Radion and Large Scale Anisotropy on the Brane

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We investigate the effect of displacements of the brane in the extra dimension. The S^1/Z_2 compactified 5D anti-de Sitter spacetime bounded by positive and negative tension branes is considered. The relative displacement of a brane is called a "radion". We study displacements caused by the "fluctuation" of a brane without the matter energy-momentum tensor on the brane. We derive the solution for a homogeneous fluctuation of an expanding brane. It is found that the homogeneous brane fluctuation interacts with an anisotropic bulk perturbation and thereby affects the anisotropy of the brane. By determining the bulk anisotropic perturbation, we calculate the homogeneous metric perturbations on the positive tension brane and find a large-scale CMB anisotropy. An interesting finding is that the radion contributes to the CMB anisotropy if the distance between the two branes is time dependent. The observational consequences of these effects are discussed.

§1. Introduction

String theory suggests the idea of confining the standard model particles to a 3-brane in a higher-dimensional spacetime. Based on this brane world idea, Randall and Sundrum proposed very interesting models with branes in a 5D anti-de Sitter (AdS) spacetime. If there is a single positive tension brane, 4D Newton gravity can be recovered on the brane although the extra dimension extends infinitely.[1)]

In brane world models, a new geometrical degree of freedom is introduced, i.e. the displacement of the brane in the extra-dimension. If we choose an appropriate coordinate gauge, the displacement of the brane is described by the scalar field existing on the brane. For example, the displacement of the Minkowski brane φ obeys the equation

$$\Box_4 \varphi = \frac{\kappa^2}{6} T, \tag{1·1}$$

where T is the trace of the energy-momentum tensor of the matter on the brane and $\Box_4$ is the d'Alembertian operator in 4D Minkowski spacetime.[3)] Thus there are two kinds of displacements of the brane, corresponding to the two kinds of solutions of Eq. (1·1), i.e. the homogeneous and the particular solutions. The homogeneous solution for Eq. (1·1) with $T = 0$ represents the "fluctuation" of the brane. The brane can fluctuate by itself, without the matter energy-momentum tensor. The other kind of displacements is the "bend" of the brane, due to the matter on the brane. The trace of the energy-momentum tensor acts as a tension, and the brane bends due to this effective tension.

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Of particular interest is the detectability of the brane fluctuation. A similar situation was considered in the analysis of the fluctuation of a thin domain wall.[4)] It was shown that the wall fluctuation cannot be seen by an interior observer on the wall, because the fluctuation does not change the curvature of the domain wall. However, this analysis was done only for a test domain wall. If we take into account gravitational perturbations, it may be possible to see the effect of the fluctuation.[5)] The same statement applies to the brane fluctuation. If there is only one brane, the bulk possesses a translational invariance, at least when the energy density of the matter on the brane is sufficiently smaller than the tension of the brane. In this case, the gravitational perturbations are not affected by the brane fluctuation. Because the brane fluctuation can be detected only through the interaction with gravitational perturbations in the bulk, the brane fluctuation has no physical degree of freedom in the one-brane model. However, if there are two branes, the gravitational perturbations are confined between the two branes. Resulting displacements of the branes change the distance between the two branes, and they should therefore affect the gravitational perturbations. Thus the brane fluctuation acquires a physical degree of freedom, and there is a possibility to detect it. If the extra dimension is Z_2 symmetric, it is compact. The brane fluctuation indeed changes the size of the extra dimension. For this reason, the brane fluctuation is called a "radion" in the literature.[6)]

Let us consider a S^1/Z_2 compactified 5D AdS spacetime bounded by two positive and negative tension branes.[2)] For such a system, it was shown that the gravity on the branes is described by the Brans-Dicke (BD) theory where the radion acts as a BD scalar.[3)] If we are living on a positive tension brane, the BD parameter can be compatible with observations if the distance between the two branes is sufficiently large.

