

FUSION03, Concluding Remarks

A. B. BALANTEKIN^{*)}

University of Wisconsin, Department of Physics, Madison, Wisconsin 53706, USA

Fusion reactions below the Coulomb barrier provide new insights into multidimensional quantum tunneling, nuclear reaction dynamics and nuclear structure. These reactions are also of considerable interest to nuclear astrophysics. In this summary recent developments in the field are reviewed and open questions related to subbarrier fusion are presented.

§1. Introduction

Fusion reactions below the Coulomb barrier provide new insights into multidimensional quantum tunneling, nuclear reaction dynamics and nuclear structure.^{1),2)} These reactions are also of considerable interest to nuclear astrophysics. The evolution of main sequence stars, in particular stellar nucleosynthesis is governed by subbarrier fusion.³⁾

It may be worthwhile to review some of the basic aspects of fusion reactions near and below the Coulomb barrier. The experimental observables are the cross section

$$\sigma(E) = \sum_{\ell=0}^{\infty} \sigma_{\ell}(E), \quad (1.1)$$

and the average angular momenta

$$\langle \ell(E) \rangle = \frac{\sum_{\ell=0}^{\infty} \ell \sigma_{\ell}(E)}{\sum_{\ell=0}^{\infty} \sigma_{\ell}(E)}. \quad (1.2)$$

The partial-wave cross sections are given by

$$\sigma_{\ell}(E) = \frac{\pi \hbar^2}{2\mu E} (2\ell + 1) T_{\ell}(E), \quad (1.3)$$

where $T_{\ell}(E)$ is the quantum-mechanical transmission probability through the potential barrier and μ is the reduced mass of the projectile and target system. For a one-dimensional barrier transmission probabilities can be evaluated using a uniform WKB approximation

$$T_{\ell}(E) = \left[1 + \exp \sqrt{\frac{2\mu}{\hbar^2}} \int_{r_{1\ell}}^{r_{2\ell}} dr \left[V_0(r) + \frac{\hbar^2 \ell(\ell + 1)}{2\mu r^2} - E \right]^{1/2} \right]^{-1}. \quad (1.4)$$

Under certain conditions⁴⁾⁻⁶⁾ we can approximate the ℓ dependence of the transmission probability at a given energy by simply shifting the energy:

$$T_{\ell} \simeq T_0 \left[E - \frac{\ell(\ell + 1)\hbar^2}{2\mu R^2(E)} \right], \quad (1.5)$$

^{*)} E-mail: baha@nucth.physics.wisc.edu

where $\mu R^2(E)$ characterizes an effective moment of inertia. In Ref. 6) it was shown that for a one-dimensional barrier $R(E)$ is a slowly varying function of energy (see Fig. 1). Using Eq. (1.5) and replacing summation in Eq. (1.1) with an integration, one obtains

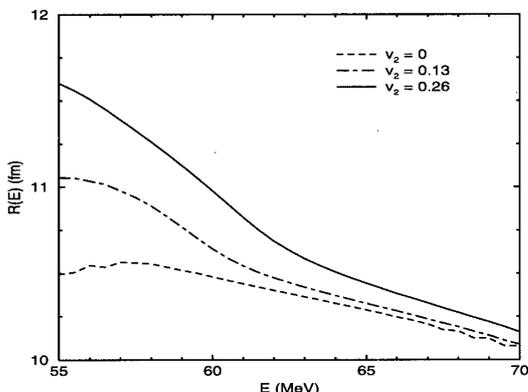


Fig. 1. $R(E)$ as a function of energy (see text).

$$E\sigma(E) = \pi R^2(E) \int_0^E dE' T_0(E'). \tag{1.6}$$

It is worth emphasizing that an $R(E)$ can be introduced not only for one-dimensional barrier penetration, but also to approximate the coupled-channels calculations. This is illustrated in Fig. 1. In this figure the lowest (dashed) line corresponds to $R(E)$ extracted from a one-dimensional calculation for the system $^{16}\text{O} + ^{154}\text{Sm}$.⁶⁾ The upper (dot-dashed and solid) lines illustrate $R(E)$ extracted from coupled

channels calculations for the same system with the solid curve corresponding to a coupling strength twice as much as the strength used in calculating the dot-dashed curve. Equation (1.5) was inverted in Ref. 4) to obtain equivalent one-dimensional barriers. The inconsistency of those results indicated the necessity of a multidimensional coupled-channels picture for fusion reactions below the Coulomb barrier.

