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応力誘起キャビテーションの粘性potential流解析

Viscous potential flow analysis of stress induced cavitation

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Cavitation in an aperture flow in a flat plate is studied using viscous potential flow. The maximum tension criterion for cavitation used here was proposed by Joseph 1995, 1998: "Liquids at atmospheric pressure which cannot withstand tension will cavitate when and where tensile stresses due to motion exceed one atmosphere. A cavity will open in the direction of the maximum tensile stress which is 45° from the plane of shearing in pure shear of a Newtonian fluid." The aperture flow is expressed using a complex potential and the stress is obtained using viscous potential flow. The viscous stress is huge near the tips of the aperture, thus cavitation could be induced.

1. Introduction

It is well known that cavitation may be induced at sharp edges of the inlet of nozzles such as those used in atomizers. It is at just such edges that the pressure of an inviscid fluid into a nozzle is minimum. At higher pressure drops (larger cavitation number) the liquid in the nozzle may break away from the nozzle wall; the flow then attaches to the sharp edge of the nozzle and is surrounded by atmospheric gas. The term incipient cavitation is used to define the situation where cavitation first appears. The term supercavitation describes the situation where there is a strong cavitation flow near the nozzle exit, which is very beneficial to atomization. Total hydraulic flip describes the situation where the liquid jet completely separates from the nozzle wall. Hydraulic flip occurs in a variety of nozzles of different cross section provided that the edge at inlet is sharp and not round. The aperture flow in a flat plate considered here (Fig.1) is a nearly perfect two dimensional model of total hydraulic flip. Experiments documenting the transition to hydraulic flip from cavitating have been presented by Bergwerk,[1] Chaves *et al.*[2] and a few others. The outstanding property of the hydraulic flip is the disappearance of any sign of the cavitation that was there before the flow detached. To our knowledge, reports of the observations of the disappearance of cavitation are for very low viscosity liquids, such as water and diesel oil. In the analysis of aperture flow which follows we find cavitation at the sharp edge for all fluids with viscosity larger than zero, but for low viscosity liquids it would be very hard to observe.

Our analysis is based on the theory of stress induced cavitation put forward by Joseph[3),4)]; the flow will cavitate at places where the principal tensile stress $T_{11} > -p_v$ where p_v is the vapor pressure. The theory of viscous potential flow allows us to compute these stresses directly and easily from the classical potential flow solution for aperture flow.

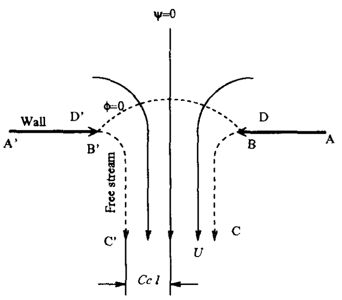


Fig.1 Flow through an aperture in a flat plate.

2. Analysis of stress induced cavitation

The aperture flow in a flat plate is shown in Fig.1. The magnitude in the resulting jet will reach some uniform value U downstream of the edges. The half-width of the jet is $C_c\ell$, where C_c is the contraction coefficient and ℓ is the half-width of the aperture. The complex potential for this flow is given implicitly by

$$f(z) = \phi + \imath\psi = -\frac{2C_c\ell U}{\pi}\ln\left\{\cosh\left[\ln\left(U\frac{\mathrm{d}z}{\mathrm{d}f}\right)\right]\right\} - \imath C_c\ell U. \quad (1)$$

The stress is calculated by $\mathbf{T} = -p\mathbf{1} + 2\mu\mathbf{D}$. The pressure can be calculated using Bernoulli's equation

$$p + \frac{\rho}{2}(u^2 + v^2) = p_d + \frac{\rho}{2}U^2 = p_u, \quad (2)$$

where p_u is the upstream pressure and p_d is the downstream pressure at a position where the velocity reaches the uniform velocity U. The velocities are evaluated using from the potential

$$u = \frac{1}{2}\left(\frac{\mathrm{d}f}{\mathrm{d}z} + \frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{z}}\right), \qquad v = \frac{\imath}{2}\left(\frac{\mathrm{d}f}{\mathrm{d}z} - \frac{\mathrm{d}\bar{f}}{\mathrm{d}\bar{z}}\right). \quad (3)$$

