論　文

Minimal Evaluation Structure for Inconsistent Multi-attributes Decision-making
—Selection and Addition Method of Evaluation Attributes—

Andreas GEHRMANN and Syohei ISHIZU

(Department of Industrial and Systems Engineering, Aoyama Gakuin University)

Abstract : In evaluation of complex systems, evaluators' preferences in design decisions become often inconsistent or incomplete. Preference relation of evaluators may not satisfy the transitivity property of ordered sets. Different evaluators will use different sets of attributes for design evaluations, depending on his point of view. We take account of rather a family of attributes sets, and represent all choices including potentially inconsistent preference relation. One of the main purposes of this paper is to present a methodology for representing inconsistent preferences of evaluators. In this paper, we focus on the problems of inconsistent preference relation. We assume that we have prepared evaluation attributes. One of our main objectives of this paper is to select important evaluation attributes and construct minimal evaluation structure under the assumption. For the case that the preference relation is not representable by prepared evaluation attributes, we give the way to construct new additional attributes based on the prepared evaluation attributes. Based on the proposed methodology we propose grid evaluation structure approach for the real questionnaire survey.

Keywords : Evaluation space, Evaluation structure, Multi-attribute decision-making, preference relation

1. INTRODUCTION

In most of management decision-making such as project planning, product planning, etc, decision-making becomes complex, and decision-makers need many different evaluation attributes in order to make rational decision. We are frequently faced with the identification problems of evaluation attributes. Indeed, in project planning or product planning, decision-makers must first identify the evaluation attributes which are appropriate to the evaluation of projects or products. In management decision-making, how to identify evaluation attributes is one of the most essential problems. Most studies of Multi-Attribute Decision-Making (MADM) have researched to identify

multi-attribute utility function or decision criteria based on the preference relation of the decision-maker (Keeney 1976) and (Saaty 1999). These studies usually assume a set of evaluation attributes to identify multi-attribute utility functions, but it is not easy to identify an appropriate set of evaluation attributes in real complex management decision-making. Even if the preference relations of the decision-makers are surveyed, decision-makers may not become conscious of the evaluation attributes that explain their preference relations.

In order to show the complex and inconsistent decision-making situations, (Ishizu 2003) showed the three types of evaluation preference relations

(1) Total order

In the case that the preference relation is a total order, one of the main problems is to identify an evaluation attribute. (Fishburn 1970) studied utility function as such an attribute.

(2) Partial order

In the case that the preference relation is a partial order, the decision-making problem will be called MADM problems. Several studies about MADM have investigated with regard to the case (2). (Keeney 1976) researched about multi-attribute utility function. (Saaty 1999) researched about AHP in order to calculate the weighting function from the results of pair-wise comparison, and proposed a concept of inconsistency ratio. (Klir 1993) investigated the fuzzy measure and proposed multi-attribute aggregation. (Ullmann 1982) and (Atzeni 1993) investigated dependency among multiple attributes. Most MADM researchers studied about the construction or identification of multi-attribute utility functions. (Pawlak 1994) and (Orlowska 1997) proposed rough set framework for MADM and tried to extract necessary attributes for the MADM situation from the standpoint of accuracy of approximation.

(3) Inconsistent preference relation

In the case that the preference relation does not satisfy transitivity and anti-symmetry properties, this preference relation may be regarded as inconsistent preference relation. In conventional MADM approaches, case (3) is considered to be inconsistent and the decision-makers must change the preference relation in order to approximate it to either partial orders or total orders. (Kelly 1963) investigated the change of evaluation attributes from a psychological view point. (Cooper 1999) proposed DEA and studied the effects of different evaluation attributes sets. (Ishizu 1992, 1995, 2003) and (Gehrmann 2001) investigated the concepts of evaluation structures and

evaluation spaces, and algorithms for constructing evaluation attributes and evaluation structure. (Ishizu 2003) pointed out the usefulness of the evaluation structure in management decision-making as follows. The preference relations of management decision-makers are frequently inconsistent. By the use of evaluation structure, we can represent the inconsistent preference relations. We usually define a set of evaluation attributes by gathering all relevant evaluation attributes, and we have too many evaluation attributes to find out desirable solution. As an evaluation structure, we can have a family of specialized and restricted sets of evaluation attributes, and then the decision-makers may easily find out desirable solution according to the specialized and restricted sets of evaluation attributes.

