Publication Office, Progress of Theoretical Physics

What Determines the Direction of Far-Field Emission in Chaotic Microcavities?

Sang Wook KIM,[1,*] Jeong-Bo SHIM,[2] Sang-Bum LEE,[2] Juhee YANG,[2] Songky MOON,[2] Jai-Hyung LEE,[2] Kyungwon AN[2] and Hai-Woong LEE[3]

[1]*Department of Physics Education and Department of Physics, Pusan National University, Pusan 609-735, Korea*
[2]*School of Physics, Seoul National University, Seoul 151-742, Korea*
[3]*Department of Physics, Korea Advanced Institute of Science and Technology, Daejon 305-701, Korea*

It is crucial to know how to determine the direction of far-field emission in deformed microcavities. We review the recent advances on this issue focusing on our works.

§1. Introduction

A laser has three key ingredients: an active medium, an external pumping, and a cavity. Usually the cavity consists of two parallel mirrors so that they give rise to multiple passages of light through the active medium to efficiently generate the stimulated emission radiation. In addition the cavity itself leads to the emitted output due to its imperfect reflection, which is indispensable for all the applications. The loss of the cavity is quantified by the so-called cavity quality Q value defined as $\omega/\Delta\omega$, where ω and $\Delta\omega$ are respectively the frequency of the laser and its linewidth. As the size of electronic devices is decreased, the micro-scale laser has also drawn much interest. In usual microlasers, Q values range from 10 to 10^3, which is extremely small in comparison with conventional macro-sized laser since the Q value depends on the size of the cavity.

To achieve much larger Q value, the so-called whispering gallery mode (WGM) in the dielectric cavity with spherical or cylindrical boundary has been extensively investigated.[1),2)] The WGM is a mode generated from the ray trapped in the cavity by total internal reflection. Even though extremely high Q value e.g. 10^7 to 10^9 has been easily achieved, the fact that the direction of far-field emission is isotropic due to the spherical or cylindrical symmetry of the cavity limits its practical application. Obvious solution is to break such symmetries, which was the original motivation to study a deformed microcavity.[3)] When a cavity becomes deformed, one immediately encounters the complicated ray dynamics described by dynamical chaos. The corresponding mode can be obtained by solving Maxwell's equations with appropriate boundary conditions. In the viewpoint of a wave equation, the mode can also be regarded as a quasi-eigenstate of a time-independent Schödinger equation describing a particle generating the dynamics of the ray. It then invokes the quantum mechanical manifestation of classical chaos, i.e. the so-called quantum chaos.[4),5)]

*) E-mail: swkim0412@pusan.ac.kr

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

Obvious from the motivation mentioned above it is crucial to know what determines the direction of far-field emission in the deformed microcavity. The existence of the emission itself also makes this problem highly non-trivial in the context of quantum chaos because the system is now open. It is necessary to develop the theory of quantum chaos for open systems. In this paper we would like to review the recent advances to explain the direction of the far-field emission observed in various deformed (or non-integrable) cavities. Definitely we cannot touch all the related references, so that we will mainly concentrate on the works closely related to ours.

§2. Chaotic ray dynamics determines the direction of far-field emission

To analyze the ray dynamics Poincaré surface of section (PSOS) by using Birkhoff coordinate is known to be quite powerful.[5] Figure 1 shows a typical PSOS of the quadrupole-deformed microcavity (QDM) described by $r(\phi) = (1 + \epsilon \cos 2\phi) / \sqrt{1+\epsilon^2/2}$, where ϕ is an azimuthal angle, and ϵ quantifies the extent of the deformation. Note that the arc length $S(\phi)$ and the angle $\sin\chi$ form respectively a coordinate and the corresponding conjugate momentum. When the angle of incidence χ is larger than the critical angle χ_c ($\sin\chi_c = 1/n$, where n is an index of refraction) the ray dynamics is equivalent to that of a particle in a billiard with perfect boundary so that the PSOS contains complete information. Once the angle of incidence becomes smaller than the critical angle, the ray escapes from the cavity giving rise to the loss. Nöckel et al. firstly pointed out that the chaotic dynamics can play an important role to understand the directionality of deformed cavities.[6] The ray bounces many times before escaping so that it exhibits slow diffusion in $\sin\chi$. Therefore the escape occurs near the critical angle, implying the ray is emitted almost tangentially.

