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Three Comments *)

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Translater's note: This article is the translated version of a proceedings report of a talk given in the workshop "Nuclear Collective Motion" held May 18-20, 1967 at the Research Institute for Fundamental Physics (now Yukawa Institute for Fundamental Physics), Kyoto University. The proceedings was published in Soryushiron Kenkyu (vol. **35** (1967), page E47-E50), in Japanese. The translation was intended to be as direct as possible. The words in the square brackets in italic (i.e., [*italic*]) are additions by the translater. Some trivial mistakes have been corrected in the translated version.

I made, in the workshop, three comments such as

- (1) on the correspondence argument of Sakai,
- (2) on the relation of the generalized coordinate method of Yoshida and Onishi to the Ikeda model,
- (3) on the effective interaction of Bando.

In this article, I shall present some notes on comment (1). Specifically, this article consists of discussions on the relation between rotational and vibrational schemes in a boson approximation, and discussions on the classification for vibrational states.

One can classify, in general, d^N configurations in terms of SU_5. For bosons, in particular, one does not need to make such a general statement, and, in fact, only fully symmetric states $[N]$ are allowed. There is a subgroup O_5 for the group SU_5. For bosons again, the irreducible representation is determined as $(\lambda, 0)$ by using the seniority λ. In Bohr's vibrational model, one can introduce variables β, γ, φ, θ, ψ instead of α_μ's. The seniority λ can then be related to n_β as $N = \lambda + 2n_\beta$, where n_β denotes the number of the nodes of the β-vibration. (Wilet Jean). Here, we include O_4 into the canonical chain, so as to classify O_5 completely. Since O_4 is isomorphic to $SU_2 \times SU_2$, the classification of O_4 can be made in terms of four parameters. In a fully symmetric case, as is the present one, one of these four parameters can be dropped, and we need only three quantum numbers. Hence, the states of the present O_5 system can be classified completely by λ, μ, L and M. One can then prove that $L = \mu$, $\mu{+}1$, $\cdots$, $2\mu - 2$, 2μ. Note that $L = 2\mu - 1$ is missing. Thus, one obtains a beautiful classification as shown in Fig. 1. When one views this figure in the horizontal direction keeping N constant, the $n_\beta = 0$ part gives rise to the classification of O_5. We shall now look at Fig. 1 differently, sweeping from the bottom upwards. As N increases one by one, we find more states, and the values of L change. One finds a clear regularity in the pattern of these variations, which actually appear to be similar to [*the pattern of*] rotational levels. I suspect that this [*similarity*] is the "correspondence" addressed by Sakai.

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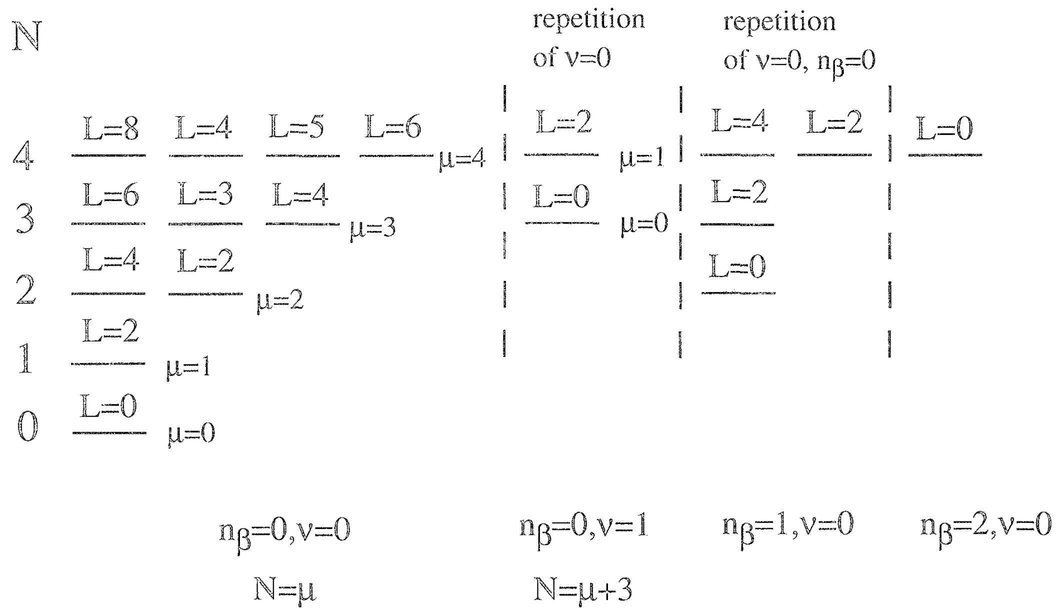

Fig. 1.