Now let us consider cosmology based on this scenario. The branes expand due to the matter energy-momentum tensor on the brane. On large scales, the brane exhibits a homogeneous fluctuation. Interestingly, the dynamics of a homogeneous and isotropic brane are determined only by the matter energy-momentum tensor on the brane if the bulk is a purely AdS spacetime. Thus a homogeneous radion does not affect the evolution of a homogeneous and isotropic brane. However, if one allows spatial anisotropy of the brane, the situation changes significantly. In this case, an anisotropic perturbation in the bulk is allowed to exist. Then, the homogeneous brane fluctuation interacts with the anisotropic bulk perturbation, and the homogeneous radion contributes to the anisotropy of the positive tension brane. Our universe has an anisotropy, which is measured by the temperature anisotropy of the cosmic microwave background (CMB). Thus it is important to clarify the contribution of the homogeneous radion to the CMB anisotropy. The aim of this paper is to derive a solution for the homogeneous radion in the case of an expanding brane and investigate its effect on the anisotropy of the positive tension brane.

Owing to the limitation on the length of this paper, we omit detailed calculations. Interested readers can find these calculations in Ref. 7).

§2. Set up of the model

We consider two branes located at the S_1/Z_2 orbifold fixed points in 5D AdS spacetime. Our system is described by the action

$$S = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\left(\mathcal{R}^5 + \frac{12}{l^2}\right) + \sum_{i=A,B}\left(-\mu^i\int d^4x\sqrt{-g_{\mathrm{brane}\,i}} + \int d^4x\sqrt{-g_{\mathrm{brane}\,i}}\mathcal{L}_{\mathrm{matter}^i}\right), \tag{2·1}$$

where $\mathcal{R}^5$ is the 5D Ricci scalar, l is the curvature radius of the AdS spacetime and $\kappa^2 = 8\pi G_5$, where G_5 is the 5D Newton constant. The brane A has positive tension μ^A, and the brane B has negative tension μ^B. These are taken as

$$\kappa^2\mu^A = \frac{6}{l}, \quad \kappa^2\mu^B = -\frac{6}{l}, \tag{2·2}$$

in order to ensure that each brane becomes Minkowski spacetime when there is no matter on it. The induced metric on the brane i is denoted by $g_{\mathrm{brane}\,i}$, and the matter that is confined to the brane i is described by the Lagrangian $\mathcal{L}_{\mathrm{matter}^i}$. We assume that we are living on the positive tension brane A and we do not explicitly express the index $i = A$ in the following.

We take the metric for the background spacetime as

$$ds^2 = e^{2\gamma(y,t)}dy^2 - e^{2\beta(y,t)}dt^2 + e^{2\alpha(y,t)}\delta_{ij}dx^i dx^j. \tag{2·3}$$

The extra coordinate y is compact and runs from $-l$ to l. Furthermore, the identification of (y, t, x^i) with $(-y, t, x^i)$ is made. Then the extra dimension becomes S^1/Z_2 compactified space. The brane A is located at $y = 0$ and the brane B is located at $y = l$. The 5D energy-momentum tensor (including the tensions of the branes) is taken as

$$T^M_N = \left[\left(-\frac{6}{\kappa^2 l}\mathrm{diag}(0,1,1,1,1) + \mathrm{diag}(0,-\rho,p,p,p)\right)\delta(y) + \left(\frac{6}{\kappa^2 l}\mathrm{diag}(0,1,1,1,1) + \mathrm{diag}(0,-\rho^B,p^B,p^B,p^B)\right)\delta(y-l)\right]. \tag{2·4}$$

We write the power series expansion of the metric near the branes as

$$\alpha(y,t) = \alpha_0(t) + \alpha_1(t)|y| + \frac{\alpha_2(t)}{2}y^2 + \cdots,$$
$$\alpha(y,t) = \alpha_0^B(t) + \alpha_1^B(t)|y-l| + \frac{\alpha_2^B(t)}{2}|y-l|^2 + \cdots. \tag{2·5}$$

