Heavy Quark Effective Theory and the Isgur-Wise Function on the Lattice

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We compute the Isgur-Wise function using the heavy quark effective theory formulated on the lattice. The kinetic energy term of the heavy quark is included to the action as well as the terms remaining in the infinite mass limit. The classical velocity of the heavy quark is renormalized on the lattice and we determine the renormalized velocity non-perturbatively using the energy-momentum dispersion relation. The slope of the Isgur-Wise function at zero recoil is obtained at $\beta=6.0$ on a $24^3\times 48$ lattice and the size of $O(1/m_Q)$-correction is estimated.

§ 1. Introduction

In the determination of the CKM (Cabibbo, Kobayashi-Maskawa) matrix element $|V_{cb}|$ from the experiment of the exclusive decay $B\to D^{(*)}l\bar{\nu}$ the heavy quark symmetry plays an essential role, because the universal form factor $\xi(v\cdot v')$, so called the Isgur-Wise function, is normalized to be one at zero recoil in the infinite heavy quark mass limit. However the differential decay width for this process disappears at the zero recoil point, so that one needs to extrapolate the experimental data to this point. The lattice computation of the slope $\xi'(1)$ enables this extrapolation in a model independent way. On the lattice Bernard, Shen and Soni[1] and the UKQCD Collaboration[2] have computed the Isgur-Wise function using the propagating quarks (Wilson fermion and clover fermion respectively) for the heavy quark. Since the heavy quark mass should be below the lattice cutoff in this framework, present simulations are performed around the charm quark mass. An alternative way of treating the heavy quark is to use the heavy quark effective theory (HQET) on the lattice. Mandula and Ogilvie formulated the Lattice HQET for the case of infinite heavy quark mass limit and applied it to a calculation of the Isgur-Wise function.[3] Their result, however, suffers from much noise so that the extraction of the ground state seems to be difficult. This fact is well known in the calculation of the heavy-light decay constant using the static approximation where one is forced to use some method to enhance the signal of the ground state[4] or to introduce $O(1/m_Q)$ terms in order to decrease the statistical noise.[5]

Our calculation is based on the lattice HQET keeping a part of the $O(1/m_Q)$ correction terms. The inclusion of the kinetic term of the heavy quark reduces, the statistical noise significantly and enables to extract the ground state reliably. We calculate the Isgur-Wise function for three values of the heavy quark mass and discuss

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$O(1/m_Q)$ effect on it.

§ 2. HQET on the lattice

The action of the lattice HQET including $O(1/m_Q)$ corrections is

$$S_h=\sum_n \bar{h}(n)\left[-iv\cdot D-\frac{1}{2m_Q}\left(D^2+\frac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}\right)\right]h(n)\,, \tag{1}$$

where $v_\mu=(iv^0, \boldsymbol{v})$ is a four-velocity of the moving heavy quark. D_μ is a lattice covariant derivative and $F_{\mu\nu}$ is a chromo-magnetic (electric) field. This action is a generalization of the non-relativistic lattice QCD (NRQCD) for finite velocities. The first term corresponds to the infinite mass limit action used by Mandula and Ogilvie and the second two terms describe the $O(1/m_Q)$ corrections. The heavy quark field $Q(n)$ is expressed by $h(n)$ as

$$Q(n)=e^{im_Q v\cdot n}\left[1-\frac{1}{2m_Q}\not{D}_\perp\right]h(n)\,, \tag{2}$$

where $D_{\perp\mu}=D_\mu+v_\mu(v\cdot D)$. For simplicity we neglect the spin dependent interaction term $(1/m_Q)\sigma_{\mu\nu}F_{\mu\nu}$ from the action and the $O(1/m_Q)$ correction term from the field redefinition. It should be noted that the $O(1/m_Q)$ corrections are not included completely with this approximation, but the kinetic term describes the motion of the heavy quark inside the meson and gives a major $O(1/m_Q)$ effect.

