Top Quark Condensate Revisited

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In the light of recent discovery of a very heavy top quark, we reexamine the top quark condensate model proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu. We first review the original MTY formulation based on the ladder Schwinger-Dyson equation and the Pagels-Stokar formula. It is particularly emphasized that the critical phenomenon gives a simple reason why the top quark can have an extremely large mass compared with other quarks and leptons. Then we discuss the Bardeen-Hill-Lindner (BHL) formulation based on the renormalization-group equation and the compositeness condition, which successfully picks up $1/N_c$-sub-leading effects disregarded by MTY. In fact BHL is equivalent to MTY at the $1/N_c$-leading order. Such a simplest version of the model predicts the top quark mass, $m_t \simeq 250$GeV (MTY) and $m_t \simeq 220$GeV (BHL), for the cutoff on the Planck scale. In this version we cannot take the cutoff beyond the Landau pole of $U(1)_Y$ gauge coupling, which yields a minimum value of the top mass prediction $m_t \simeq 200$GeV. We then propose a "top mode walking GUT": The standard gauge groups are unified into a ("walking") GUT so that the cutoff can be taken to infinity thanks to the renormalizability of the four-fermion theory coupled to "walking" gauge theory. The top and Higgs mass prediction is then controlled by the Pendleton-Ross infrared fixed point at GUT scale and can naturally lead to $m_t \simeq m_H \simeq 180$GeV.

§1. Introduction

As it stands now, the standard model (SM) is a very successful framework for describing elementary particles in the low energy region, say, less than 100 GeV. However one of the most mysterious parts of the theory, *the origin of mass*, has long been left unexplained. Actually, mass of *all* particles in the SM is attributed to a *single* order parameter, the vacuum expectation value of the Higgs doublet ($\simeq 250$ GeV). Thus the problem of the origin of mass is simply reduced to understanding dynamics of the Higgs sector.

Recently the elusive top quark has been finally discovered and found to have a mass of about 180 GeV,[1)] roughly on the order of weak scale 250 GeV. This is extremely large compared with mass of all other quarks and leptons and seems to suggest a special role of the top quark in the electroweak symmetry breaking, *the origin of mass*, and hence a strong connection with the Higgs boson itself.

Such a situation can be most naturally understood by the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY)[2),3)] and by Nambu[4)] independently. This entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate. The Higgs boson emerges as a $\bar{t}t$ bound state and hence is deeply connected with the top quark itself. Thus the model may be called "top mode standard model"[3)] in contrast to the SM (may be called "Higgs mode standard model"). The model was further developed by the renormalization-group (RG) method.[5),6)]

Once we understand that the top quark mass is of the weak scale order, then the question is why other quarks and leptons have very small mass compared with the weak scale. Actually, the Yukawa coupling is dimensionless and hence naturally expected to be of $O(1)$. This is the question that MTY [2),3)] solved in the top quark condensate through the amplification of the symmetry violation in the critical phenomenon.

MTY [2)] introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation [7),8)] and the Pagels-Stokar (PS) [9)] formula, MTY predicted the top quark mass to be about 250 GeV (for the Planck scale cutoff), which actually coincides with the weak scale. MTY also found that even if all the dimensionless four-fermion couplings are of $O(1)$, only the coupling larger than the critical coupling yields non-zero (large) mass, while others do just zero masses. This is a salient feature of the *critical phenomenon.* It should be emphasized that the MTY prediction (receipt date: Jan. 3, 1989) [2)] was made when the lower bound of the top quark mass through direct experiment was only 28 GeV (TRISTAN value) and many theorists (including SUSY enthusiasts) were still expecting the value below 100 GeV. It in fact appeared absurd at that time to claim a top mass on the order of weak scale. Thus such a large top mass was really a *prediction* of the model.

The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) [6)] in the SM language, based on the RG equation and the compositeness condition. BHL incorporated composite Higgs loop effects as well as the $SU(2)_L \times U(1)_Y$ gauge boson loops. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV, a somewhat smaller value but still on the order of the weak scale. Although the prediction appears to be substantially higher than the experimental value mentioned above, there still remains a possibility that (at least) an essential feature of the top quark condensate idea may eventually survive.

In this talk we reexamine the simplest version of the top quark condensate in view of the recent discovery [1)] of a heavy top quark. We first review the top quark condensate model based on the explicit four-fermion interactions introduced by MTY. [2),3)] Combined with the standard gauge interactions, dynamics of the model becomes a gauged Nambu-Jona-Lasinio (NJL) model. We then explain the MTY analysis of the model done in the ladder SD equation at the $1/N_c$ leading order. We shall emphasize how a *critical phenomenon* implied by the solution naturally explains why quarks and leptons other than the top quark can have extraordinarily small mass compared with the weak scale.

As to concrete mass prediction, solution of the SD equation should be combined with the PS formula [9)] for the decay constant of the composite Nambu-Goldstone (NG) bosons, $F_\pi \simeq 250\text{GeV}$, which determines the overall scale of the solution, namely the top mass. We then explain the BHL [6)] formulation based on the RG equation and the compositeness condition. It essentially incorporates $1/N_c$ sub-leading effects such as those of the composite Higgs loops and $SU(2)_L \times U(1)_Y$ gauge boson loops which were disregarded by the MTY formulation. We shall explicitly see that BHL is in fact equivalent to MTY at $1/N_c$-leading order. As far as the cutoff is below the Planck scale, the top quark mass prediction has a lower bound: $m_t \simeq 250\text{GeV}$ (MTY) or $m_t \simeq 220\text{GeV}$ (BHL).

We shall next experiment with the idea of taking the cutoff beyond the Planck scale. In this simplest version, even if we were allowed to ignore the quantum gravity effects, we cannot take the cutoff beyond the Landau pole of $U(1)_Y$ gauge coupling, which actually yields an absolute minimum value of the top mass prediction $m_t \simeq 200\text{GeV}$. However, if the standard gauge groups are unified into a ("walking")[*)] GUT, we may take the cutoff to infinity thanks to the renormalizability arguments[12)-17)] of the gauged NJL model with "walking" gauge coupling. We shall consider this possibility ("top mode walking GUT")[18)] in which the top and Higgs mass prediction is controlled by the Pendleton-Ross (PG) infrared fixed point[19)] at GUT scale and can naturally lead to $m_t \simeq m_H \simeq 180\text{GeV}$.

