

Black Holes and Relativistic Jets

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There is strong observational evidence that AGN, Galactic X-ray transients and (probably) γ -ray bursts are associated with black holes, and that these sources are able to form collimated, ultrarelativistic outflows. There is much interest in trying to understand how these prime movers are able to release energy from accreting gas and their own spin energy. Electromagnetic field plays a large role in many of the mechanisms under active consideration. In this article, several of the many possible “metabolic pathways” through which mass, angular momentum and energy can flow around and away from black hole magnetospheres are discussed. Particular importance is attached to the interactions between the inflowing disk, the outflowing wind, the black hole and the jet. Some important unresolved questions are identified and it is argued that large scale numerical computation will almost certainly be necessary to address them.

§1. Introduction

As is well known, one of the first proposals that was made, soon after the discovery of quasars in 1963, was that they were powered by accretion onto massive black holes.^{1),2)} The fundamental reason why this proposal was made was that quasars were known to be prodigiously powerful, with luminosities equivalent to hundreds of galaxies, and that up to $\sim 0.1c^2 \equiv 10^{20}$ erg g⁻¹ of energy per unit mass could be released by lowering matter close to a black hole. This efficiency could be over a hundred times that traditionally associated with nuclear power. Since this time, we have also learned about black holes with masses $\sim 5 - 10 M_\odot$ in Galactic binary systems, gamma ray bursts and ultra-luminous X-ray sources which, with decreased confidence, we also associate with black holes, primarily on energetic grounds.

Simultaneously, remarkable progress was made in understanding the theoretical physics of black hole spacetime and in the decade following the discovery of the Kerr metric in 1963, the geometrical and kinematic properties of the exterior spacetime were comprehensively understood. As a consequence, any physical process that could be described in a flat spacetime could also, in principle, be computed around a spinning black hole. In an almost literal sense, the stage was set for interpreting the extraordinary observational discoveries that followed and the fact that we still do not possess confident and widely accepted answers to most of the most basic questions concerning AGN, X-ray transients and GRBs is that we are still wrestling with trying to understand non-relativistic processes, most notably those associated with electromagnetic field and gas dynamics.

Astrophysical black holes form a two parameter family. They can be completely characterized by their mass m and specific angular momentum in geometrical

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units a .⁴⁾ The mass simply provides a scale of length, time and energy and provides the basis for our discussing stellar and massive holes simultaneously. By contrast black holes are almost surely spinning quite rapidly as their surroundings are always endowed with far more specific angular momentum than the maximum that holes can possess and this second parameter introduces some qualitatively new features. (Charged black holes can also be described and are proving to be extremely valuable to the theoretical physicist; however they are thought to discharge almost immediately under astrophysical conditions. In addition, there remains the slim possibility that general relativity needs to be modified in these environments and astronomers are obligated to test the theory whenever they can. Here, though, I shall assume that the Kerr metric is all that is required.)

In this article, I would like to present a partial summary of some general ways by which gravity, spin and electromagnetic field can combine to power some of the most extensively observed high energy phenomena in astronomy. I shall first give a quick list of some of the more pertinent pieces of observational evidence concerning black holes. Then I shall discuss the four elements, the disk, the outflow, the hole and the jet where electromagnetic fields are believed to be relevant and I shall go on to describe the crucial electromagnetic interactions between these four elements.

I should make two apologies in advance. Firstly, if this article is read by anyone who actually attended the meeting, they may notice that it contains material that I did not discuss. This is because I have lost my notes and viewgraphs! Secondly, the literature on this subject has become quite large and I apologize for only referring to a few representative articles which, I hope will provide ongoing access to further research.

§2. Observed black holes

2.1. AGN

Although the first direct evidence for the existence of black holes was garnered in 1972, the observational database has grown impressively over the past decade. It now appears that most normal galaxies with the possible exception of types later than Sbc, harbor massive black holes in their nuclei with masses from a million to several billion solar masses measured dynamically through the motions of surrounding stars and gas. It appears that the mass of the hole is correlated with the properties of the surrounding galaxy, most recently with its central velocity dispersion σ ⁵⁾

$$M_H \sim 1.3 \times 10^8 \left(\frac{\sigma}{200 \text{ km s}^{-1}} \right)^{3.65}. \quad (2.1)$$

Likewise, it has been possible to measure nearly twenty black hole masses in accreting (often transiently) binary systems. We really have no idea as to the number of single black holes orbiting our galaxy although candidate objects have been reported in microlensing surveys. More recently, a several compact X-ray sources have been reported in nearby galaxies radiating well above the Eddington limit for stellar mass holes. These are conjectured to be intermediate holes with masses $> 100 M_\odot$, possibly formed at very early cosmological times. Finally, although the evidence for this

is purely theoretical, most contemporary models for gamma ray bursts involve either the formation or the augmentation of a stellar mass.

There have been advances towards the important goal of measuring the second parameter, the spin. Firstly, observations of some nearby, lower power AGN at X-ray wavelengths have revealed very broad iron lines.⁶⁾ These are believed to originate in fluorescing gas on the surface of an accretion disk and the strong gravitational plus Doppler redshifts that are observed indicate that the disk is located very close to the event horizon. Now it is known that Keplerian orbits are only stable beyond a radius $r = 6m$ for a non-spinning, Schwarzschild hole and disks exterior to this radius would only produce narrow lines. Therefore, it is argued, the hole must be rapidly spinning as this allows the disk to exist right up to the horizon. Although there are concerns that gas on unstable orbits plunging into the black hole might also fluoresce, the conclusion seems reasonable. (Note that this method furnishes the ratio a/m and does not give the mass separately.)

2.2. *X-ray binaries*

Another approach, that has been developed in recent years, involves the measurement of quasi-periodic oscillations from black hole X-ray binaries.⁷⁾ Here the $\sim 1 - 10$ percent, $Q \sim 10 - 500$ X-ray variations are thought to come from modes associated with the accretion disk. (To be more precise, the disk is thought to be the timekeeper; the very hard spectrum of the emission, strongly suggests that it originates in a coronal region that is excited by the pulsating disk.) The exact nature of these modes remains controversial however. Local waves involving horizontal and vertical epicyclic motion, trapped normal modes that may require reflection from an inner edge and magnetic modes have all been invoked. What is not at issue, though, is that all of these mode frequencies depend significantly upon the spin of the hole in addition to the mass, and as soon as we can agree upon the type of wave that we are seeing, it should also be possible to measure the spin frequency of black holes as well as their mass. In addition, there is the challenge of using “diskoseismology” to test general relativity, just like helioseismology has been used to test stellar physics.