The heavy quark propagator is obtained by solving the evolution equation

$$G(n+\hat{t})=U_4^\dagger(n)\left(1-\frac{1}{n}H\right)^n G(n,0)+(\text{source term}) \tag{3}$$

and

$$H=\frac{1}{v^0}\left\{-i\boldsymbol{v}\cdot\boldsymbol{D}-\frac{1}{2m_Q}\left(\frac{-1}{(v^0)^2}(\boldsymbol{v}\cdot\boldsymbol{D})^2+\boldsymbol{D}^2\right)\right\}, \tag{4}$$

where we used the equation of motion in the infinite mass limit

$$(v^0\cdot D_4-i\boldsymbol{v}\cdot\boldsymbol{D})h(n)=0 \tag{5}$$

to remove the $D_4{}^2$ term from the evolution equation. The cost for the computation of this deterministic equation is much less than one of obtaining the propagator for the Wilson fermion using some iterative solver.

The parameter n is introduced to stabilize the unphysical high frequency modes. Considering the evolution equation for the free field, the stability condition to assure the convergence of the equation is

$$\left|1-\frac{1}{n}H_0(\boldsymbol{k})\right|<1\,,\quad (\text{for any } \boldsymbol{k}) \tag{6}$$

$$H_0(k)=\frac{1}{v_0}\left\{\boldsymbol{v}\cdot\tilde{\boldsymbol{k}}+\frac{1}{2m_Q}\left(\hat{\boldsymbol{k}}^2-\left(\frac{\boldsymbol{v}}{v^0}\cdot\hat{\boldsymbol{k}}\right)^2\right)\right\}, \tag{7}$$

where $\tilde{k}_i=\sin k_i$ and $\hat{k}_i=2\sin(k_i/2)$. Similar condition is required in NRQCD,[6] but in the present case it depends on the heavy quark velocity as well as m_Q and n. Because of the presence of term $(1/v^0)\boldsymbol{v}\cdot\tilde{\boldsymbol{k}}$ high frequency modes of residual momentum do not converge with time evolution for a large value of velocity unless n is taken to be sufficiently large. On the other hand the $O(a)$ effect becomes larger for larger n, then much large n is not preferable. Fixing n for a value of m_Q, magnitude of velocities are restricted below a certain value. For our simulation parameters mentioned later, $\sum_i|v_i|$ is restricted to be smaller than 0.25.

Multiplying the projection operator to $G(n)$, the heavy quark propagator in 4-spinor representation is obtained as

$$S_Q(n)=G(n)\otimes\frac{1-i\not{v}}{2}.$$

The Isgur-Wise function is extracted from the three point correlation function

$$\langle 0|B(v')V_4(v', v)B(v)^\dagger|0\rangle\,, \tag{8}$$

where $V_4(v', v)$ is a temporal component of the vector current $\overline{h_{v'}}\gamma_4 h_v$ and $B(v)$ is a local interpolating field for an initial or a final meson state $\bar{q}\gamma_5 h_v$ for which we take the same mass for both initial and final mesons. The three point correlation function is related to the Isgur-Wise function $\xi(v\cdot v')$ as

$$\begin{aligned}G_{v\to v'}(t_f, t_s, t_i)&=\sum_{n_s}\sum_{n_f}\langle 0|B(v')V_4(v', v)B(v)^\dagger|0\rangle\\&=\sum_{n_s}\sum_{n_f}\langle \mathrm{Tr}[S_q(n_f, t_i)^\dagger S_{h_{v'}}(n_f, n_s)\gamma_4 S_{h_v}(n_s, t_i)]\rangle\\&\propto m_B\xi(v\cdot v')(v+v')_0 e^{-E_f(t_f-t_s)-E_i(t_s-t_i)}\,,\quad (\text{for } t_f\gg t_s\gg t_i)\end{aligned} \tag{9}$$