§2. Top mode standard model

2.1. *The model*

Let us first explain the original version of the top quark condensate model (top mode standard model) proposed by MTY[2),3)] based on explicit four-fermion interactions. The model consists of the standard three families of quarks and leptons with the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge interactions but *without Higgs doublet.* Instead of the standard Higgs sector MTY introduced $SU(3)_C \times SU(2)_L \times U(1)_Y$-invariant *four-fermion interactions among quarks and leptons*, the origin of which is expected to be a new physics not specified at this moment. The new physics specifies the ultraviolet (UV) scale (cutoff Λ) of the model, in contrast to the infrared (IR) scale (weak scale $F_\pi \simeq 250\text{GeV}$) determined by the mass of W/Z bosons.

The explicit form of such four-fermion interactions reads:[2),3)]

$$\mathcal{L}_{4f} = \Big[G^{(1)} (\bar{\psi}_L^i \psi_R^j)(\bar{\psi}_R^j \psi_L^i) + G^{(2)} (\bar{\psi}_L^i \psi_R^j)(i\tau_2)^{ik}(i\tau_2)^{jl}(\bar{\psi}_L^k \psi_R^l) + G^{(3)} (\bar{\psi}_L^i \psi_R^j)(\tau_3)^{jk}(\bar{\psi}_R^k \psi_L^i) \Big] + \text{h.c.}, \tag{2·1}$$

where i,j,k,l are the weak isospin indices and $G^{(1)}$, $G^{(2)}$ and $G^{(3)}$ are the four-fermion coupling constants among top and bottom quarks $\psi \equiv (t,b)$. It is straightforward[2),3)] to include other families and leptons into this form.

The symmetry structure (besides $SU(3)_C$) of the four-fermion interactions, $G^{(1)}$, $G^{(2)}$ and $G^{(3)}$, is $SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$, $SU(2)_L \times SU(2)_R \times U(1)_V$ and $SU(2)_L \times U(1)_Y \times U(1)_V \times U(1)_A$, respectively. The $G^{(2)}$ term is vital to the mass of the bottom quark in this model.[2),3)] In the absence of the $G^{(2)}$-term, (2·1) possesses a $U(1)_A$ symmetry which is explicitly broken only by the color anomaly and plays the role of the Peccei-Quinn symmetry.[3)]

Let us disregard the $G^{(2)}$ term for the moment, in which case the MTY Lagrangian (2·1) simply reads

$$\mathcal{L}_{4f} = G_t (\bar{\psi}_L t_R)^2 + G_b (\bar{\psi}_L b_R)^2 + \text{h.c.} \tag{2·2}$$

*) Nowadays, a walking[10)] coupling means a *very* slowly running coupling with $A = c/b \gg 1$, where b, c are coefficients of the one-loop beta function and the anomalous dimension, respectively; $\beta(g) = -bg^3$, $\gamma(g) = cg^2$. In the context of renormalizability of the gauged NJL model, we here use "walking" for $A > 1$ (slow running) instead of $A \gg 1$ (very slow running, or "standing").[11)]

with $G_t \equiv G^{(1)} + G^{(3)}$ and $G_b \equiv G^{(1)} - G^{(3)}$. The above MTY Lagrangian with $G_b = 0$ was the starting point of BHL[6] but setting $G_b = 0$ overlooks an important aspect of the top quark condensate, as we will see in the followings.

2.2. *Why $m_t \gg m_{b,\,c,\,\cdots}$?*

We now explain one of the key points of the model, i.e., explicit dynamics which gives rise to a large isospin violation in the condensate $\langle \bar{t}t \rangle \gg \langle \bar{b}b \rangle$ ($m_t \gg m_b$), or more generally, naturally explains why only the top quark has a very large mass. MTY[2),3)] found that critical phenomenon, or theory having nontrivial UV fixed point with large anomalous dimension, is actually such a dynamics, based on the spontaneous-chiral-symmetry-breaking (SχSB) solution of the ladder SD equation for the gauged NJL model. For the $SU(3)_c \times SU(2)_L \times U(1)_Y$-gauged NJL model, the ladder SD equation becomes simpler in the large N_c limit: Rainbow diagrams of the $SU(2)_L \times U(1)_Y$ gauge boson lines are suppressed compared with those of the QCD gluon lines.

For simplicity we first consider the ladder SD equation with the *non-running* QCD coupling and four-fermion coupling (2·2). Without $G^{(2)}$ term, the top and bottom quarks satisfy decoupled SD equations. (We can easily find a solution for the SD equation with the $G^{(2)}$ term.)[3] In Euclidean space, the ladder SD equation for each quark propagator $S_i^{-1}(p) = A_i(p^2)(\not{p} - \Sigma_i(p^2))(i = t, b)$ in Landau gauge takes the form (after angular integration):

$$\Sigma_i(x) = \frac{g_i}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y + \Sigma_i(y)^2} + \int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y + \Sigma_i(y)^2} K(x, y), \tag{2·3}$$

where $x \equiv p^2$, $K(x, y) = \lambda / \max(x, y)$ and Λ is a UV cutoff (a scale of new physics). We have defined *dimensionless* four-fermion couplings $g_i \equiv (N_c \Lambda^2/4\pi^2) G_i$ and $\lambda \equiv (3C_2(F)/4\pi)\alpha_{\rm QCD}$, with $N_c(=3)$ and $C_2(F) = (N_c^2 - 1)/2N_c(= 4/3)$ being the number of color and the quadratic Casimir of the fermion color representation $F(=\mathbf{3})$, respectively. (Note that $A_i(p^2) = 1$ in Landau gauge in the ladder approximation.) The dynamical mass function is normalized as $\Sigma_i(m_i^2) = m_i$.

Before discussing reality, let us look at a simplified case with the QCD coupling being switched off ($K(x, y) = 0$). Then this SD equation is simply reduced to the *gap equation* of the NJL model[20] in the large N_c limit. It is well known that the NJL model has a nontrivial solution $\Sigma(p^2) \equiv \text{const} = m \neq 0$ for $g > g^* = 1$. In the critical region ($0 < m/\Lambda \ll 1$) the solution reads ("scaling relation"):

$$\left(\frac{m}{\Lambda}\right)^2 \simeq \frac{1}{g^*} - \frac{1}{g} \tag{2·4}$$

up to logarithm. In view of the existence of such a critical coupling $g = g^* = 1$, it is easy to see that our four-fermion interactions (2·2) can give a *maximal isospin violation* in dynamical mass; $m_t \neq 0$ and $m_b = 0$, if $g_t > g^* = 1 > g_b$ (*not necessarily* $g_t \gg g^* \gg g_b$). Equation (2·4) shows that the dynamical mass sharply rises to the order of cutoff, $m = O(\Lambda)$, as the coupling moves off the critical point.[*)] Thus the critical phenomenon distinguishes the top quark ($g_t > g^*$) from all others ($g < g^*$)

[*)] See [*)] on page 23.

qualitatively: $m_t \neq 0, m_{\text{others}} = 0$, even if all the couplings are $O(1)$.

