Ordinary and Exotic Baryons, Strange and Charmed, in the Relativistic Mean Field Approach

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All baryon resonances can be systematized from one and the same point of view, according to what they would look like if the number of colors N_c was large. Although in the real world N_c is only three, the consequences of the large-N_c classification are visible in the observed spectrum. At large N_c baryon resonances are collective excitations about intrinsic one-quark or particle-hole states in a certain mean field. Exotic pentaquark baryons appear as 'Gamov–Teller' particle-hole excitations. A charmed $\mathcal{B}_c$ pentaquark baryon is predicted which may well be so light that it is stable with respect to strong decays.

§1. Introduction

Recently a classification of baryon resonances was suggested, according to what they would look like if the number of colors N_c was large.[1),2)] While in the real world N_c is only three, we do not expect qualitative difference in the baryon spectrum with the large-N_c limit. The hope is that if one develops a clear picture at large N_c, its imprint will be visible at $N_c = 3$.

At large N_c, the N_c quarks constituting a baryon can be considered in a mean (non-fluctuating) field which does not change as $N_c \to \infty$.[3)] At the microscopic level quarks experience only color interactions, however gluon field fluctuations are not suppressed if N_c is large: the mean field can be only 'colorless'. An example how originally color interactions are Fierz-transformed into interactions of quarks with mesonic fields is provided by the instanton liquid model.[4)] A non-fluctuating confining 'bag' is also an example of a 'colorless' mean field.

The advantage of the large-N_c approach is that at large N_c baryon physics simplifies considerably, which enables one to take into full account the important relativistic and field-theoretic effects that are often ignored. Baryons are not just three (or N_c) quarks but contain additional quark-antiquark pairs, as it is well known experimentally. Baryon resonances may be formed not only from quark excitations as in the non-relativistic quark models, but also from particle-hole excitations and "Gamov–Teller" transitions. At large N_c these effects become transparent and tractable. At $N_c = 3$ it is a mess called "strong interactions".

We shall thus assume that quarks in the large-N_c baryon obey the Dirac equation in a background mesonic field since there are no reasons to expect quarks to be non-relativistic, especially in excited baryons. All 'intrinsic' quark Dirac levels in the mean field are stable in N_c. All negative-energy levels should be filled in by N_c quarks in the antisymmetric state in color, corresponding to the zero baryon number state. Filling in the lowest positive-energy level makes a baryon. Exciting higher quark levels or making particle-hole excitations produces baryon resonances. The baryon mass is $\mathcal{O}(N_c)$, and the excitation energy is $\mathcal{O}(1)$. When one excites one quark the change of the mean field is $\mathcal{O}(1/N_c)$ that can be neglected to the first approximation.

The approach can be illustrated by the chiral quark soliton model[5] or by the chiral bag model[6] but actually the arguments of this paper are much more general. Dynamics is not considered here, which today would require adopting a model. A concrete model would say what is the intrinsic relativistic quark spectrum in baryons. It may get it approximately correct, or altogether wrong. Instead of calculating the intrinsic spectrum from a model, we extract it from the known baryon spectrum by interpreting baryon resonances as collective excitations about the ground state and about the one-quark and particle-hole transitions.

§2. Relativistic quarks in a mean field

In the mean field approximation, justified at large N_c, one looks for the solutions of the Dirac equation for single quark states in the background mean field. In a most general case the background field couples to quarks through all five Fermi variants. If the mean field is stationary in time, it leads to the Dirac eigenvalue equation for the u, d, s quarks in the background field, $H\psi = E\psi$, the Dirac Hamiltonian being schematically

$$H = \gamma^0\left(-i\partial_i\gamma^i + S(\boldsymbol{x}) + P(\boldsymbol{x})i\gamma^5 + V_\mu(\boldsymbol{x})\gamma^\mu + A_\mu(\boldsymbol{x})\gamma^\mu\gamma^5 + T_{\mu\nu}(\boldsymbol{x})\frac{i}{2}[\gamma^\mu\gamma^\nu]\right), \tag{2·1}$$

where S, P, V, A, T are the mean fields that are matrices in flavor. In fact, the one-particle Dirac Hamiltonian (2·1) is generally nonlocal, however that does not destroy symmetries in which we are primarily interested. We include the current and the dynamically-generated quarks masses into the scalar term S.

The key issue is the symmetry of the mean field. From the large-N_c point of view, the current strange quark mass is very small, $m_s = \mathcal{O}(1/N_c^2)$,[1] and therefore a good starting point is exact $SU(3)$ flavor symmetry implying baryons appear in degenerate $SU(3)$ multiplets **8**, **10**,...; the splittings inside $SU(3)$ multiplets can be determined later on as a perturbation in m_s (see e.g. Ref. 7)).

A natural assumption, then, would be that the mean field is flavor-symmetric, and spherically symmetric. However we know that baryons are strongly coupled to pseudoscalar mesons ($g_{\pi NN} \approx 13$). It means that there is a large pseudoscalar field inside baryons; at large N_c it is a classical mean field. There is no way of writing down the pseudoscalar field (it must change sign under inversion of coordinates) that would be compatible with the $SU(3)_{\rm flav} \times SO(3)_{\rm space}$ symmetry. The minimal extension of spherical symmetry is to write the "hedgehog" *Ansatz* "marrying" the isotopic and space axes:[*)]

$$\pi^a(\boldsymbol{x}) = \begin{cases} n^a F(r), \quad n^a = \frac{x^a}{r}, & a = 1,2,3, \\ 0, & a = 4,5,6,7,8. \end{cases} \tag{2·2}$$

*) Historically, this *Ansatz* for the pion field in a nucleon appears for the first time in a 1942 paper by Pauli and Dancoff[8] (I thank A. Hosaka for bringing my attention to that early work), and reappears in 1961 in the seminal papers by Skyrme.[9]

This *Ansatz* breaks the $SU(3)_{\rm flav}$ symmetry. Moreover, it breaks the symmetry under independent space $SO(3)_{\rm space}$ and isospin $SU(2)_{\rm iso}$ rotations, and only a simultaneous rotation in both spaces remains a symmetry, since a rotation in the isospin space labeled by a, can be compensated by the rotation of the space axes. The *Ansatz* (2·2) implies a spontaneous (as contrasted to explicit) breaking of the original $SU(3)_{\rm flav}\times SO(3)_{\rm space}$ symmetry down to the $SU(2)_{\rm iso+space}$ symmetry. It is analogous to the spontaneous breaking of spherical symmetry by the ellipsoid form of many nuclei.