Classically the tunneling probability is given by

$$T_0(E) = \begin{cases} 1 & E \geq V_B \\ 0 & E < V_B \end{cases}, \tag{1.7}$$

where V_B is the barrier height. Hence replacing $R(E)$ in Eq. (1.6) with a constant, usually taken to be position of the barrier height r_0 , one obtains

$$E\sigma(E) = \begin{cases} \pi r_0^2(E - V_B) & E \geq V_B \\ 0 & E < V_B \end{cases}. \tag{1.8}$$

Equation (1.8) was widely used to describe above the barrier fusion in the 1970's. From Eq. (1.8) one can calculate the second derivative

$$\frac{d^2}{dE^2}[E\sigma(E)] = \pi r_0^2 \delta(E - V_B). \tag{1.9}$$

Quantum mechanically this sharp peak is broadened as the transmission probability smoothly changes from zero at energies far below the barrier to unity at energies far above the barrier. When a number of channels are included the quantity in Eq. (1.9) can be interpreted as the barrier distribution⁷⁾ with the proviso that the energy dependence of $R(E)$ is not dominant.

§2. Coupled-channel calculations

The multidimensional barrier penetration problem inherent in subbarrier fusion needs to be addressed either in the coupled-channels⁸⁾ or the physically equivalent path-integral approach.⁹⁾ For numerical calculations coupled-channels formulation is the preferred method. In the earlier period of the study of subbarrier fusion, simplified coupled-channel codes such as CCFUS,¹⁰⁾ CCDEF¹¹⁾ and CCMOD¹²⁾ were widely used. The precision of the current data requires a more accurate treatment of the channel-coupling problem going well beyond the approximations utilized in these codes. Currently newer codes are available to experimentalists, such as CC-FULL,¹³⁾ which try to eliminate at least some of these approximations. Erroneous interpretations of the data still appear in the literature. Most of these originate from using much older schemes, for example using a parabola to approximate the nuclear plus centrifugal potential. I would urge the experimental groups to analyze their data using the most precise coupled-channels code available at the moment of their analysis.

Various aspects of coupled-channels calculations were discussed in this conference by a number of speakers. Esbensen presented an analysis of the fusion of ^{27}Al on a series of germanium isotopes.¹⁴⁾ He found that one needs to utilize nonlinear couplings for a reasonable description of the data, consistent with earlier results of other authors.^{15),16)} In the channel coupling problem one usually assumes that at low energies nuclei start from their ground states. Takigawa discussed the implications of starting from an excited state.¹⁷⁾ He showed that a significant number of transitions from the excited to the ground state may take place during fusion. A long-standing question in the coupled-channel calculations is to separate the effects of the surface excitations from those of the particle transfer reactions. Sensitivity to surface properties was explored by several speakers. Takigawa investigated the effects of double-folding potentials on the nuclear surface.¹⁸⁾ Pollarolo, who analyzed the data for the fusion of ^{40}Ca ¹⁹⁾ and ^{48}Ca ²⁰⁾ with Zr isotopes also found that tunneling is dominated by the surface modes.²¹⁾ The consequences of taking different deformation parameters for neutrons and protons in coupled-channels calculations were discussed by Takigawa.^{18),22)}

§3. The nuclear potential

A long-standing puzzle in the study of subbarrier fusion is the large values of the surface diffuseness parameter in the nuclear potential required to fit the data. In fact the value of the diffuseness one needs to fit the fusion data is typically 1.5 to 2 times the value of the diffuseness required to fit the elastic scattering data.^{23),24)} One should emphasize that elastic scattering and fusion at subbarrier energies are complementary processes. While the outer part of the potential is probed both by elastic scattering and fusion processes, the inner part affects only the fusion (see Fig. 2). The inner turning point of the potential is much more sensitive to the value of the diffuseness parameter. Hence it may not be entirely surprising that fusion and elastic scattering can be fitted with somewhat different potentials.