Now, a similar argument applies to the gauged NJL model, (2·3). The same type of equation as (2·3) was first studied by Bardeen, Leung and Love[21] in QED for the strong gauge coupling region $\lambda > \lambda_c = 1/4$. A full set of spontaneous chiral symmetry breaking (SχSB) solutions in the whole (λ, g) plane and the *critical line* were found by Kondo, Mino and Yamawaki and independently by Appelquist, Soldate, Takeuchi and Wijewardhana.[7]

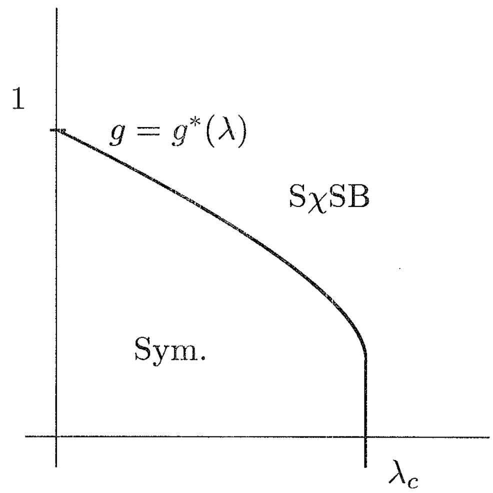

Fig. 1. Critical line in (λ, g) plane. It separates spontaneously broken (SχSB) phase and unbroken phase (Sym.) of the chiral symmetry.

The critical line in the (λ, g) plane is a generalization of the critical coupling in NJL model. It is the line of the second-order phase transition separating spontaneously broken ($m/\Lambda \neq 0$) and unbroken ($m/\Lambda = 0$) phases of the chiral symmetry (Fig.1):[7]

$$g = \frac{1}{4}(1+\omega)^2 \equiv g^*, \qquad \omega \equiv \sqrt{1-\lambda/\lambda_c}\ , \qquad (0 < \lambda < \lambda_c)$$
$$\lambda = \lambda_c\ . \qquad \left(g < \frac{1}{4}\right) \tag{2·5}$$

The asymptotic form of the solution of the ladder SD equation (2·3) takes the form:[7]

$$\Sigma(p^2) \underset{p \gg m}{\simeq} m \left(\frac{p}{m}\right)^{-1+\omega} \underset{\lambda \ll 1}{\simeq} m \left(\frac{p}{m}\right)^{-2\lambda}, \tag{2·6}$$

which is reduced to a constant mass function $\Sigma(p^2) \equiv \text{const} = m$ in the pure NJL limit ($\lambda \to 0$) as it should be. Through operator product expansion and RG equation,

$$\Sigma(p^2) \underset{p \gg m}{\simeq} \frac{m^3}{p^2} \left(\frac{p}{m}\right)^{\gamma_m}, \tag{2·7}$$

such a slowly damping solution (2·6) actually corresponds to a large anomalous dimension:[8]

$$\gamma_m = 1 + \omega\ , \qquad (0 < \omega < 1) \tag{2·8}$$

at the critical line for $0 < \lambda < \lambda_c$.

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[*] This implies that we need a fine-tuning of bare coupling $1/g^* - 1/g \ll 1$ in order to guarantee a hierarchy $m \ll \Lambda$. In the pure NJL model the limit $\Lambda/m \to \infty$ leads to a trivial (non-interacting) theory and hence this fine-tuning is not connected with a finite renormalized theory. In the gauged NJL model (with "walking" gauge coupling, $A > 1$), on the other hand, this fine-tuning is traded for a renormalization procedure arriving at a finite continuum (renormalized) theory defined at the UV fixed point with large anomalous dimension.[12),13)] We shall later return to this renormalizability of the gauged NJL model.[12)-17)]

Here the overall mass scale m at $0 < \lambda < \lambda_c$ satisfies the form similar to (2·4):[7)]

$$\left(\frac{m}{\Lambda}\right)^{2\omega} \simeq \frac{1}{g^*} - \frac{1}{g}. \tag{2·9}$$

As in the pure NJL model the dynamical mass m sharply rises as we move away from the critical coupling. Now the critical coupling of g on the critical line does depend on the value of gauge coupling λ and vice versa. This means that *even a tiny difference (symmetry violation) of λ (g) for the same g (λ) can cause amplified effects on the dynamical mass; $m = 0$ (below the critical line) or $m \neq 0$ (above the critical line).*

Returning to the top quark condensate, we note that our SD equations (2·3) separately include isospin-violating four-fermion couplings $g_t \neq g_b$ $(g^{(3)} \neq 0)$. In view of the critical line (2·5) and the critical behavior (2·9), MTY[2),3)] indeed found *amplified isospin symmetry violation* for a small (*however small*) violation in the coupling constants. Thus we have an SχSB solution with *maximal isospin violation*, $m_t \neq 0$ and $m_b = 0$, when

$$g_t > g^* = \frac{1}{4}(1+\omega)^2 > g_b \tag{2·10}$$

(g_t is above the critical line and g_b is below it). As already mentioned, we need *not* to set $G_b = 0$ in the four-fermion interactions (2·2) to obtain $m_b = 0$. Thus, even if we assume that all the dimensionless couplings are $O(1)$, the *critical phenomenon* naturally explains why only the top quark can have a large mass, or more properly, *why other fermions can have very small masses*: $m_t \gg m_{b,c,\cdots}$. It is indeed realized if only the top quark coupling is above the critical coupling, while all others below it: $g_t > g^* > g_{b,c,\cdots} \Longrightarrow m_t \neq 0, m_{b,c,\cdots} = 0$. Note that other couplings do *not* need to be zero nor *very* small.