When we consider the perturbations, the explicit form of the bulk metric must be known. In addition, we also need to solve the bulk evolution equation for perturbations. It is generally difficult to exactly solve the evolution equations in the bulk. For this reason, we solve the evolution equation in the bulk by assuming that the system is nearly static. To do so, we assume that the energy density of the matter on the brane is sufficiently smaller than the tension of the brane

$$\kappa^2 l\rho \ll 1, \quad \kappa^2 l\rho^B \ll 1. \tag{2·6}$$

From the Friedmann equation on the brane, the time derivative of the metric $\dot{\alpha}$ is of the order $(\kappa^2 l^{-1}\rho)^{1/2}$. Also, from the junction condition, the y derivative of the metric α' is of the order l^{-1} for $\kappa^2 l\rho \ll 1$. Then the time derivative of the metric is much smaller than the y-derivative of the metric:

$$\left(\frac{\partial_t \alpha}{\partial_y \alpha}\right)^2 \sim \kappa^2 l\rho \ll 1. \tag{2·7}$$

The bulk metric can be obtained by solving the Einstein equation and junction conditions perturbatively in terms of $\kappa^2 l\rho$. The leading order solutions are obtained as[8)]

$$\alpha = -b(t)\frac{y}{l} + \alpha_0(t), \;\; \beta = -b(t)\frac{y}{l}, \;\; \gamma = \log b(t), \tag{2·8}$$

where $b(t)$ is the function that describes the time evolution of the physical distance between two branes:

$$\int_0^l dy e^{\gamma(y,t)} = lb(t). \tag{2·9}$$

We assume that the time dependence of $b(t)$ is also weak, i.e. $(\dot{b}/l)^2 \sim \kappa^2 l\rho \ll 1$. The behavior of the scale factor α_0 and the distance between the two branes $b(t)$ are determined by the next order equations. It should be noted that the function $b(t)$ is also called the "radion" in the literature.[8)] In order to avoid the confusion, we use the term "radion" only in the reference to the displacements of the brane in this paper.

§3. Brane fluctuation

In this section, we find the solution for the brane fluctuation. The brane fluctuation is not coupled to the energy-momentum tensor of the matter on the brane. Because the evolution of the brane universe is determined solely by the matter energy-momentum tensor, the brane fluctuation does not change the evolution of the brane universe. Therefore, we will find the displacement of the brane that does not affect the evolution of the homogeneous and isotropic brane.

Let us consider a brane A that is located at $y=0$. Now, suppose that the location of the brane is displaced infinitesimally. The displaced brane is denoted by $\hat{A}$. The displaced brane is no longer located at $y=0$. It is convenient to perform an infinitesimal coordinate transformation and go to the coordinate system in which the displaced brane $\hat{A}$ is located at $\hat{y}=0$. For this purpose, we perform the coordinate transformation

$$x^M \to x^M + \xi^M, \quad \xi^M = (\xi^y(y,t), \xi^t(y,t), 0), \tag{3·1}$$

which preserves the homogeneity and isotropy of the brane. By choosing an appropriate ξ^t, we can impose the normal condition $\hat{g}_{y0}=0$. This choice is

$$\xi^t(y,t)=\int_0^y dy e^{2(\gamma-\beta)}\dot{\xi}^y(y,t)+T_0(t), \tag{3·2}$$

where $T_0(t)$ is the residual gauge transformation which depends only on time t. Then, the induced metric on the displaced brane $\hat{A}$ is given by

$$ds^2_{\text{brane}A}=-e^{2\hat{\beta}_0(t)}dt^2+e^{2\hat{\alpha}_0(t)}\delta_{ij}dx^i dx^j, \tag{3·3}$$

where

$$\begin{aligned} e^{2\hat{\beta}_0}&=e^{2\beta_0}(1+2\dot{T}_0+2\dot{\beta}_0T_0+2\beta_1\xi^y_0),\\ e^{2\hat{\alpha}_0}&=e^{2\alpha_0}(1+2\dot{\alpha}_0T_0+2\alpha_1\xi^y_0). \end{aligned} \tag{3·4}$$