2.3. *Jets*

Undoubtedly the largest body of observational data, that pertains to the effects I have been asked to discuss, is associated with relativistic jets. These are commonly found to accompany a minority of active galactic nuclei, especially those associated with elliptical galaxies. The ultrarelativistic outflow speeds, have Lorentz factors that are typically $\gamma \sim 10$ (and which could be much larger), strongly suggested that the outflows are formed quite deep in the potential well. This inference has received support from VLBI observations of M87 which show that collimation takes place within $\sim 100m$.⁸⁾ Observations of this source also demonstrate that the jet power can apparently exceed the bolometric power of the accreting gas and, in general the jet phenomenon has to be seen, on energetic grounds, as an intrinsic part of the accretion process. In the case of Cygnus A, for example, the jet power which inflates the two radio lobes, located well outside the optical image of the galaxy, must exceed $\sim 10^{45} \text{ erg s}^{-1}$, comparable with the bolometric luminosity of a typical quasar.⁹⁾ Jets

are increasingly being found connected to accreting Galactic black holes, the so-called Galactic superluminal sources.¹⁰⁾ As these probably involve similar processes to those occurring in extragalactic sources, they are well worth studying because the cycle times are very short and give a far better sampled dataset for studying accretion processes.

Our most detailed measurements of jets come from observations of young stellar objects.¹¹⁾ Here, we are able to infer speed and density as well as observe the strong spatial inhomogeneity. Of course, there is no black hole, but the presence of a disk, with a measured magnetic field, as well as a source of free energy associated with the relative angular velocity between the inner disk and the more slowly spinning hole, may well allow it to function in an essentially similar manner to the AGNs.

2.4. *Solar corona*

Another source of indirect, though no less promising, intelligence is the solar corona.¹²⁾ There has been a revolution in solar physics, due largely to observations by YOHKOH, SOHO and TRACE. Observations of the behavior of the active corona should allow us to divine the true behavior of cosmic magnetic fields in the bulk. Fundamental processes like topological rearrangement, reconnection and the formation of shock waves, can be monitored on a daily basis and ought to be milder versions of similar processes taking place above the surface of a more rapidly, and differentially, rotating accretion disk.

§3. The four elements: the hole, the disk, the outflow and the jet

We have emphasized that there is still no agreement about the fundamentals of the flow of mass, angular momentum and energy around black holes. Part of the problem is that these mechanisms may vary considerably from one class of objects to the next. After all, we need to explain why broad absorption line quasars differ from powerful radio galaxies and so on. It may be helpful to take a “systems engineering” approach and suppose that the four essential elements are the hole (surely spinning), the inflow (almost certainly in the form of a disk or a torus), the outflow (a wind derived from the disk) and the relativistic jet (that derives from the region around the black hole). The difficulty of the problem is that there are interactions between all pairs of elements and most of the controversy comes about in assessing the character and strength of these interactions. However let us first consider the individual elements (considered whimsically using an appeal to Greek philosophy) in turn.

3.1. *“Earth”—the hole*

As mentioned above black hole astrophysics is almost exclusively studied in the context of the Kerr metric. This is pragmatic because it is much harder to amass evidence from observing black holes that general relativity failed in this regime than it is to interpret what we see assuming that it is correct.

3.1.1. Reducible mass

As is well known, a non-rotating hole introduces an event horizon with radius $r_+ = 2m$ ($G = c = 1$) into the spacetime across which radiation and plasma can only cross inwards. In order to describe the spacetime surrounding the hole, we must introduce a coordinate system. The easiest one to use, Schwarzschild coordinates, has a pathology at the horizon and kinematic thought experiments have to be interpreted carefully, often with the aid of alternative, non-singular coordinate systems.⁴⁾ For example, particles remain outside the horizon after the Schwarzschild time coordinate — essentially time measured at infinity — has advanced by an infinite amount. For this reason, black holes were once called “frozen stars”. However, this is not a useful way to view the physics. According to a clock attached to an infalling particle, it only takes a finite proper time to cross the horizon.

Rotating black holes exhibit the effects of spin. The radius of the event horizon in Boyer-Lindquist coordinates — the generalization of Schwarzschild coordinates — changes to

$$r_+ = m + (m^2 - a^2)^{1/2}, \quad (3.1)$$

where a measures the angular momentum per unit mass. In addition, the spacetime is dragged in the direction of rotation. In practice, what this means is that a material particle, that orbits the hole at fixed latitude and radius, must rotate with respect to infinity when it is within a region called the ergosphere which is defined by $r < r_{\text{ergo}} = m + (m^2 - a^2 \cos^2 \theta)^{1/2}$. To be more precise, the angular velocity of these particles (which require non-gravitational forces to keep them on their trajectories) must lie between two limits

$$\Omega_{\min} < \Omega < \Omega_{\max} \quad (3.2)$$

and $\Omega_{\min} > 0$ within the ergosphere. In the limit as $r \rightarrow r_+$, $\Omega_{\min} \rightarrow \Omega_{\max} \rightarrow \Omega_H = a/(r_+ + a^2)$ which is defined to be the angular frequency of the hole. (Of course this angular frequency cannot be measured directly as the spacetime is axisymmetric. However, it can be measured indirectly using gyroscope precession, etc.)

One of the most perceptive theoretical discoveries that was made about black holes was that, when they spin, a fraction of their mass could be ascribed to rotational energy and was, in principle, extractable.³⁾ This is most convincingly demonstrated by observing that there exist orbits of test particles with negative total energy, (including their rest mass), within the ergosphere. This implies that if, for example, a cloud of plasma is attached to magnetic field lines, anchored at large distance and this magnetic field drags the cloud backwards relative to the rotation of the hole so that it is placed onto a negative energy orbit, then the work that was performed in placing the cloud onto this orbit can be thought of as energy that has been extracted from the spin of the black hole.¹³⁾ After the cloud has crossed the event horizon, the gravitational mass of the hole will decrease by the mass equivalent of the work that was performed. (Note that this is a purely classical process, in contrast to Hawking radiation.) Obviously, this requires that we define energy (and angular momentum) in a consistent fashion, and this happens naturally when one uses the full machinery of general relativity.

