Omori Law after Large-Scale Destruction of Production Network

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After the two giant earthquakes, Kobe 1995 and East Japan 2011, a large-scale destruction took place on supplier-customer network. These primary and exogenous shocks were propagated on the production network, which caused a secondary effect resulting in chained bankruptcies. By employing data of bankrupcies occuring in a neighbor to the primarily damaged firms or regional economy, we show that the number of neighboring failures obeys a Omori law, a power-law relaxation. This finding implies that the recovery from such a huge shock on production network is much more sluggish than one can naively expect.

§1. Introduction

Several complex systems have abnormally long-time relaxation after extreme events. An example is what we call *Omori law* today, which is a power-law decay of aftershock activity with time after a large earthquake. This law was discovered in the pioneering work of seismology by Fusakichi Omori during the 1890's.[1] His discovery was based on the Great Nobi (Mino-Owari) Earthquake in 1891, the largest inland earthquake in the historical records of Japan, two other earthquakes and their aftershocks. After the centenary of his discovery, an accumulated data of aftershocks due to the Nobi event until today reveals the validity of Omori law, surprisingly long relaxation time for more than 10^4 days after the earthquake (see Fig. 1 in the review paper[2]).

Social and economic systems as well as natural systems have been known to exhibit long-time relaxation processes similar to the Omori law. Examples include financial market crashes,[3] book sales ranking dynamics accompanying shocks,[4] and so forth. See Refs. 3) and 4) and the references therein for example. These empirical findings are addressing the same question: how is the dynamics of such a system affected and reacting to extreme events or shocks? This question is important to the understanding the system, predicting recovery processes after shocks and, in some cases, precursory signals.

The Great East Japan Disaster on March 11, 2011 was a huge shock to the domestic and overseas economy. The supplier-customer or production network was severely damaged at a nation-wide scale. Firstly, a mass destruction of firms and a set of intervention in many industrial sectors and geographical locations took place in such a devastating way that a number of firms instantly ceased their production activity. Secondly, financially fragile firms went into bankruptcies and defaults. If such failed firms are irreplaceable nodes in supplier-customer links, many places in the production network were severely damaged. Thirdly, such a damage was so serious that other firms in upstream or downstream side of the network eventually went into bankruptcies or financially ill-conditions as a kind of *Tsunami* effect.

In this paper, we shall show a preliminary result that Omori law holds for the "aftershocks" following after a large-scale destruction of production network. Aftershocks are defined and measured by bankruptcies, which were directly caused by other failed firms due to such a large-scale shock. The identification of causes for bankruptcies was done carefully by credit research agencies in their datasets. By using such datasets, failed *and* linked nodes can be identified so that one can count the number of failed nodes in the *neighbor* of the nodes damaged by the initial shock, which is our definition of aftershock. We employ two episodes of nation-wide shocks, the Kobe Earthquake in 1995 and the Great East Japan Earthquake in 2011 for empirical tests.

Section 2 describes aftershocks defined by bankruptcies and a basic idea behind the method of identifying neighboring failures. In §3, we shall examine the validity of Omori law by using two episodes of the Kobe and East Japan. The aftershocks are quantitatively characterized by the cumulative number of chained bankruptcies, which are shown to satisfy a modified Omori formula. This finding would provide potentially useful estimation on how long and large those aftershocks of defaults can last. We shall discuss these points as well as things to be investigated further in §4. Section 5 summarizes the paper.

§2. Data of neighboring bankruptcies as aftershocks

According to the 2011 White Paper on Small and Medium Enterprises in Japan,[7] a large number of firms were directly affected by the East Japan Disaster due to the earthquake, tsunami, nuclear and other disasters, especially along the north-east coast. This primary and exogenous shock resulted in business failures, mostly temporary but often permanent, in a considerable fraction in the economy.

Because more than 0.7 million firms were present in the region among 2 million firms in the entire country, the suppliers and customers, who depend their business on the damaged firms and the regional economy, were affected afterward. For example, suppliers had a delay, or often a loss, in the receipt of accounts receivable, causing an abrupt drop in sales which may have deteriorated their financial conditions subsequently.

To measure such secondary effects, we shall focus on bankruptcies. A bankruptcy or business failure is a critical financial state of a firm; its debt dominates its balance-sheet so that it has little equity, and the firm cannot no longer manage its business. Because the secondary effects propagate along the supplier-customer network, a bankruptcy may be regarded as a fracture under a stress strengthened in the neighbor of preceding increase of stress. We assume that the process of stress propagation can be traced by observing such neighboring bankruptcies on the production network.

We employ two datasets by leading credit research agencies in Tokyo, which carefully identified the causes of bankruptcies in exhaustive lists of domestic failures.[5],[6] Let us illustrate the method of identification in the dataset of Tokyo Shoko Research, Inc.[6] It is exhaustive in the sense that all bankruptcies with debt exceeding 10 million yen (roughly equal to 0.1 million dollars or euros) are recorded. Causes for bankruptcies are investigated and classified as follows.