Let us find the coordinate transformation ξ^y that does not change the evolution of the brane. It should be noted that the evolution of the brane is determined only by $\alpha_0,\beta_0,\alpha_1$ and β_1. Hence, we first demand that the metric on the brane be unchanged by the coordinate transformation; that is,

$$\hat{\alpha}_0=\alpha_0, \quad \hat{\beta}_0=\beta_0. \tag{3·5}$$

Then $T_0(t)$ is determined by ξ^y_0 as

$$T_0=-\frac{\alpha_1}{\dot{\alpha}_0}\xi^y_0, \quad \dot{T}_0=-\beta_1\xi^y_0-\dot{\beta}_0T_0. \tag{3·6}$$

Hence, the physical displacement of the brane $\varphi=e^{\gamma_0}\xi^y_0$ should satisfy

$$\varphi-\frac{\alpha^2_{0,\tau}}{\alpha_{0,\tau\tau}}\left(\frac{1}{\alpha_{0,\tau}}\varphi_{,\tau}-\varphi\right)=0, \tag{3·7}$$

where τ is the cosmic time on the brane. Next, we consider the first derivatives of the metric with respect to y. In order to ensure that the evolution of the metric $\hat{\alpha}_0=\alpha_0$ and $\hat{\beta}_0=\beta_0$ on the displaced brane $\hat{A}$ is the same as the evolution of the metric on the brane A, the junction conditions should be the same as the junction conditions for the brane A. These conditions also give the equation for φ,

$$\varphi_{,\tau\tau}+(2+3c_s^2)\alpha_{0,\tau}\varphi_{,\tau}-\left(3\alpha^2_{0,\tau}+2\alpha_{0,\tau\tau}+\frac{\alpha^2_{0,\tau\tau}}{\alpha_1^2e^{-2\gamma_0}}+3c_s^2\alpha^2_{0,\tau}\right)\varphi=0. \tag{3·8}$$

It is very interesting that this equation has a conserved quantity ζ_*, where

$$\zeta_*=-\alpha_1e^{-\gamma_0}\left[\varphi-\frac{\alpha^2_{0,\tau}}{\alpha_{0,\tau\tau}}\left(\frac{1}{\alpha_{0,\tau}}\varphi_{,\tau}-\varphi\right)\right]. \tag{3·9}$$

If $\zeta_*=0$, the condition (3·9) is compatible with the condition (3·7). For $w=c_s^2=$ *const*, we can integrate Eq. (3·7) and obtain the solution for φ. At high energies ($\kappa^2l\rho\gg 1$) and at low energies ($\kappa^2l\rho\ll 1$), we obtain the solution as

$$\begin{aligned} \varphi(t)=f(t), \quad f(t)&=f_*e^{-(3w+2)\alpha_0}, \quad (\text{for } \kappa^2l\rho\gg 1)\\ f(t)&=f_*e^{-\frac{3w+1}{2}\alpha_0}, \quad (\text{for } \kappa^2l\rho\ll 1) \end{aligned} \tag{3·10}$$

where f_* is the integration constant. These are the solutions for the brane fluctuation. The same argument holds for the fluctuation of the brane B. We will denote the fluctuation of the brane B as $f^B(t)$.

§4. Radion and large scale anisotropy on the brane

4.1. *Anisotropic shear in the brane world*

In the previous section, we found the solution for the brane fluctuation $f(t)$ which does not change the evolution of a homogeneous and isotropic universe. In this case, the evolution of the brane fluctuation is determined locally, and it is not necessary to find the geometry of the whole 5D spacetime. However, once we take into account the anisotropy of the brane, the situation changes significantly.