When we generalize this idea, we find that associated with a black hole of mass m is an irreducible mass m_0 related to m through

$$m = m_0(1 - \beta^2)^{-1/2}, \quad (3.3)$$

where $\beta = a/2m_0 = (a\Omega_H)^{1/2} < 2^{-1/2}$. The reducible mass, $m - m_0$ is available for powering high energy emission.

3.2. “Water” — the disk

The accreting gas almost certainly has sufficient angular momentum to form a disk or a torus, if the pressure support is substantial. Several different modes of accretion have been proposed, dependent upon the rate of gas supply and possible other factors, like the spin of the hole or the stellar environment (binary companion, massive star envelope, dense star cluster and so on).

3.2.1. Thin disks

In the simplest and most standard type of accretion disk, the flow is supposed to be stationary and efficiently radiative. One peculiarity of thin, radiative disks, that has consequences for what follows, is that when the torque, $G(r)$, that drives the inflow is local, then three times as much energy is radiated from an annular ring (well removed from the disk boundaries and assuming a steady state) than is released by the infalling gas. The reason for this is that the torque that the inner disk exerts on the outer disk and which balances the inflow of angular momentum $\dot{M}(mr)^{1/2}$ (where \dot{M} is the accretion rate) does work at a rate $G\Omega$ which overcompensates the inflow of binding energy. The divergence of the energy flux is $-Gd\Omega/dr$ which equals three times the rate of release of binding energy. Energy is conserved overall as the boundary conditions ensure that there is a shortfall in the energy radiated close to the inner boundary of the disk.

3.2.2. Adiabatic accretion

At sufficiently low accretion rates the energy that is created by viscous dissipation may be almost completely taken up by the ions. (There are some plasma physical arguments that this is indeed what happens.) Under these circumstances, the gas cannot cool and maintains a pressure scale height comparable with its radius. However, this cannot happen too close to the symmetry axis where horizontal, centrifugal force cannot oppose the vertical pull of gravity and a funnel is expected to form. There should still be a viscous torque carrying angular momentum outward. It turns out that the energy dissipated by this torque is sufficient to unbind the gas. It has been proposed¹⁴⁾ that, under these conditions, inflow into the hole should be accompanied by prodigious mass loss from larger radii driven by the binding energy released by gas as it crosses the horizon. (Under these circumstances, the gas is likely to be convective.) The outflow is most commonly argued to be magnetically driven.²⁰⁾

There are, at least, two alternative views. Under the “ADAF” prescription,¹⁵⁾ the flow is conservative and the released energy is advected inward across the horizon in a quasi-spherical inflow. Under the accretion torus model,¹⁶⁾ the inner surface of the disk has a very small binding energy from which gas falls ballistically onto the

hole and the overall radiative efficiency is very small.

These matters have become important in interpreting recent Chandra observations of massive black holes in dormant galactic nuclei. These show that the holes are spectacularly underluminous, especially in relation to the estimated rate of mass supply. The best studied case is surely our Galactic center, where we know the hole mass to be $2.6 \times 10^6 M_\odot$ and the rate of mass supply is roughly $\sim 10^{22} \text{ g s}^{-1}$ while the bolometric luminosity is $\sim 10^{36} \text{ erg s}^{-1}$.¹⁷⁾ The detailed X-ray observations suggest that the emission is due to nonthermal emission from close to the hole.

There is another limit where these considerations may be very important. This is when the rate of mass supply is much greater than the Eddington rate and there is no difficulty in emitting radiation. The problem is in allowing these photons to escape from the Thomson thick inflowing gas. Again, it is likely that the inflow is effectively adiabatic, (though recent intriguing suggestions as to how radiation may escape through magnetized channels challenge this view.¹⁸⁾) However, if we suppose that super-Eddington accretion is adiabatic, then it too may be accompanied by prodigious mass loss. Perhaps this is what is happening in broad absorption line quasars (BALQ).

3.3. “Air” — the outflow

Having made the case that accretion at both high and low rates is accompanied by powerful winds, it is worth thinking about the implications. We already know that these winds are a prominent feature of accretion onto young stellar objects and they may well carry off much of the angular momentum and even much of the liberated energy.

As we discuss further below, these outflows may or may not collimate. If they do, then they become jets and are likely to be able to propagate to large distances from their sources as is observed in radio galaxies, quasars and, especially young stellar objects. If they do not, then they are likely to become supersonic and pass through a strong shock when their momentum flux falls to match the ambient pressure. These shocks could be sources of nonthermal emission as they are likely to accelerate relativistic electrons. In addition they may have an important role in shaping the accretion flow.

The BALQ outflows are particularly interesting as it is highly likely that they are subject to additional radiative acceleration due to the momentum flux carried by the “absorbed” (actually scattered) resonance line photons.¹⁹⁾ Naive arguments hold that the flow is highly inhomogeneous and composed of clouds with dimension given by the local pressure scale height $\sim r/M^2 \sim 10^{10} \text{ cm}$, where $M \sim 30000 \text{ kms}^{-1}/10 \text{ kms}^{-1}$ is the Mach number. It is possible that these clouds might be the result of a radiation-driven instability. (These remarks might also be relevant to SS433.)

3.4. “Fire” — the jet

Finally, there is the region extending from around the black hole and the inner edge of the accretion disk and extending out along the symmetry axis to form a jet. We presume that the plasma density will be very low and particle acceleration is

very efficient. This is also a region where the optical depth to Thomson scattering is likely to be quite low and bright ionizing radiation may be able to escape to form an ionizing cone.

Following successful detections by the EGRET telescope on CGRO, it has been possible to trace relativistic jets down to what are inferred to be even smaller radii than the “smoke” observed using VLBI. There is almost universal agreement that the gamma rays that are observed are produced by inverse Compton scattering by GeV-TeV electrons in a collimated relativistic outflow. However there has been controversy concerning the next level of description. In particular, it is not known whether or not the plasma comprises electrons and positrons or if it is a normal electron-ion plasma. The former option seems to be the more likely on the same general grounds that lead one to this conclusion in the case of pulsars. However it is quite hard to give a compelling observational demonstration. One approach is to try to estimate the power carried by the jets by observing them on large scales and then comparing this with the power carried by relativistic electrons derived on the basis of their radio emission.