This simple idea is extended in their following paper[7] so as to consider the dynamical flow pattern of the rays in the PSOS. Since the time scale of dynamics in $\sin\chi$ is much slower than that in S, the ray dynamics is described as series of motions around the so-called adiabatic curve given by $\sin\chi(\phi) = [1-(1-A^2)\kappa(\phi)^{2/3}]^{1/2}$, where A is the average of $\sin\chi$ and $\kappa(\phi)$ is the curvature of the interface. So to speak the ray slowly diffuse among the adiabatic curves. This curve exhibits a typical shape in the PSOS as shown in Fig. 1, explaining the reason why the direction of far-field emission depends strongly on the index of refraction.[8]

So far the completely chaotic dynamics has been paid attention to. The ray dynamics in a deformed cavities, especially a QDM, shows generic mixed phase space consisting of both chaotic and regular motion. The existence of the stable islands near the critical angle gives rise to dramatic influence on the far-field direction because the rays in the chaotic region cannot penetrate into the stable island. Namely the stable island forms an obstacle against the dynamical flow of the ray in the chaotic sea.[9] On the other hand the rays in the stable island cannot go out of the island itself if the tunneling is ignored, and form the quasi-eigenmode localized on the island. The directional output observed in the quantum cascade semiconductor microlaser was successfully explained by introducing the so-called bow-tie mode of period-4 stable islands emerging from the bifurcation of the diametral orbit near the

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

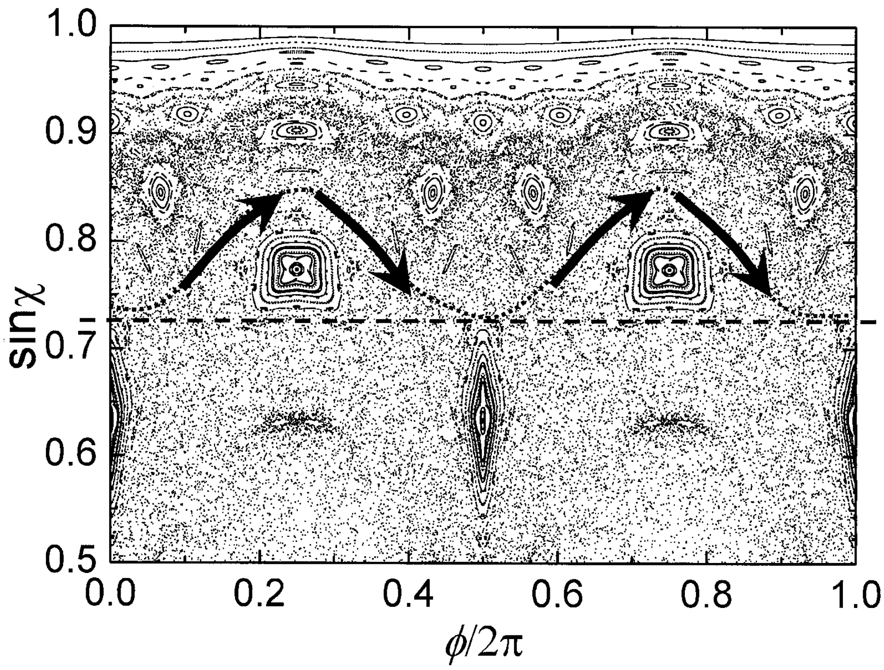

Fig. 1. Ray dynamics inside a QDM visulaized through a Poincaré surface of section. The dotted line represents an example of adiabatic curve explained in text. The thick arrows show the dynamical flow of the rays and the horizontal dashed line represents the critical angle.

critical angle.[10)]