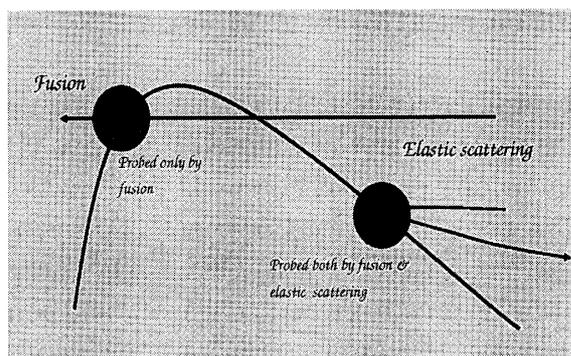


Fig. 2. $R(E)$ as a function of energy (see text).

ism also require a large value of the surface diffuseness parameter.^{26), 27)} Very steep fall-off of the fusion data for the system $^{60}\text{Ni} + ^{89}\text{Y}$ at extreme subbarrier energies, described in this conference,²⁸⁾ is hard to understand using the canonical value of the diffuseness,²⁹⁾ but may be explained with a larger diffuseness parameter.²⁴⁾ Such a large diffuseness parameter yields a shallow potential pocket, truncating the fusion cross section at energies below the minimum of the potential pocket when incoming-wave boundary conditions are used.

§4. Inhibition of fusion

Another set of interesting results concerning fusion dynamics presented in this conference was studies of the fusion inhibition. Theoretically, when the product of the electric charges of the colliding nuclei exceeds about $Z_1 Z_2 \sim 1600$, one expects³⁰⁾ that the large electrostatic energy of the system results in quasi-fission, inhibiting the fusion process. Experimental data, however, indicate that fusion is inhibited even when $Z_1 Z_2$ is much smaller. By studying three systems that lead to the compound nucleus ^{216}Ra , namely $^{12}\text{C} + ^{204}\text{Pb}$, and $^{19}\text{F} + ^{197}\text{Au}$, and $^{30}\text{Si} + ^{186}\text{W}$, the Australian National University group found³¹⁾ model-independent evidence for both quasi-fission and reduced fusion. Similarly fusion suppression and the presence of quasi-fission were observed in the systems $^{48}\text{Ca} + ^{168,170}\text{Er}$ ³²⁾ and $^{48}\text{Ca} + ^{154}\text{Sm}$ ³³⁾ at the Legnaro Laboratory. The latter groups found that the effect is more pronounced for more symmetric combinations and the target deformation may play a role in quasi-fission. In this conference a group from JAERI also reported fusion hindrance for the system $^{82}\text{Se} + ^{138}\text{Ba}$.³⁴⁾ It is worth emphasizing that while the fusion of asymmetric stiff systems is well-understood, symmetric systems present major theoretical challenges. For asymmetric systems the electromagnetic interaction is weaker, hence it takes the very tail of the nuclear potential to turn it around to form the potential barrier. The system tunnels through this barrier before nuclei get very close, hence a coupled-channels description in the basis of the truncated nuclear levels using incoming-wave boundary conditions is a good description of the underlying physics. In contrast, for heavy symmetric systems the barrier may be where the nuclei start touching each other, a truncated (as opposed to complete) nuclear level basis is inappropriate; other degrees of freedom such as transfer may come in.

In this conference Dasgupta raised the possibility that this large value of the surface diffuseness may be a result of the dissipative effects ignored in the coupled-channels codes.²⁵⁾ One should emphasize that calculations using an entirely different, algebraic, approach where nuclear structure effects are described by the interacting boson model and barrier penetration is calculated within a group-theoretical formal-