This can be made a bit more precise in a self-absorbed synchrotron source, close to the inverse Compton limit. The energies of the emitting electrons at the radio photosphere must be ~ 300 MeV while lower energy electrons are essentially invisible. However, if they have proton partners, the lower cut off in the electron distribution function must be ~ 30 MeV, otherwise there would typically be too much jet kinetic energy, carried by the protons. Alternatively, if the partners are positrons, the pair distribution function can extend down to mildly relativistic energy without producing large jet powers. Similar conclusions are drawn from observations of strong linear polarization which limits the internal Faraday rotation within the source. Finally the detection of persistent circular polarisation has been used to rule in favor of a pair plasma, on the grounds that the frequency dependence is better fit by Faraday conversion in a pair plasma than intrinsic circular polarisation in a proton plasma.²¹⁾ Note that, if the radiation mechanism is coherent cyclotron radiation, then the fact that the polarisation is not even stronger than observed may be an indication that there is a pair plasma present.

One thing that is very clear is that jets cannot only comprise pairs close to the black hole. This is because pairs are subject to strong radiative drag and their outflow speeds would be limited to no more than mildly relativistic values. If protons are precluded then this suggests that the jet momentum is carried by electromagnetic field, at least initially. This is really not a very unusual conclusion. It is just what is thought to happen in the case of radio pulsars, like the one in the Crab Nebula.

Another controversy concerns the origin of the soft photons that are Compton-scattered by relativistic jets as γ -rays. One idea is that the soft photons originate within the jet as synchrotron emission, the so-called synchrotron self-Compton process. The other is that they originate outside the jet and are scattered into it. The resolution of this controversy appears to be that external scattering dominates in the most powerful sources and the self-Compton process is most important in weaker sources.

Many radio jets are also observed using HST at optical wavelengths and, quite

recently at X-ray energies using Chandra. The emission mechanisms seem to be quite varied. In some cases, like the X-ray knots of M87, it appears that synchrotron emission by > 3 TeV electrons are responsible for the emission. This places impressive demands on the acceleration mechanism within the jet. In other cases the emission is attributable to inverse Compton scattering of internal, external or microwave background photons.²²⁾

§4. The fifth element — magnetic field

I now turn to the *interactions* between these four elements (Fig. 1). (This is, after all, a meeting to celebrate the science of Professor Yukawa!) As I have emphasized, these are likely to be quite varied and complex. However they have the common feature that magnetic fields seem likely to be playing a major role. From a theoretical perspective, most attention has been paid to understanding the “magnetorotational instability” (MRI).²³⁾ This is a dynamical instability which, in its simplest form has a weak, vertical magnetic field becoming unstable with a growth time of order the orbital period. What it is really saying is that weakly magnetized, conducting disks are not viable. The only possibility is that disks be imbued with magnetic field of nonlinear strength which is responsible for the internal torque. The saturation field amplitude is determined by a balance between nonlinear growth, dissipative processes like reconnection and buoyant escape and can really only be estimated through careful, three dimensional, numerical simulations which are just now becoming possible.

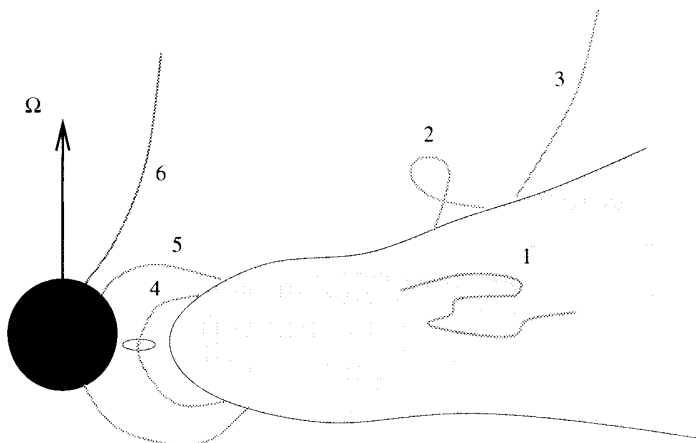


Fig. 1. Different types of magnetic field line discussed in the text. 1. The interior torque is contributed by magnetic field amplified through the magnetorotational instability. 2. Short loops of toroidal field will energize the disk corona, through having their footpoints being twisted in opposite senses and creating small scale flares. 3. Open field lines that connect the disk to the outflow may drive a hydromagnetic wind. Loops of field from the inner disk that connect to plunging gas (4) or the event horizon (5) of the hole can effectively remove energy from the hole. 6. Open field lines that cross the event horizon can power a relativistic jet, which may be collimated by and possibly decelerated by the outflow.

4.1. *Disk-outflow connection*

If the sun and the simulations, are any guide, then magnetic field that is growing in the disk will also escape into the corona. We expect coronal arches, as well as larger scale magnetic structures to be quite common and to be regenerated on an orbital timescale. If the footpoints of an arch are at different orbital radii in the disk then they will separate tangentially in a single period. Field lines will be stretched across the disk surface and will quickly be forced to reconnect. I therefore expect that most of the magnetic field that connects the disk to the corona will be toroidal and that most of the magnetic flux is “open” and escapes into the outflow. However, differential rotation acts in opposite senses on the footpoints of a single flux loop and this will cause the loop to twist and probably to undergo some topological rearrangement, just like a speeded-up solar flare. This provides one mechanism for heating a disk corona and perhaps for driving an outflow through thermal heating. Although these tangential flux loops may be regenerated by buoyancy, they will also be stretched and pulled back into the disk. Alternatively, it has been proposed that they will form an inverse cascade, creating larger and larger loops.²⁴⁾

Magnetized disks can also launch supersonic outflows.²⁵⁾ The disk may be threaded by a vertical field that is unidirectional, at least over an octave of radius. Differential rotation will also make the field approximately axisymmetric. Not only will this create a very effective coupling between the disk and the outflow but it can also facilitate the outflow by launching it either centrifugally or through the magnetic pressure associated with coiled up toroidal field. The outflow may be neither stable, nor even stationary. Again, if the field filaments into individual flux tubes, then twisting of the footpoints may lead to additional dissipation. One criticism that has been leveled against this mechanism is that any large scale magnetic flux threading the disk will migrate outward faster than it is advected inward if the effective magnetic Prandtl number is of order unity.