We list below all possible structures in the S, P, V, A, T fields, compatible with the $SU(2)_{\rm iso+space}$ symmetry and with the C, P, T quantum numbers of the fields. Since $SU(3)$ symmetry is broken, all fields can be divided into three categories:

I. Isovector fields acting on u, d quarks

$$\begin{aligned}\text{pseudoscalar}: & \; P^a(\boldsymbol{x}) = n^a\, P_0(r),\\ \text{vector}: & \; V_i^a(\boldsymbol{x}) = \epsilon_{aik}\, n_k\, P_1(r),\\ \text{axial}: & \; A_i^a(\boldsymbol{x}) = \delta_{ai}\, P_2(r) + n_a n_i\, P_3(r),\\ \text{tensor}: & \; T_{ij}^a(\boldsymbol{x}) = \epsilon_{aij}\, P_4(r) + \epsilon_{bij}\, n_a n_b\, P_5(r).\end{aligned} \tag{2·3}$$

II. Isoscalar fields acting on u, d quarks

$$\begin{aligned}\text{scalar}: & \quad S(\boldsymbol{x}) = Q_0(r),\\ \text{vector}: & \quad V_0(\boldsymbol{x}) = Q_1(r),\\ \text{tensor}: & \quad T_{0i}(\boldsymbol{x}) = n_i\, Q_2(r).\end{aligned} \tag{2·4}$$

III. Isoscalar fields acting on s quarks

$$\begin{aligned}\text{scalar}: & \quad S(\boldsymbol{x}) = R_0(r),\\ \text{vector}: & \quad V_0(\boldsymbol{x}) = R_1(r),\\ \text{tensor}: & \quad T_{0i}(\boldsymbol{x}) = n_i\, R_2(r).\end{aligned} \tag{2·5}$$

All the rest fields and components are zero as they do not satisfy the $SU(2)_{\rm iso+space}$ symmetry and/or the needed discrete C, P, T symmetries. The 12 'profile' functions $P_{0,1,2,3,4,5}$, $Q_{0,1,2}$ and $R_{0,1,2}$ should be eventually found self-consistently from the minimization of the mass of the ground-state baryon. We shall call Eqs. (2·3)–(2·5) the hedgehog *Ansatz.* However, even if we do not know those profiles, there are important consequences of this *Ansatz* for the baryon spectrum.

§3. Baryons made of u, d, s quarks

Given the $SU(2)_{\rm iso+space}$ symmetry of the mean field, the Dirac Hamiltonian for quarks actually splits into two: one for s quarks and the other for u, d quarks.[1)] It should be stressed that the energy levels for u, d quarks on the one hand and for s quarks on the other are completely different, even in the chiral limit $m_s \to 0$.

The energy levels for s quarks are classified by half-integer J^P where $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$ is the angular momentum, and are $(2J+1)$-fold degenerate. The energy levels for

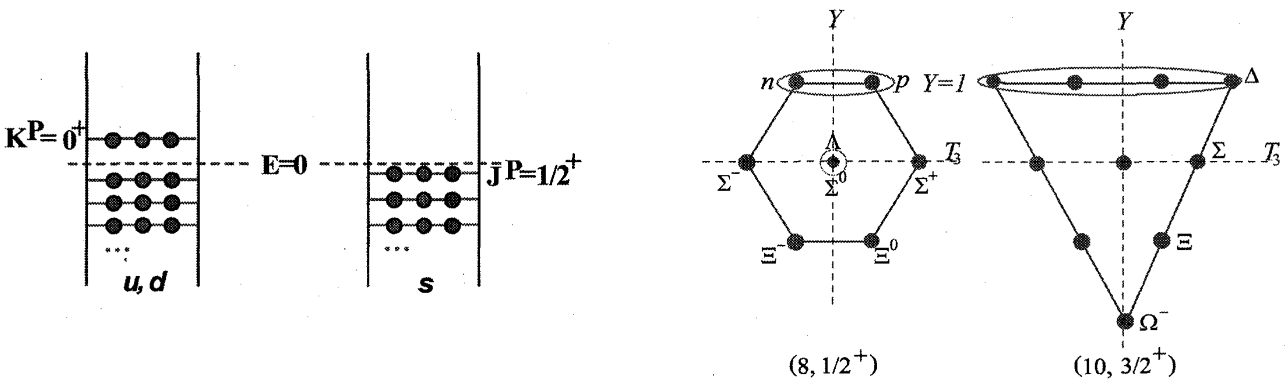

Fig. 1. Filling u, d and s shells for the ground-state baryon (left), and the two lowest baryon multiplets that follow from quantizing the rotations of this filling scheme (right).

u, d quarks are classified by integer K^P where $\boldsymbol{K} = \boldsymbol{T} + \boldsymbol{J}$ is the 'grand spin' (T is isospin), and are $(2K+1)$-fold degenerate.

All energy levels, both positive and negative, are probably discrete owing to confinement. Indeed, a continuous spectrum would correspond to a situation when quarks are free at large distances from the center, which contradicts confinement. One can model confinement e.g. by forcing the effective quark masses to grow linearly at infinity, $S(\boldsymbol{x}) \to \sigma r$.

According to the Dirac theory, all *negative*-energy levels, both for s and u, d quarks, have to be fully occupied, corresponding to the vacuum. It means that there must be exactly N_c quarks antisymmetric in color occupying all degenerate levels with J_3 from $-J$ to J, or K_3 from $-K$ to K; they form closed shells. Filling in the lowest level with $E > 0$ by N_c quarks makes a baryon[1),5)] (see Fig. 1). A similar picture arises in the chiral bag model.[6)]

The mass of a baryon is the aggregate energy of all filled states, and being a functional of the mesonic field, it is proportional to N_c since all quark levels are degenerate in color. Therefore quantum fluctuations of mesonic field in baryons are suppressed as $1/N_c$ so that the mean field is indeed justified.

