

## Remarks on $N_c$ Dependence of Decays of Exotic Baryons

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We calculate the  $N_c$  dependence of the decay widths of exotic eikosiheptaplet within the framework of Chiral Quark Soliton Model. We also discuss generalizations of regular baryon representations for arbitrary  $N_c$ .

### §1. Introduction

One of the most puzzling results of the chiral quark-soliton model ( $\chi$ QSM) for exotic baryons consists in a very small hadronic decay width,<sup>1)</sup> governed by the decay constant  $G_{\overline{10}}$ . While the small mass of exotic states is rather generic for all chiral models<sup>1)–3)</sup> the smallness of the decay width appears as a subtle cancellation of three different terms that contribute to  $G_{\overline{10}}$ . Decay width in solitonic models<sup>4)</sup> is calculated in terms of a matrix element  $\mathcal{M}$  of the collective axial current operator corresponding to the emission of a pseudoscalar meson  $\varphi$ <sup>1)</sup> — see Ref. 5) for criticism of this approach:

$$\hat{O}_\varphi^{(8)} = 3 \sum_{i=1}^3 \left( G_0 D_{\varphi i}^{(8)} - G_1 d_{ibc} D_{\varphi b}^{(8)} \hat{S}_c - \frac{G_2}{\sqrt{3}} D_{\varphi 8}^{(8)} \hat{S}_i \right) \times p_\varphi^i. \quad (1.1)$$

For notation see Ref. 1). Constants  $G_{0,1,2}$  are constructed from the so-called *moments of inertia* that are calculable in  $\chi$ QSM. The decay width is given as

$$\Gamma_{B \rightarrow B' + \varphi} = \frac{1}{8\pi} \frac{p_\varphi}{M M'} \overline{\mathcal{M}^2} = \frac{1}{8\pi} \frac{p_\varphi^3}{M M'} \overline{\mathcal{A}^2}. \quad (1.2)$$

The “bar” over the amplitude squared denotes averaging over initial and summing over final spin (and, if explicitly indicated, over isospin).

For  $B^{(\overline{10})} \rightarrow B'^{(8)} + \varphi$  for spin “up” and  $\vec{p}_\varphi = (0, 0, p_\varphi)$  we have

$$\mathcal{M} = \langle 8_{1/2}, B' | \hat{O}_\varphi^{(8)} | \overline{10}_{1/2}, B \rangle = -\frac{3G_{\overline{10}}}{\sqrt{15}} \left( \begin{array}{cc|c} 8 & 8 & \overline{10} \\ \varphi & B' & B \end{array} \right) \times p_\varphi \quad (1.3)$$

and

$$G_{\overline{10}} = G_0 - G_1 - \frac{1}{2} G_2. \quad (1.4)$$

In order to have an estimate of the width (1.2) the authors of Ref. 1) calculated  $G_{\overline{10}}$  in the nonrelativistic limit<sup>6)</sup> of  $\chi$ QSM and got  $G_{\overline{10}} \equiv 0$ . It has been shown that this

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cancellation between terms that scale differently with  $N_c$  ( $G_0 \sim N_c^{3/2}$ ,  $G_{1,2} \sim N_c^{1/2}$ ) is in fact consistent with large  $N_c$  counting,<sup>7)</sup> since

$$G_{\overline{10}} = G_0 - \frac{N_c + 1}{4} G_1 - \frac{1}{2} G_2, \quad (1.5)$$

where the  $N_c$  dependence comes from the  $SU(3)$  Clebsch-Gordan coefficients calculated for large  $N_c$ . In the nonrelativistic limit (NRL):

$$G_0 = -(N_c + 2)G, \quad G_1 = -4G, \quad G_2 = -2G, \quad G \sim N_c^{1/2}. \quad (1.6)$$

In this paper we ask whether the similar cancellation takes place for the decays of 27 of spin 1/2 and 3/2. We also discuss the possible modifications of the  $N_c$  dependence of the decay width due to the different choice of the large  $N_c$  generalizations of regular  $SU(3)$  multiplets.