As mentioned above, a strong vertical field will not only drive an outflow but may even have implications for the evolution of the disk. It is possible that the dominant torque acting on disk be due to this large scale global magnetic field as oppose to the internal torque due to the nonlinear MRI. This torque removes energy and angular momentum in the ratio Ω just as required by energy and angular momentum conservation in the disk and so there need be no dissipation in the disk associated with an external torque of this type.

If magnetic winds exist, then they are likely to be strongly collimating as demonstrated by axisymmetric numerical simulations.²⁰⁾ The basic collimating action is due to magnetic field lines that are wrapped around the symmetry axis and which exert a tension leading to a radial force density.

4.2. *Disk-hole connection*

There is also a possible magnetic connection between the disk and the hole.^{26)-29),32),33)} Again there are several possibilities.³⁰⁾ If the field is confined to the disk plane and the hole is spinning rapidly, then gas that falls in from the inner disk edge, presumably located close to the marginally stable particle orbit, will

slide along magnetic field lines and allow magnetic energy flux to be transmitted outward in the form of a torque, thereby increasing the energy per unit mass that can be dissipated from the disk from that associated with the binding energy of the marginal stable orbit.

However, magnetic field is unlikely to be confined to the equatorial plane and, besides, this is where reconnection should be very actively disconnecting the inflow from the disk.³¹⁾ The field lines that leave the surface of the inner disk, go up to high latitude and then, connect with either the gas plunging into the hole or the event horizon of the black hole are likely to be more significant and they, too, can extract energy and angular momentum. They have the further advantage that they are likely to propagate in a region where the Alfvén speed is large and there is causal contact between the inflow and the disk from closer to the hole.

4.2.1. Black hole magnetosphere

In fact it is quite likely that, except in the region close to the disk or the infalling matter, the Alfvén speed will become relativistic which may allow substantial energy to be extracted from the black hole itself.^{37)–42)} Before we discuss this, though, let us make two key points adopting, for illustration, a typical $10^8 M_\odot$ hole accreting $\sim 10^{-2} M_\odot \text{ yr}^{-1}$. Firstly, if we assume a fairly thick disk and a viscosity parameter $\alpha \sim 0.01$, then the gas density in the disk will be $\sim 10^{-11} \text{ g cm}^{-3}$ and the pressure $\sim 10^8 \text{ dyne cm}^{-2}$. This can support a field of strength $\sim 10^4 \text{ G}$. It is important to realize that it is necessary for there to be a heavy disk present to contain the field. The electrical currents that act as a source of this field are external to the hole and the stresses that the field exerts on them are matched by pressure and gravity. Secondly, the gas density in the high latitude region around the hole, which we call the black hole magnetosphere, although likely to be quite low is not going to vanish altogether. This is important because only a tiny density of plasma, in this case $\sim 10^{-25} \text{ g cm}^{-3}$ is needed to supply enough charge to short out the potential differences along the magnetic field lines. It is hard to imagine that plasma could be excluded from the magnetosphere so efficiently that $\vec{E} \cdot \vec{B}$ is not effectively zero. Cross-field diffusion and pair production happen all too readily. Conversely, provided that the magnetospheric density is $\ll 10^{-14} \text{ g cm}^{-3}$ then the formal Alfvén speed will be ultrarelativistic so that the conditions are effectively electromagnetic and the matter will have almost no dynamical role. It is not clear if and when this second condition is satisfied, but it does not seem unreasonable to hypothesize that black hole magnetospheres are, in this regard similar to pulsar magnetospheres. The work function of a classical event horizon is infinite! If so, then electromagnetic field in the magnetosphere must be relativistically force-free.³⁵⁾ That is to say,

$$\rho \vec{E} + \vec{j} \times \vec{B} = 0. \quad (4.1)$$

In addition, we also require that the Lorentz invariant $B^2 - E^2 > 0$. The relativistic force-free condition is mathematically equivalent to perfect, relativistic MHD, where it is assumed that the electric field vanishes in the center of momentum frame, of the plasma, moving with velocity \vec{v} so that $\vec{E} + \vec{v} \times \vec{B} = 0$, in the limit that the inertia of the plasma can be ignored.

These two points cast into doubt some schemes for extracting energy from strongly charged black holes and magnetospheres where the electromagnetic field is supposed to satisfy the Einstein-Maxwell equations *in vacuo*.⁴³⁾

4.2.2. A digression on pulsars and causality

In order to explain, in more detail, how this energy extraction is thought to work under electromagnetic conditions, it may be helpful to start with an axisymmetric model of a pulsar.³⁴⁾ The model is simple but known to be incomplete — at the very least, pulsars are not axisymmetric. Nonetheless, it is helpful for explaining some principles. Under this model, we have a magnetic field that is frozen into the spinning neutron star and we can think of the field lines as moving with the same angular velocity $\vec{\Omega}$ as that of the star, provided that, as we have asserted above, $\vec{E} \cdot \vec{B} = 0$. What we mean by this statement is that the electric field vanishes in any local Lorentz frame moving with speed $\vec{v} = \kappa \vec{B} + \vec{\Omega} \times \vec{r}$, where κ is only limited by the requirement $v < c$. In the inertial frame, the electric field has a divergence which is satisfied by a charge density and currents flow along the magnetic field, driven by relatively small residual and quite possibly transient electric field. The poloidal component of the current is the source for a toroidal magnetic field whereas the electric field $\vec{E} = -(\vec{\Omega} \times \vec{r}) \times \vec{B}$ is poloidal. There is a surface, known as the light cylinder, with cylindrical radius c/Ω , beyond which plasma, which is tied to the moving magnetic field lines, must move outward. The combination of the poloidal electric field and the toroidal magnetic field leads to a radial Poynting flux that carries energy away from the star at the expense of its rotational kinetic energy; the combination of the poloidal magnetic field and the poloidal electric field carry away angular momentum and the ratio of the two quantities is $dE/dL = \Omega$ which is identical to the ratio of the energy to the angular momentum lost from the pulsar. The currents, which carry no net charge away from the star, complete their circuit at large distance from the pulsar. In order to do so, there must be either or inertial effects dissipation which allows electromagnetic energy to be converted into kinetic energy or heat. Most plausibly this would happen at a distant shock front, which we call the load.

