A Covariant Approach to Noncommutative M5-Branes

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We briefly review how to discuss noncommutative (NC) M5-branes and intersecting NC M5-branes from κ-invariance of an open supermembrane action with constant three-form fluxes. The κ-invariance gives rise to possible Dirichlet brane configurations. We shortly summarize a construction of projection operators for NC M5-branes and some intersecting configurations of NC M5-branes. A strong flux limit of them is also discussed.

§1. Introduction

Supermembrane theory in eleven dimensions[1),2)] is closely related to the M-theory formulation,[3)] where open membranes[4),5)] can be considered as well as closed ones. Open membranes can end on a p-dimensional Dirichlet p-brane for $p = 1, 5$ and 9[6),7)] just like an open string can attach to D-branes. The $p = 5$ case corresponds to M5-brane and the $p = 9$ is the end-of-world 9-brane in the Horava-Witten theory.[8)]

The Dirichlet branes can be investigated from the κ-symmetry argument.[6),9)] It is a covariant way and a specific gauge-fixing such as light-cone gauge is not necessary. Then it is sufficient to consider a single action of open string or open membrane, rather than each of D-brane actions. It is moreover easy to find what configurations are allowed to exist for rather complicated D-brane setups such as intersecting D-branes or less supersymmetric D-branes, which are difficult to discuss within a brane probe analysis. The method is not restricted to a flat spacetime and can be generalized to curved backgrounds.[10)]

§2. The κ-symmetry argument

The Green-Schwarz action of a supermembrane in flat spacetime is composed of the Nambu-Goto (NG) part and the Wess-Zumino (WZ) part[1)]

$$S = \int_{\Sigma} d^3\sigma \left[\mathcal{L}_{\rm NG} + \mathcal{L}_{\rm WZ}\right] .$$

Since the bulk action is κ-invariant, the κ-variation of the action $\delta_\kappa S$ leaves only surface terms. The NG part does not give rise to any surface terms. Thus it is

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sufficient to examine the κ-variation of the WZ part,

$$\delta_\kappa S_{\rm WZ}| = \int_{\partial\Sigma} d^2\xi \left[\mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)}\right] ,$$

$$\mathcal{L}^{(2)} = -i\left[\bar\theta\Gamma_{\bar A\bar B}\delta_\kappa\theta + \mathcal{H}_{\bar A\bar B\bar C}\bar\theta\Gamma^{\bar C}\delta_\kappa\theta\right]\dot X^{\bar A}X'^{\bar B} , \tag{2·1}$$

$$\mathcal{L}^{(4)} = \left[-\frac{3}{2}\bar\theta\Gamma^A\delta_\kappa\theta\;\bar\theta\Gamma_{A\bar B} + \frac{1}{2}\bar\theta\Gamma_{A\bar B}\delta_\kappa\theta\;\bar\theta\Gamma^A\right](\theta'\dot X^{\bar B} - \dot\theta X'^{\bar B}), \tag{2·2}$$

where the sixth order part $\mathcal{L}^{(6)}$ disappears due to the Fierz identity. Here we have already utilized bosonic boundary conditions.[11] In order to ensure the κ-invariance these surface terms should vanish. Thus the problem of finding possible Dirichlet branes is boiled down to constructing the projection operators to make (2·1) and (2·2) vanish. It can be performed by constructing a gluing matrix M satisfying $\theta = M\theta$ on the boundary.

§3. Noncommutative M5-brane

A single NC M5-brane (012345) with $\mathcal{H}_{012}$ and $\mathcal{H}^{345}$ is characterized, for example, by the gluing matrix,[12]

$$M = h_0\Gamma^{012345} + h_1\Gamma^{012} . \tag{3·1}$$

For M to define a projection, $M^2 = 1$ should be satisfied. Then we obtain the following condition,

$$h_0^2 + h_1^2 = 1 . \tag{3·2}$$

We can see that (2·1) may vanish by imposing the conditions

$$h_1 - \mathcal{H}_{012} = 0 , \quad h_1 - h_0\mathcal{H}^{345} = 0 . \tag{3·3}$$

It is easy to see that (2·2) also becomes zero, and the gluing matrix (3·1) with the two conditions (3·2) and (3·3) gives a possible M5-brane configuration.

Then let us consider the interpretation of the solution constructed above. By substituting (3·3) for (3·2), we obtain the following condition,

$$\frac{1}{(\mathcal{H}^{345})^2} - \frac{1}{(\mathcal{H}_{012})^2} = -1 .$$

This is nothing but the self-dual condition[13] of the gauge field on the M5-brane.[14] That is, we have reproduced the information on the NC M5-brane from the κ-symmetry argument for the open supermembrane action. Thus we recognize that the projection operator should describe the NC M5-brane.

Let us consider a commutative limit and a strong flux limit. The conditions (3·2) and (3·3) are solved by using an angle variable φ,

$$h_0 = \cos\varphi , \quad h_1 = \sin\varphi , \quad \mathcal{H}_{012} = \sin\varphi , \quad \mathcal{H}^{345} = \tan\varphi . \qquad (0 \le \varphi \le \pi/2)$$

Then the gluing matrix M is written as

$$M = e^{\varphi \Gamma^{345}} \Gamma^{012345} .$$

For a commutative limit $\varphi \to 0$, the NC M5 reduces to commutative M5 (012345), since $\mathcal{H} \to 0$ and $M \to \Gamma^{012345}$.