## §2. Baryons in large $N_c$ limit

Soliton is usually quantized as quantum mechanical symmetric top with two moments of inertia  $I_{1,2}$ :

$$M_B^{(\mathcal{R})} = M_{\text{cl}} + \frac{1}{2I_1} S(S+1) + \frac{1}{2I_2} \left( C_2(\mathcal{R}) - S(S+1) - \frac{N_c^2}{12} \right) + \delta_B^{(\mathcal{R})}. \quad (2.1)$$

Here  $S$  denotes baryon spin,  $C_2(\mathcal{R})$  the Casimir operator for the  $SU(3)$  representation  $\mathcal{R} = (p, q)$ :

$$C_2(\mathcal{R}) = \frac{1}{3} (p^2 + q^2 + pq + 3(p+q)) \quad (2.2)$$

and quantities  $\delta_B^{(\mathcal{R})}$  denote matrix elements of the  $SU(3)$  breaking hamiltonian:

$$\hat{H}' = \frac{N_c}{3} \sigma + \alpha D_{88}^{(8)} + \beta Y + \frac{\gamma}{\sqrt{3}} D_{8A}^{(8)} \hat{J}_A. \quad (2.3)$$

Model parameters that can be found in Ref. 8)

$$\alpha = -\frac{N_c}{3}(\sigma + \beta), \quad \beta = -m_s \frac{K_2}{I_2}, \quad \gamma = 2m_s \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right), \quad \sigma = \frac{2}{N_c} \frac{m_s}{m_u + m_d} \Sigma_{\pi N}$$

scale with  $N_c$  in the following way:

$$i_{1,2} = 3I_{1,2}/N_c \quad \text{where} \quad i_{1,2} \sim \mathcal{O}(N_c^0), \quad \sigma, \beta, \gamma \sim \mathcal{O}(m_s N_c^0). \quad (2.4)$$

Here  $\Sigma_{\pi N}$  is pion-nucleon sigma term and  $m_q$  denote current quark masses. Numerically  $\sigma > |\beta|, |\gamma|$ .

So far we have specified *explicit*  $N_c$  dependence (2.4) that follows from the fact that model parameters are given in terms of the quark loop. Another type of the  $N_c$  dependence comes from the constraint<sup>9)</sup> that selects  $SU(3)_{\text{flavor}}$  representations  $\mathcal{R} = (p, q)$  containing states with hypercharge  $Y_R = N_c/3$ . Therefore for arbitrary

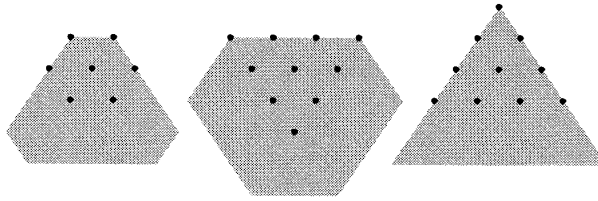


Fig. 1. Standard generalization of  $SU(3)$  flavor baryon representations for arbitrary  $N_c$ .

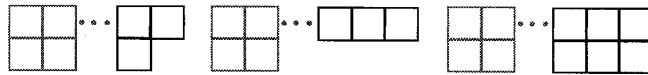


Fig. 2. Adding  $\bar{3}$  diquarks to regular  $SU(3)$  baryon representations 8, 10 and  $\bar{10}$  corresponds to the representation set of Fig. 1.

$N_c$  ordinary baryon representations have to be extended and one has to specify which states correspond to the physical ones. Usual choice<sup>10)</sup>

$$“8” = (1, (N_c - 1)/2), \quad “10” = (3, (N_c - 3)/2), \quad “\bar{10}” = (0, (N_c + 3)/2), \quad (2.5)$$

depicted in Fig. 1 corresponds — in the quark language — to the case when each time when  $N_c$  is increased by 2, a spin-isospin singlet (but charged)  $\bar{3}$  diquark is added, as depicted in Fig. 2.

