Energy spectrum of superfluid turbulence: Numerical Analysis of the Gross-Pitaevskii equation with the small scale dissipation

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流体力学最大の問題の１つである乱流現象を要素還元的に理解するための理想的モデルとして、超流動乱流が近年非常に注目を浴びている。我々はこの超流動乱流のダイナミクスを記述する最も簡単なモデルである Gross-Pitaeskii 方程式の解析を行い、通常の古典乱流との関係を調べた。超流動乱流の要素である量子渦の、渦芯よりも小さなスケールにのみ働く散逸を記述する散逸項を導入したところ、超流動乱流が古典乱流の最も重要な統計則である Kolmogorov 則を示すことを見出した。

Abstract

We investigated dynamics of superfluid turbulence by the Gross-Pitaevskii equation and its relation to conventional classical turbulence. We introduced the dissipation term which is effective only in the scale smaller than the core size of quantized vortices, and found that the energy spectrum of superfluid turbulence was consistent with the Kolmogorov law which is one of the most important statistical laws of classical turbulence.

The steady state for fully developed turbulence of an incompressible classical fluid follows the Kolmogorov law

$$E(k) = C\epsilon^{2/3}k^{-5/3}. \tag{1}$$

for the energy spectrum in the inertial range [1]. Here the energy spectrum $E(k)$ is defined as $E = \int \mathrm{d}k\, E(k)$, where E is the kinetic energy per unit mass, k is the wave number from the Fourier transformation of the velocity field and C is the dimensionless Kolmogorov constant. Recently, there are many discussions about whether this law can also support superfluid turbulence which consists of topological defects, "quantized vortices" [2]. By using Gross-Pitaevskii

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equation which can describe the dynamics of superfluid turbulence, Nore *et al.* studied the energy spectrum of superfluid turbulence [3]. The spectrum of the incompressible kinetic energy is temporarily consistent with the Kolmogorov law. However, the consistency becomes weak in the late stage when dynamics is dominated by small scale excitations created through reconnections of quantized vortices [4].

In this work, we introduced a dissipation term that works only on scales smaller than the core size of vortices ξ and solved the following Fourier transformed Gross-Pitaevskii equation

$$[\mathrm{i}-\gamma(k)]\frac{\partial}{\partial t}\Phi(\boldsymbol{k},t)=[k^2-\mu]\Phi(\boldsymbol{k},t)+\frac{g}{V^2}\sum_{\boldsymbol{k}_1,\boldsymbol{k}_2}\Phi(\boldsymbol{k}_1,t)\Phi^*(\boldsymbol{k}_2,t)\times\Phi(\boldsymbol{k}-\boldsymbol{k}_1+\boldsymbol{k}_2,t), \qquad (2)$$

where V is the volume of the system, μ is the chemical potential, g is the coupling constant between particles and $\Phi(\boldsymbol{k},t)$ is the Fourier component of the macroscopic wave function of Bose condensation with the wave number $\boldsymbol{k}$. $\gamma(k)=\gamma_0\theta(k-2\pi/\xi)$ is the introduced dissipation term and effective only in small scale $k>2\pi/\xi$.

Figure 1 (a) is one example of superfluid turbulence given by the simulation of Eq. (2). Figure 1 (b) is the spectrum of the incompressible kinetic energy and shows that our superfluid turbulence satisfies the Kolmogorov law in the region of small k (inertial range).

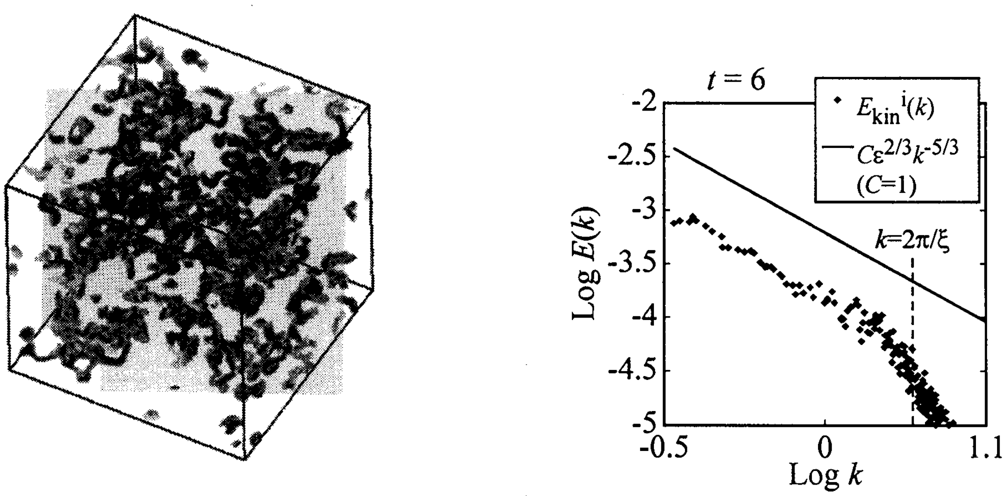

図 1: (a) Plot of quantized vortices in the case of $g=1$. (b) Comparison between the Kolmogorov law and the spectrum of incompressible kinetic energy $E^{\rm i}_{\rm kin}(k)$.

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