§3. Wave nature of light influences the direction of far-field emission

Even though the classical description of the direction of far-field emission is quite successful, it is incomplete because the wave nature of light is ignored. In completely chaotic regime the dynamics of the ray is almost stochastic except following the adiabatic curves, so that the direction of far-field of every existing modes should be simply determined from theory by Nöckel et al.,[8)] which is referred to as a chaotic WGM. In addition the spectrum might be explained by random matrix theory (RMT); it probably presents quite irregular pattern. In order to check the spectral properties of a chaotic WGM we devised a deformation tunable microcavity,[11)] whose basic idea is as follows. A liquid jet containing fluorescent dye molecules is laterally forced so that a 2D microcavity with *variable* quadrupole deformation is created. By adjusting the lateral force we can control the degree of deformation accurately. The cavity is optically pumped that any persisting WGM with low loss would undergo laser oscillations. What surprised us is the regular spectrum with almost equal spacing obtained in experiment in highly deformed cavity whose classical dynamics shows complete chaos.[11)] Subsequently the far-field direction shows apparent deviation from that of chaotic WGM's.[12)]

Even in fully chaotic regime a dense set of unstable periodic orbits (UPO's) are still embedded in the chaotic orbits.[4)] Although UPO's are found with zero probability in the classical dynamics, in quantum mechanics they manifest themselves in the eigenstates of the system. There exist extra and unexpected concentrations,

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

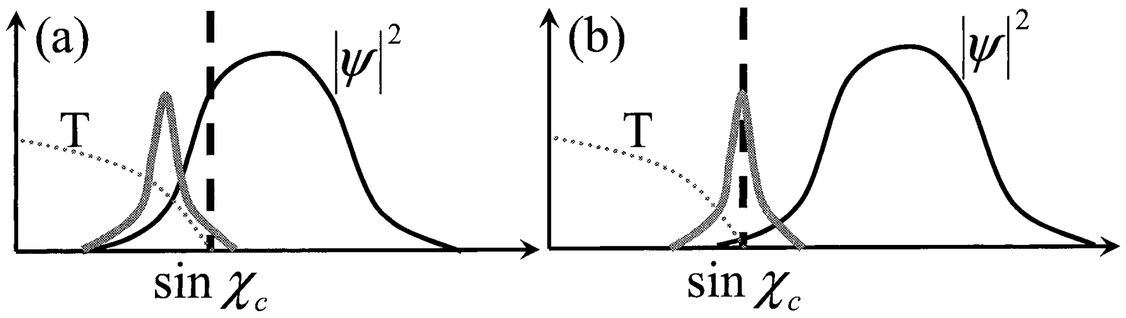

Fig. 2. Schematic picture explaining Fresnel filtering effect. The solid, the dotted and the thick solid curve represent the wavefunction $|\psi|^2$, the Fresnel coefficient T and their multiplication, i.e. the output power, respectively, as a function of the incident angle χ of ray on the boundary. The vertical thick dashed line represents the critical angle. (a) Strong FFE for a low Q mode. (b) Negligible FFE for a high Q mode.

so-called *scars*, of eigenstate density near UPO's.[13] In the end we found that the scarred modes corresponding to hexagonal UPO was excited in our experiment. The emission direction is well explained assuming the emission occurs tangentially at the four bouncing position of the hexagonal UPO (the other two with larger incident angle is ignored).[12]

At the same time Rex et al. also reported the scarred mode lasing in a *semiconductor* (GaN) QDM, where the direction of the lasing output is tremendously deviated from the direction tangential to the corner of the underlying UPO on the cavity surface.[14] They explained such deviation using the so-called Fresnel filtering effect (FFE).[15]

Let us briefly explain what the FFE is. The outgoing wave in a given mode can be determined from the multiplication of the transmission probability, determined mainly from Fresnel coefficients, and the intensity of the wavefunction. It is shown in Fig. 2(a) that when the wavefunction of the mode leaks into the evanescent region with considerable amount of tail the direction of the far field pattern is quite deviated from the tangential one originating from the critical angle. Recently it is reported that besides the FFE the so-called Goos-Hänchen effect can also play an important role in the emission direction.[16]