Quantum numbers of the lightest baryons are determined from the quantization of the rotations of the mean field, leading to specific $SU(3)$ multiplets that reduce at $N_c = 3$ to the octet with spin $\frac{1}{2}$ and the decuplet with spin $\frac{3}{2}$ (see e.g. 10)). Witten's quantization condition $Y' = \frac{N_c}{3}$[11)] follows trivially from the fact that there are N_c u, d valence quarks each with the hypercharge $\frac{1}{3}$.[7)] Therefore, the ground state shown in Fig. 1 entails in fact 56 rotational states. The splitting between the centers of the multiplets $(\mathbf{8}, \frac{1}{2}^+)$ and $(\mathbf{10}, \frac{3}{2}^+)$ is $\mathcal{O}(1/N_c)$, and the splittings inside multiplets can be determined as a perturbation in m_s.[7)]

The lowest baryon resonance beyond the rotational excitations of the ground state is the singlet $\Lambda(1405, \frac{1}{2}^-)$. Apparently, it can be obtained only as an excitation of the s quark, and its quantum numbers must be $J^P = \frac{1}{2}^-$[1)] (see transition *1* in Fig. 2).

The existence of an $\frac{1}{2}^-$ level for s quarks automatically implies that there is a particle-hole excitation of this level by an s quark from the $\frac{1}{2}^+$ level. I identify

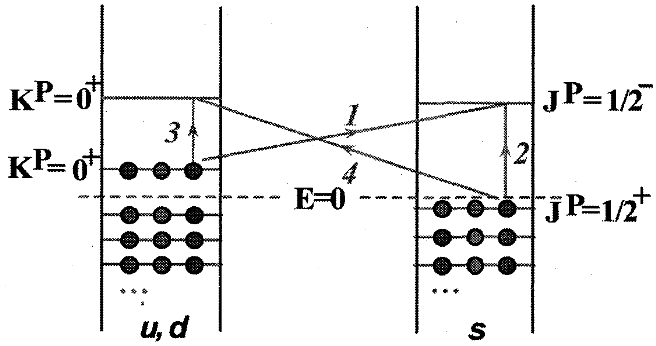

Fig. 2. The existence of the two lowest excited levels — one for the u, d quarks and the other for the s quarks — implies four resonances shown by arrows. The transitions correspond to: *1*: $\Lambda(1405, 1/2^-)$, *2*: $N(1535, 1/2^-)$, *3*: $N(1440, 1/2^+)$, *4*: $Theta^+(1530, 1/2^+)$. Each transition generally entails its own rotational band of $SU(3)$ multiplets.

this transition *2* with $N(1535, \frac{1}{2}^-)$.[1] At $N_c = 3$ it is predominantly a pentaquark state $u(d)uds\bar{s}$ (although it has also a nonzero three-quark Fock component). This explains its large branching ratio in the ηN decay,[12] a long-time mystery. We also see that, since the highest filled level for s quarks is lower than the highest filled level for u, d quarks, $N(1535, \frac{1}{2}^-)$ must be *heavier* than $\Lambda(1405, \frac{1}{2}^-)$: the opposite prediction of the non-relativistic quark model has been always of some concern. Subtracting $1535 - 1405 = 130$, I find that the $\frac{1}{2}^+$ s-quark level is approximately 130 MeV lower in energy than the valence 0^+ level for u, d quarks. This is an important number which will be used below. The transition entails its own rotational band discussed in Ref. 2).

The low-lying Roper resonance $N(1440, \frac{1}{2}^+)$ requires an excited one-particle u, d state with $K^P = 0^+$ (or 1^+)[1] (see transition *3*). Just as the ground state nucleon, it is part of the excited $(\mathbf{8}', \frac{1}{2}^+)$ and $(\mathbf{10}', \frac{3}{2}^+)$ split as $1/N_c$. Such identification of the Roper resonance solves another problem of the non-relativistic model where $N(1440, \frac{1}{2}^+)$ must be heavier than $N(1535, \frac{1}{2}^-)$. In our approach they are unrelated.

Given that there is an excited 0^+ level for u, d quarks, one can put there a quark taking it as well from the s-quark $\frac{1}{2}^+$ shell (see transition *4*). It is a particle-hole excitation with the valence u, d level left untouched, its quantum numbers being $S = +1$, $T = 0$, $J^P = \frac{1}{2}^+$. At $N_c = 3$ it is a pentaquark state $uudd\bar{s}$, precisely the exotic Θ^+ baryon predicted in Ref. 13) from related but somewhat different considerations. The quantization of its rotations produces the antidecuplet $(\overline{\mathbf{10}}, \frac{1}{2}^+)$. In our original prediction the $\mathcal{O}(1)$ gap between Θ^+ and the nucleon was due to the rotational energy only, whereas here the main $\mathcal{O}(1)$ part of that gap is due to the one-particle levels, while the rotational energy is $\mathcal{O}(1/N_c)$. Methodologically, it is now more satisfactory.

In nuclear physics, excitations generated by the axial current $j^{\pm}_{\mu 5}$, when a neutron from the last occupied shell is sent to an unoccupied proton level or *vice versa.* are known as Gamov–Teller transitions.[14] Thus our interpretation of the Θ^+ is that it is a Gamov–Teller-type resonance long known in nuclear physics.