where the component proportional to $(v-v')_0$ is neglected since its form factor is $O(\Lambda_{\mathrm{QCD}}/m_Q)$ and $(v-v')_0$ is small itself in our velocity region. Taking the following ratio of the three-point functions the exponential factor and the renormalization factor for the vector current cancel and we obtain

$$\begin{aligned}R_{v,v'}(t_f, t_s, t_i)&=\frac{G_{v\to v'}(t_f, t_s, t_i)G_{v'\to v}(t_f, t_s, t_i)}{G_{v\to v}(t_f, t_s, t_i)G_{v'\to v'}(t_f, t_s, t_i)}\\&\to|\xi(v\cdot v')|^2\frac{(v_0+v_0')^2}{4v_0v_0'}\,.\end{aligned} \tag{10}$$

We use this relation for the calculation of the Isgur-Wise function.

§ 3. Simulation

We used 120 configurations of $24^3\times 48$ lattice at $\beta=6.0$ in the quenched approximation. Each configuration is separated by 2,000 pseudo-heat bath sweeps after 20,000 sweeps for thermalization. For the light quarks we used the Wilson fermion with hopping parameters 0.153 and 0.155. The boundary condition for the light quark propagator is periodic and Diriclet for spatial and temporal direction respectively.

The critical hopping parameter and the inverse lattice spacing determined from light rho mass are $\kappa_c=0.156986(50)$ and $a^{-1}=2.341(91)$ GeV. Generation of gauge configurations and matrix inversions to obtain the light quark propagators are performed on Intel Paragon XP/S (56 nodes).

Heavy quark masses and stabilization parameters we used are

$$\begin{pmatrix} m_Q \\ n \end{pmatrix}=\begin{pmatrix} 1.8 \\ 3 \end{pmatrix},\begin{pmatrix} 2.5 \\ 2 \end{pmatrix},\begin{pmatrix} 5.0 \\ 1 \end{pmatrix},$$

where $m_Q=1.8$ nearly corresponds to the bottom quark mass. As mentioned above, heavy quark velocities should satisfy the stability condition (6) which leads $\sum_i|v_i|\le 0.25$ for our sets of m_Q and n. This corresponds to the region $1\le v\cdot v'\le 1.12$.

The mean-field improvement[7] is applied when we compute the heavy quark evolution equation. Link variables are altered as $U_\mu(n)\to U_\mu(n)/u_0$ where $u_0=\langle(1/3)U_{\rm plaq.}\rangle^{1/4}$. $U_{\rm plaq}$ is the product of link variables along plaquette. We used the value $u_0=0.8776$ measured on our configurations.

Before we proceed to the calculation of the Isgur-Wise function we mention about the determination of masses and velocities of the heavy-light mesons. For this purpose we use the dispersion relation for the heavy-light meson

$$e^{-(E(k)-E_0v_0)}-1=-\frac{1}{v_R{}^0}\left\{v_R\cdot\tilde{k}+\frac{f}{2m_P}\left(\hat{k}^2-\left(\frac{v_R}{v_R{}^0}\cdot\hat{k}\right)^2\right)\right\}, \tag{11}$$

where E_0 is the binding energy and m_P is the heavy-light meson mass. The velocity of the moving heavy-light meson v_R could differ from the one of the bare heavy quark v for the lattice HQET because of the violation of the space-time $O(4)$ symmetry.[8),9)] We extract the 'renormalized' velocity v_R using the relation

$$\frac{e^{-E(k)+E(0)}-e^{-E(-k)+E(0)}}{2}=-\frac{v_{Ri}}{v_R{}^0}\cdot\sin k_i\,. \tag{12}$$