Let us consider a homogeneous but slightly anisotropic 5D spacetime. The metric for this 5D spacetime is taken as

$$ds^2 = e^{2\gamma(y,t)}dy^2 + e^{2\beta(y,t)}dt^2 + e^{2\alpha(y,t)}(\delta_{ij} + \Pi_{ij}(y,t))dx^i dx^j, \tag{4·1}$$

where

$$\begin{aligned} \Pi_{ij}(y,t) &= 0, \quad (\text{for} \ \ i=j) \\ &= \Pi(y,t). \quad (\text{for} \ \ i \neq j) \end{aligned} \tag{4·2}$$

The branes are again located at $y=0$ and $y=l$, respectively. For a linear anisotropic shear $\Pi(y,t) \ll 1$, the evolution equation for $\Pi(y,t)$ is given by

$$-(\Pi'' + (3\alpha' + \beta' - \gamma')\Pi') + e^{-2(\beta-\gamma)}(\ddot{\Pi} + (3\dot{\alpha} - \dot{\beta} + \dot{\gamma})\dot{\Pi}) = 0. \tag{4·3}$$

Let us investigate the effect of the brane fluctuation on the anisotropy of the brane. Now, suppose that the brane is displaced due to the brane fluctuation. We again perform an infinitesimal coordinate transformation and consider the coordinate system in which the brane is located at $\hat{y}=0$:

$$x^M \to x^M + \xi^M, \quad \xi^M = (Y(y,t,x^i), T(y,t,x^i), X^i(y,t,x^i)). \tag{4·4}$$

In an anisotropic spacetime, a coordinate transformation that depends on the spatial coordinate is allowed. We take the coordinate transformation function to be

$$Y = \xi^y(y,t)\omega(x^i), \;\; T = \xi^t(y,t)\omega(x^i), \;\; X^i = \xi(y,t)\sigma^i(x^i), \tag{4·5}$$

where $\omega(x^i)$ and $\sigma^i(x^i)$ are functions of the spatial coordinates given by

$$\begin{aligned} &l^2\omega(x^i) = l^2 + (x^1x^2 + x^2x^3 + x^3x^1), \\ &l\sigma^1(x^i) = x^2 + x^3, \quad l\sigma^2(x^i) = x^3 + x^1, \quad l\sigma^3(x^i) = x^1 + x^2. \end{aligned} \tag{4·6}$$

The functions $\omega(x^i)$ and $\sigma^i(x^i)$ were determined so that the spatial homogeneity of the universe is preserved after the coordinate transformation. As in the isotropic universe, we can find a coordinate transformation that does not change the evolution of the metric by choosing $\xi^y_0 = e^{-\gamma_0}f(t)$ and

$$\xi^t(y,t) = \int_0^y dy e^{2(\gamma-\beta)}\dot{\xi}^y(y,t) + T_0(t), \tag{4·7}$$

where T_0 is given by Eq. (3·6). The normal condition $\hat{g}_{yi} = 0$ and the condition $\hat{g}_{i0} = 0$ on the brane A can be satisfied by choosing an appropriate X^i:

$$\xi(y,t) = -l^{-2}\int_0^y dy e^{2(\gamma-\alpha)}\xi^y(y,t) + l^{-1}X_0(t),$$
$$\dot{X}_0(t) = l^{-1}e^{2(\beta_0-\alpha_0)}T_0(t). \tag{4·8}$$

The trace part of g_{ij} is not transformed. However, the traceless part of g_{ij}, namely the anisotropic shear $\Pi(y,t)$, is transformed as

$$\hat{\Pi}(y,t) = \Pi(y,t) - 2l^{-2}\int_0^y e^{2(\gamma-\alpha)}\xi^y(y,t) + 2l^{-1}X_0(t). \tag{4·9}$$

Thus the junction condition for Π is given by

$$\hat{\Pi}_1 = 0 = \Pi_1 - 2e^{(\gamma_0-2\alpha_0)}l^{-2}f(t). \tag{4·10}$$

Thus, now the problem is to solve the wave equation for $\Pi(y,t)$, i.e. Eq. (4·3) with the boundary conditions (4·10) and to calculate the anisotropic shear $\hat{\Pi}_0(t) = \Pi_0(t) + 2l^{-1}X_0(t)$ on the displaced brane. If the solution for $\hat{\Pi}_0$ is different from $\Pi_0(t)$, we conclude that the brane fluctuation affects the anisotropy of the brane.

