String Tension and Glueball Masses of $SU(2)$ QCD from Perfect Action for Monopoles and Strings

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(Received October 11, 1999)

We study the perfect monopole action and string model as an infrared effective theory of $SU(2)$ QCD. It is transformed exactly into a lattice string model. The strong coupling expansion of string model shows that the quantum fluctuation is small. The classical string tension and the glueball mass are estimated analytically, and we see that it is very close to the quantum one in the $SU(2)$ QCD.

The infrared effective theory of QCD is important for the analytical understanding of hadron physics. Abelian monopoles which appear after abelian projection of QCD [1] seem to be relevant dynamical degrees of freedom for infrared region. [2] Shiba and Suzuki [3] derived the monopole action from vacuum configurations obtained in Monte-Carlo simulations.

In order to obtain the effective infrared action of $SU(2)$ gluodynamics we performed the following steps. (I) The abelian monopole action is extracted from the $SU(2)$ gauge fields in the Maximal Abelian projection using Monte-Carlo simulations. In order to study the infrared behaviour of the monopole, we also considered the renormalized monopole action $\mathcal{S}[k]$ performing block spin transformations numerically, and found that scaling for fixed physical length b looks good. [4] If the action $\mathcal{S}[k]$ also satisfies the continuum rotational invariance, then we can regard $\mathcal{S}[k]$ as a good approximation of the renormalized trajectory (RT). In order to check this, we have to determine the correct form of physical operators as well as the action on the coarse lattice, because naive lattice action does not reproduce the continuum rotational invariance. (II) We start from the monopole action composed of two-point interactions between magnetic monopole currents formulated on an infinite lattice with very small lattice constant a. Note that such an action with quadratic interactions alone is dominant in the infrared region of QCD. We also construct the monopole contribution to the potential between static abelian electric charges. Performing a block spin transformation for monopole currents analytically, we obtain the expectation value of the Wilson loop and the effective action on the coarse lattice ($b = n \cdot a$). [5] This can be done exactly because we consider only quadratic interactions alone for the monopole action. (III) Since the (numerically obtained) effective monopole action for $SU(2)$ QCD in the infrared region is well dominated by quadratic interactions, we regard the renormalization flow obtained in (II) as a projection of RT to the quadratic-interaction plane. We determined the free parameters in (II) from the monopole action obtained by inverse Monte Carlo method (I). (IV) The above monopole action can be transformed exactly into that of the string

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model using BKT transformation.[6),7)] We find finally the correct form of Wilson loop operator on the coarse lattice.[5)]

It turns out that the monopole action on the dual lattice is in the weak coupling region for large b, namely the infrared region of pure $SU(2)$ QCD. Then the string model on the original lattice is in the strong coupling region. The strong coupling expansion on the lattice can be performed easily and quantum fluctuations terms which include more plaquettes become small.[8)] Thus we consider only classical part. As a result the potential takes only the linear form and **the continuum rotational invariance is recovered completely even for the nearest-neighbour sites**.

The string tension is evaluated from the static potential and we have found that it is consistent with the analytical results.[9)] It occurs that the *classical* string tension in the string model is same as the string tension in *quantum* $SU(2)$ gluodynamics within 30%.

The glueball mass spectrum can be obtained by computing the correlation functions of gauge invariant local operators or Wilson loops, and extract the particle poles. In our model, we take the operator $\mathcal{O} = 2(1 - \mathrm{Re}W_m(\mathcal{C}))/a^4$ on the a-lattice. We can show this operator coincides with abelian counterpart of $\mathcal{O} = \mathrm{Tr}(F^2)$ when lattice constant a goes to zero. The result is almost consistent with the recent lattice results.[10)] For details, we refer to Ref. 11)

T. S. acknowledges financial support from JSPS Grant-in Aid for Scientific Research (B) (No. 10440073 and No. 11695029).

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