On the other hand, for $\varphi \to \pi/2$, we see that $\mathcal{H}^{345} \to \infty$ and so the gluing condition reduces to $M \to \Gamma^{012}$ with a critical flux $\mathcal{H}_{012} = 1$. It seems that the resulting projection operator should describe a critical M2-brane (012). Eventually this limit is nothing but the OM limit[15] and it should correspond to one of infinitely many M2-branes dissolved on the M5-brane. This is analogous to the D2-D0 system where a D2-brane with a flux reduces to a D2-brane with infinitely many D0-brane in a strong magnetic flux limit.

As is well known, the $p = 2$ case is not allowed as a projection operator in the case without fluxes. Hence it is a non-trivial problem whether the resulting projection operator for a critical M2 is consistent to the κ-symmetry. The $p = 2$ case is actually special among other p, and due to some identities intrinsic to $p = 2$, (2·1) vanishes when

$$\mathcal{H}_{012} = 1 \, . \tag{3·4}$$

It is easy to show that (2·2) disappears. Thus we have checked that the κ-variation surface terms should vanish for an M2-brane with the critical $\mathcal{H}$ (3·4). Although the κ-symmetry is maintained for the M2-brane, the charge conservation[4] requires the existence of M5-brane behind M2-branes. That is, a NC M5-brane should be regarded as a bound state of M5 and M2.

§4. Intersecting noncommutative M5-branes

In comparison to the case of a single NC M5-brane, a configuration of intersecting NC M5-branes is characterized by two gluing matrices,[16]

$$M_1 = e^{\varphi_1 \Gamma^{A_0 A_1 A_2}} \Gamma^{A_0 \cdots A_5} , \quad M_2 = e^{\varphi_2 \Gamma^{B_0 B_1 B_2}} \Gamma^{B_0 \cdots B_5} , \quad [M_1, M_2] = 0 \, .$$

The requirement $[M_1, M_2] = 0$ leads to the four possibilities for the projection. As an example, let us focus upon one of these cases, NC M5⊥NC M5(3) described by

$$\begin{aligned} M_1 &= e^{\varphi_1 \Gamma^{235}} \Gamma^{012345} , \quad \mathcal{H}_{014} = \sin\varphi_1 , \quad \mathcal{H}^{235} = \tan\varphi_1, \qquad (0 \le \varphi_1 \le \pi/2) \\ M_2 &= e^{\varphi_2 \Gamma^{137}} \Gamma^{012367} , \quad \mathcal{H}_{026} = -\sin\varphi_2 , \quad \mathcal{H}^{137} = \tan\varphi_2 . \qquad (0 \le \varphi_2 \le \pi/2) \end{aligned}$$

It reduces to a commutative M5 (012345)⊥M5 (012367)[17)–19)] in the limit $\varphi_{1,2} \to 0$.

M2⊥M5 (1)[17),19)] can be realized from the NC M5⊥NC M5 (3) by taking a strong flux limit. In the limit $\varphi_2 \to \pi/2$, we obtain a NC version of M2⊥M5 (1),

$$M_1 = e^{\varphi_1 \Gamma^{235}} \Gamma^{012345} , \qquad M_2 = -\Gamma^{026} .$$

Further letting $\varphi_1 \to \pi/2$, we obtain M2 (014)⊥M2 (026).[17),19)] The other way is possible and the two sequences of the strong flux limits are depicted in Fig. 1. It is also possible to discuss NC M5⊥C M5 (1).[12)]

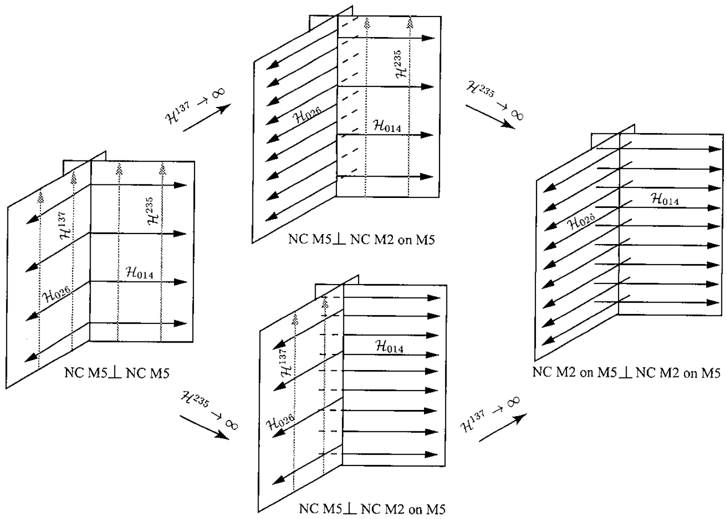

Fig. 1. Two sequences of the strong flux limits of NC M5⊥NC M5 (3).

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