The question naturally arises: Why the FFE was not observed in our experiment? The discrepancy on the far-field directionality originates mainly from huge difference in their Q values. The FFE strongly depends on how much the wavefunction of the mode overlaps with the evanescent region. Such overlap is associated with the cavity loss of the mode, which is nothing but the Q value. The Q obtained in our experiment reached almost 10^6 causing rather smaller tail as shown in Fig. 2(b) so that the outgoing wave is mainly obtained from the tangential emission. Even though Ref. 14) did not give any number of their Q, one can expect it is at best order of 10^3 since a semiconductor laser usually has stronger absorption loss and purer quality of boundary surface compared with our liquid jet laser.[6] It implies the considerable amount of the FFE takes place. This resolves the discrepancy of the far field patterns of the scar modes observed in two different systems. We also measured the near-field pattern of the lasing mode in QDM made of the liquid jet newly

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

updated.[17] The observed scarred mode is shown to be associated with a hexagonal UPO of ray undergoing total internal reflections with its direction *tangential* to the cavity surface. It confirms our argument related to FFE.

The main message of this section is that the wave nature of light (or quantum nature of the ray in terms of quantum chaos) considerably modifies the characteristics of the far-field direction. However, this is not the end of the story.

§4. Universal direction of far-field emission

With more improved experimental setup[18),19] we are able to obtain much finer spectrum, where five distinct mode groups are identified.[20] The spectral peaks in each mode group have equivalent spacing while those belonging to different groups do not. Moreover the Q values differ by two or three orders of magnitudes implying that they form qualitatively distinct quasi-eigenmodes. Surprisingly the far-field emission of all these five mode groups exhibit almost *identical* direction, which is unlikely considering that their Q values are distinctly different from each other. We also obtain the corresponding quasi-eigenmodes numerically using boundary element method,[21),22] where the five distinct modes are also found. Even though the Husimi plots of those five modes are different from each other, the far-field emission numerically obtained exhibits almost similar direction. It also surprises us because it has been believed that the intracavity mode distribution makes a considerable influence on the far-field emission direction, e.g. the scarred modes, the bow-tie modes, and so on. We call this identical far-field direction *universal.*

A hint comes from the observation that the far-field emission pattern obtained from ray dynamics in classical limit for the QDM is similar to those from the wave calculation and the experiment. For an open system, long time ray dynamics in chaotic region are predominantly determined by the so-called unstable manifolds[4),23] as shown in Fig. 3(a). Since the rays escape from an open cavity before reaching completely ergodic limit, the ray dynamics is usually restricted in limited phase space and thus follows a few dominant unstable manifolds. It was recognized by Schwefel et al.[24] and Lee et al.[25] that the unstable manifolds can play a crucial role in determining the far-field emission direction. Even in quantum mechanics such a faint structure of the unstable manifold manifests itself. When we enhance the intensity near the line of critical angle by 100 times, we can see a few tail-like faint structures extending from the regions of wave concentration to the region far below the line of critical angle. The probability associated with these structures below the line of critical angle is negligibly small compared to that of the total mode distribution as shown in Fig. 3(b). No matter how small their probability is, they are the only direct routes to outside world by refraction. Without them, light would have to tunnel through a substantial distance in phase space from the region of wave concentration to outside across the line of critical angle, and thus the probability of such tunneling would be extremely low. For the other mode groups similar faint structures also exist, providing routes to refractive escape of light.[20] Note that the importance of the unstable manifold in the far-filed directionality is also confirmed in stadium cavities containing nonlinear lasing medium.[26]