2.3. *Running QCD coupling*

One can easily take account of *running effects* of the QCD coupling in the ladder SD equation ("improved ladder SD equation")[22)] by replacing λ in (2·3) by the one-loop running one $\lambda(p^2)$ parameterized as follows:

$$\lambda(p^2) = \begin{cases} \lambda_\mu, & (p^2 < \mu_{\rm IR}^2) \\ \frac{A/2}{\ln(p^2/\Lambda_{\rm QCD}^2)}, & (p^2 > \mu_{\rm IR}^2) \end{cases} \tag{2·11}$$

where $A = c/b = 18C_2(F)/(11N_c - 2N_f)$ $(= 24/(33-2N_f))$ and λ_μ $(= (A/2) / \ln(\mu_{\rm IR}^2/\Lambda_{\rm QCD}^2))$ are constants and $\mu_{\rm IR} (= O(\Lambda_{\rm QCD})$ an artificial "IR cutoff" of otherwise divergent running coupling constant. (We choose $\lambda_\mu > 1/4$ so as to trigger the SχSB already in the pure QCD.) Then the SD equation takes the form

$$\Sigma_i(x) = \frac{g_i}{\Lambda^2}\int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i^2(y)} + \int_0^{\Lambda^2} dy \frac{y\Sigma_i(y)}{y+\Sigma_i^2(y)} K(x,y), \tag{2·12}$$

where $K(x,y) \equiv \lambda(\max(x,y,\mu_{\rm IR}^2))/\max(x,y)$. Note that the non-running case is regarded as the "standing" limit $A \to \infty$ (with $\lambda_\Lambda \equiv \lambda(\Lambda^2)$ fixed) of the walking coupling $(A \gg 1)$.[11)]

The SχSB solution of (2·12) is logarithmically damping,[8] essentially the same as (2·6) with the small power $\lambda(\sim \lambda_\Lambda) \ll 1$:

$$\Sigma(p^2) \simeq m\left[\frac{\lambda(p^2)}{\lambda(m^2)}\right]^{\frac{A}{2}}, \quad A = \frac{8}{7}. \quad (N_f = 6) \tag{2·13}$$

In the case of pure QCD ($g = 0$), such a very slowly damping solution ("irregular asymptotics") is the *explicit* chiral-symmetry-breaking solution due to the quark bare mass.[23),22)] However, Miransky and Yamawaki[8)] pointed out that it can be the SχSB solution *in the presence of an additional four-fermion interaction.* The solution corresponds to a very large anomalous dimension $\gamma_m \simeq 2 - 2\lambda_\Lambda$ (compare with (2·8)) near the "critical line"[24),12)]

$$g = g^* \simeq 1 - 2\lambda_\Lambda \tag{2·14}$$

at $\lambda_\Lambda \ll 1$. (There is no critical line in the rigorous sense in this case, since SχSB takes place in the whole coupling region due to pure QCD dynamics, yielding dynamical mass $m = m_{\rm QCD} = O(\Lambda_{\rm QCD})$.) Note that (2·14) coincides with the critical line in the non-running case (2·5), $g = \frac{1}{4}(1+\omega)^2 \simeq 1 - 2\lambda$, at $\lambda \ll 1$. Actually, we can obtain exact expression for "critical line", which becomes identical with the entire critical line (2·5) in the limit $A \to \infty$.[12)]

In view of the "critical line", we again have an SχSB solution with *maximal isospin violation*, $m_t \neq 0, m_b = 0$ (apart from $m_{\rm QCD}$) , under a condition similar to (2·10); $g_t > g^*(\simeq 1 - 2\lambda_\Lambda) > g_b$.

§3. Top quark mass prediction

3.1. *SD equation plus PS formula (MTY)*

Now we come to the central part of the model, namely, relating the dynamical mass of the condensed fermion (top quark) to the mass of W/Z bosons.

The top quark condensate $\langle \bar{t}t \rangle$ indeed yields a standard gauge symmetry breaking pattern $SU(2)_L \times U(1)_Y \to U(1)_{em}$ to feed the mass of W and Z bosons. Actually, the mass of W and Z bosons in the top quark condensate is generated via dynamical Higgs mechanism as in the technicolor:

$$m_W^2 = \left(\frac{g_2}{2}F_{\pi^\pm}\right)^2, \qquad m_Z^2 \cos^2\theta_W = \left(\frac{g_2}{2}F_{\pi^0}\right)^2, \tag{3·1}$$

where g_2 is the $SU(2)_L$ gauge coupling, and $F_{\pi^\pm}$ and F_{π^0} are the decay constants of the composite NG bosons $\pi^\pm, \pi^0$ to be absorbed into W and Z bosons, respectively. $F_\pi(\simeq 250\text{GeV})$ determines the IR scale of the model and plays a central role in fixing the top quark mass.

Decay constants of those composite NG bosons may be calculated in terms of the Bethe-Salpeter (BS) amplitude of the NG bosons determined by the BS equation, which must be solved consistently with the SD equation for the fermion propagator.[25)] Instead of solving the BS equation, however, here we use the famous PS formula[9)] which expresses the decay constants in terms of dynamical mass function $\Sigma(p^2)$ of the condensed fermion, i.e., a solution of the ladder SD equation (2·13). The PS formula was generalized by MTY [2] to the $SU(2)$-asymmetric case $m_t \neq m_b$ and $m_{t,b} \neq 0$:

$$F_{\pi^\pm}^2 = \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} dx x \times \frac{(\Sigma_t^2+\Sigma_b^2) - \frac{x}{4}(\Sigma_t^2+\Sigma_b^2)' + \frac{x}{2}(\Sigma_t^2-\Sigma_b^2)\left[\frac{1+(\Sigma_t^2)'}{x+\Sigma_t^2} - \frac{1+(\Sigma_b^2)'}{x+\Sigma_b^2}\right]}{(x+\Sigma_t^2)(x+\Sigma_b^2)}, \tag{3·2}$$

$$F_{\pi^0}^2 = \frac{N_c}{8\pi^2}\int_0^{\Lambda^2} dx x \left[\frac{\Sigma_t^2 - \frac{x}{4}(\Sigma_t^2)'}{(x+\Sigma_t^2)^2} + \frac{\Sigma_b^2 - \frac{x}{4}(\Sigma_b^2)'}{(x+\Sigma_b^2)^2}\right]. \tag{3·3}$$

Let us consider the extreme case, the maximal isospin violation mentioned above, $\Sigma_t(p^2) \neq 0$ and $\Sigma_b(p^2) = 0$. We further take a "toy" case switching off the gauge interactions: $\Sigma_t(p^2) \equiv$ const (pure NJL limit). Then (3·2) and (3·3) are both *logarithmically divergent* at $\Lambda/m_t \to \infty$ *with the same coefficient*:

$$F_{\pi^\pm}^2 = \frac{N_c}{8\pi^2} m_t^2 \left[\ln\frac{\Lambda^2}{m_t^2} + \frac{1}{2}\right], \tag{3·4}$$

$$F_{\pi^0}^2 = \frac{N_c}{8\pi^2} m_t^2 \ln\frac{\Lambda^2}{m_t^2}. \tag{3·5}$$

Now, we could predict m_t by fixing $F_{\pi^\pm} \simeq 250$GeV so as to have a correct m_W through (3·1). Actually, (3·4) determines m_t as a *decreasing function of cutoff* Λ. The largest physically sensible Λ (new physics scale) would be the Planck scale $\Lambda \simeq 10^{19}$GeV at which we have a minimum value $m_t \simeq 145$GeV. If we take the limit $\Lambda \to \infty$, we would have $m_t \to 0$, which is nothing but triviality (no interaction) of the pure NJL model: $y_t \equiv \sqrt{2}m_t/F_\pi \to 0$ at $\Lambda \to \infty$.