It follows that the rate of strain tensor is

$$2\mathbf{D} = \begin{pmatrix} \left(\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} + \frac{\mathrm{d}^2\bar{f}}{\mathrm{d}\bar{z}^2}\right) & \imath\left(\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} - \frac{\mathrm{d}^2\bar{f}}{\mathrm{d}\bar{z}^2}\right) \\ \imath\left(\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} - \frac{\mathrm{d}^2\bar{f}}{\mathrm{d}\bar{z}^2}\right) & -\left(\frac{\mathrm{d}^2 f}{\mathrm{d}z^2} + \frac{\mathrm{d}^2\bar{f}}{\mathrm{d}\bar{z}^2}\right) \end{pmatrix}. \quad (4)$$

To use the maximum tension criterion for cavitation, the principal axes coordinates in which $2\mathbf{D}$ is diagonalized need to be found. In the two-dimensional case under consideration here, the diagonalized rate of strain tensor is

$$2\mathbf{D} = \begin{pmatrix} \lambda & 0 \\ 0 & -\lambda \end{pmatrix}, \quad \text{where} \quad \lambda = 2\left|\frac{\mathrm{d}^2 f}{\mathrm{d}z^2}\right|. \quad (5)$$

Thus the maximum tension T_{11} is given by $T_{11} = -p + \mu\lambda = -p_u + \frac{\rho}{2}(u^2 + v^2) + \mu\lambda$, and the cavitation threshold is given by $T_{11} = -p_v$. Combining these, we obtain $T_{11} + p_v = -p_u + p_v + \frac{\rho}{2}(u^2 + v^2) + \mu\lambda = 0$. We use $\frac{\rho}{2}U^2$ to render this dimensionless

$$\frac{\tau}{\frac{\rho}{2}U^2} = \frac{T_{11} + p_v}{\frac{\rho}{2}U^2} = \frac{-p_u + p_v}{p_u - p_d} + \frac{u^2 + v^2}{U^2} + \frac{\mu\lambda}{\frac{\rho}{2}U^2}$$
$$= -\frac{1+K}{K} + \left|\alpha\left(\frac{\phi}{\ell U}, \frac{\psi}{\ell U}\right)\right|^2 + \frac{1}{R_e}\frac{2\pi}{C_c}\left|\beta\left(\frac{\phi}{\ell U}, \frac{\psi}{\ell U}\right)\right| = 0, \quad (6)$$

where the dimensionless parameters are defined as

$$\text{Cavitation number} \quad K = (p_u - p_d)\,/\,(p_d - p_v)\,, \tag{7}$$

$$\text{Reynolds number} \quad R_e = \rho \ell U/\mu, \tag{8}$$

and the complex functions α and β are given by

$$\alpha = \left[e^{\gamma} \pm \sqrt{e^{2\gamma}-1}\right]^{-1}, \quad \beta = \left[e^{\gamma} \pm \frac{e^{2\gamma}}{\sqrt{e^{2\gamma}-1}}\right]\alpha^3, \tag{9}$$

with

$$\gamma = -\frac{f + \imath C_c \ell U}{2C_c \ell U}\pi = -\frac{\pi}{2C_c}\frac{\phi}{\ell U} - \frac{\imath\pi}{2C_c}\frac{\psi}{\ell U} - \frac{\imath\pi}{2}. \tag{10}$$

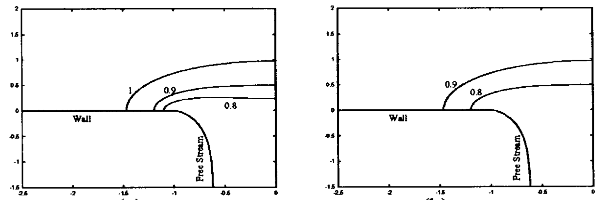
The definition (7) for the cavitation number follows Bergwerk.[1] For a flow with given cavitation number and Reynolds number, equation (6) gives the positions where cavitation inception occurs in terms of $\phi/\ell U$ and $\psi/\ell U$.

3. Cavitation threshold

We use water as an example to illustrate the stress induced cavitation. The vapor pressure of water at 20°C is 2339 Pa and we assume that the downstream pressure is the atmospheric pressure: $p_d = p_a = 10^5$Pa. We calculate the pressure using the Bernoulli's equation (2). The pressure does not depend on R_e and we show the pressure distribution for different cavitation numbers in Fig.2.