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

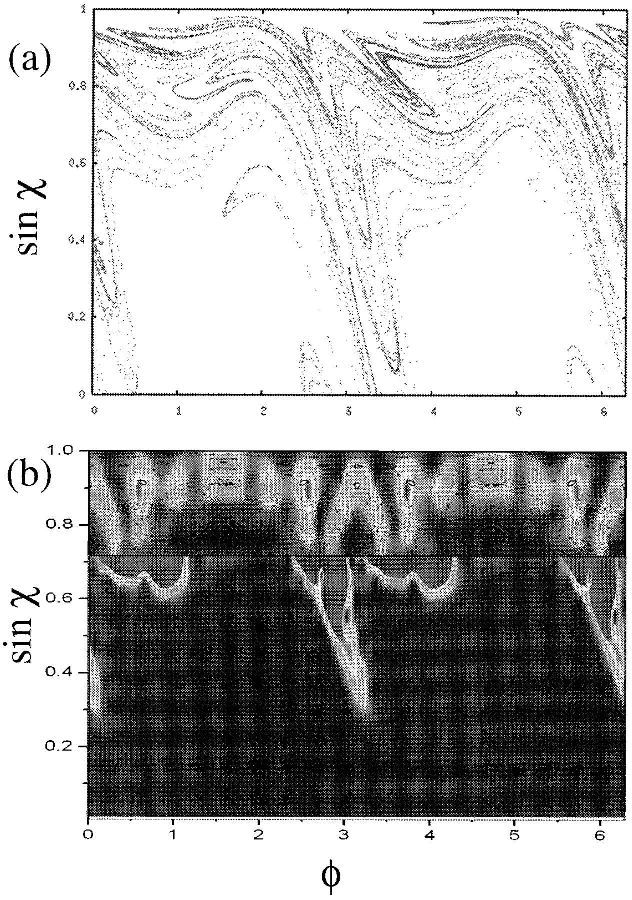

Fig. 3. (a) The complicated structure of the unstable manifolds obtained from short time dynamics of the rays staring from the localized initial condition. (b) Husimi plot of a certain quasi-eigenmode. Irrespective of the shape above the critical angle the probability is mostly localized along the unstable manifolds. The probability below the critical angle is enhanced by 100 times otherwise it cannot be seen.

It is noted that for a mode with rather low Q the Husimi distribution has significant overlap with the region below the critical angle and thus the output directionality is mainly determined by the intracavity mode distribution as usual.[27] In addition, for high-Q modes for rather small nkr, namely ~ 50, which happens to be the size parameter at which many other theoretical studies have been performed on the role of unstable manifolds in output directionality,[24] the faint structure corresponding to the unstable manifolds has not been observed in our numerical studies.[27]

§5. Concluding remarks

Finally we mention that one of the five mode groups obtained both numerically and experimentally in our work is found to correspond to the scarred mode on the hexagonal UPO whose far-field direction happens to be coincident with the universal directionality. Regardless of the characteristics of the intracavity mode structure including scarred modes the far-filed direction exhibits universality when the Q value is large enough.

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

What determines the direction of far-field emission is still under intensive investigation for various non-integrable cavities such as spirals,[28)–31)] stadiums,[32)–34)] ovals,[35)] polygons[36)] and others. We all know that chaotic dynamics is quite robust so that any tiny shape perturbation in billiards does not affect physics so much. However, this is not applicable to the direction of far-field emission since it depends sensitively on the geometry of the cavity.[24)] Our work[20)] unambiguously reveals that even though the far-field direction itself is quite sensitive on the perturbation of the shape one can systematically predict it by considering the unstable manifold which is a robust dynamical structure irrespective of small variation of cavity.

Acknowledgements

SWK thanks Takahisa Harayama and Katsuhiro Nakamura for their kind invitation to QMC2006. This work was supported by Korea Research Foundation Grant (KRF-2006-005-J02804) and by Korea Science and Engineering Foundation Grant (R01-2005-000-10678-0). SNU group was supported by National Research Laboratory Grant and by KRF Grant (2005-070-C00058).