One might naively expect a disastrous weak isospin violation for the maximal isospin-violating dynamical mass, $m_t \neq 0$ and $m_b = 0$. However, for $\Lambda \gg m_t$, (3·4) and (3·5) yield $F_{\pi^\pm} \simeq F_{\pi^0}$ and

$$\delta\rho \equiv \frac{F_{\pi^\pm}^2 - F_{\pi^0}^2}{F_{\pi^\pm}^2} = \frac{N_c m_t^2}{16\pi^2 F_{\pi^\pm}^2} \simeq \frac{1}{2\ln\frac{\Lambda^2}{m_t^2}} \ll 1. \tag{3·6}$$

Then the problem of weak isospin relation can in principle be solved *without custodial symmetry*. Actually, the isospin violation $F_{\pi^\pm} \neq F_{\pi^0}$ in (3·2) and (3·3) solely comes from the different propagators having different $\Sigma_i(p^2)$, essentially the IR quantity, which becomes less important for $\Lambda \gg m$, since the integral is UV dominant. This is the essence of the "dynamical mechanism" of MTY to save the isospin relation $\rho \simeq 1$ without custodial symmetry.[*)]

Now in the gauged NJL model, QCD plus four-fermion interaction (2·2), essentially the same mechanism as the above is operative. Based on the very slowly damp-

*) In the alternative formulation made by BHL [6] this dynamical consequence is tacitly incorporated into their assumption to take the *renormalizable form* for the effective theory of the composite Higgs (pure Higgs sector). Actually, it is impossible to write down a renormalizable pure Higgs Lagrangian having isospin violation $F_{\pi^\pm} \neq F_{\pi^0}$ (it is possible in the nonlinear sigma model).

ing solution of the ladder SD equation (2·13) and the PS formulas, (3·2) and (3·3), MTY[2),3)] predicted m_t and $\delta\rho$ as the *decreasing function of cutoff* Λ. For the Planck scale cutoff $\Lambda \simeq 10^{19}$GeV, we have:[2),3),*)]

$$m_t \simeq 250\text{GeV}, \tag{3·7}$$

$$\delta\rho \simeq 0.02 \ll 1. \tag{3·8}$$

This is compared with the pure NJL case $m_t \simeq 145$GeV: The QCD corrections are quantitatively rather significant. (As we will see later, presence of the gauge coupling will also change the qualitative feature of the theory from a nonrenormalizable/trivial theory into a renormalizable/nontrivial one.)[12)-17)]

It will be more convenient to write an *analytical* expression for F_π. Neglecting the derivative terms with $\Sigma_t(x)'$ and using (2·13), we may approximate (3·2) as

$$\begin{aligned} F_\pi^2 &\simeq \frac{N_c}{8\pi^2}\int_{m_t^2}^{\Lambda^2} dx \frac{\Sigma_t^2}{x} \\ &\simeq \frac{N_c m_t^2}{16\pi^2}\frac{A}{A-1}\frac{(\lambda(m_t^2))^{A-1} - (\lambda(\Lambda^2))^{A-1}}{(\lambda(m_t^2))^{A}}. \end{aligned} \tag{3·9}$$

This analytic expression was obtained by Marciano[5)] in the case $A = 8/7$ ($N_f = 6$), which actually reproduces the MTY prediction.[2),3)]

3.2. *RG equation plus compositeness condition (BHL)*

Now, we explain the BHL formulation[6)] of the top quark condensate, which is based on the RG equation combined with the compositeness condition. BHL starts with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level:

$$\mathcal{L}_{\text{Higgs}} = -y_t(\bar\psi_L^i t_R \phi_i + \text{h.c.}) + \left(D_\mu\phi^\dagger\right)(D^\mu\phi) - m_H^2\phi^\dagger\phi - \lambda_4\left(\phi^\dagger\phi\right)^2, \tag{3·10}$$

where y_t and λ_4 are Yukawa coupling of the top quark and quartic interaction of the Higgs, respectively. BHL imposed "compositeness condition" on y_t and λ_4 in such a way that (3·10) becomes the *MTY Lagrangian* (2·2) (with $G_b = 0$):

$$\frac{1}{y_t^2} \to 0, \quad \frac{\lambda_4}{y_t^4} \to 0 \quad \text{as} \quad \mu \to \Lambda, \tag{3·11}$$

where μ is the renormalization point above which the composite dynamics are integrated out to yield an effective theory (3·10). Thus the compositeness condition implies divergence at $\mu = \Lambda$ of both the Yukawa coupling of the top quark and the quartic interaction of the Higgs.

Now, in the one-loop RG equation, the beta function of y_t is given by

$$\beta(y_t) = \frac{y_t^3}{(4\pi)^2}\left(N_c + \frac{3}{2}\right) - \frac{y_t}{(4\pi)^2}\left(3\frac{N_c^2-1}{N_c}g_3^2 + \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2\right), \tag{3·12}$$

*) One may substitute into (2·13) the *numerical* solution (instead of the analytical one (2·13)) of the ladder SD equation (2·12), the result being the same as (3·7).[26)]

where g_1, g_2 and g_3 are the gauge couplings of $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively. BHL solved the RG equation for the beta function (3·12) combined with the compositeness condition (3·11) as a boundary condition at $\mu = \Lambda$.