The pressure criterion for cavitation is that cavitation occurs when the pressure is lower than the vapor pressure. The minimum pressure in the aperture flow is the downstream pressure and $p_d = 10^5\text{Pa} > p_v = 2339\text{Pa}$. Thus the pressure criterion predicts no cavitation for the case under consideration. However, we will show that cavitation occurs in the aperture flow according to the tensile stress criterion (Joseph[3),4)]).

Next we include the viscous part of the stress and consider the maximum tension T_{11}. The cavitation criterion is that cavitation occurs when $\tau \geq 0$. T_{11} depends on both the Reynolds number and the cavitation number. We show the contour plot for $\tau/(\rho U^2/2)$ with different R_e and K in Fig.3. Although the velocity is continuous everywhere in the aperture flow, its derivative, and therefore the viscous stress, are singular at the sharp edge. Thus at the sharp edge for all fluids with viscosity larger than zero, τ is always larger than zero and cavitation occurs. In our analysis here, we shall avoid the singular points and calculate the stresses at points very close to the edges. This is partially justified by the fact that in reality the edges are not perfectly sharp. As Chaves *et al.*[2)] noted "... Microscopic pictures of the nozzle inlet still show however small indentations of the corner, i.e. less than 5 micrometers." (The diameter of the nozzle in their experiments was 0.2 mm or 0.4 mm).



(a) (b)

Fig.2 Contour plot for $(p - p_v)/(\rho U^2/2)$ in the $(x/\ell, y/\ell)$ plane, (a) $K = 10$; (b) $K = 1000$. In the flow field, $p - p_v > 0$ everywhere. Thus there is no cavitation according to the pressure criterion.

In Fig.3, the curves on which $\tau = 0$ are the thresholds for cavitation. On the side of a $\tau = 0$ curve which is closer to the sharp edge, $\tau > 0$ and cavitation appears; on the other side of the $\tau = 0$ curve, $\tau < 0$ and there is no cavitation.

4. Conclusions

We use the potential flow through an aperture as a 2D model to study the hydraulic flip observed in injection flows at a nozzle. The pressure in the flow field is computed using Bernoulli's equation and the viscous stress is evaluated on the potential. The stress tensor is transformed to the principal axes coordinates and the principal stress T_{11} is obtained. If T_{11} is larger than the negative value of the vapor pressure p_v, the flow will cavitate. We find that cavitation occurs for all fluids with viscosity larger than zero at the sharp edges of the aperture. The region in which cavitation occurs depends on the Reynolds number R_e and the cavitation number K. The cavitation region is larger if R_e is smaller and K is larger. The cavitation is confined to very small regions near the edges of the aperture when R_e is larger and K is smaller.

Researchers do not observe cavitation in hydraulic flip. The reason may be that the Reynolds numbers in nozzle flows are usually very high (in the order of thousands and tens of thousand). Thus even if cavitation occurred at the edge of the nozzle, the cavities would collapse quickly outside the small cavitation region (the time for cavities to collapse is in the order of microseconds according to Chaves *et al.*[2)]) and would be very difficult to observe. The effects of liquid viscosity on cavitation are apparently not known; we could not find an evaluation of these effects in the literature. The results obtained here and in Joseph[4)] suggest that an increase in viscosity lowers the threshold to stress induced cavitation.

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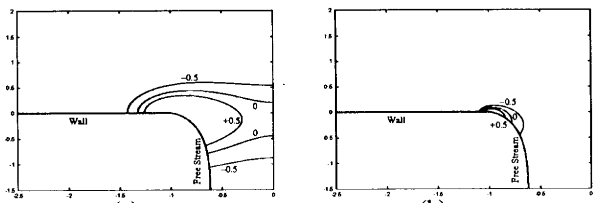

(a) (b)

Fig.3 Contour plot for $\tau/(\rho U^2/2)$ in the $(x/\ell, y/\ell)$ plane, (a) $K = 1$ and $R_e = 1$; (b) $K = 1$ and $R_e = 5$. The curve on which $\tau = 0$ is the threshold for cavitation; cavitation occurs inside this curve.