Extension (2.5) leads to (1.5). It implies that mass differences between centers of multiplets scale differently with  $N_c$ :

$$\Delta_{10-8} = \frac{3}{2I_1} \sim \mathcal{O}(1/N_c), \quad \Delta_{\bar{10}-8} = \frac{N_c + 3}{4I_2} \sim \mathcal{O}(1). \quad (2.6)$$

The fact that  $\Delta_{\bar{10}-8} \neq 0$  in large  $N_c$  limit, triggered recently discussion on the validity of the semiclassical quantization for exotic states.<sup>11)</sup> Since in the chiral limit the momentum  $p_\varphi$  of the outgoing meson scales according to (2.6), overall  $N_c$  dependence of the decay width is strongly affected by its third power (1.2):

$$\Gamma_{B \rightarrow B' + \varphi} \sim \frac{1}{N_c^2} \mathcal{O}(\overline{\mathcal{A}}^2) \mathcal{O}(p_\varphi^3). \quad (2.7)$$

Phenomenologically, however, scaling (2.6) is not sustained. Indeed, meson momenta in  $\Delta$  and  $\Theta$  decays are almost identical (assuming  $M_\Theta^{(\bar{10})} \simeq 1540$  MeV):

$$p_\pi \simeq 225 \text{ MeV}, \quad p_K \simeq 268 \text{ MeV}. \quad (2.8)$$

Unfortunately, going off  $SU(3)_{\text{flavor}}$  limit does not help. Explicitly

$$\begin{aligned} \delta^{(8)} &= \frac{N_c}{3} \sigma + \frac{(N_c - 3)}{3} \beta + \frac{(N_c - 2)\alpha + \frac{3}{2}\gamma}{N_c + 7} + \left( \beta + \frac{3(N_c + 2)\alpha - \frac{1}{2}(2N_c + 9)\gamma}{(N_c + 3)(N_c + 7)} \right) Y \\ &\quad + \frac{(6\alpha + (N_c + 6)\gamma)}{(N_c + 3)(N_c + 7)} \left( \frac{Y^2}{4} - I(I + 1) \right) = 3\sigma + 2\beta - \sigma Y + \dots, \\ \delta^{(10)} &= \frac{N_c}{3} \sigma + \frac{(N_c - 3)(N_c + 4)}{(N_c + 1)(N_c + 9)} \alpha + \frac{N_c - 3}{3} \beta + \frac{5(N_c - 3)}{2(N_c + 1)(N_c + 9)} \gamma \end{aligned} \quad (2.9)$$

$$+ \left( \beta + \frac{3(N_c - 1)\alpha - \frac{5}{2}(N_c + 3)\gamma}{(N_c + 1)(N_c + 9)} \right) Y = 3\sigma + 2\beta - \sigma Y + \dots, \quad (2.10)$$

$$\begin{aligned} \delta^{(\overline{10})} &= \frac{N_c}{3}\sigma + \frac{N_c(N_c - 3)}{(N_c + 3)(N_c + 9)}\alpha + \frac{N_c - 3}{3}\beta - \frac{3(N_c - 3)}{2(N_c + 3)(N_c + 9)}\gamma \\ &+ \left( \beta + \frac{6N_c\alpha - 9\gamma}{2(N_c + 3)(N_c + 9)} \right) Y = 5\sigma + 4\beta - \sigma Y + \dots, \end{aligned} \quad (2.11)$$

where  $\dots$  denote terms  $\mathcal{O}(1/N_c)$ ,  $Y$  and  $I$  denote *physical* hypercharge and isospin.