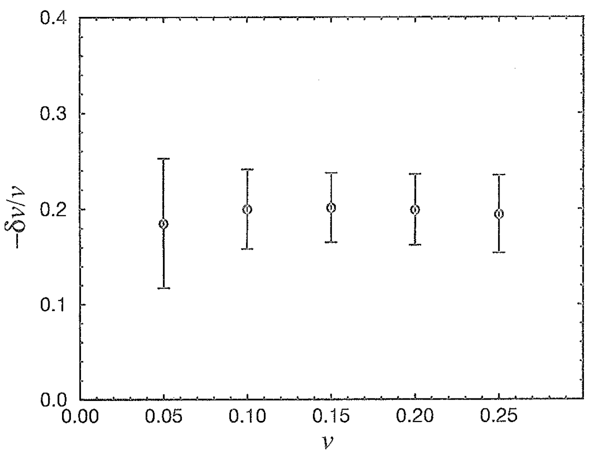

Fig. 1. Nonperturbative determination of the renormalized velocity. $-\delta v/v=-(v_R-v)/v$ is shown as a function of v for $m_Q=1.8$.

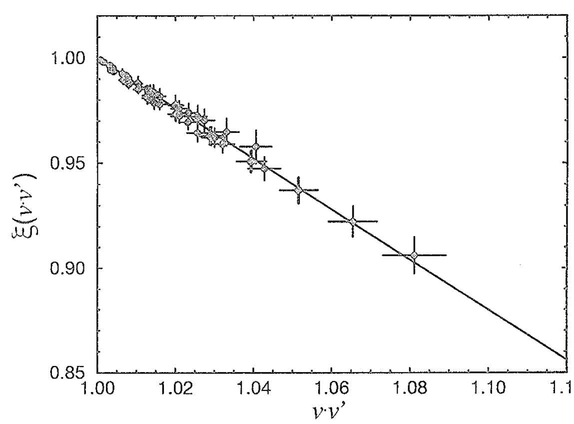

Fig. 2. The Isgur-Wise function $\xi(v_R\cdot v_R')$ at $m_Q=1.8$ and $\kappa=0.153$.

Masses can be obtained similarly using the relation

$$\frac{e^{-E(k)+E(0)}+e^{-E(-k)+E(0)}}{2}=1-\frac{1}{2m_P v_R{}^0}\left(\hat{\boldsymbol{k}}^2-\left(\frac{\boldsymbol{v}_R}{v_R{}^0}\cdot\hat{\boldsymbol{k}}\right)^2\right). \tag{13}$$

The obtained results for v_R/v and m_P are almost independent on the velocity as shown in Fig. 1 about v_R/v.

In the calculation of the three point function we set $t_i=8$ and $t_s=20$ where it seems to reach the ground state of initial meson moving with velocity v. Beyond $t_f \approx 24$ asymptotic signal is observed. For each v and v' we fit $R_{v,v'}$s at $t_f=24-27$ as the value of form factor at $v\cdot v'$. For $m_Q=5.0$, signals are noisy so that we cannot observe clear plateau. Nevertheless we treat them in the same manner as $m_Q=1.8$ and 2.5 with $t_f=24-25$.

In Fig. 2 we show the form factor $\xi(v_R\cdot v_R')$ for $m_Q=1.8$ at $\kappa=0.153$. Error bars in the horizontal direction are coming from the statistical uncertainty in the determination of the renormalized velocity.

In our $v\cdot v'$ region the form factors have almost linear behavior where we extract the slope of $\xi(v_R\cdot v_R')$ at $v_R\cdot v_R'=1$, which is usually denoted as ρ^2, from a fit of our data to the form

$$\xi(v_R\cdot v_R')=1-\rho^2(v_R\cdot v_R'-1)\,. \tag{14}$$

ρ^2 for each m_Q and κ are given in Table I with the values extrapolated to the critical hopping parameter for the light quark. $m_Q=1.8$ roughly corresponds to the B meson mass and the data should be compared with the experimental data obtained by CLEO II.[10)]

$$\rho^2=1.11\pm0.28 \quad \text{this work}\,, \tag{15}$$

$$=0.84\pm0.15 \quad \text{CLEO II}\,. \tag{16}$$

Our data is slightly larger than that of CLEO, but still consistent considering the large statistical uncertainty.