§5. Fusion of unstable nuclei

The study of the fusion of unstable nuclei is a promising endeavor with an increasing number of new radioactive beam facilities coming on line. Preliminary efforts along this direction include a study of the coupling of the translational motion of the nuclei to a resonance state⁹⁾ and investigating the possibility of a molecular bond formation in the (experimentally difficult to achieve) system of ${}^9\text{Li} + {}^{11}\text{Li}$.³⁵⁾ Key issues in the fusion of halo nuclei are separating complete and incomplete fusion and understanding the effects of transfer processes and positive Q -values on the barrier penetration. For such nuclei one needs to understand better the effect of the break-up process on the fusion cross section. There are a number of puzzles in the experimental data presented in this conference. For example ${}^{11,9}\text{Be} + {}^{209}\text{Bi}$ data reported³⁶⁾ are similar to the ${}^{10}\text{Be}$ data even though ${}^{11}\text{Be}$ is a halo nucleus. Similarly there seems to exist a stripping break-up mechanism for the ${}^6\text{Li} + {}^{208}\text{Pb}$ system.^{37),38)} On the other hand a very large subbarrier fusion enhancement was observed for the ${}^{132}\text{Sn} + {}^{64}\text{Ni}$ system.³⁹⁾ Hagino showed that⁴⁰⁾ although the continuum-continuum couplings reduce fusion above the barrier as compared to the no break-up case, they do not significantly change at energies below the barrier. Clearly much theoretical work needs to be done, in particular coupling to resonances should be better understood.

§6. Electron screening

Laboratory studies of subbarrier fusion of light nuclei are needed as input into the calculation of dynamics and evolution of various astrophysical objects. Experimental methods are constantly improving: it is now possible to measure the fusion rates at Gamow peak energies in underground laboratories with a very low background.³⁾ One current puzzle in these experiments is associated with electron screening; at very low energies appropriate for stellar conditions the atomic electrons of the fusing nuclei screen the Coulombic contribution to the potential. This contribution is usually calculated in the adiabatic approximation where one obtains a constant energy shift to the Coulomb potential.⁴¹⁾ However the experimental data seem to require even larger shifts than those calculated in the adiabatic approximation. This is a long-standing puzzle as the adiabatic approximation is typically thought to over-predict. This discrepancy suggests some physics is excluded from the calculations. So far attempts to locate this missing physics have failed. Effects such as vacuum polarization, relativity, bremsstrahlung, and atomic polarization are very small.⁴²⁾ It was suggested that virtual photon emission during tunneling may increase penetration probability.⁴³⁾ The radiation field can be integrated out in the path-integral formalism; it turns out that this enhancement is extremely small.⁴⁴⁾ Any enhancement of the probability due to the break-up of the nuclei is also too small.⁴⁵⁾ In this conference Kimura showed that the screening potential may exhibit a radial variation in the tunneling region,⁴⁶⁾ which may increase the screening energy to beyond the adiabatic limit. Unfortunately this is not a satisfactory solution since this result depends on the excited state components. Discrepancy in screening continues to be

a puzzle. In this conference Bertulani⁴⁷⁾ explored if the stopping power is correctly extrapolated. Itahashi explored if experiments can be reconfigured to be free of screening corrections. In this regard the so-called Trojan horse method may help. This is a procedure to extract the astrophysical S-factor for two-body reactions by studying a closely-related three-body reaction under quasi-free scattering conditions. After an outline of this promising technique by Baur⁴⁸⁾ successful applications of this method to the ${}^6\text{Li}(p,\alpha){}^3\text{He}$ reaction⁴⁹⁾ and p-p elastic scattering⁵⁰⁾ were discussed in this conference.

§7. Fusion in astrophysical settings

Nuclear fusion reactions play a very important role in astrophysical settings. Kajino⁵¹⁾ discussed nucleosynthesis in a core-collapse supernova and presented beautiful r-process nuclei abundance data obtained with the SUBARU telescope. Fiorentini⁵²⁾ stressed that the Sun can be used as a laboratory to do fundamental physics and presented limits obtained on the p-p fusion reaction at very low energies using helioseismological observations.

Nuclear reactions in a medium may be very different than in the laboratory (in the vacuum). This was highlighted by Kasagi⁵³⁾ who discussed low-energy nuclear reactions in metals. Along those lines Rolfs discussed a very interesting possibility. It looks like some deuterated materials give very high screening potentials. This is reminiscent of the Debye screening and raises the possibility of metals being similar to plasmas in this regard.

§8. Conclusions

This conference was held in one of the three famous scenic locations of Japan: The beautiful Matsushima Bay. The well-known poet Basho, who was a contemporary of Newton, visited Matsushima during his trip to the north of Japan. Literary lore tells us that he was speechless when he encountered the bay dotted with many islands covered with pine trees. His traveling companion Sora, however, was able to compose this Haiku:

Matsushima ya
Tsuru ni mi wo kare
Hototogisu.