I now present the following discussions in order to clarify the relation of the level structure mentioned above to the rotational one. Assuming that the quantity N_0 [*,which is to be defined,*] has an upper limit, I introduce a hypothetical s-boson so that the total number of bosons is conserved. Then, the above arguments can be applied to the classification of the states, $s^{N_0-N}d^N$. I introduce further a boson-boson interaction of the Q-Q type between bosons, and then diagonalize it. It turns out that the classification in terms of the SU_3 group is more convenient, producing [*the level scheme shown in*] Fig. 2. In this figure, $N_0 = 4$ is taken. In general, for fully symmetric states in the $(s,d)^{N_0}$ configuration, the following sets of the SU_3 irreducible representations emerge;

(1) $(2N_0,\ 0)$, $(2N_0-4,\ 2)$, $(2N_0-8,\ 4)$, $\cdots$,

(2) $(2N_0-6,\ 0)$, $(2N_0-10,\ 2)$, $(2N_0-14,\ 4)$, $\cdots$,

(3) $(2N_0-12,\ 0)$, $(2N_0-16,\ 2)$, $\cdots$,

etc. The values of L belonging to these (λ,μ) irreducible representations are given, accordingly to Elliott, by

$$K = \min(\lambda,\mu),\ \min(\lambda,\mu)-2,\ \cdots,\ 0 \text{ or } 1$$

$$L = K,\ K+1,\ K+2,\ \cdots,\ K+\max(\lambda,\mu),$$

where $L = 0,\ 2,\ 4,\ \cdots\ ,\ \max(\lambda,\mu)$ for $K = 0$.*) Therefore, if $N_0 = 4$ (the total number of states is certainly the same between Figs. 1 and 2),

*) In the original version, this sentence was followed by an additional statement " or 1, 3, 5, $\cdots$, $\max(\lambda,\mu)$ ". This was omitted in this version in order to avoid confusion, because this argument is about the general property of $SU(3)$.

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	(λ, μ)	K	L							
(1)	(8, 0)	0	0	0	2		4		6	8
	$(4, 2)^*$	0	0	0	2		4			
		2		2	2	3	4	5	6	
	(0, 4)	0	0	0	2		4			
(2)	(2, 0)	0	0	0	2					

Since the eigenvalue of $(Q \cdot Q)$ is $\frac{\lambda^2+\lambda\mu+\mu^2}{9} + \frac{\lambda+\mu}{3} - \frac{1}{12}L(L+1)$, one ends up with [*the level scheme in*] Fig. 2. A remarkable correspondence between Figs. 1 and 2 is noticed in the region of low excitation energy. A model with s and d bosons has also been discussed by Taruishi of the Tokyo University of Education [*now Tsukuba University*].

What consequences can we obtain if we suppose that the states in Fig. 2 represent those of a deformed nucleus? Once one accepts this hypothesis, one can find a way

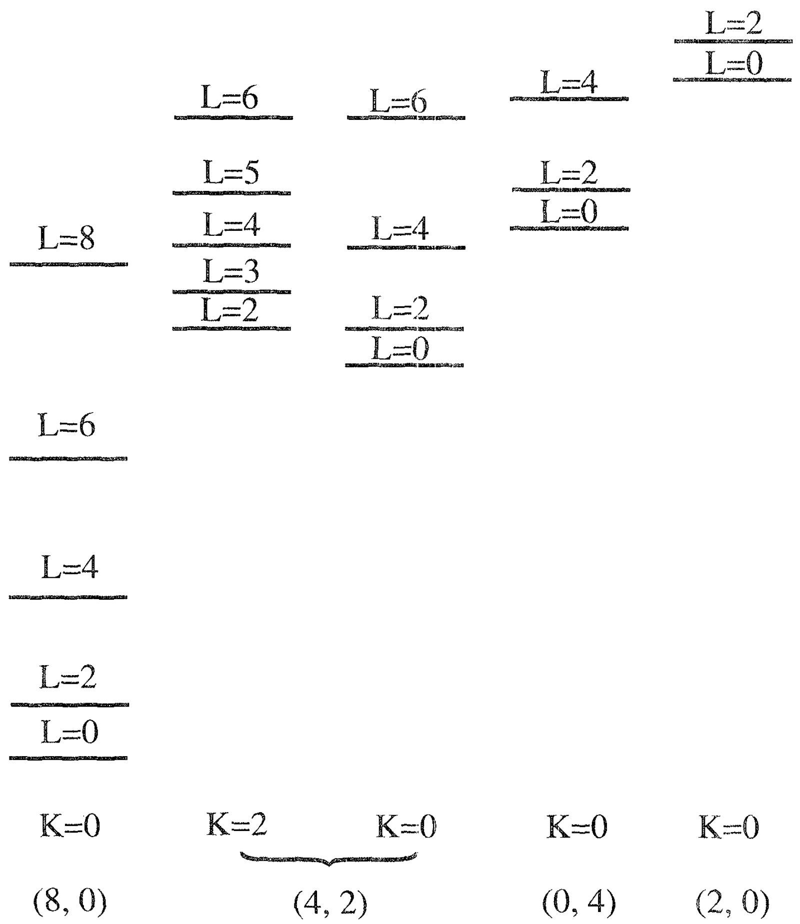

Fig. 2.

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of moving from the vibrational situation in Fig. 1 to the rotational one in Fig. 2, by increasing [*the strength of the*] (Q-Q) [*interaction*] compared to $\hbar\omega_d$ (i.e., the [*single-particle*] energy of the d boson). For instance, [*the spin sequence*] 0, 2, 4, 6, $\cdots$ starting from $N = 0$ 0^+ [*in Fig. 1*] seems to correspond to the ground state band of the rotational case [*in Fig. 2*], and [*the spin sequence*] 2, 3, 4, $\cdots$ built on the 2 phonon [*state*] [*in Fig. 1*] corresponds to the γ band [*in Fig. 2*], $\cdots$. Now let me ask whether or not the 3-phonon sequence 0, 2, 4, $\cdots$ starting from the state of $n_\beta = 0$, $\nu = 1$ and $L = 0$ becomes the $K = 0$ δ-band. *)

*) The δ-band is mistyped as γ-band in the original version.

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