1. "solo" failure
   - poor performance in business, which includes business depression, excessive competition and extrinsic shocks
   - loose management, which includes failure of speculative investment, internal conflict and lack of efficient management
   - long-term accumulated deficit
   - insufficient working capital
   - accidental causes (disasters etc.)
   - deterioration of products in inventory
   - excessive investment in facilities and equipment
2. "link" effect
   - secondary effect from bankruptcy of customer, subsidiary or collateral companies and failure of business-related firms
   - failure of accounts receivable
3. others
   - refusal of credit by financial institutions
   - unclassified

A bankruptcy is caused by two or more combination of these classified effects; the most dominant cause is recorded in such a case. See Ref. 8) for more details.

In addition, they carefully investigated the causes of bankruptcies after the Kobe Earthquake in 1995 and the East Japan Earthquake in 2011. The solo failures and link effects may have been originated from business depression and other bankruptcies in the primary shock. A typical case of solo failures is an abrupt drop of sales due to an exogenous shock in a damaged region, when the firm depends crucially on the region for its sales. Typical link effects are what we already mentioned above. We assume that such cases are *neighboring bankruptcies* in secondary effects due to the primary shock.

The two credit research agencies had accumulated monthly numbers of bankruptcies after the Kobe Earthquake, and are accumulating after the East Japan Earthquake. The data are publicly available, which we shall examine in the following section.

§3. Omori law after large-scale shocks of production network

The Omori law states that the frequency of aftershocks per unit time, $n(t)$ at time t decays as

$$n(t) = K\,(t+c)^{-p}, \tag{3·1}$$

where K is a positive constant which determines the magnitude of $n(t)$ and c is a positive constant to avoid divergence at the origin, $t = 0$. $p > 0$ is the exponent of the power-law decay. The original proposal corresponds to the case $p = 1$, and was later modified into the above form, a modified Omori formula. This is a power-law decay

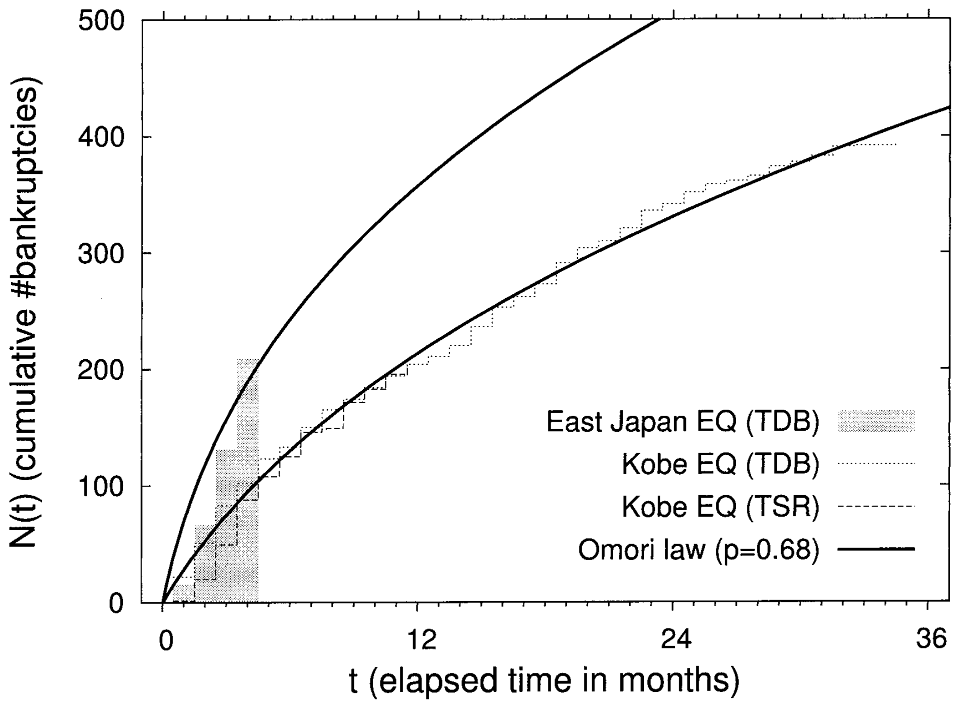

Fig. 1. Cumulative number of bankruptcies $N(t)$ and elapsed time t since the events of two earthquakes, Kobe (1995) and East Japan (2011). Dotted lines are two data $N(t)$ based on investigation of the Kobe case for three years by two independent agencies, TDB and TSR (the latter is scaled with its number for comparison). Gray bars are a few monthly data of TDB for the East Japan case. The two lines are modified Omori formula with the same parameter $p = 0.68$, which is estimated by least-square fit for the Kobe data.

with exponent p, so an extremely long relaxation in the aftershock earthquakes. The value of p ranges from 0.6 to 2.5 with median 1.1 in the survey of more than 200 observations during 33 years of earthquakes (see Ref. 2)).

Equivalently, but more suitable for comparison with data, the cumulative number $N(t)$ can be written by the following:

$$N(t) = \int_0^t dt' n(t') = \begin{cases} K\ \left((t+c)^{1-p} - c^{1-p}\right)/(1-p) & \text{for } p \neq 1, \\ K\ \ln(t/c+1) & \text{for } p = 1. \end{cases} \tag{3·2}$$

Because $N(t)$ is accumulation of $n(t)$, the statistical fluctuation and measurement errors in $N(t)$ are much reduced compared to them in $n(t)$. It is a custom to measure $N(t)$ to estimate the parameters of p and K, c.