Interestingly in all cases in the large  $N_c$  limit,  $m_s$  splittings are proportional to the hypercharge differences only. In this limit  $\Sigma - \Lambda$  splitting in the octet is zero and this degeneracy is lifted in the next order at  $\mathcal{O}(1/N_c)$ . This explains the smallness of  $\Sigma - \Lambda$  mass difference. Additionally  $\delta_N^{(8)} \simeq \delta_\Delta^{(10)}$  up to higher order terms  $\mathcal{O}(1/N_c^2)$ , however  $\delta_\Theta^{(\overline{10})} - \delta_N^{(8)} \simeq \sigma + 2\beta > 0$ . This implies that

$$\begin{aligned} M_\Theta^{(\overline{10})} - M_N^{(8)} &= \frac{3}{2I_2} - \frac{1}{20}\alpha + \beta - \frac{3}{40}\gamma \rightarrow \frac{3}{4i_2} + \sigma + 2\beta + \mathcal{O}(1/N_c), \\ M_\Delta^{(10)} - M_N^{(8)} &= \frac{3}{2I_1} - \frac{7}{40}\alpha - \frac{21}{80}\gamma \rightarrow \mathcal{O}(1/N_c). \end{aligned} \quad (2.12)$$

The first equation shows that the  $\Theta - N \neq 0$  in the large  $N_c$  limit even if  $m_s$  corrections are included. We will come back to this problem in the last section.

### §3. Decay constants of twentysevenplet for large $N_c$

In this section we shall consider decays of eikosiheptaplet (27-plet)

$$\text{“27”} = (2, (N_c + 1)/2) \quad (3.1)$$

that can have either spin 1/2 or 3/2, the latter being lighter. Mass differences read

$$\begin{aligned} \Delta_{27_{3/2}-8} &= \frac{3}{2I_1} + \frac{N_c + 1}{4I_2} \sim \mathcal{O}(1), \quad \Delta_{27_{1/2}-8} = \frac{N_c + 7}{4I_2} \sim \mathcal{O}(1), \\ \Delta_{27_{3/2}-10} &= \frac{N_c + 1}{4I_2} \sim \mathcal{O}(1), \quad \Delta_{27_{1/2}-10} = -\frac{3}{2I_1} + \frac{N_c + 7}{4I_2} \sim \mathcal{O}(1), \\ \Delta_{27_{3/2}-\overline{10}} &= \frac{3}{2I_1} - \frac{1}{2I_2} \sim \mathcal{O}(1/N_c), \quad \Delta_{27_{1/2}-\overline{10}} = \frac{1}{I_2} \sim \mathcal{O}(1/N_c). \end{aligned} \quad (3.2)$$

Matrix elements for the decays of eikosiheptaplet (with  $S_3 = 1/2$ ) read:

$$\begin{aligned} \mathcal{A}(B_{27_{3/2}} \rightarrow B'_8 + \varphi) &= 3 \left( \begin{array}{cc|c} 8 & \text{“8”} & \text{“27”} \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{8(N_c + 5)}{9(N_c + 3)(N_c + 9)}} \times G_{27}, \\ \mathcal{A}(B_{27_{3/2}} \rightarrow B'_{10} + \varphi) &= -3 \left( \begin{array}{cc|c} 8 & \text{“10”} & \text{“27”} \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{(N_c - 1)(N_c + 7)}{9(N_c + 1)(N_c + 3)(N_c + 9)}} \times F_{27}, \\ \mathcal{A}(B_{27_{3/2}} \rightarrow B'_{\overline{10}} + \varphi) &= 3 \left( \begin{array}{cc|c} 8 & \text{“\overline{10}”} & \text{“27”} \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{2(N_c + 1)(N_c + 7)}{3(N_c + 3)(N_c + 9)}} \times E_{27}, \end{aligned} \quad (3.3)$$

and

Decay	Large $N_c$ NRL	Scaling in NRL
$27_{3/2} \rightarrow 8_{1/2}$	$G_{27} = G_0 - \frac{N_c-1}{4}G_1 = -3G$	$N_c^{1/2}$
$27_{3/2} \rightarrow 10_{3/2}$	$F_{27} = G_0 - \frac{N_c-1}{4}G_1 - \frac{3}{2}G_2 = 0$	0
$27_{3/2} \rightarrow \overline{10}_{1/2}$	$E_{27} = G_0 + G_1 = -(N_c + 6)G$	$N_c^{3/2}$