3.3. *BHL versus MTY*

Let us first demonstrate[14)] that *in the large N_c limit* BHL formulation[6)] is equivalent to that of MTY,[2),3)] both based on the same MTY Lagrangian (2·2). In the $N_c \to \infty$ limit for (3·12), we may neglect the factor 3/2 in the first term (composite Higgs loop effects) and g_2^2 and g_1^2 in the second term (electroweak gauge boson loops), which corresponds to the similar neglection of $1/N_c$ sub-leading effects in the ladder SD equation in the MTY approach. Then (3·12) becomes simply:

$$\frac{dy_t}{d\mu} = \beta(y_t) = N_c \frac{y_t^3}{(4\pi)^2} - \frac{3N_c y_t g_3^2}{(4\pi)^2}. \tag{3·13}$$

Within the same approximation the beta function of the QCD gauge coupling reads

$$\frac{dg_3}{d\mu} = \beta(g_3) = -\frac{1}{A}\frac{3N_c g_3^3}{(4\pi)^2}. \tag{3·14}$$

Solving (3·13) and (3·14) by imposing the compositeness condition at $\mu = \Lambda$, we arrive at

$$y_t^2(\mu) = \frac{2(4\pi)^2}{N_c}\frac{A-1}{A}\frac{\left(\lambda(\mu^2)\right)^A}{\left(\lambda(\mu^2)\right)^{A-1} - \left(\lambda(\Lambda^2)\right)^{A-1}}. \tag{3·15}$$

Noting the usual relation $m_t^2 = \frac{1}{2}y_t^2(\mu = m_t)v^2$ $(v = F_\pi)$, we obtain

$$\frac{m_t^2}{F_\pi^2} = \frac{y_t^2(m_t)}{2} = \frac{(4\pi)^2}{N_c}\frac{A-1}{A}\frac{\left(\lambda(m_t^2)\right)^A}{\left(\lambda(m_t^2)\right)^{A-1} - \left(\lambda(\Lambda^2)\right)^{A-1}}. \tag{3·16}$$

This is precisely the same formula as (3·9) obtained in the MTY approach based on the SD equation and the PS formula.*) Thus we have established

$$\mathrm{BHL}\left(\frac{1}{N_c}\mathrm{leading}\right) = \mathrm{MTY}. \tag{3·17}$$

Having established equivalence between MTY and BHL in the large N_c limit, we now comment on the relation between them in more details. Note that MTY formulation is based on the nonperturbative picture, ladder SD equation and PS formula, which is valid at $1/N_c$ leading order, or the NJL bubble sum with ladder-type QCD corrections (essentially the leading log summation). MTY extrapolated this $1/N_c$ leading picture all the way down to the low energy region where the sub-leading effects may become important.

*) Alternatively, we may define $F_\pi^2(\mu^2) \equiv 2m_t^2/y_t^2(\mu)$ which coincides with the integral (3·9) with the IR end m_t^2 simply replaced by μ^2. Then the compositeness condition (3·11) reads $F_\pi^2(\mu^2 = \Lambda^2) = 0$ (no kinetic term of the Higgs).

On the other hand, BHL is crucially based on the perturbative picture, one-loop RG equation, which can easily accommodate $1/N_c$ sub-leading effects in (3·12) such as the loop effects of composite Higgs and electroweak gauge bosons. However, BHL formalism must necessarily be combined with the compositeness condition (3·11). The compositeness condition is obviously inconsistent with the perturbation and is a purely nonperturbative concept based on the same $1/N_c$ leading NJL bubble sum as in the MTY formalism. Thus the BHL perturbative picture breaks down at high energy near the compositeness scale Λ where the couplings y_t and λ_4 blow up as required by the compositeness condition.

So there must be a certain matching scale $\Lambda_{\text{Matching}}$ such that the perturbative picture (BHL) is valid for $\mu < \Lambda_{\text{Matching}}$, while only the nonperturbative picture (MTY) becomes consistent for $\mu > \Lambda_{\text{Matching}}$. Such a point may be defined by the energy region where the two-loop contributions dominate over the one-loop ones. A simple way to do such a matching is to use the BHL perturbative formalism for $\mu < \Lambda_{\text{Matching}}$, while using the MTY formalism (or equivalently the BHL at $1/N_c$ leading order) for $\mu > \Lambda_{\text{Matching}}$.*) Thus the reality may in principle be expected to lie in between BHL and MTY. From $1/N_c$ sub-leading terms in (3·12) we can see that the composite Higgs loops push down the Yukawa coupling at low energy, while somewhat smaller effects of the electroweak gauge boson loops make contributions in the opposite direction. As a result we would expect that BHL value is smaller than MTY one:

$$m_t(\text{BHL}) < m_t < m_t(\text{MTY}). \tag{3·18}$$

However, thanks to the presence of a *quasi-infrared fixed point*, [28] BHL prediction is *numerically* quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. Then we expect $m_t \simeq m_t(\text{BHL}) = \frac{1}{\sqrt{2}} y_t(\mu = m_t) v \simeq \frac{1}{\sqrt{2}} \bar{y}_t v$ within 1-2 %, where $\bar{y}_t$ is the quasi-infrared fixed point given by $\beta(\bar{y}_t) = 0$ in (3·12). The composite Higgs loop changes $\bar{y}_t^2$ by roughly the factor $N_c/(N_c + 3/2) = 2/3$ compared with the MTY value, i.e., 250GeV $\rightarrow 250 \times \sqrt{2/3} =$ 204GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value [6] is then given by

$$m_t = 218 \pm 3\text{GeV} \quad \text{at} \quad \Lambda \simeq 10^{19}\text{GeV}. \tag{3·19}$$

The Higgs boson was predicted as a $\bar{t}t$ bound state with a mass $M_H \simeq 2m_t$ [2)-4)] based on the pure NJL model calculation. [20] Its mass was also calculated by BHL [6] through the full RG equation of λ_4, the result being

$$M_H = 239 \pm 3\text{GeV}(M_H/m_t \simeq 1.1)\text{GeV} \quad \text{at} \quad \Lambda \simeq 10^{19}\text{GeV}. \tag{3·20}$$

If we take only the $1/N_c$ leading terms, we would have the mass ratio $M_H/m_t \simeq \sqrt{2}$, which is also obtained through the ladder SD equation. [29]

*) Of course, the $1/N_c$ leading picture might be subject to ambiguity such as the possible higher dimensional operators, cutoff procedures, etc., all related with the nonrenormalizability of the NJL model. [27] These problems will be conceptually solved and phenomenologically tamed, when coupled to the ("walking" ($A > 1$)) gauge interactions (renormalizability of the gauged NJL model) [12)-17)] to be discussed later. Here we just comment that even if there might be such an ambiguity, the $1/N_c$ picture (MTY) is the only consistent way to realize the compositeness condition as was done by the BHL paper itself.