There is one more point that should be made here, and it is important for what follows. This concerns the nature of the small amplitude, short wavelength wave modes in this system. There are two types of wave in the rest frame of a plasma where $\vec{E} \cdot \vec{B} = 0$ and particle inertia is ignorable. One is a fast mode with $\delta \vec{E} \propto \vec{k} \times \vec{B}$ which is indistinguishable from a vacuum electromagnetic wave; the other is an intermediate (or Alfvén) mode, with $\delta \vec{B} \propto \vec{k} \times \vec{B}$, which travels with phase speed $\hat{k} \cdot \hat{B}$ along \vec{k} and group velocity 1 along \vec{B} .

Let us look at these modes in the non-rotating, global inertial frame. What we find is that, if we restrict attention to axisymmetric modes, then the fast mode has a toroidal magnetic perturbation and the intermediate mode has a poloidal magnetic perturbation. This means that information about the toroidal magnetic field (and, correspondingly, the poloidal current, can propagate inward at effectively the speed of light across magnetic field lines even beyond the light cylinder. In other words, if we change the conditions in the outer dissipation region — for example

by changing the resistance in the load in the electrical circuit — then this will eventually react back upon the current flowing through the pulsar and change the Poynting flux. However, it will not change the EMF appreciably. By contrast, if we consider the intermediate mode, then the disturbances are constrained to flow along the magnetic field direction and can only propagate inward within the light cylinder. These disturbances essentially carry information about the toroidal current and the poloidal magnetic field.

I am belaboring these points in what, I hope, is a non-controversial context, because essentially the opposite conclusion has been drawn in a recent book by Pun-
sly⁴⁵⁾ (and earlier references cited therein). There are many points of disagreement but one key difference is that Punsly argues that the intermediate mode propagates information about the global charge and current density because they involve non-zero perturbations to the current density. I believe this argument to be incorrect. Just as sound waves do not physically transport mass despite having a density fluctuation, intermediate waves do not physically transport charge. Actually, in making the force-free approximation, Eq. (4.2), we are implicitly saying that, although the electromagnetic field evolves according to Maxwell's equations, $(\partial \vec{B}/\partial t = -\nabla \times \vec{E}$, $\partial \vec{E}/\partial t = \nabla \times \vec{B} - \mu_0 \vec{j}$, $\partial \rho/\partial t = -\nabla \cdot \vec{j}$), there is no corresponding evolution equation for the current, density which might be determined by regions where inertia or dissipation are important. The use of such relations is tantamount to saying that the timescale on which charge adjusts locally to the electromagnetic field is very short compared with the time it takes light to cross the circuit. In this way, an axisymmetric pulsar can act as a battery with negligible internal resistance and can drive current around a circuit in a manner that is eventually responsive to changes in the physical conditions in the load, well beyond the light cylinder.

4.2.3. Extraction of energy from the hole by the disk

Returning to the matter at hand, the field lines that connect the inner disk to the gas plunging into the hole, may be able to transport energy and angular momentum outward in a time-dependent manner. The field will quickly be dragged into the hole while its footpoints will be (temporarily) anchored in the disk and will rotate with the disk angular velocity Ω . Now, it is probably the case that the angular velocity of the black hole exceeds that of all of the disk. (If the disk terminates at the marginally stable orbit, then this is true for $\Omega_H > 0.093/m$.) Under these circumstances, the Einstein-Maxwell (plus relativistic force free) equations can be solved subject to suitable boundary conditions at the horizon (essentially that the electromagnetic field as measured by an infalling observer remain finite). It is found that the magnetic field will trail the hole and that, if we adopt axisymmetry, a slender equatorial ring, threaded by flux Φ , will experience a torque

$$G = \frac{I\Phi}{2\pi} = \frac{(\Omega_H - \Omega)\Phi^2}{4\pi^2 \Delta R_H}, \quad (4.2)$$

where I is the current circulating through the ring and into and out of the hole. In using this equation we need to know that the effective resistance of the horizon is effectively the impedance of free space, multiplied by geometrical factors, $\Delta R_H \sim$

$60\Delta\theta/\sin\theta\ \Omega$. Also, note that current can flow through the hole simply by having positive charges preferentially cross the horizon at high latitude and negative charges cross it at low latitude or vice versa.

This torque will do mechanical work on the disk at a rate $G\Omega$ and the energy transfer will ultimately be dissipated by viscous processes in the accretion disk in the form of heat. If we consider the causal structure of this electromagnetic configuration, then there will be an inner light surface, the counterpart of the pulsar light cylinder, within which intermediate waves cannot propagate outwards and there will be a fast mode surface, very close indeed to the horizon.^{39), 33)} The details of what happens close to the horizon are, as usual, unimportant as they are redshifted away. This electrodynamic configuration is, in no essential respect, different from what happens if we connect the flux lines to gas endowed with substantial resistivity orbiting in the equatorial plane between the disk and the horizon.⁴⁷⁾

Note though that the disk is actually extracting energy from the hole rather than the other way around. To see why this is possible, we must return to the properties of spinning holes. When we work in Boyer-Lindquist coordinates, we are exploiting the fact that the spacetime is stationary and axisymmetric and the metric is independent of time and azimuth. Associated with these two symmetries, are two conservation laws — those of energy and angular momentum. If we look at the electromagnetic Poynting flux in the Boyer-Lindquist frame, then the energy and angular momentum fluxes are conserved from the horizon outward, despite the fact that a physical observer within the ergosphere must move with respect to this coordinate system. If we transform to a frame in which physical observers orbit with angular velocity lying in $[\Omega_{\min}, \Omega_{\max}]$ then we find that the energy and angular momentum fluxes must be transformed and the energy flux must change sign close to the horizon. From the point of view of physical observers, the hole is a sink not a source of energy. The reason for this strange behavior is intrinsically relativistic and can be traced to the existence of the component $g_{0\phi}$ in the metric tensor. The hole's spin is communicated to the world exterior to its event horizon not through outwardly propagating electromagnetic disturbances emanating from close to the event horizon but through the metric tensor. It is really the spacetime around the hole, not the event horizon, from which the energy is being extracted.