For  $S = 1/2$  and  $S_3 = 1/2$  we have:

$$\begin{aligned}
 \mathcal{A}(B_{27_{1/2}} \rightarrow B'_8 + \varphi) &= -3 \left( \begin{array}{cc|c} 8 & \text{"8"} & \text{"27"} \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{(N_c+1)(N_c+5)}{9(N_c+3)(N_c+7)(N_c+9)}} \times H_{27}, \\
 \mathcal{A}(B_{27_{1/2}} \rightarrow B'_{10} + \varphi) &= -3 \left( \begin{array}{cc|c} 8 & \text{"10"} & \text{"27"} \\ \varphi & B' & B \end{array} \right) \sqrt{\frac{8(N_c-1)}{9(N_c+3)(N_c+9)}} \times G'_{27}, \\
 \mathcal{A}(B_{27_{1/2}} \rightarrow B'_{\overline{10}} + \varphi) &= 3 \left( \begin{array}{cc|c} 8 & \text{"}\overline{10}\text{"} & \text{"27"} \\ \varphi & B' & B \end{array} \right) \frac{N_c+4}{\sqrt{9(N_c+3)(N_c+9)}} \times H'_{27}, \quad (3.4)
 \end{aligned}$$

Decay	Large $N_c$ NRL	Scaling in NRL
$27_{1/2} \rightarrow 8_{1/2}$	$H_{27} = G_0 - \frac{N_c+5}{4}G_1 + \frac{3}{2}G_2 = 0$	0
$27_{1/2} \rightarrow 10_{3/2}$	$G'_{27} = G_0 - \frac{N_c+5}{4}G_1 = 3G$	$N_c^{1/2}$
$27_{1/2} \rightarrow \overline{10}_{1/2}$	$H'_{27} = G_0 + \frac{2N_c+5}{2N_c+8}G_1 + \frac{3}{2N_c+8}G_2 = -\frac{(N_c+3)(N_c+7)}{N_c+4}G$	$N_c^{3/2}$

In order to calculate the  $N_c$  behavior of the width we have to know the  $N_c$  dependence of the flavor Clebsch-Gordan coefficients that depend on the states involved. For the decays into 8 and 10 the only possible channels are  $\Theta_{27} \rightarrow N(\Delta) + K$ , and the pertinent Clebsches do not depend on  $N_c$ . For the decays into  $\overline{10}$  we have  $\Theta_{27} \rightarrow \Theta_{\overline{10}} + \pi$  that scales like  $\mathcal{O}(1)$  and  $\Theta_{27} \rightarrow N_{\overline{10}} + K$  that scales like  $\mathcal{O}(1/\sqrt{N_c})$ . The resulting scaling of  $\Gamma_{\Theta_{27} \rightarrow B' + \varphi}$  calculated from Eq. (2.7) reads as follows:

decay of	$N_c$ scaling		decay of	$N_c$ scaling	
$\Theta_{27_{3/2}}$	exact	NRL	$\Theta_{27_{1/2}}$	exact	NRL
$\rightarrow N_8 + K$	$\mathcal{O}(1)$	$\mathcal{O}(1/N_c^2)$	$\rightarrow N_8 + K$	$\mathcal{O}(1)$	0
$\rightarrow \Delta_{10} + K$	$\mathcal{O}(1)$	0	$\rightarrow \Delta_{10} + K$	$\mathcal{O}(1)$	$\mathcal{O}(1/N_c^2)$
$\rightarrow N_{\overline{10}} + K$	$\mathcal{O}(1/N_c^3)$	$\mathcal{O}(1/N_c^3)$	$\rightarrow N_{\overline{10}} + K$	$\mathcal{O}(1/N_c^3)$	$\mathcal{O}(1/N_c^3)$
$\rightarrow \Theta_{\overline{10}} + \pi$	$\mathcal{O}(1/N_c^2)$	$\mathcal{O}(1/N_c^2)$	$\rightarrow \Theta_{\overline{10}} + \pi$	$\mathcal{O}(1/N_c^2)$	$\mathcal{O}(1/N_c^2)$