§4. Top mode walking GUT

As we have seen, the top quark condensate naturally explains, through the critical phenomenon, why only the top quark mass is much larger than that of other quarks and leptons: $m_t \gg m_{b,c,\ldots}$. It further predicts the top mass on the order of weak scale. However, the predicted mass 220GeV is somewhat larger than the mass of the recently discovered top quark, $176\text{GeV} \pm 13\text{GeV}$ (CDF) and $199 + 38/-36\text{GeV}$ (D0).[1] Here we shall discuss a possible remedy of this problem within the simplest model based on the MTY Lagrangian (2·2).[18]

4.1. *Landau pole scenario*

First we recall that the top mass prediction is a *decreasing* function of the cutoff Λ. Then the simplest way to reduce the top mass would be to raise the cutoff as much as possible. Let us assume that quantum gravity effects would not change drastically the physics described by the low energy theory without gravity. Then we may raise the cutoff Λ beyond the Planck scale up to the Landau pole $\Lambda \simeq 10^{41}\text{GeV}$ where the $U(1)_Y$ gauge coupling g_1 diverges and the SM description itself stops to be self-consistent. In such a case the top and Higgs mass prediction becomes:

$$m_t \simeq 200\text{GeV}, \qquad M_H \simeq 209\text{GeV} \quad \text{at} \quad \Lambda \simeq 10^{41}\text{GeV} \tag{4·1}$$

which is the absolute minimum value of the prediction within a simplest version of the top quark condensate.

If it is really the case, it would imply composite $U(1)_Y$ gauge boson and composite Higgs generated *at once by the same dynamics*, since the Landau pole then may be regarded as a BHL compositeness condition also for the vector bound state as well as the composite Higgs. Actually, we can formulate the BHL compositeness condition for vector-type four-fermion interactions (Thirring-type four-fermion theory) as a *necessary condition* for the formation of a vector bound state. The possibility that both the Higgs and $U(1)_Y$ gauge boson can be composite by the same dynamics may be illustrated by an explicit model, the Thirring model in $D(2 < D < 4)$ dimensions. Reformulated as a gauge theory through hidden local symmetry, the Thirring model was shown to have the dynamical mass generation, which implies that a composite Higgs and a composite gauge boson are generated at the same time.[30]

At any rate, the prediction of this scenario $m_t \simeq 200\text{GeV}$ still seems to be a little bit higher than the experimental value, although the situation is not very conclusive yet.

4.2. *Renormalizability of gauged NJL model*

Then we shall consider another possibility, namely, taking the cutoff to infinity: $\Lambda \to \infty$. In order to do this we should first discuss the renormalizability of the gauged NJL model with "walking" gauge coupling ($A > 1$).[12)-17]

This phenomenon was first pointed out by Kondo, Shuto and Yamawaki[12] through the convergence of F_π in the PS formula for the solution of the SD equation (2·13) in the four-fermion theory plus QCD. Contrary to the logarithmic divergence of (3·4) in the pure NJL model, it was emphasized that for $A > 1$ we have a *convergent* integral for F_π and hence a nontrivial (interacting) theory $y_t \equiv m_t/F_\pi \neq 0$ in the continuum limit: Namely, the presence of "walking" ($A > 1$) gauge interaction changes the trivial/nonrenormalizable theory (pure NJL model) into a nontrivial/renormalizable theory (gauged NJL model).[12)]

As to the non-running (standing) case ($A \to \infty$), the integral for F_π^2 is more rapidly convergent, since $\Sigma(p^2)$ is power damping, (2·6), instead of logarithmic damping. In this case the renormalization procedure was performed explicitly by Kondo, Tanabashi and Yamawaki[13)] through the effective potential in the ladder approximation. The fine-tuning of the bare coupling $1/g^* - 1/g \ll 1$ in (2·9) corresponds to the continuum limit $\Lambda/m \to \infty$, which now defines a finite renormalized theory explicitly written in terms of renormalized quantities, in sharp contrast to the pure NJL model where the similar fine-tuning in (2·4) has nothing to do with a finite renormalized theory. This renormalization led to the beta function and the anomalous dimension:

$$\beta(g) = 2\omega g \left(1 - \frac{g}{g^*}\right), \tag{4·2}$$

$$\gamma_m(g) = 1 - \omega + 2\omega \frac{g}{g^*}, \tag{4·3}$$

where both functions take the same form in either bare or renormalized coupling g. These expressions are valid both in the SχSB and unbroken phases. It is now clear that the critical line $g = g^* = \frac{1}{4}(1+\omega)^2$ is a UV fixed line where the anomalous dimension takes the large value (2·8):

$$1 < \gamma_m(g = g^*) = 1 + \omega < 2. \tag{4·4}$$

This result was first obtained by Miransky and Yamawaki[8)] for the bare coupling in the SχSB phase, and was further shown[13)] to hold in both phases and also for the renormalized coupling, based on the effective potential.

The essence of the renormalizability now resides in the fact that this dynamics possesses a large anomalous dimension $\gamma_m > 1$ but not too large, $\gamma_m < 2$.[13)] It in fact implies that the four-fermion interactions are relevant operators, $2 < d_{(\bar\psi\psi)^2} = 2(3 - \gamma_m) = 4 - 2\omega < 4$.[8)] Accordingly, possible *higher dimensional interactions*, $(\bar\psi\psi)^4$, $\partial_\mu(\bar\psi\psi)\partial^\mu(\bar\psi\psi)$, etc., are *irrelevant operators* ($d > 4$ due to $d_{\bar\psi\psi} > 1$), in contrast to the case without gauge interactions where these operators are marginal ones ($d = 4$ due to $d_{\bar\psi\psi} = 1$).

Returning to the "walking" coupling, we note that the anomalous dimension is given as $\gamma_m \simeq 2 - 2\lambda_\Lambda$ which is very close to 2 but less than 2 by only a logarithmic factor. Then the above arguments for the standing coupling become rather delicate in this case. In order to discriminate between $A > 1$ and $A < 1$, we again discuss the finiteness of F_π, or equivalently finiteness of effective Yukawa coupling, $y_t \equiv \sqrt{2}m_t/F_\pi > 0$, in the continuum limit $\Lambda \to \infty$. The analytical expression of the effective Yukawa coupling is already given by (3·9) (MTY), which is equivalent to (3·16) obtained as a solution of the RG equation with a compositeness condition at $1/N_c$ leading (BHL). From this expression it was noted[14)] that *iff* $A > 1$ ("walking" gauge coupling with $N_c \sim N_f \gg 1$), then the effective Yukawa coupling remains finite, $y_t > 0$, in the continuum limit $\Lambda \to \infty$. This is in sharp contrast to the triviality of the pure NJL model in which $y_t \to 0$ in the continuum limit as was mentioned earlier.

