangle_N)-\frac{1}{8}\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\right\rangle_N-\frac{1}{2}\left\langle\frac{\alpha_s}{\pi}\mathcal{S}(G_{0\nu}G_0{}^{\nu})\right\rangle_N\Big\}\frac{1}{\omega^4}\Big]. \quad (23)$$

From Eqs. (21) and (23) we obtain

$$2\frac{m_\varphi{}^2}{M_B{}^2}f_\varphi{}^2\exp\left(-\frac{m_\varphi{}^2}{M_B{}^2}\right)=\frac{3}{8\pi^2}(m_1+m_2)^2-\frac{1}{M_B{}^2}\langle\mathcal{O}_2\rangle_0-\frac{(m_1+m_2)^2}{8M_B{}^4}\langle\mathcal{O}_3\rangle_0\,,$$

$$2\frac{m_\varphi{}^2}{M_B{}^6}f_\varphi{}^2T_{\varphi N}\exp\left(-\frac{m_\varphi{}^2}{M_B{}^2}\right)=-\frac{m_\varphi{}^3}{M_B{}^2}\langle\mathcal{O}_1\rangle_N+\frac{m_\varphi{}^2}{M_B{}^2}\langle\mathcal{O}_2\rangle_N+\left(1-\frac{m_\varphi{}^2}{M_B{}^2}\right)\langle\mathcal{O}_4\rangle_N\,, \quad (24)$$

where

$$\mathcal{O}_1=q_1^\dagger q_1-q_2^\dagger q_2\,,$$

$$\mathcal{O}_2=(m_1+m_2)(\bar{q}_1q_1+\bar{q}_2q_2)\,,$$

$$\mathcal{O}_3=\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\,,$$

$$\mathcal{O}_4=(m_1+m_2)^2\Big[m_\varphi(q_1^\dagger q_1-q_2^\dagger q_2)-\Big\{2(i\bar{q}_1\mathcal{S}(\gamma_0D_0)q_1+i\bar{q}_2\mathcal{S}(\gamma_0D_0)q_2)$$
$$-\frac{1}{8}\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}-\frac{1}{2}\frac{\alpha_s}{\pi}\mathcal{S}(G_{0\nu}G_0{}^{\nu})\Big\}\Big]. \quad (25)$$

In this paper we concentrate on the leading order terms. The effects of the higher order terms are discussed in Ref. 6). We also neglect higher order terms in $m_\pi{}^2/M_B{}^2$.

In the pion-nucleon channel the T-matrices are obtained as

$$T_{\pi N}^{(+)}=\frac{(m_u+m_d)\langle\bar{u}u+\bar{d}d\rangle_N}{f_\pi{}^2}=\frac{\sigma_{\pi N}}{f_\pi{}^2}\,,$$

$$T_{\pi N}^{(-)}=-\frac{m_\pi\langle u^\dagger u-d^\dagger d\rangle_p}{f_\pi{}^2}=-\frac{m_\pi}{2f_\pi{}^2}\,, \quad (26)$$

where $T_{\pi N}^{(\pm)}=(T_{\pi^- p}\pm T_{\pi^+ p})/2=(T_{\pi^+ n}\pm T_{\pi^- n})/2$ and $\sigma_{\pi N}$ is the pion-nucleon sigma term.

The leading order term, the dimension-three operator, contributes to the isospin-odd component of the T-matrix and gives the Tomozawa-Weinberg term.[7),8)] The next-to-leading order term, the dimension-four operator, contributes to the isospin-even component of the T-matrix and gives the sigma term, which is the same as that obtained by using the PCAC and current algebra at the Weinberg point.[12)]

It is interesting that the quark number in the nucleon determines the leading-order form of the T-matrix.