An unambiguous feature of our picture is that the exotic pentaquark Θ^+ is a consequence of the existence of three well-known resonances and must be light. Indeed, the Θ^+ mass can be estimated from the apparent sum rule following from Fig. 2: $m_\Theta \approx 1440+1535-1405 \approx 1570\,\mathrm{MeV}$.[1] Since the $N(1440)$ and $N(1535)$ resonances are broad such that their masses are not well defined, there is a numerical uncertainty in this equation. For example, if one uses the pole positions of the resonances the equation reads $m_\Theta \approx 1365+1510-1405 \approx 1470\,\mathrm{MeV}$. Therefore, it is fair to say that the sum rule predicts $m_\Theta = 1520 \pm 50\,\mathrm{MeV}$. This is in remarkable agreement with the claimed masses of the Θ^+: $m_\Theta = 1524 \pm 2 \pm 3\,\mathrm{MeV}$,[15] $1537 \pm 2\,\mathrm{MeV}$,[16] $1523 \pm 2 \pm 3\,\mathrm{MeV}$,[17] $1521.5 \pm 1.5 \pm 2.8/1.7\,\mathrm{MeV}$,[18] and $1528 \pm 2.6 \pm 2.1\,\mathrm{MeV}$.[19] For a possible explanation why Θ^+ is seen in some experiments while not observed in others see Refs. 20) and 21).

To account for higher baryon resonances one has to assume that there are higher one-particle levels, both in the u, d- and s-quark sectors, to be published elsewhere.[22]

§4. Baryon resonances from rotational bands

A filling scheme of one-particle quark levels by itself does not tell us what are the quantum numbers of the state. The filling scheme treats u, d quarks and s quarks differently and therefore violates the $SU(3)_{\mathrm{flav}}$ and also $SO(3)_{\mathrm{space}}$ symmetries. Only the $SU(2)_{\mathrm{iso+space}}$ symmetry of simultaneous isospin and compensating space rotations is preserved. In the chiral limit (which I assume for the time being) an arbitrary $SU(3)_{\mathrm{flav}}$ rotation of the mean field and hence of what we call u, d, s quarks does not change the energy of the state. The same is true for the $SO(3)_{\mathrm{space}}$ rotation. However, if $SU(3)_{\mathrm{flav}}$ and $SO(3)_{\mathrm{space}}$ rotations are slowly dependent on time, they generate a shift in the energy of the system; it is called the rotational energy. Being quantized according to the general quantization rules for rotations, it produces states with definite $SU(3)_{\mathrm{flav}}$ quantum numbers and spin.

Thus the original $SU(3)_{\mathrm{flav}} \times SO(3)_{\mathrm{space}}$ symmetry broken spontaneously by a 'hedgehog' *Ansatz* of the mean field, is restored when flavor and space rotations are accounted for. Each transition in Fig. 2 generally entails "rotational bands" of $SU(3)$ multiplets with definite spin and parity. The short recipe of getting them is: Find the hypercharge Y' of the given excitation from the number of u, d, s quarks involved; only those multiplets are allowed that contain this Y'. Take an allowed multiplet and read off the isospin(s) T' of particles at this value of Y'. The allowed spin of the multiplet obeys the angular momentum addition law:

$$\boldsymbol{J} = \boldsymbol{T}' + \boldsymbol{J_1} + \boldsymbol{J_2}, \tag{4·1}$$

where $J_{1,2}$ are the initial and final momenta of the s shells involved in the transition. (If nonzero K shell is involved in the transition the quantization rule is more complex.[22]) The mass of the center of an allowed rotational multiplet does not depend on $\boldsymbol{J}$ but only on $\boldsymbol{T}'$ according to the relation[10]

$$\mathcal{M} = \mathcal{M}_0 + \frac{C_2(p,q) - T'(T'+1) - \frac{3}{4}Y'^2}{2I_2} + \frac{T'(T'+1)}{2I_1}, \tag{4·2}$$

where $C_2(p,q) = \frac{1}{3}(p^2+q^2+pq)+p+q$ is the quadratic Casimir eigenvalue of the $SU(3)$ multiplet characterized by (p,q), and $I_{1,2} = \mathcal{O}(N_c)$ are moments of inertia.

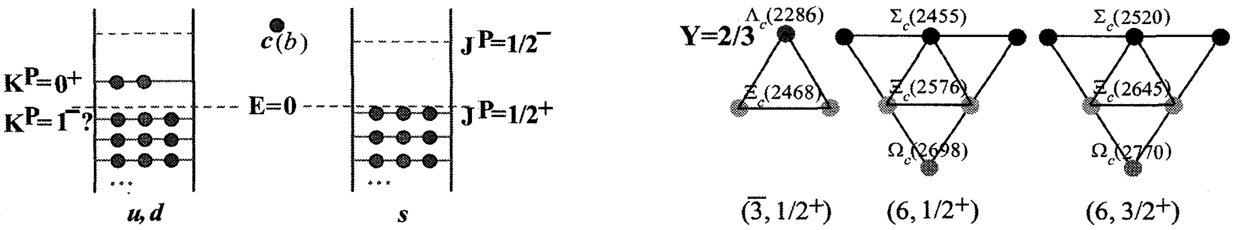

Fig. 3. Filling u, d and s shells for the ground-state charmed baryons (left), and $SU(3)$ multiplets generated by this filling scheme (right): $(\overline{\mathbf{3}}, 1/2^+)$, $(\mathbf{6}, 1/2^+)$ and $(\mathbf{6}, 3/2^+)$.

After the rotational band for a given transition is constructed, one has to check if the rotational energy of a particular multiplet is $\mathcal{O}(1/N_c)$ and not $\mathcal{O}(1)$ (see the Appendix of Ref. 2)), and if it is compatible with Fermi statistics at $N_c=3$: some *a priori* possible multiplets drop out. One gets a satisfactory description of all light baryon resonances up to about 2 GeV, to be published separately.[22)]

§5. Charmed and bottom baryons, the lowest multiplets

If one of the light quarks in a light baryon is replaced by a heavy b or c quark, there are still N_c-1 light quarks left. At large N_c, they form *the same* mean field as in light baryons, with the same sequence of Dirac levels, up to $1/N_c$ corrections. The heavy quark contributes to the mean $SU(3)_{\rm flav}$ symmetric field but it is a $1/N_c$ correction, too. It means that at large N_c one can *predict* the spectrum of the $Qq\ldots q$ (and $Qq\ldots qq\bar{q}$) baryons from the spectrum of light baryons. At $N_c=3$ one does not expect qualitative difference with the $N_c\to\infty$ limit, although $1/N_c$ corrections should be kept in mind. I consider the heavy quark as a non-relativistic particle having spin $J_h=\frac{1}{2}$. $SU(4)_{\rm flav}$ symmetry is badly violated and is of no guidance.