In this paper we focus on the problems of inconsistent preference relation and follow the framework of the evaluation structure. Hence (Ishizu 2003) proposed new evaluation attributes and evaluation structure, the new evaluation attributes is difficult to identify the meaning of them. In the most questionnaire survey we may prepare many of evaluation attributes. It is better to use prepared evaluation attributes for constructing evaluation structure. We assume that we have many evaluation attributes before the questionnaire survey. Our main target is to select important evaluation attributes and select minimal evaluation structure under the assumption. For the case that the preference relation is not represented by prepared evaluation attributes, we give the way to construct new additional attributes based on the prepared evaluation attributes.

In making minimal evaluation structure, we delete redundant evaluation attributes. But minimal evaluation structure may not be unique. Depending the way of deletion, different evaluation structure will be

made. From the practical viewpoint, constructing an appropriate minimal evaluation structure is important problem. In order to recognize appropriate evaluation attributes for decision-maker, we apply evaluation grid method. The evaluation grid method is a way of questionnaire survey (Kelly 1963). In Section 4, we propose Grid Evaluation Structure Approach (GESA) as a tool for the questionnaire survey of complex management decision-making. GESA is based on both algorithm of minimal evaluation structure and evaluation grid method.

In order to illustrate the essence of this paper, we introduce an example of an inconsistent preference relation. This example is an evaluation preference about 3 cars x, y, z. The characteristics of the cars are shown in Figure 1 (1). The car x is a sports car, and it is preferred because of engine power of the car, but it is poor in providing cabin space for its passengers. The car y is a family car, and is preferred because of good fuel efficiency but it is poor in engine power. And the car z is a transport car, and is preferred because of providing cabin space but it is poor in fuel efficiency. In order to know the preference relation of evaluator, a survey of pair-wise comparison may be performed. The result of the pair-wise comparison may be represented by the form of Figure 1 (2). Figure 1 (2) shows the inconsistent preference relation over the cars $\{x,y,z\}$. Each mark × shows a preference pair. For example, mark × at row y and column x in (2) means that the car y is preferred to the car x.

2. SELECTION AND ADDITION METHOD OF EVALUATION ATTRIBUTES

In this section, first we show the concepts of empirical preference relation of a decision-maker, and concepts of evaluation structure and evaluation space that represents the empirical preference relation. In addition, we define the representability of the value system in terms of the prepared evaluation attributes.

Definition 1 Value system

Let X be a set of decision-making entities or objects, and let R be a preference relation on X (that is $R \subset X \times X$). The pair $<X,R>$ is called a value system.

X is a set of entities, which are evaluated by the decision-maker. X may be a set of options, which the decision-maker will consider to apply, or a set of results which will occur when the decision-maker applies the options. R is a binary relation on X and shows the preference relation on X. If the decision-maker decides that y is better than x, then (x,y) is in R. In the case of Figure 1, the set of entities X is the set of $\{$car x, car y, car $z\}$, and the preference relation $R=\{(x,y),(y,z),(x,x),(y,y),(z,z)\}$, where x, y, z indicates car x, car y, and car z, respectively.

	fuel efficiency	cabin space	engine power
car x	2	3	1
car y	1	2	3
car z	3	1	2

(1) Evaluation objects and attributes

(2) inconsistent

Figure 1 Example of inconsistent preference relation

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Definition 2 Evaluation space

$<\Pi\{X_a|a\in A\},ES,\{T_a|\alpha\in A\}>$ is called an evaluation space.

Where,

   A: set of evaluation attributes,

   X_a: set of values of evaluation attribute a $(a\in A)$,

   $\Pi\{X_a|a\in A\}$: Cartesian product of $\{X_a|a\in A\}$ (referred to as an attributes system),

   $P(A)$: power set of A,

   $ES\subset P(A)$: family of subsets of A (referred to as an evaluation structure).