References

1) Y. Yamamoto and R. E. Slusher, Phys. Today **46** (1993), 66.
2) *Optical Processes in Microcavities*, ed. R. K. Chang and A. K. Campillo (World Scientific, Singapore, 1996).
3) A. D. Stone, Physica Scripta **T90** (2003), 248.
4) L. E. Reichl, *The Transition to Chaos in Conservative Classical Systems: Quantum Manifestations* (Springer-Verlag, New York, 1992).
5) H.-J. Stoeckmann, *Quantum Chaos: an introduction* (Cambridge Univ. Press, Cambridge, 1999).
6) J. U. Nöckel, A. D. Stone and R. K. Chang, Opt. Lett. **19** (1994), 1693.
7) J. U. Nöckel, A. D. Stone, G. Chen, H. Grossman and R. K. Chang, Opt. Lett. **21** (1996), 1609.
8) J. U. Nöckel and A. D. Stone, Nature **385** (1997), 45.
9) S. Chang, R. K. Chang, A. D. Stone and J. U. Nöckel, J. Opt. Soc. Am. B **17** (2000), 1828.
10) C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, G. J. Faist, D. L. Sivco and A. Y. Cho, Science **280** (1998), 1556.
11) H. J. Moon, K. H. Ko, Y. C. Noh, G. H. Kim, J.-H. Lee and J. S. Chang, Opt. Lett. **22** (1997), 1739.
12) S.-B. Lee, J.-H. Lee, J. S. Chang, H. J. Moon, S. W. Kim and K. An, Phys. Rev. Lett. **88** (2002), 033903.
13) E. J. Heller, Phys. Rev. Lett. **53** (1984), 1515.
14) N. B. Rex, H. E. Tureci, H. G. L. Schwefel, R. K. Chang and A. D. Stone, Phys. Rev. Lett. **88** (2002), 094102.
15) H. E. Tureci and A. D. Stone, Opt. Lett. **27** (2002), 7.
16) H. Schomerus and M. Hentschel, Phys. Rev. Lett. **96** (2006), 243903.
17) S.-B. Lee et al., unpublished.
18) J. Yang, S. Moon, S.-B. Lee, J.-H. Lee, K. An, J.-B. Shim, H.-W. Lee and S. W. Kim, Rev. Sci. Instrum. **77** (2006), 083103.
19) S.-B. Lee, J. Yang, S. Moon, J.-H. Lee, K. An, J.-B. Shim, H.-W. Lee and S. W. Kim, Appl. Phys. Lett. **90** (2007), 041106.
20) S.-B. Lee, J. Yang, S. Moon, J.-H. Lee, K. An, J.-B. Shim, H.-W. Lee and S. W. Kim, Phys. Rev. A **75** (2007), 011802(R)
21) S. Kagami and I. Fukai, IEEE Trans. Microwave Theory and Techniques **32** (1984), 455.

NII-Electronic Library Service

Publication Office, Progress of Theoretical Physics

22) J. Wiersig, J. Opt. A **5** (2003), 53.
23) A. J. Lichtenberg and M. A. Lieberman, *Regular and Stochastic Motion* (Springer-Verlag, New York, 1983).
24) H. G. Schwefel, N. B. Rex, H. E. Tureci, R. K. Chang and A. D. Stone, J. Opt. Soc. Am. B **21** (2004), 923.
25) S.-Y. Lee, J.-W. Ryu, T.-Y. Kwon, S. Rim and C.-M. Kim, Phys. Rev. A **72** (2005), 061801(R).
26) S. Shinohara, T. Harayama, H. E. Tureci and A. D. Stone, Phys. Rev. A **74** (2006), 033820.
27) J.-B. Shim, H.-W. Lee, S.-B. Lee, J. Yang, S. Moon, J.-H. Lee, K. An and S. W. Kim, arxiv.org/physics/0603221.
28) M. Kneissl, M. Teepe, N. M. Johnson, G. D. Chern and R. K. Chang, Appl. Phys. Lett. **84** (2004), 2485.
29) T. Ben-Messaoud and J. Zyss, Appl. Phys. Lett. **86** (2005), 241110.
30) F. Courvoisier, V. Boutou, J. P. Wolf, R. K. Chang and J. Zyss, Opt. Lett. **30** (2005), 738.
31) S.-Y. Lee, S. Rim, J.-W. Ryu, T.-Y. Kwon, M. Choi and C.-M. Kim, Phys. Rev. Lett. **93** (2004), 164102.
32) T. Harayama, T. Fukushima, S. Sunada and K. S. Ikeda, Phys. Rev. Lett. **91** (2003), 073903.
33) T. Harayama, S. Sunada and K. S. Ikeda, Phys. Rev. A **72** (2005), 013803.
34) M. Lebental, J. S. Lauret, R. Hierle and J. Zyss, Appl. Phys. Lett. **88** (2006), 031108.
35) T. Tanaka, M. Hentschel, T. Fukushima and T. Harayama, Phys. Rev. Lett. **98** (2007), 033902.
36) J. Wiersig, Phys. Rev. A **67** (2003), 023807.

NII-Electronic Library Service