The filling of Dirac levels for the ground-state c (or b) baryon is shown in Fig. 3, left: there is a hole in the 0^+ shell for u, d quarks as there are only N_c-1 quarks there, in an antisymmetric state in color. Adding the heavy quark makes the full state 'colorless'.

As in the case of light baryons, the filling scheme by itself does not tell us what are the quantum numbers of the state: they arise from quantizing the $SU(3)_{\rm flav}$ and $SO(3)_{\rm space}$ rotations of the given filling scheme. Let us do it for the ground-state baryons.

First of all, we determine the hypercharge of the filling scheme: in this case it is $Y'=\frac{1}{3}(N_c-1)$ since there are N_c-1 u, d quarks each having hypercharge one third. At $N_c=3$ one has $Y'=\frac{2}{3}$. There are two $SU(3)$ multiplets containing particles with hypercharge $\frac{2}{3}$: the anti-triplet $\overline{\mathbf{3}}$ $(p=0, q=1)$ and the sextet $\mathbf{6}$ $(p=2, q=0)$. Therefore these are the allowed multiplets (see Fig. 3, right). What are their spins?

In the $\overline{\mathbf{3}}$ representation, there is one particle with $Y'=\frac{2}{3}$ and hence its isospin $T'=0$. The possible spin of the multiplet is found from Eq. (4·1) which needs to be modified to include the spin of the heavy quark J_h:

$$\boldsymbol{J}=\boldsymbol{T}'+\boldsymbol{J_1}+\boldsymbol{J_2}+\boldsymbol{J_h}. \qquad (5\cdot1)$$

In this case $J_1=J_2=0$ since s quarks are not involved, $T'=0$, and $J_h=\frac{1}{2}$.

Therefore, the only possible spin of the anti-triplet is $\frac{1}{2}$, and parity plus. Its rotational energy is, according to Eq. (4·2),

$$E_{\rm rot}^{(\bar{\mathbf{3}})} = \frac{1}{2I_2}. \tag{5·2}$$

In the **6** representation, there are three particles with $Y' = \frac{2}{3}$ and hence their isospin $T' = 1$. From Eq. (5·1) one finds then that there are *two* sextets, one with spin $\frac{1}{2}$ and the other with spin $\frac{3}{2}$. They are degenerate in the leading order as the rotational energy (4·2) depends only on T' but not on the spin:

$$E_{\rm rot}^{(\mathbf{6})} = \frac{1}{2I_2} + \frac{1}{I_1}. \tag{5·3}$$

Thus the filling scheme in Fig. 3, left, implies in fact three $SU(3)$ multiplets: $(\bar{\mathbf{3}}, \frac{1}{2}^+)$, $(\mathbf{6}, \frac{1}{2}^+)$ and $(\mathbf{6}, \frac{3}{2}^+)$ (see Fig. 3, right). The last two are degenerate (but the degeneracy is lifted in the next $1/N_c^2$ order and also from the $1/m_h$ corrections) whereas the center of the anti-triplet is separated from the center of the sextets by the rotational energy $\Delta E_{\rm rot} = \frac{1}{I_1}$. The splitting *inside* multiplets owing to the explicit violation of $SU(3)$ by the strange quark mass is $\mathcal{O}(m_s N_c)$. If m_s is treated as a small perturbation, $m_s = \mathcal{O}(1/N_c^2)$, as I claim it should,[1] the splitting inside the sextet must be equidistant to a good accuracy. Let us confront these predictions with current data.

There are good candidates for the above ground-state multiplets: $\Lambda_c(2286)$ and $\Xi_c(2468)$ for $(\bar{\mathbf{3}}, 1/2^+)$; $\Sigma_c(2455)$, $\Xi_c(2576)$ and $\Omega_c(2698)$ for $(\mathbf{6}, 1/2^+)$; finally $\Sigma_c(2520)$, $\Xi_c(2645)$ and $\Omega_c(2770)$ presumably form $(\mathbf{6}, 3/2^+)$ (see Fig. 3, right). Strictly speaking the J^P quantum numbers of most of these baryons are not measured directly but there is not much doubt they differ from the above assignments. Assuming they are correct, the observed parity-plus charmed baryons form precisely those multiplets that follow from the collective quantization.

The splittings inside the two sextets are equidistant to high accuracy, confirming that m_s can be treated as a small perturbation. Were m_s "not small", there would be substantial $\mathcal{O}(m_s^2)$ corrections to the masses, which would violate the equidistant character of the sextets spectrum.

The centers of the three multiplets are at

$$\begin{aligned} m\left(\bar{\mathbf{3}}, 1/2^+\right) &= \frac{2287 + 2 * 2468}{3} = 2408 \text{ MeV}, \\ m\left(\mathbf{6}, 1/2^+\right) &= \frac{3 * 2455 + 2 * 2576 + 2698}{6} = 2536 \text{ MeV}, \\ m\left(\mathbf{6}, 3/2^+\right) &= \frac{3 * 2520 + 2 * 2645 + 2770}{6} = 2603 \text{ MeV}. \end{aligned} \tag{5·4}$$

Although the two sextets are not exactly degenerate, their splitting 67 MeV (an unaccounted $1/N_c^2$ effect) is much less than the splitting between the anti-triplet and the mean mass of the sextets, which is

$$\frac{2536 + 2603}{2} - 2408 = 162 \text{ MeV} = E_{\rm rot}^{(\mathbf{6})} - E_{\rm rot}^{(\bar{\mathbf{3}})} = \frac{1}{I_1} = \mathcal{O}(1/N_c). \tag{5·5}$$