4.2. *Anisotropic shear induced by brane fluctuation*

As mentioned in §2, we solve the bulk evolution equation using the assumption of a nearly static configuration. The bulk metric is given by Eq. (2·8). Then the wave equation for Π in the bulk is given by

$$\Pi'' - b(t)\frac{4}{l}\Pi' - b(t)^2 e^{2b(t)y/l}\left(\ddot{\Pi} + \left(\frac{\dot{b(t)}}{b(t)} + 3\dot{\alpha}_0 - 2\dot{b}(t)\frac{y}{l}\right)\dot{\Pi}\right) = 0. \tag{4·11}$$

Here, the time dependence of Π is assumed to be weaker than the y dependence of Π:

$$\left(\frac{\partial_t \Pi}{\partial_y \Pi}\right)^2 = \mathcal{E} \ll 1. \tag{4·12}$$

Within this approximation, we obtain the following solution for $\dot{\hat{\Pi}}_0$ that depends on $f(t)$:

$$\dot{\hat{\Pi}}_0 = -2e^{-2\alpha_0}\frac{f(t)}{\dot{\alpha}_0}l^{-3}N(b(t)), \quad N(b(t)) = \left(\frac{e^{-b(t)}}{2\sinh b(t)}\right). \tag{4·13}$$

Thus one can see that the brane fluctuation becomes a source of anisotropic shear. However, if we take $b \to \infty$ yielding the one-brane model, we get $N(b(t)) \to 0$. Hence, in this case the brane fluctuation does not affect the evolution of the anisotropic shear. Thus, the brane fluctuation has no physical degree of freedom in the one-brane model.

§5. Conclusion and discussion

In this paper, we investigated the effect of displacements of a brane in the extra dimension. We considered the S^1/Z_2 compactified 5D AdS spacetime. The positive and negative tension branes are located at the orbifold fixed points. It is assumed that we are living on the positive tension brane. We showed that a homogeneous fluctuation of this brane $f(t)$ evolves as $f(t) \propto e^{-(3w+1)\alpha_0/2}$ at low energies, where $e^{\alpha_0(t)}$ is the scale factor of the brane. Such a homogeneous brane fluctuation can interact with an anisotropic perturbation of the bulk. In this way, the anisotropy of the brane is affected by the fluctuation. We derived the anisotropic shear induced by the brane fluctuation. In order to derive this solution, it is necessary to solve the bulk evolution equation. We solved this equation with the assumption that the system is nearly static.

An interesting point is that the CMB anisotropy due to the Sachs-Wolfe effect is given by

$$\frac{\Delta T}{T} = -\frac{1}{2} e^{\alpha_0} \frac{d}{dt}\left[e^{\alpha_0} \dot{\hat{\Pi}}_0 \right] = \frac{\dot{N}(b(t))}{\dot{\alpha}_0} l^{-1} f(t). \tag{5·1}$$

Then, if the distance $b(t)$ is time dependent, the brane fluctuation will indeed affect the CMB anisotropy. However, if the distance between two branes is time independent, the anisotropic shear induced by the brane fluctuation will not contribute to the CMB anisotropy. The time variation of the distance between the two branes makes the effective 4D Newton constant vary with time. This time variation of the 4D Newton constant is constrained by observations. If we consider only the displacement of our brane, the modification induced by the time variation of the distance is suppressed by the factor $\dot{N}(b(t))/\dot{\alpha}_0$, which is of order 10^{-6} at the decoupling. Because the brane fluctuation $f(t)$ is a decreasing function of time in a dust dominated universe, it is difficult to detect the fluctuation of our brane.

If we consider displacements of the hidden brane, the situation becomes more complicated. Displacements of the hidden brane appear in the CMB anisotropy on our brane in a non-trivial manner. It would be very interesting to perform detailed calculations of their effects with some specific models.

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