   $T_a\subset X_a\times X_a$: total order on X_a $(a\in A)$,

Evaluation space $<\Pi\{X_a|a\in A\},ES,\{T_a|a\in A\}>$ consists of an attributes system $\Pi\{X_a|a\in A\}$, an evaluation structure ES, and a set of total orders $\{T_a|a\in A\}$. Where, $\Pi\{X_a|a\in A\}$ is a Cartesian product of totally ordered sets X_a $(a\in A)$. In the case of Figure 1, $A=\{$fuel efficiency, cabin space, engine power$\}$ is an attributes set. $ES=\{\{$fuel efficiency, cabin space$\},\{$cabin space, engine power$\}\}$ is a family of attributes sets and is called an evaluation structure. For each attribute $a\in A$, $X_a=\{1, 2, 3\}$ is a set of attribute values. Total order is defined naturally on each X_a. Figure 1 (1) is regarded as a subset of Cartesian product $\Pi\{X_a|a\in A\}$.

Proposition 1 Representability of the value system

Let $<X,R>$ be a value system and assume that $\Delta\subset R$, where $\Delta=\{(x,x)|x\in X\}$. Then there exists an evaluation space $<\Pi\{X_a|a\in A\},ES,\{T_a|a\in A\}>$ and a function $f:X\to\Pi\{X_a|a\in A\}$ which satisfy the following Condition 1. We say value system $<X,R>$ is multi-attributes representable by evaluation space $<\Pi\{X_a|a\in A\},ES,\{T_a|a\in A\}>$.

Condition 1: For any $x,y\in X$:

   $[(x,y)\in R]\leftrightarrow[\exists I\in ES: \forall a\in I:((f(x))_a,(f(y))_a)\in T_a]$.

   Where $(\ )_a$ is a projection operator from $\Pi\{X_b|b\in A\}$ to X_a.

(Proof of Proposition 1 is given by Ishizu 2003)

$\Delta\subset R$ means R is reflexive, i.e., for any $x\in X$ $(x,x)\in R$. Proposition 1 shows that any reflexive preference relation R can be multi-attributes representable by some evaluation space. Since reflexive property is very popular and most of preference relations may satisfy the reflexive property, Proposition 1 shows high representability of evaluation space.

Hereafter we assume that value system $<X,R>$, evaluation attributes set A, an attributes system $\Pi\{X_a|a\in A\}$, a set of total orders $\{T_a|a\in A\}$, and function $f:X\to\Pi\{X_a|a\in A\}$ are given. This situation is that the decision-maker has value system, related evaluation attributes and attributes system, but he or she does not have clear evaluation structure which represents preference relation. In most management decision-making, this situation may be frequently occurred. Based on the above assumption, we develop an algorithm for constructing an evaluation structure ES such that $<X,R>$ is multi-attributes representable by ES. In other words, we assume $<X, R, f, A,\Pi\{X_a|a\in A\}, \{T_a|a\in A\}>$ is given, and the problem is how to construct ES which satisfies Condition 1. If the assumed evaluation attributes A is not enough to represent $<X,R>$, then we can not construct evaluation structure ES which satisfies Condition 1. In such case we must add the necessary evaluation attributes so that we can construct ES. In order to introduce the concepts for constructing ES, we use the following notations.

   $XT_a=\{(x,y)\in X\times X|(f(x)_a,f(y)_a)\in T_a\}$.

   $A(x,y)=\{a|a\in A, (x,y)\in XT_a\}$, where $(x,y)\in X\times X$.

   $RI=\cap\{XT_a|a\in I\}$, where $I\subset A$.

XT_a is an order naturally induced on X by the evaluation attribute a. Since T_a is total order, XT_a is

pre-total order, where pre-total order is binary relation which satisfies reflexivity, transitivity and comparability. $A(x,y)$ is a set of evaluation attributes compatible with the preference (x,y). And RI is intersection of order induced by attribute $a \in I$. In other words, RI is a preference relation naturally induced by a set of attributes I. $RA(x,y)$ is intersection of all induced orders XT_a, which compatible with the preference (x,y).

In the case of cars in Figure 1 (1), XT_a is shown in Figure 2. XT_{fe}, XT_{cs}, and XT_{ep} are orders induced by evaluation attribute "fuel efficiency," "cabin space," and "engine power" respectively. A(car x, car y) is a set of evaluation attributes compatible with the preference pair (car x, car y). Since (car x, car y) $\in XT_{fe}$, (car x, car y) $\in XT_{cs}$, and (car x, car y) $\notin XT_{ep}$, so A(car x, car y)={fuel efficiency, cabin space}. It is easy to show that A(car y, car z)={cabin space, engine power}.