Let us put the monthly data of neighboring bankruptcies described in §2 into the cumulative number $N(t)$, where t is the time of occurrence of the extreme shocks. The result is shown in Fig. 1.

For the Kobe Earthquake in 1995, the neighboring bankruptcies were recorded for three years by Teikoku Data Bank, Inc. (TDB),[5] for a year by Tokyo Shoko Research, Inc. (TSR).[6] We fit the data of TDB by (3·2) in a nonlinear least-square fitting. The parameter p was estimated as $p = 0.68 \pm 0.18$ with the standard error. Although the two data have different magnitude of $N(t)$ based on independent investigation, they simply give different scales of the parameter K if they obey the same power-law. The TSR data is rescaled vertically in Fig. 1 which can be fitted well by the power-law with the same exponent p.

Similarly, although the data for the East Japan Earthquake has only a half-year elapsed time, it is observed from the fitting is satisfactory with respect to the same power-law (see gray bars for the actual data and a line for the Omori law). One can also see that the East Japan case has a much larger impact in terms of the overall scale in the number of failures. The parameters p and K are to be determined by new data available in coming months.

Note that while the TDB data was for three years, longer investigation might reveal that the effect is longer than the first three years. Actually, if one naively extrapolates the Omori law, there is a possibility that neighboring bankruptcies had been still going on beyond into the fourth or even longer years.

We have shown that the Omori law holds for the number of aftershocks measured by neighboring bankruptcies after the exogenous shocks of the two large earthquakes. Let us discuss the implication of this finding and also necessary verification to be done in our future work.

§4. Discussion

While our result in the preceding section is encouraging, there are several issues to be investigated. Let us discuss them in this section.

1. Other methods of measuring aftershocks, especially with "magnitudes":
   We used the number of neighboring bankruptcies on the production network in order to measure aftershocks, because a bankruptcy is the most clear evidence that a firm is in a so critically deteriorated state that it fails under a stress caused by the neighboring failures. In a more mild case, such a firm may be affected seriously in its financial state but yet does not go into bankruptcy. If one measures the magnitude of stress by an abrupt drop of sales, a sudden increase of debt and so forth, for example, one is able to quantify the extent of aftershocks including their magnitude. These methods will add the validity of our finding.
2. Other economic shocks:
   By using the methods of measuring, the one in this paper and others described above, one will be able to test the Omori law in other large-scale destruction or depression of the economy rather than the exogenous and natural disasters. The Lehman shock and the present and past financial crises are good examples.
3. Implication of Omori law:
   The fact that the relaxation process obeys a power-law implies that the influence of secondary effects or "Tsunami" after the primary damage in the network is extremely sluggish. Figure 1 indicates two things — duration of relaxation and its extent. Suppose that one can observe the first half-year, for instance, then by fitting the data by a modified Omori formula with a set of parameters, one can predict how long the relaxation will take and how large effects it will bring about under the hypothesis of power-law relaxation. What is more important than such a prediction is the possibility that one can identify industrial sectors

and geographical regions that are fragile or robust under such a propagation of failures. Such information would be useful to plan a recovery of and investment into sectors and regions.

4. Origin of Omori law:
The presence of a power-law relaxation seems to be ubiquitous in a variety of complex systems. The origin of Omori law even in the case of earthquake is not fully understood. Nevertheless, many proposed mechanisms are based on the idea that the entire region of aftershocks is composed of a number of small and heterogeneous regions containing faults, among which the stress of main earthquake is released subsequently with delayed shocks. This idea can be possibly applied to our case. Because the production network is not homogeneous but very heterogeneous with hierarchical tightly-knitted clusters or modules (see Ref. 9) for more details). Such a hierarchical and heterogeneous network structure quite likely provides an arena where a delayed release of stress propagation in terms of financial deterioration of firms. This approach of understanding the origin of Omori law seems promising, because this is common to other phenomena with power-law relaxation in natural and social systems.

These open issues are to be investigated in a future work.

§5. Summary

We investigated "aftershocks" after a large-scale destruction of supplier and customer network. The aftershock is defined and measured by neighboring bankruptcies on the production network. The neighborhood is determined in two independent investigations of credit research agencies on exhaustive lists of bankruptcies by the criterion that the cause of bankruptcy is due to the primarily damaged firms or regional economy. Assuming that the relaxation process in secondary effects after the primary exogenous shock can be traced by such neighboring bankruptcies, we found that the modified Omori law holds for the decay in the number of neighboring bankruptcies in the country by using two cases of the Kobe Earthquake in 1995 and the East Japan Earthquake in 2011.

At this preliminary stage, our finding in this paper implies that the recovery from such a huge shock on production network is much more sluggish than one can naively expect. There are several open problems, which include other methods of quantifying aftershocks, consideration of magnitudes of aftershocks, study of economic disasters rather than natural ones, the origin of Omori law, and so forth. These issues are to be investigated further in future.

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