In the kaon-nucleon channel we take into account the contribution of $\Lambda(1405)$ which exists below the $\bar{K}N$ threshold. Having done this modification the T-matrices are obtained as

$$T_{Kp}^{(+)}=\frac{\sigma_{Kp}}{f_K{}^2}+\frac{g_{\Lambda^*}^2}{2}\frac{m_K{}^2}{m_K+M_N-M_{\Lambda^*}}\left(\frac{1}{M_{\Lambda^*}-M_N}\right)^2,$$

$$T_{Kn}^{(+)}=\frac{\sigma_{Kn}}{f_K{}^2},$$

$$T_{Kp}^{(-)}=-\frac{m_K}{f_K{}^2}+\frac{g_{\Lambda^*}^2}{2}\frac{m_K{}^2}{m_K+M_N-M_{\Lambda^*}}\left(\frac{1}{M_{\Lambda^*}-M_N}\right)^2,$$

$$T_{Kn}^{(-)}=-\frac{m_K}{2f_K{}^2}, \tag{27}$$

where $T_{KN}^{(\pm)}=(T_{K\text{-}N}\pm T_{K^+N})/2$, σ_{KN} is the kaon-nucleon sigma term, g_{Λ^*} is the $\bar{K}N\Lambda(1405)$ coupling constant and M_{Λ^*} is the mass of $\Lambda(1405)$. In Eq. (27) we neglected higher order terms in the binding energy, $m_K+M_N-M_{\Lambda^*}$.

The results for the kaon-nucleon channel are similar to those for the pion-nucleon channel except for the contribution from $\Lambda(1405)$: the dimension-three operator gives the Tomozawa-Weinberg term to $T^{(-)}$ and the dimension-four operator gives the sigma term to $T^{(+)}$. Though the error of the $\Lambda(1405)$ contribution due to the experimental uncertainty of the $\bar{K}N\Lambda(1405)$ coupling constant is quite large, the inclusion of the $\Lambda(1405)$ contribution seems to improve the agreement with the observed scattering lengths which are determined by the scattering experiments.

§ 5. Relation to sum rules at finite density

In Ref. 9) Drukarev and Levin pointed out that if the condensate of the operator $\mathcal{O}$ in the nuclear medium is expanded in the baryon number density, ρ, the coefficient of the term linear in ρ is given by the matrix element of $\mathcal{O}$ with respect to the one-nucleon state:

$$\mathcal{O}=\mathcal{O}_0+\mathcal{O}_N\rho+o(\rho). \tag{28}$$

Then in Ref. 5) it was pointed out that if the correlation function is expanded similarly in ρ, the coefficient of the term linear in ρ is given by the correlation function in which the expectation value is taken with respect to the one-nucleon state, Π_N, on the one hand,

$$\Pi_\rho=\Pi_0+\Pi_N\rho+o(\rho), \tag{29}$$

and the same coefficient is related to the self energy of the baryon, B, in the nuclear medium, Σ, on the other hand,

$$\Pi_\rho=\frac{1}{\not{q}-M_B}+\frac{1}{\not{q}-M_B}\frac{\partial\Sigma}{\partial\rho}\frac{1}{\not{q}-M_B}\rho+o(\rho). \tag{30}$$

By comparing the second-order pole position of Π_N in Eqs. (10) and (30), and noting the relation,

$$\delta M_B=\frac{\partial\Sigma}{\partial\rho}\rho+o(\rho), \tag{31}$$

we finally obtain

$$\delta M_B=\begin{cases}-2\pi\dfrac{M_B+M_N}{M_BM_N}a_{BN}\rho+o(\rho)\,, & (B\neq N)\\ -3\pi\dfrac{1}{M_N}a_{NN}\rho+o(\rho)\,. \end{cases} \tag{32}$$

This clearly shows that the calculation of the effective mass in the linear density approximation is nothing but the calculation of the scattering length.

§ 6. Summary

To summarize, recent works on the application of the QCD sum rule to the calculation of hadron-nucleon scattering lengths are reported. The formalism to calculate scattering lengths in the QCD sum rule is explained and then it is applied first to the nucleon-nucleon and hyperon-nucleon channels and secondly to the pion-nucleon and kaon-nucleon channels. It was found that the quark densities in the nucleon determines the hadron-nucleon scattering lengths in the leading order and that the calculated scattering lengths are in qualitative agreement with experimentally known scattering lengths.

Finally, I should mention that recently Furnstahl and Hatsuda made critical comments on our work[20] but we disagree with them.[21]

In conclusion the application of the QCD sum rule to the hadronic interactions is promising. But, clearly much work has to be done in order to confirm the present results and clarify existing discrepancies.

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