Interestingly, we see that whenever the exact scaling is  $\mathcal{O}(1)$  the nonrelativistic cancellation (exact or partial) lowers the power of  $N_c$ , whereas in the case when the width has *good* behavior for large  $N_c$ , there is no NRL cancellation.

#### §4. Alternative choices for large $N_c$ multiplets

So far we have only considered the “standard” generalization (2.5) of baryonic  $SU(3)_{\text{flavor}}$  representations for large  $N_c$ . This choice is based on the requirement that generalized baryonic states have physical spin, isospin and strangeness, however their hypercharge and charge are not physical.<sup>10)</sup> Moreover the generalization of the octet is not selfadjoint and antidecuplet is not complex conjugate of decuplet. Some years ago it has been proposed to consider alternative schemes.<sup>12)</sup>

If we require the generalized octet to be self-adjoint we are led to the following set of representations:

$$“8” = (N_c/3, N_c/3), \quad “10” = ((N_c + 6)/3, (N_c - 3)/3), \quad “\overline{10}” = “10”^* \quad (4.1)$$

that are depicted in Figs. 3 and 4. This means that we enlarge  $N_c$  in steps of 3 adding each time a  $uds$  triquark. Generalized states have physical isospin, hypercharge (and charge), but unphysical strangeness and spin that is of the order of  $N_c$ . With this choice both  $\Delta_{10-8}$ ,  $\Delta_{\overline{10}-8} \neq 0$  in large  $N_c$  limit:

$$\Delta_{10-8} = (N_c/6 - 1)/I_1, \quad \Delta_{\overline{10}-8} = (N_c/6 - 1)/I_2. \quad (4.2)$$

With this power counting we can calculate large  $N_c$  approximation of the meson momenta in the decays of  $\Delta$  and  $\Theta$ :

$$\begin{aligned} \Delta \rightarrow N \quad p_\pi &= \sqrt{(M_\Delta - M_N)^2 - m_\pi^2} = 256 \text{ MeV}, \\ \Theta \rightarrow N \quad p_K &= \sqrt{(M_\Theta - M_N)^2 - m_K^2} = 339 \text{ MeV} \end{aligned} \quad (4.3)$$

that are much closer to the physical values (2.8) than (2.6).

Finally let us mention a third possibility in which we require generalized decuplet to be a completely symmetric  $SU(3)_{\text{flavor}}$  representation for arbitrary  $N_c$ . This leads to (see Figs. 5 and 6):

$$“8” = (N_c - 2, 1) \quad “10” = (N_c, 0) \quad “\overline{10}” = (N_c - 3, 3). \quad (4.4)$$

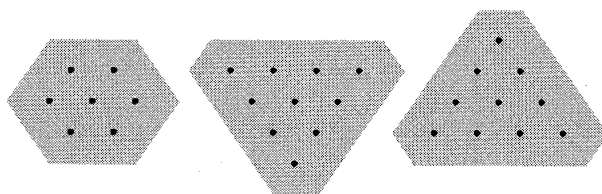


Fig. 3. Generalization of  $SU(3)$  flavor representations in which octet is selfadjoint.

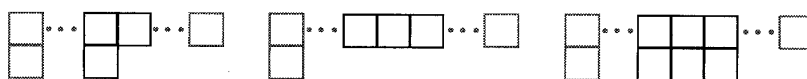


Fig. 4. Adding triquarks to regular  $SU(3)$  baryon representations 8, 10 and  $\overline{10}$  corresponds to the representation set of Fig. 3.