Definition 3 Multi-attributes representability of preference pair

Let (x,y) be a preference pair ($(x,y) \in R$) and A be a given set of evaluation attributes. If the condition $RA(x,y) \subset R$ is satisfied, then we call that the **preference pair** (x,y) **is multi-attributes representable by** A.

In the case of car example shown by Figure 1 and 2, RA(car x, car y)$\subset R$, so preference pair (car x, car y) is

multi-attributes representable. Moreover RA(car y, car z)$\subset R$ and RA(car x, car y)$\cup RA$(car y, car z)=R. We can get ES ={A(car x, car y), A(car y, car z)}= {{fuel efficiency, cabin space}, {cabin space, engine power}}.

Proposition 2

If for each preference $(x,y) \in R$, preference (x,y) is multi-attributes representable by A, then the value system $<X,R>$ is multi-attributes representable.

(Proof of Proposition 2 is given in the Appendix)

Definition 4 Attributes set A is complete

Attributes set A is called complete, if for every pre-total order T on X, there exists an attribute $a \in A$ such that $T=XT_a$.

Proposition 3

If R is reflexive relation and A is complete, then $<X,R>$ is multi-attributes representable on A.

(Proof of Proposition 3 is given in the Appendix)

The above proposition shows that if we can make evaluation attribute for any pre-total order on X then reflexive preference relation R will be multi-attributes representable. But if prepared A is not complete, then $<X,R>$ may not be multi-attributes representable. In such case, we need the way to construct new additional evaluation attributes. The next proposition shows the condition that new additional evaluation attributes must satisfy.
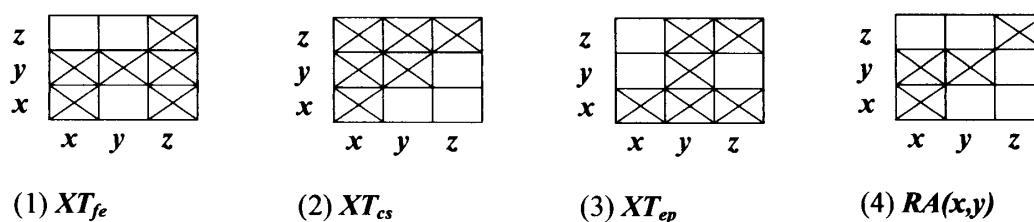

(1) XT_{fe} (2) XT_{cs} (3) XT_{ep} (4) $RA(x,y)$

Figure 2 Examples XT_a and $RA(x,y)$

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Proposition 4

Let (x,y) be a preference pair $((x,y) \in R)$ and A be a given set of evaluation attributes. Let (x,y) be not multi-attributes representable by evaluation attributes set A. If there exists a set of new evaluation attributes K $(A \cap K = \phi)$ which satisfies the following condition, then (x,y) is multi-attributes representable by evaluation attributes set $A \cup K$.

Condition 2: $\forall (v,w) \in RA(x,y)\text{-}R: \exists a \in K: [(v,w) \notin XT_a$ and $(x,y) \in XT_a]$

(Proof of Proposition 4 is given in the Appendix)

In the above proposition, we assume that there are $\{X_a | a \in K\}$, $\{T_a | a \in K\}$, and function $\{f_a : X \to X_a | a \in K\}$ related to evaluation attributes K. Based on this assumption, we can introduce $<X, R, f', A \cup K, \Pi\{X_a | a \in A \cup K \}, \{T_a | a \in A \cup K \}>$, and discuss about the representability on $A \cup K$. Where for each $a \in A \cup K$ and $x \in X, f'$ is defined as follows.

$(f'(x))_a = (f(x))_a$ if $a \in A$,

$(f'(x))_a = f_a(x)$ if $a \in K$.

Definition 4 Degree of anti-transitivitiy

Degree of anti-transitivity of R is defined as total number of triple (x,y,z) which satisfies the following condition

Condition 3: $(x,y) \notin R$ and $(x,z) \in R$ and $(z,y) \in R$

By using the above definitions and propositions, we propose an algorithm for constructing minimal evaluation structure ES for the preference relation R. Using this algorithm, an evaluation structure ES is obtained from a value system $<X,R>$ under the condition that the evaluation set A is given. The algorithm is shown in Figure 3.