Sora

This roughly translates into English as

At Matsushima
Borrow your plumes from cranes
O nightingales.*)

*) It is amusing to note that this haiku can be translated into Turkish keeping its meter intact:

Matsushima'da
Turnanın sorgucunu
Alsın bülbüller

Nightingales' own plumes were not sufficiently magnificent to do justice to the beauty of Matsushima. One feels the same way looking at the beautiful data presented in this conference. Many techniques we use to analyze the data are very simple-minded; the quality of the present data demands a better treatment with at least with full coupled-channels calculations, preferably using more microscopic approaches whenever it is possible.

Acknowledgements

I would like to thank the organizers of the FUSION03 Conference for their hospitality and K. Hagino for his comments on the manuscript. This work was supported in part by the U. S. National Science Foundation Grant No. PHY-0244384 and in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation.

References

- 1) A. B. Balantekin and N. Takigawa, *Rev. Mod. Phys.* **70** (1998), 77; nucl-th/9708036.
- 2) M. Dasgupta, D. J. Hinde, N. Rowley and A. M. Stefanini, *Annu. Rev. Nucl. Part. Sci.* **48** (1998), 401.
- 3) E. G. Adelberger et al., *Rev. Mod. Phys.* **70** (1998), 1265; astro-ph/9805121.
- 4) A. B. Balantekin, S. E. Koonin and J. W. Negele, *Phys. Rev. C* **28** (1983), 1565.
- 5) A. B. Balantekin and P. E. Reimer, *Phys. Rev. C* **33** (1986), 379.
- 6) A. B. Balantekin, A. J. DeWeerd and S. Kuyucak, *Phys. Rev. C* **54** (1996), 1853; nucl-th/9607002.
- 7) N. Rowley, G. R. Satchler and P. H. Stelson, *Phys. Lett. B* **254** (1991), 25.
- 8) C. H. Dasso, S. Landowne and A. Winther, *Nucl. Phys. A* **405** (1983), 381.
- 9) A. B. Balantekin and N. Takigawa, *Ann. of Phys.* **160** (1985), 441.
- 10) C. H. Dasso and S. Landowne, *Comput. Phys. Commun.* **46** (1987), 187.
- 11) J. Fernandez-Niello, C. H. Dasso and S. Landowne, *Comput. Phys. Commun.* **54** (1989), 409.
- 12) M. Dasgupta, A. Navin, Y. K. Agarwal, C. V. K. Baba, H. C. Jain, M. L. Jhingan and A. Roy, *Nucl. Phys. A* **539** (1992), 351.
- 13) K. Hagino, N. Rowley and A. T. Kruppa, *Comput. Phys. Commun.* **123** (1999), 143; nucl-th/9903074.
- 14) H. Esbensen, *Phys. Rev. C* **68** (2003), 034604.
- 15) A. B. Balantekin, J. R. Bennett and S. Kuyucak, *Phys. Rev. C* **48** (1993), 1269.
- 16) K. Hagino, N. Takigawa and S. Kuyucak, *Phys. Rev. Lett.* **79** (1997), 2943; nucl-th/9703029.
- 17) S. Kimura and N. Takigawa, *Phys. Rev. C* **66** (2002), 024603.
- 18) N. Takigawa, Tamanna Rumin, T. Masamoto, T. Takehi, K. Washiyama, S. Ayik and S. Kimura, *Proceedings of the 10th International Conference on Nuclear Reaction Mechanisms, Varenna 9-13 June, 2003*, ed. E. Gadioli (Universita Degli Studi Di Milano), p. 393.
- 19) G. Montagnoli et al., *Eur. Phys. J. A* **15** (2002), 351.
- 20) F. Scarlassara, *Prog. Theor. Phys. Suppl. No. 154* (2004), 31.
- 21) G. Pollarolo, *Prog. Theor. Phys. Suppl. No. 154* (2004), 201.
- 22) N. Takigawa, T. Masamoto, T. Takehi and Tamanna Rumin, *J. Korean Phys. Soc.* **43** (2003), S91.
- 23) K. Hagino, M. Dasgupta, I. I. Gontchar, D. J. Hinde, C. R. Morton and J. O. Newton, nucl-th/0110065.
- 24) K. Hagino, N. Rowley and M. Dasgupta, *Phys. Rev. C* **67** (2003), 054603; nucl-th/0302025.
- 25) M. Dasgupta, D. J. Hinde, J. O. Newton and K. Hagino, *Prog. Theor. Phys. Suppl. No. 154* (2004), 209.
- 26) A. B. Balantekin, J. R. Bennett and S. Kuyucak, *Phys. Rev. C* **49** (1994), 1079.