It was further pointed out by Kondo, Tanabashi and Yamawaki [13] that this renormalizability is equivalent to existence of a PR infrared fixed point [19] for the gauged Yukawa model. The PR fixed point is given by the solution of $\frac{d(y_t/g_3)}{d\mu} = 0$ with (3·13) and (3·14):

$$y_t^2 = \frac{(4\pi)^2}{N_c}\frac{A-1}{A}\lambda, \tag{4·5}$$

where $\lambda = 3C_2(F)g_3^2/(4\pi)^2$. Similar argument was recently developed more systematically by Harada, Kikukawa, Kugo and Nakano. [17]

4.3. *Top mode walking GUT*

In view of the renormalizability of the gauged NJL model with "walking" gauge coupling, we may take the $\Lambda \to \infty$ limit of the top quark condensate. However, in the realistic case we actually have the $U(1)_Y$ gauge coupling which, as it stands, grows at high energy to blow up at Landau pole and hence invalidates the above arguments of the renormalizability. Thus, in order to apply the above arguments to the top quark condensate, we must remove the $U(1)_Y$ gauge interaction in such a way as to unify it into a GUT with "walking" coupling ($A > 1$) beyond GUT scale. Then the renormalizability requires that the GUT coupling at GUT scale should be determined by the PR infrared fixed point. [18]

For a simple-minded GUT with SU-type group, the PR fixed point takes the form similar to (4·5):

$$y_t^2(\Lambda_{\rm GUT}) = \frac{3C_2(F)}{N_c}\frac{A-1}{A}g_{\rm GUT}^2(\Lambda_{\rm GUT}) \simeq \frac{3}{2}g_{\rm GUT}^2(\Lambda_{\rm GUT}), \tag{4·6}$$

$$\lambda_4(\Lambda_{\rm GUT}) = \frac{6C_2(F)}{N_c}\frac{(A-1)^2}{A(2A-1)}g_{\rm GUT}^2(\Lambda_{\rm GUT}) \simeq \frac{3}{2}g_{\rm GUT}^2(\Lambda_{\rm GUT}), \tag{4·7}$$

where we assumed $N_c \gg 1$ and $A \gg 1$ ($N_f \sim N_c \gg 1$) for simplicity. Then the top Yukawa coupling at GUT scale is essentially determined by the GUT coupling at GUT scale up to some numerical factor depending on the GUT group and the representations of particle contents. Using "effective GUT coupling" including such possible numerical factors, we may perform the BHL full RG equation analysis for $\mu < \Lambda_{\rm GUT} \simeq 10^{15}$GeV with the boundary condition of the above PR fixed point at GUT scale.

For typical values of the effective GUT coupling $\alpha_{\rm GUT} \equiv g_{\rm GUT}^2/4\pi = 1/40, 1/50, 1/60$, prediction of the top and Higgs masses reads:

$$(m_t, M_H) \simeq (189, 193)\text{GeV},\ (183, 183)\text{GeV},\ (177, 173)\text{GeV}, \tag{4·8}$$

respectively. Note that these PR fixed point values at GUT scale are somewhat smaller than the coupling values at GUT scale which focus on the quasi-infrared fixed point in the low energy region. Thus the prediction is a little bit away from the quasi-infrared fixed point. This would be the simplest extension of the top quark condensate consistent with the recent experiment on the top quark mass.

§5. Conclusion

In the light of recent discovery of a very heavy top quark, we have reexamined the top quark condensate (top mode standard model). A salient feature of the model is to give a simple explanation of an extremely large mass of the top quark compared with other quarks and leptons *as a critical phenomenon.*[2] See Eq. (2·9). Even if dimensionless four-fermion couplings $g_t, g_b, \cdots$ are all $O(1)$, we still can have a large hierarchy $m_t \gg m_{b,c,\cdots}$ *iff* there exists nonzero critical coupling (critical line) g^* such that $g_t > g^* > g_{b,c,\cdots}$ (not necessarily $g_t \gg g^* \gg g_{b,c,\cdots}$). This is an amplification mechanism of symmetry violation characteristic to the critical phenomenon (or, dynamical symmetry breaking having a nontrivial UV fixed point/line g^* and a large anomalous dimension).

The original MTY[2] formulation predicted $m_t \simeq 250$GeV (for Planck scale cutoff), based on a purely nonperturbative picture of the large N_c limit of the ladder SD equation and the PS formula, i.e., the bubble sum with leading log QCD corrections. On the other hand, the BHL[6] formulation reduced the above MTY value to 220GeV, incorporating the $1/N_c$ sub-leading effects such as the composite Higgs loops and electroweak gauge boson loops, based on a combination of the (perturbative) RG equation and the (nonperturbative) compositeness condition. In fact, if we pick up only the $1/N_c$ leading order in BHL formulation, then it becomes equivalent to MTY. The perturbative picture of BHL breaks down in the high energy region near the compositeness scale where the couplings blow up due to the very condition of the compositeness condition: The compositeness condition can only be justified nonperturbatively through the large N_c argument. Thus the BHL formulation must in principle be switched over to its $1/N_c$ leading part, or equivalently the MTY formulation, in high energies. As far as actual numerical prediction is concerned, however, the above BHL value is quite insensitive to this switch-over, thanks to the quasi-infrared fixed point.

Then we experimented with a heretic idea to raise the cutoff scale beyond the Planck scale, ignoring all possible effects of the quantum gravity which we do not know at present anyway. First we simply placed the cutoff scale on the Landau pole of the $U(1)_Y$ gauge coupling, $\Lambda \simeq 10^{41}$GeV, the largest scale for which the top quark condensate with the SM gauge couplings can be self-consistent. This yields $m_t \simeq 200$GeV, which is absolutely the smallest value of the top mass within such a simplest version of the top quark condensate.

Next we considered a drastic possibility that the cutoff may be taken to infinity, based on the renormalizability of the gauged NJL model. In order to make the model renormalizable, we should remove the $U(1)_Y$ factor by unifying the SM gauge groups into a GUT with "walking" coupling ($A > 1$). In this renormalizable "top mode walking GUT" in the infinite cutoff limit, the couplings y_t, λ_4 at GUT scale are essentially given in terms of the GUT gauge coupling through the PR infrared fixed point, which can naturally predict somewhat lower values of the top and Higgs masses: $m_t \simeq M_H \simeq 180$GeV.

Although the situation about top quark mass is still not yet conclusive, we hope that at least essence of the idea of the top quark condensate may eventually survive in the sense that the *origin of mass* is deeply related to the top quark mass.

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