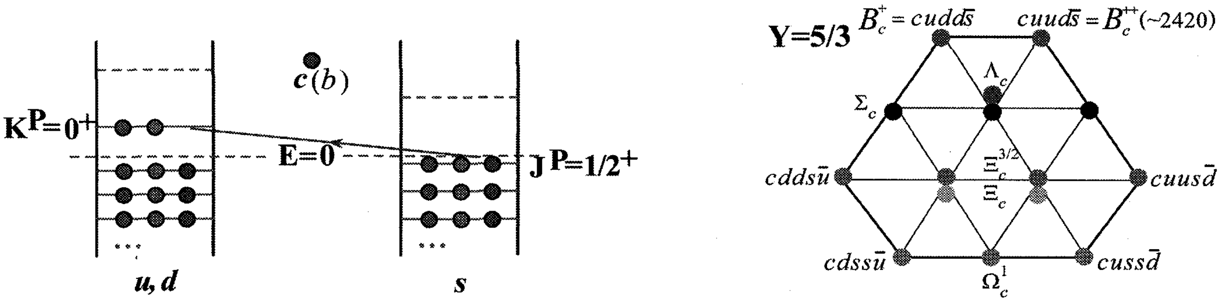

Fig. 4. The arrow shows the lowest Gamov–Teller excitation (left) leading to charmed pentaquarks forming $\overline{\mathbf{15}}$ (right).

Furthermore, this number should be compared with the moment of inertia following from the splitting between *light* baryons, $(\mathbf{10}, \frac{3}{2}^+)$ and $(\mathbf{8}, \frac{1}{2}^+)$, yielding $1/I_1 = 153\,\mathrm{MeV}$. The proximity of the two completely different determinations of the moment of inertia supports the basic idea that it is reasonable to view both light and heavy baryons from the same large-N_c perspective.*)

§6. Charmed and bottom baryons, exotic states

Our new observation is that there is a Gamov–Teller-type transition when the axial current annihilates a strange quark in the $\frac{1}{2}^+$ shell, and creates an u or d quark in the 0^+ shell (see Fig. 4, left), like in the case of the Θ^+. In heavy baryons it is even more trivial as there is a hole in the 0^+ valence shell from the start. Filling in this hole means making charmed (or bottom) pentaquarks which I name "Beta baryons": $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$, and $\mathcal{B}_b^{+} = buud\bar{s}$, $\mathcal{B}_b^{0} = budd\bar{s}$.**) While the existence of Θ^+ requires an excited ('Roper') one-particle level, the existence of the $\mathcal{B}_{c,b}$ baryons needs only the ground-state level which is undoubtedly there. In this sense, the $\mathcal{B}_{c,b}$ baryons are more basic than the Θ^+.

What are the $SU(3)$ multiplets corresponding to this excitation? The hypercharge is $Y' = 3 \times \frac{1}{3} - (-\frac{2}{3}) = \frac{5}{3}$. The lowest $SU(3)$ representation containing particles with $Y' = \frac{5}{3}$ is the *anti-decapenta* $(\overline{\mathbf{15}})$-plet $(p = 1, q = 2)$ (see Fig. 4, right). Therefore, this is an allowed multiplet generated by the transition. There are two particles with $Y' = \frac{5}{3}$, and hence their isospin is $T' = \frac{1}{2}$. The allowed spin is given by Eq. (5·1) where one puts $J_1 = \frac{1}{2}$, $J_2 = 0$ and obtains that the possible spins of the multiplets are $\frac{1}{2}$ (twice) and $\frac{3}{2}$, parity plus. All of them are degenerate

*) The relation $m(\mathbf{6}) - m(\bar{\mathbf{3}}) = \frac{2}{3}(m(\Delta) - m(N))$ has been first derived in Ref. 23) from the application of the Skyrme model to heavy baryons, an approach being similar in spirit to the present one.

**) On the naming: After I proposed the name Θ^+ for the pentaquark $uudd\bar{s}$, its heavy counterparts $uudd\bar{c}$ and $uudd\bar{b}$ conjectured by Karliner and Lipkin[24] have been named Θ_c and Θ_b, respectively, according to the tradition to denote baryons with subscripts c, b when the strange quark in a light baryon is replaced by a heavy one. The $uudd\bar{c}$ pentaquark with positive parity was first considered by Stancu.[25] Pentaquarks of the type $qqqs\bar{Q}$ have been hypothesized by Gignoux, Silvestre-Brac and Richard[26] and Lipkin[27] and denoted as $P_{\bar{c}s}$. I propose here a very different type of pentaquarks $qqqQ\bar{s}$ which I suggest to call "Beta baryons" and denote as $\mathcal{B}_{c,b}$ (calligraphic 'Bee' in LaTeX). The implication is that "Alpha baryons" are mainly the three-quark ones, of course.

in the leading order in $1/N_c$ but split in the next-to-leading order.

Thus the Gamov–Teller-type transition shown in Fig. 4, left, induces three almost degenerate multiplets: $2 \times (\overline{\mathbf{15}}, 1/2^+)$ and $(\overline{\mathbf{15}}, 3/2^+)$.*)

The six baryons at the corners of the hexagon in Fig. 4, right, are explicitly exotic: their quantum numbers cannot be achieved from 3-quark states. The rest 9 baryons are crypto-exotic: they are mainly pentaquarks but have the quantum numbers of the ground-state baryons belonging to $\overline{\mathbf{3}}$ and $\mathbf{6}$ representations, and can mix with them. The mixing is an $SU(3)$ violating effect, the mixing angle being $\theta = \mathcal{O}(m_s N_c^2/\Lambda)$ where $\Lambda \sim 1\,\mathrm{GeV}$ is a typical scale in strong interactions. Actually the isotopic quadruplet $\Xi_c^{3/2}$ and the triplet Ω_c^1 mix up with the corresponding members of the $\overline{\mathbf{3}}$ and $\mathbf{6}$ only through isospin breaking, and therefore this mixing can be neglected. The mixing of Λ_c, Σ_c and Ξ_c leads to a shift in the physical baryon masses, that is quadratic in m_s; it is of the order of $m_s^2 N_c^3/\Lambda$. The fact that baryons in the sextets are almost equidistant means that in practice the mixing is numerically small. Probably more important is the mixing between the two $(\overline{\mathbf{15}}, 1/2^+)$-plets with identical quantum numbers: one goes up, and the other goes down. The splitting of the $\overline{\mathbf{15}}$-plet due to nonzero m_s has been considered in Ref. 2).