- 27) A. B. Balantekin and S. Kuyucak, *J. of Phys. G* **23** (1997), 1159; nucl-th/9706068.
 - 28) C. L. Jiang, et al., *Phys. Rev. Lett.* **89** (1992), 052701.
 - 29) C. L. Jiang, H. Esbensen, B. B. Back, R. V. F. Janssens and K. E. Rehm, *Phys. Rev. C* **69** (2004), 014604.
 - 30) J. P. Blocki, H. Feldmeier and W. J. Swiatecki, *Nucl. Phys. A* **441** (1986), 145.
 - 31) A. C. Berriman, D. J. Hinde, M. Dasgupta, C. R. Morton, R. D. Butt and J. O. Newton, *Nature* **413** (2001), 144.
 - 32) R. N. Sagaidak et al., *Phys. Rev. C* **68** (2003), 014603.
 - 33) M. Trotta et al., *Prog. Theor. Phys. Suppl. No. 154* (2004), 37.
 - 34) H. Ikezoe et al., *Prog. Theor. Phys. Suppl. No. 154* (2004), 45.
 - 35) C. A. Bertulani and A. B. Balantekin, *Phys. Lett. B* **314** (1993), 275.
 - 36) C. Signorini et al., *Eur. Phys. J. A* **2** (1998), 227.
 - 37) C. Signorini et al., *Phys. Rev. C* **67** (2003), 044607.
 - 38) Y. W. Wu et al., *Phys. Rev. C* **68** (2003), 044605.
 - 39) J. F. Liang et al. nucl-ex/0304002.
 - 40) K. Hagino and A. Vitturi, nucl-th/0401012.
 - 41) K. Langanke, in *Advances in Nuclear Physics*, ed. J. W. Negele and E. Vogt (Plenum, New York, 1993), Vol. 21, Chapter 2.
 - 42) A. B. Balantekin, C. A. Bertulani and M. S. Hussein, *Nucl. Phys. A* **627** (1997), 324; nucl-th/9706081.
 - 43) V. V. Flambaum and V. G. Zelevinsky, *Phys. Rev. Lett.* **83** (1999), 3108; nucl-th/9812076.
 - 44) K. Hagino and A. B. Balantekin, *Phys. Rev. C* **66** (2002), 055801; nucl-th/0208032.
 - 45) K. Hagino, M. S. Hussein and A. B. Balantekin, *Phys. Rev. C* **68** (2003), 048801; nucl-th/0307030.
 - 46) S. Kimura, N. Takigawa, M. Abe and D. M. Brink, *Phys. Rev. C* **67** (2003), 022801; nucl-th/0211080.
 - 47) C. A. Bertulani, nucl-th/0401007.
 - 48) G. Baur and S. Typel, nucl-th/0401054.
 - 49) C. Spitaleri et al., *Phys. Rev. C* **63** (2001), 055801.
 - 50) M. G. Pellegriti, *Prog. Theor. Phys. Suppl. No. 154* (2004), 349.
 - 51) T. Kajino, *Prog. Theor. Phys. Suppl. No. 154* (2004), 301.
 - 52) G. Fiorentini, *Prog. Theor. Phys. Suppl. No. 154* (2004), 309.
 - 53) J. Kasagi, *Prog. Theor. Phys. Suppl. No. 154* (2004), 365.
 - 54) C. Rolfs, *Prog. Theor. Phys. Suppl. No. 154* (2004), 373.
- See also,
G. Fiorentini, C. Rolfs, F. L. Villante and B. Ricci, *Phys. Rev. C* **67** (2003), 014603; astro-ph/0210537.