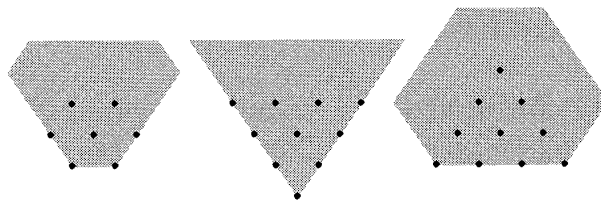


Fig. 5. Generalization of  $SU(3)$  flavor representations in which decuplet is fully symmetric  $(0, q)$ .

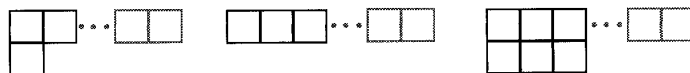


Fig. 6. Adding sextet diquarks to regular  $SU(3)$  baryon representations 8, 10 and  $\overline{10}$  corresponds to the representation set of Fig. 5.

Interestingly this choice has a smooth limit to the one flavor case. In the quark language it amounts to adding a symmetric diquark to the original  $SU(3)_{\text{flavor}}$  representation when increasing  $N_c$  in steps of 2. As seen from Fig. 5 physical states are situated at the bottom of infinite representations (4.4) and therefore have unphysical strangeness, charge (hypercharge) and also spin.

The mass splittings for this choice read

$$\Delta_{10-8} = N_c / 2I_1, \quad \Delta_{\overline{10}-8} = 3 / 2I_2. \quad (4.5)$$

Here the generalized decuplet remains split from the “8”, while  $\Delta_{\overline{10}-8} \rightarrow 0$  for large  $N_c$ . The phase space factor for  $\Theta$  decay is therefore suppressed with respect to the one of  $\Delta$ .

## §5. Summary

In this short note we have shown that very small width of exotic baryons — if they exist — cannot be explained by the standard  $N_c$  counting alone. Certain degree of *nonrelativisticity* is needed to ensure cancellations between different terms in the decay constants. This phenomenon observed firstly for antidecuplet, is also operative for the decays of eikosiheptaplet. We have shown that in  $\chi$ QSM in the nonrelativistic limit all decays are suppressed for large  $N_c$ . Exact cancellations occur for  $\Theta_{27_{3/2}} \rightarrow \Delta_{10} + K$  and  $\Theta_{27_{1/2}} \rightarrow N_8 + K$ , leading  $N_c$  terms cancel for  $\Theta_{27_{3/2}} \rightarrow N_8 + K$  and  $\Theta_{27_{1/2}} \rightarrow \Delta_{10} + K$ . For  $27 \rightarrow \overline{10}$  there are no cancellations, but the phase space is  $N_c^{-3}$  suppressed.

We have also briefly discussed nonstandard generalizations of regular baryon representations for arbitrary  $N_c$ . For  $N_c > 3$  baryons are no longer composed from 3 quarks and therefore they form large  $SU(3)_{\text{flavor}}$  representations that reduce to octet, decuplet and antidecuplet for  $N_c = 3$ . The standard way to generalize regular baryon representations is to add antisymmetric antitriplet diquark when  $N_c$  is increased in intervals of 2. This choice fulfils many reasonable requirements; most importantly for  $SU(2)_{\text{flavor}}$  these representations form regular isospin multiplets. However, representations (2.5) do not obey conjugation relations characteristic for regular representations. Therefore we have proposed generalization (4.1) that sat-

ifies conjugation relations. Most important drawback of (4.1) is that spin  $S \sim N_c$  that contradicts semiclassical quantization. Nevertheless as a result meson momenta emitted in  $10$  and  $\overline{10}$  decays scale in the same way with  $N_c$  (4.3), consistently with “experimental” values (2.8), whereas for (2.5) the scaling is different (2.6).

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