The lightest member of the $\overline{\mathbf{15}}$-plet is the exotic doublet $\mathcal{B}_c$, and the heaviest is the exotic triplet Ω_c^1. Since we know the separation between the $1/2^+$ level for s quarks and the 0^+ level for u, d quarks from fitting the light baryon resonances (it is 130 MeV, see §3), and assuming that it does not change for heavy baryons (as it would be at $N_c \to \infty$), I estimate the mass of the $\mathcal{B}_c^{++,+}$ pentaquarks at about $m(\Lambda_c) + 130\,\mathrm{MeV} = 2420\,\mathrm{MeV}$. The corresponding bottom pentaquarks $\mathcal{B}_b^{+,0}$ mass is about $m(\Lambda_b) + 130\,\mathrm{MeV} = 5750\,\mathrm{MeV}$. These are very light masses.

The accuracy of this prediction is $\mathcal{O}(1/N_c) \sim 150\,\mathrm{MeV}$ but there is still a 360 MeV margin below the threshold for strong decays $\mathcal{B}_c \to \Lambda_c K$ (2780 MeV), $\mathcal{B}_b \to \Lambda_b K$ (6110 MeV). If the mass is above the threshold the charmed pentaquark can be observed as a narrow peak in the $\Lambda_c K^+$ and $\Lambda_c K_s$ mass distribution. However, more likely it is below the threshold meaning that it can decay only weakly.

Charmed pentaquarks have been considered by Wu and Ma in another approach;[28] however, these authors get far larger masses and in addition pentaquarks with $\bar{c}$ quarks appear almost degenerate with those made of c quarks. This is not the case in the present scheme where *anti*-charmed pentaquarks are about 500 MeV heavier than the charmed ones.[2] Allowing even for a 360 MeV uncertainty in numerics, Beta baryons $\mathcal{B}_{b,c}$ remain below the threshold for strong decays!

§7. Production rate of Beta baryons, and decay signatures

In principle, $\mathcal{B}_{b,c}$ baryons can be produced whenever charm (bottom) is produced. However, the production rate is expected to be very low. It is affected by the general

*) In $SU(3)$, there are four representations with dimension 15: $\mathbf{15}$ $(p=2, q=1)$, $\overline{\mathbf{15}}$ $(p=1, q=2)$, $\mathbf{15}'$ $(p=4, q=0)$, $\overline{\mathbf{15}}'$ $(p=0, q=4)$. The tradition is to call the multiplet *anti* if the number of particles with highest hypercharge is less than those with lowest hypercharge, cf. $\mathbf{3}$ vs $\overline{\mathbf{3}}$, $\mathbf{6}$ vs $\overline{\mathbf{6}}$, $\mathbf{10}$ vs $\overline{\mathbf{10}}$. The representation $\overline{\mathbf{15}}'$ formally also contains particles with hypercharge $\frac{5}{3}$ but it can be minimally built from four antiquarks and is thus irrelevant.

suppression of charm (bottom) production, and by the small coalescence factor specific for the production of objects built of many constituents. Therefore, high-energy, high-luminosity machines like LHC, Belle and BaBar have better chances.

It is very difficult to make a reliable estimate of the production rate, say, at LHC, therefore I make a pessimistic estimate.*) The number of charmed baryons produced in the central rapidity range (where it is maximal) is estimated as $dN/dy \sim 10^{-3}$. For bottom quarks it is several times less. The number of anti-deuterons produced at LHC is expected at the level of $dN/dy \sim 10^{-4}$. Deuterons are 6 quarks so the rate gives an idea of the coalescence factor for a 5-quark system, too. This suppression factor is roughly consistent with the pentaquark Θ^+ production cross section of 10 μb claimed by the SVD collaboration.[17] To get the lower bound for the production rate for the pentaquark $\mathcal{B}_c$ baryons I am inclined to multiply the two probabilities and obtain for the LHC

$$\frac{dN^{\mathcal{B}_c}}{dy} \sim 10^{-7}, \qquad y \approx 0. \tag{7·1}$$

This is low enough but one looses even more when a specific channel is chosen to trigger the decay of $\mathcal{B}_c$. From the experience with 'ordinary' charmed baryons we know that there are very many decay channels, the largest branching ratios being at the level of 1%. Therefore, it is important to choose a decay channel with as low background as possible, rather than seeking for a dominant decay mode. $\mathcal{B}_c^{++}$ has a remarkable decay into $p\pi^+$ proceeding through the Cabibbo-unsuppressed annihilation $c\bar{s} \to u\bar{d}$. However, this decay has probably a large background even if events are selected with protons spatially displaced from the reaction vertex. I expect that the $\mathcal{B}_c$ lifetime is of the same order as that of normal charmed baryons, i.e. 10^{-13} s, meaning that its decay can be resolved in a vertex detector. In addition, the in-flight Cabibbo-unsuppressed decay $c \to su\bar{d}$ is probably faster than annihilation.

The $\mathcal{B}_c^{++} \to \bar{s}s\bar{d}duuu$ intermediate state is interesting because it can further proceed into $\Lambda K^+\pi^+$ or to $pK^+\bar{K}^0$ or, via a narrow resonance ϕ, to $p\phi\pi^+ \to pK^+K^-\pi^+$. These channels may balance the branching ratio and background conditions. In fact, a similar channel $pK^+K^-\pi^-$ has been used by E791 in the search for the neutral anti-charm pentaquark $P_{\bar{c}s}$[29] but with the trigger that four charged particles have the total zero charge. Here it must be +2. The $\mathcal{B}_c^+$ can decay into three-prong final states $p\phi \to pK^+K^-$ or ΛK^+.

Returning to the production rate (7·1) it should be multiplied by a typical branching ratio 10^{-2} to a particular observation channel, yielding a tiny observation rate of 10^{-9}. Given that the total number of events at LHC is 10^{15}/year, it still promises 10^6 registrations of $\mathcal{B}_c$ per year. Respectively, there could be as much as 10^5 $\mathcal{B}_b$ events per year. At Fermilab the rate is 3 orders of magnitude less but still probably accessible. It is interesting that a good fraction of $\mathcal{B}_b$ decays must be into $\mathcal{B}_c$ plus pions since the dominant weak decay is $b \to cd\bar{u}$.