4.2.4. Quasi periodic oscillations

How much of the above actually happens and how important it is quantitatively depend upon the details of the gas flow and the magnetic field configuration that develops. However, if the hole-disk connection is important, then it does open up the possibility that quasi-periodic oscillations might be excited by interaction with the hole.²⁷⁾

4.3. Hole-jet connection

Having explained how to connect a disk to a hole and extract energy, we can now explain how to connect the hole directly to a jet.^{37), 46)} (We do, however, still need a disk to retain the poloidal magnetic field.) Let us do this in two stages. Suppose that magnetic field lines which cross the horizon are frozen into a highly conducting

ring orbiting the hole at high latitude at a radius beyond the ingoing light cylinder. Now make charges available at the inner surface of the ring and allow current to flow around the circuit and energy to be extracted from the hole, just as we did with the disk. Next make charges available on the other side of the ring and allow it to behave like an axisymmetric pulsar and lose energy to a distant load. In general, the rate of energy (and angular momentum) gain from the hole at the interior surface of the ring will not balance the loss from the exterior surface to the load. However, if we adjust the angular frequency of the ring and the magnetic field lines which are frozen into it to a suitable compromise, between that of the hole and the load, determined by a simple circuit analysis,

$$\Omega = \frac{\Omega_H \Delta R_L + \Omega_L \Delta R_H}{\Delta R_L + \Delta R_H}, \quad (4.3)$$

where $\Omega_{H,L}$, $\Delta R_{L,H}$ are the effective angular velocity and resistance of the load and the hole respectively, then we should be able to achieve balance. Furthermore, this equilibrium should be stable. At this point, we can remove the conductor. Provided that we are still able to make charges available in the region between the two light surfaces, as we have argued will be the case, then currents can flow all the way around a circuit spanning the event horizon and the load. Energy will be lost by the hole and dissipated in the load. In fact there is generally some dissipation in the hole and although its gravitational mass may decrease, its irreducible mass will increase.

This is the basis of the proposal that spinning black holes power jets and, possibly, gamma ray bursts. The power is created as a large scale Poynting flux or equivalently as a battery-driven current flowing around an electrical circuit. (For the example quoted, the power evaluates to $\sim 10^{43}$ erg s $^{-1}$ and the order of magnitude estimate of the EMF and the current are $\sim 10^{19}$ V and $\sim 10^{17}$ A.) This neatly avoids the problem of catastrophic radiative drag close to the jet origin, and allows the terminal jet Lorentz factors to be very large as observations indicate is the case.

One objection to this model is that the flux threading the black hole may be small,^{37), 56)} (especially if the hole is slowly rotating) so that far more power is extracted from the inner disk, or, more plausibly the region between the inner disk and the horizon than from the hole directly. I emphasize the word directly because when energy is removed from gas in the ergosphere, the torques force that gas onto a lower energy, perhaps even a negative energy orbit, than it would ordinarily follow. This means that when the mass is eventually captured by the hole, its gravitational mass increases by less than it would do so in the absence of magnetic stresses. (It may even decrease.) We can therefore think of the energy as having been derived from the spin of the hole even when the torque is applied to accreting gas.

One way to concentrate much more flux onto the horizon of a rapidly spinning hole is to replace a thin disk with a thick toroidal flow, such as is thought likely to develop when the gas does not radiate effectively.^{48), 52)} It may even be necessary that this happen in order to form ultrarelativistic jets as this will minimize the effects of radiative drag.

4.3.1. Development of the jet

I have argued that a relativistic jet begins life close to a spinning black hole in an essentially purely electromagnetic form. However, what we observe are high energy Compton-scattered gamma rays and lower frequency synchrotron radiation. Given the large electric fields present in the jets, it is very easy to imagine pairs being created copiously near the hole. Now, the pair energy density is unlikely to become very large here because it will be limited by annihilation. Further from the hole, though, when the energy density diminishes, pairs and gamma rays will carry a progressively larger fraction of the energy flux.

4.3.2. Gamma-ray bursts

As more is being learned about gamma ray bursts, the more they appear similar to AGN. The inferred bulk Lorentz factors of GRBs and AGN are separated by less than an order of magnitude. GRBs are argued to have a low baryon fraction just like radio jets. Studies of jets increasingly show them to be episodic phenomena as opposed to continuous flows. Finally, the appearance of achromatic breaks in the development of GRB afterglows has been interpreted as indicating that they too are jet flows beamed towards us, (though these observations may also be associated with the trans-relativistic evolution of spherical blast waves). If GRBs are mostly jets, then this reduces the energy per burst by two or three orders of magnitude at the expense of increasing their overall frequency.

Many models of GRBs also involve the extraction of large amounts of energy from stellar mass black holes and electromagnetic processes occurring at or near the horizon of the black hole have commonly been invoked. The magnetic fields and EMFs involved ($\sim 10^{16}$ G, $\sim 10^{23}$ V) are much larger than those associated with AGN but this should create no difficulty of principle.

4.4. Jet-outflow connection

We can now complete the cycle and consider the interaction of the jet with the less collimated and slower outflow associated with the disk. The most important feature of this interaction is, of course, that the wind may be responsible for collimating the jet. It may provide an effectively invisible sheath that protects the rather fragile jet outflow from interaction with its environment.

A second feature of the jet-outflow interaction is that there will surely be some entrainment of the electron-ion plasma and this should ultimately show up in the polarization observations which can, in principle distinguish a pair plasma from a protonic plasma. Entrainment will be promoted by linear instabilities that can grow at the jet surface.⁴⁴⁾

Thirdly, there can be an exchange of linear momentum with the surrounding outflow, even if there is minimal exchange of mass. This can, in turn, have two consequences. The jet, itself, is likely to develop a velocity profile so that different parts move with different Lorentz factors. This implies that a radio observer is likely to infer a value for the Lorentz factor from the rate of superluminal expansion that depends upon the inclination i of the line of sight to the jet axis. Making the simplest assumptions, we expect that the measured Lorentz factor will be $\sim i^{-1}$. This could

lead to some very strong biases in interpreting the statistical properties of a sample of compact radio sources.

These features could account for the difference between Fanaroff-Riley Type I and Type II sources. Perhaps the former arise when the outflow has more linear momentum than the jet which is ultimately decelerated to speeds of a few hundred km s^{-1} , comparable with that of the outflow emanating from the outer disk. Conversely, if the jet carries more linear momentum, it can drag along the outflow as presumably happens in the Type II sources, with little in the way of observed consequences.