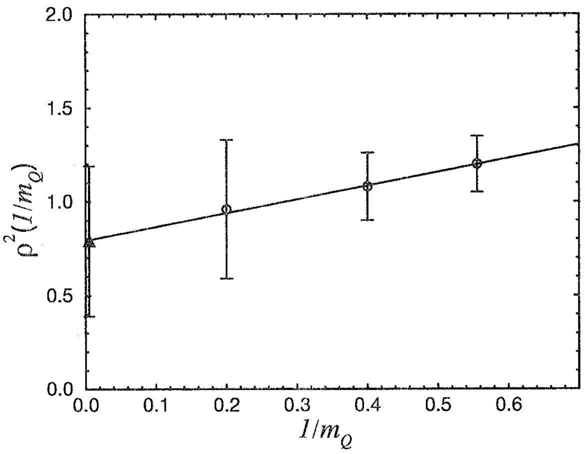

Fig. 3. Extrapolation of ρ^2 to the vanishing $1/m_Q$ limit.

Table I. Results of $\rho^2\equiv-\xi'(1)$ for each m_Q and κ. They were obtained by fitting simulation data with the form $\xi(v_R\cdot v_R')=1-\rho^2(v_R\cdot v_R'-1)$. Results at $m_Q=\infty$ are obtained from extrapolation in $1/m_Q$ and the values at the critical hopping parameter ($\kappa_c=0.156986(50)$) are obtained by linear extrapolation in $1/\kappa$.

	m_Q			
κ	1.8	2.5	5.0	∞
0.153	1.20(15)	1.08(18)	0.96(37)	0.79(40)
0.155	1.15(21)	1.02(26)	0.94(58)	0.76(61)
κ_c	1.11(28)	0.97(35)	0.93(80)	0.74(88)

In order to see the $O(1/m_Q)$ effect we extrapolate our ρ^2 to the vanishing $1/m_Q$ limit as shown in Fig. 3. ρ^2 becomes smaller when we approach the infinite mass limit. We need, however, more statistics to quantify the size of the $O(1/m_Q)$ effect and to extrapolate our data to the charm quark mass.

§4. Conclusion

We calculated the Isgur-Wise function near the zero recoil point using HQET including the kinetic term of the heavy quark. $O(1/m_Q)$ effect is partially included in this approximation. We obtained rather clear signals compared with the infinite mass limit. The obtained value for the slope parameter ρ^2 is consistent with the experimental value, but the statistical error is still large. A large amount of this statistical error is due to the large statistical uncertainty in the determination of the renormalized classical velocity. It is essential for reducing the statistical error to improve the signal for finite momenta using the smearing technique for example.

It is important to include the $O(1/m_Q)$ terms completely and to quantify the size of violations of the heavy quark symmetry in several semi-leptonic form factors. The lattice HQET is also applicable for studies of heavy to light semi-leptonic decays ($B\to\rho l\nu$, etc.) which can be used for extraction of V_{ub} from the exclusive decay width in a model independent way.

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References

1) C. Bernard, Y. Shen and A. Soni, Phys. Lett. **B317** (1993), 164.
2) UKQCD Collaboration (K. C. Bowler et al.), Phys. Rev. **D52** (1995), 5067.
3) J. Mandula and M. C. Ogilvie, Phys. Rev. **D45** (1992), 2183 and hep-lat/9408006.
4) C. Bernard, Nucl. Phys. **B** (Proc. Suppl.) **34** (1994), 47.
5) S. Hashimoto, Phys. Rev. **D50** (1994), 4639.
6) B. A. Thacker and G. P. Lepage, Phys. Rev. **D43** (1991), 196.
7) G. P. Lepage and P. B. Mackenzie, Phys. Rev. **D48** (1993), 2250.
8) U. Aglietti, Nucl. Phys. **B421** (1994), 191.
9) J. Mandula and M. C. Ogilvie, Nucl. Phys. **B** (Proc. Suppl.) **42** (1995), 424.
10) CLEO Collaboration, Phys. Rev. **D51** (1995), 1014.