Since the main b-quark decay is into c quarks, $\mathcal{B}_c$ can be looked for at B-factories, Belle and BaBar. As a conservative estimate of the $\mathcal{B}_c$ production probability I would take the product of the probability to create a charmed baryon of comparable mass

*) I am indebted to Ya. Azimov, Yu. Shabelsky and M. Strikman for their input in this discussion.

(say, $\Sigma_c(2455)$ or $\Xi_c(2468)$), and of the probability to create a deuteron, making about 10^{-5}. Therefore, from a sample of 10^9 $c\bar{c}$ events at Belle, one could expect about 100 $\mathcal{B}_c$ decays in a given channel which makes its observation feasible.

§8. Conclusions

If the number of colors N_c is treated as a free algebraic parameter, baryon resonances are classified in a simple way. At large N_c all baryon resonances are basically determined by the "intrinsic" quark spectrum which takes certain limiting shape at $N_c \to \infty$. This spectrum is the same in light baryons $q \ldots qq$ with N_c light quarks q, and in heavy baryons $q \ldots qQ$ with $N_c{-}1$ light quarks and one heavy quark Q, since the difference is a $1/N_c$ effect.

One can excite quark levels in various ways called either one-particle or particle-hole excitations; in both cases the excitation energy is $\mathcal{O}(1)$. On top of each one-quark or quark-antiquark excitation there is generically a band of $SU(3)$ multiplets of baryon resonances, that are rotational states of a baryon as a whole. Therefore, the splitting between multiplets is $\mathcal{O}(1/N_c)$. The rotational band is terminated when the rotational energy reaches $\mathcal{O}(1)$. Some multiplets which differ only by spin are degenerate in the leading order but become split in the next $\mathcal{O}(1/N_c^2)$ order.

In reality N_c is only 3, and the above idealistic hierarchy of scales is somewhat blurred. Nevertheless, a close inspection of the spectrum of baryon resonances reveals certain hierarchy schematically summarized as follows:

- Baryon mass: $\mathcal{O}(N_c)$, numerically 1200 MeV, the average mass of the ground-state octet.
- One-quark and particle-hole excitations in the intrinsic spectrum: $\mathcal{O}(1)$, typically 400 MeV, for example the excitation of the Roper resonance.
- Splitting between the centers of $SU(3)$ multiplets arising as rotational excitations of a given intrinsic state: $\mathcal{O}(1/N_c)$, typically 133 MeV.
- Splitting between the centers of rotational multiplets differing by spin, that are degenerate in the leading order: $\mathcal{O}(1/N_c^2)$, typically 44 MeV.
- Splitting inside a given multiplet owing to the nonzero strange quark mass: $\mathcal{O}(m_s N_c)$, typically 140 MeV.

In practical terms, the lowest light baryon multiplets $(\mathbf{8}, 1/2^+)$ and $(\mathbf{10}, 3/2^+)$ form the "rotational band" about the ground state, with the splitting between their centers being $\frac{3}{2I_1} = 230\,\mathrm{MeV} = \mathcal{O}(1/N_c)$. The ground state of a heavy baryon (where one light quark is replaced by a heavy one so that there is a hole in the light quarks valence shell) generates a rotational band of three multiplets, $(\bar{\mathbf{3}}, 1/2^+)$, $(\mathbf{6}, 1/2^+)$ and $(\mathbf{6}, 3/2^+)$. These are precisely the observed multiplets, and the prediction is that the two sextets are degenerate in the leading order whereas the splitting between the $\bar{\mathbf{3}}$ and $\mathbf{6}$ is $\frac{1}{I_1} = 153\,\mathrm{MeV}$. In reality the two sextets are not degenerate but their splitting 67 MeV (an $1/N_c^2$ and $1/m_c$ effect) is substantially less than the splitting between the mean mass of the sextets and the anti-triplet, which is 162 MeV, off by only 6% from the large-N_c prediction.

This coincidence encourages to look what is the *lowest non-rotational excitation* of a heavy baryon in the large-N_c limit. Apparently, it is the particle-hole excitation where one takes an s quark from the highest filled shell and puts an u or d quark at the lowest u, d valence shell, filling in the hole there (see Fig. 4). The corresponding baryon resonances have the (penta) quark content $\mathcal{B}_c^{++} = cuud\bar{s}$, $\mathcal{B}_c^{+} = cudd\bar{s}$ with mass $m(\Lambda_c) + 130\,\text{MeV} = 2420\,\text{MeV}$ and $\mathcal{B}_b^{+} = buud\bar{s}$, $\mathcal{B}_b^{0} = budd\bar{s}$ with mass $m(\Lambda_b) + 130\,\text{MeV} = 5750\,\text{MeV}$. I call them "Beta baryons" (implying, of course, that "Alpha baryons" are the standard, mainly three-quark baryons). Actually, $\mathcal{B}_{b,c}$ baryons are part of the larger $\overline{\mathbf{15}}$ multiplet of pentaquarks, and there must be three of them: two with spin-parity $1/2^+$ and one with $3/2^+$. The splitting of these $\overline{\mathbf{15}}$-plets is expected to be less than 100 MeV.

The arithmetic for the masses would be exact in the limit of infinite N_c, however in reality $\mathcal{O}(1/N_c) \sim 150\,\text{MeV}$ corrections are allowed. However, there is still quite some room below the threshold for strong decays, which is at 2780 MeV. Therefore, I believe that at least one but maybe two or even three exotic pentaquarks $\mathcal{B}_{b,c}$ are stable with respect to strong decays. This makes their discovery feasible, despite that the production rate is probably very low (see §7).

I think that the case presented for the heavy $\mathcal{B}_{b,c}$ pentaquarks is even stronger than that it has been for the Θ^+ pentaquark,[13] whose mass I confirm here from a new, unified point of view.

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