§5. Numerical simulations

Much of what I have summarized is qualitative and conjectural and the debates that I have highlighted revolve largely around different prejudices as to how magnetized, three dimensional flows behave in strong gravitational fields. There are serious issues of theory that need to be settled independent of what guidance we get from observations of astrophysical black holes and those of other sources, like the solar corona, where magnetic field holds sway. (I might also remark that there is the strong prospect of laboratory experiment being highly relevant in teaching us the true laws of MHD.)

The best prospects probably lie with performing numerical simulations or experiments. Numerical MHD is coming of age.^{49)-51), 53)} The sophistication of the difference schemes and visualization techniques is growing apace with the speed and especially the memory of large parallel-processing computers. Well — resolved 3-D (and even 4-D) simulations are becoming increasingly common and they rarely fail to surprise us. The symmetry-breaking involved in transitioning from 2-D to 3-D is crucial and leads to qualitatively new phenomena.

The key to using simulations productively is to isolate questions that can realistically be addressed and where we do not know what the outcome will be and then to analyze the simulations so that we can learn what is the correct way to think about the problem and to describe it in terms of elementary principles. Simulations in which the input physics is so circumscribed that they merely illustrate existing prejudice are of less value!

Let me frame some of these issues using a series of questions, many of which are already being tackled, as we have heard at this meeting.

- *What is the global evolution of the magnetorotational instability?* The re-discovered linear instability is so important that it is irrelevant! The non-linear evolution has been followed in 2- and 3-D, mostly in shearing boxes, particularly with regard to following the development of stress. Is there a cascade of wave energy to small scale through an inertial range turbulence spectrum or does most of the energy follow an inverse cascade to form large loops as has also been conjectured? Another question is “What is the role of buoyant escape of magnetic flux as well as the negative buoyancy associated with magnetic tension as described above?”. Existing simulations suggest that it is too slow to be important. However only a few regimes have been studied. Most important of all, we would like to know if large scale magnetic fields develop or can be

sustained in Keplerian disks. One approach to this problem is to use simulations performed for a few dynamical time scales to try to measure the effective average resistivity for large scale fields. Another approach is look for signs of an inverse cascade developing in the presence of strong differential rotation.

- *What is the nature of the dissipation in magnetized flows?* It is also necessary to understand the flow of energy and, in particular, the nature of the dissipation, seen from a fluid perspective. This is a pre-requisite to understanding dissipation from a kinetic perspective which, as the above makes clear, is crucial to understanding the radiative and consequently the dynamical properties of adiabatic flows. The most common assumption is that the dynamical magnetic field creates a magnetosonic wave spectrum that dissipates on some inner scale, small compared with the thickness of the disk. However it is also possible that magnetic reconnection contributes to the dissipation in addition to being a switch to bring about topological re-arrangement of the field. If reconnection heating is significant, then is it the occasional large events that dominate the heating or is it the ongoing “nanoflares” that are most important. (This is an issue of active debate in studies of solar coronal heating.)
- *How do magnetic fields develop in radiation-dominated disks?* One interesting possibility is that they are essential in allowing a super-Eddington radiation flux to escape without blowing away the gas by creating large density inhomogeneities. Understanding the strength of the torques in the region where most of the energy is released is of central importance in modeling quasar spectra.
- *How and where are disk coronae heated?* X-ray observations of Seyfert galaxies are commonly and fairly convincingly interpreted in terms of a model in which a coronal source, in the form of a Comptonized power law extending up to ~ 100 keV illuminates a disk. The reflection spectra create the curved continuum and the fluorescent iron lines. It has generally been supposed that the source is powered magnetically. However, it is again not clear if this is due to a few large flares at high altitude or a multitude of small flares occurring close to the disk. In this case the sun may not be a very good guide, because accretion disks are fast rotators in the sense that centrifugal force is more important than pressure gradients whereas the rotation that ultimately drives solar flares is only a small perturbation. Understanding the origin of the X-ray power law continua is also crucial for assessing if it will ever be possible to perform reverberation mapping at X-ray energies.
- *If large scale fields develop in accretion disks, how strong are the outflows and which mechanism dominates their formation?* Several possibilities have been discussed in addition to centrifugally-driven outflows. These include thermally-driven winds, outflows driven by the pressure of a strongly coiled magnetic field and outflows in which the field is basically poloidal. A major issue is the stability of these outflows and here, again, three dimensional simulations make all the difference. Preliminary studies of centrifugal winds show that they are surprisingly resilient.
- *Can electromagnetic power really be extracted from a spinning black hole and is it enough to power relativistic jets?* Here there are two questions. The first

is one of physics principle, as I have discussed. What is probably necessary to settle the dispute alluded to above is to perform a time-dependent calculation in a Kerr metric. I have argued that it is sufficient to perform this calculation in the electromagnetic limit; there is no need to carry the unwanted baggage of inertial terms in the limit that these are very small as must be the case if it is intended to account for ultrarelativistic jets. The second question is how much flux can be concentrated onto the black hole relative to that which threads the inner disk? This must be a hydromagnetic calculation.

- *How do jets propagate, collimate and dissipate?* There has been steady progress in describing disk winds numerically. What is really needed now is to produce hybrid simulations that combine an ultrarelativistic jet core with a subrelativistic wind. The flows may become unstable, especially as toroidal field starts to dominate, but this does not imply that jets destroy themselves catastrophically. Indeed many of the maps from VLBI monitoring programs look like helically unstable flows.
- *Can ultrarelativistic jets really be formed inside stars?* As of this writing, the collapsar model appears to be the leading candidate for explaining the long duration GRBs. This is not on the face of it the most propitious environment to create an ultrarelativistic, baryon-starved jet. I have argued that electromagnetic effects, similar to those already invoked for AGN jets, seem to be the most promising way to create the enormous powers required. What needs to be demonstrated is that the outflow is not “poisoned” by baryons by the time that it reaches the surface of the star. It is not, in my view necessary to collimate the jet very tightly or achieve high bulk Lorentz factor as the flow leaves the stellar surface. As long as the emergent flow has a high enthalpy per baryon, it will expand on a Mach cone and achieve its high terminal speed some distance from the star. This, incidentally, provides a natural explanation for the otherwise puzzling initial absence of causal contact across an expanding ultrarelativistic jet — similar to the famous “hello-goodbye-hello” behavior of an inflationary universe.

It is easier to make such lists than to implement them.

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