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鉛直矩形ダクト内における内部加熱流れの線形安定性

Linear stability of pressure-driven flow through a straight duct with internal heating

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We have investigated the linear stability of flow in a vertical rectangular duct subject to a driving pressure gradient and homogeneous internal heating. This problem has been solved numerically by means of a Galerkin method based upon modified Chebyshev polynomials and the standard QZ algorithm. In the square duct, a new class of unstable modes with high propagation velocity has been revealed. The unstable region corresponds to basic flow profiles exhibiting additional inflection lines as compared to isothermal flow.

1. Introduction

The hydrodynamic mechanism of transition from laminar developed flow to turbulence in a straight duct with rectangular cross-section is still poorly understood. Linear studies of Tatsumi and Yoshimura(2) and Theofilis *et al.*(3) have established that the flow in geometries with aspect ratios below a critical value of 3.2 (in the range from unity to infinity) are stable with respect to all infinitesimal disturbances. Furthermore, to our knowledge no investigations of the non-linear stability of this configuration have appeared to date.

In the present study we consider the flow in a vertical duct which is pressure-driven *and* internally heated. Our objectives are the following: (1) By adding thermal effects it is expected that the flow in the square duct becomes linearly unstable (cf. the work of Nagata and Generalis(1) for the analogue flow in a plane channel). These perturbations, in turn, will provide a suitable initial guess for a subsequent non-linear study. A continuation technique can then be used in an attempt to track the solution back to the original problem (i.e. the isothermal flow). (2) We wish to elucidate the stability characteristics of the internally heated case in its own right since a number of important applications involve homogeneously distributed heat sources: e.g. in nuclear engineering and geophysics.

2. Methodology

Invoking the Boussinesq hypothesis and scaling the system of momentum, continuity and temperature equations leaves us with three non-dimensional parameters: the Reynolds number Re (indicating the strength and direction of the mean pressure gradient), the Grashof number Gr (indicating the intensity of the heat source) and the Prandtl number Pr. At the walls the no-slip condition and a constant (reference) temperature is imposed. The perturbation component of the field is expanded in normal modes with the wavenumber α in the streamwise direction, x, and modified Chebyshev polynomials in the cross-stream plane, y-z. A full Galerkin method is employed and the linear eigensystem is analyzed by means of the QZ algorithm.

3. Results

Our results reveal a new class of unstable modes in the square duct, appearing at finite values of the Prandtl number. When plotted in the (Re, Gr)-plane the unstable region is roughly wedge-shaped (cf. figure 1). It is also closely related to the region where the basic flow exhibits 9 additional disjoint inflection lines near the center of the duct when the action of buoyancy and the mean pressure gradient are opposed (already one inflection line can be found in each corner in the isothermal case). The striking match displayed in figure 1 shows the importance of the inflectional properties of the basic flow for its stability. The perturbations have a positive growth rate over a broad range of streamwise wavenumbers α and are characterized by a high phase velocity (comparable in size to the maximum basic flow velocity). The corresponding eigenfunctions are of a bulge-like shape with their highest intensities located near the inflectional lines in the center of the cross-section.

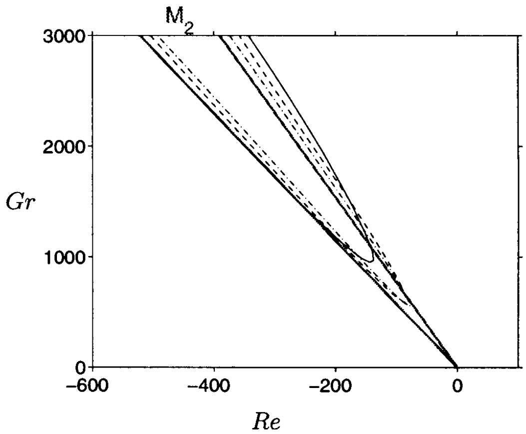

Fig. 1 Neutral curves for heated and pressure-driven square duct flow at finite Prandtl number Pr=7 and various values for the streamwise wavenumber: ——— α=0.5; ---- α=1; —·— α=1.5. In the zone M_2 (indicated by red lines) the basic flow exhibits 9 additional inflection lines.

References

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