Algorithm for minimal evaluation structure

Step 1: Selection of a preference pair

Select a preference pair (x,y) in R.

Step 2: Check of representability of a selected pair

Check whether the preference pair (x,y) is multi-attributes representable by prepared attributes set A (Definition 3). If a pair (x,y) is not

```
0.Pair wise comparison
          |
          v
1.Selection of a preference pair  <---  3.Addition of new
          |                                  evaluation attributes
          v                                       ^
        < >  --- no --------------------------------|
          |
          v yes                           2.Check of representability of a
4.Selection of minimal attributes set        selected pair
          |
          v
5.Selection of a preference pair  <---|
          |                            |
          v                            |
6.Construction of minimal attributes   |
  set which represents selected        |
  preference pair                      |
          |                            |
          v                            |
        < >  --- no -------------------|
          |
          v yes                        7.Check of representability
8.Selection of minimal ES                of all preference pair of
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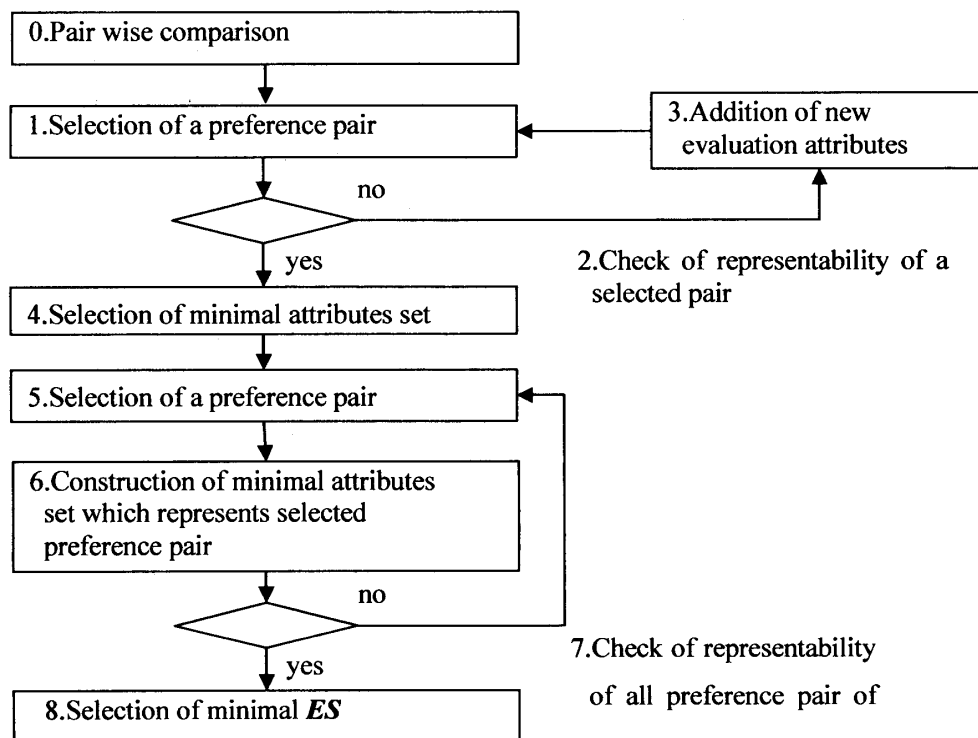
Figure 3 Algorithm for minimal evaluation structure

multi-attributes representable, then go to **Step 3**. If all pairs are multi-attributes representable, then go to **Step 4** (Proposition 2).

Step 3: Addition of new evaluation attributes

Find additional evaluation attributes set K which satisfies the condition of Proposition 4, and make new evaluation attributes set $A'= A \cup K$. Go to **Step 1**.

Step 4: Selection of minimal attributes set by which the preference is representable

Select minimal subset of evaluation attributes A'' $(A'' \subset A')$ by which R is representable.

Let i be $i=1$, and $R_i = R$.

Step 5: Selection of a preference pair

Select arbitrary preference pair (x_i, y_i) in R_i

Step 6: Construction of minimal attributes set which represents selected preference pair

Make minimal subset I_i of $A(x_i, y_i)$ which satisfies $RI_i \subset R$.

Step 7: Check of representability of all preference pair of R

Let $R_{i+1} = R_i - RI_i$. If $R_{i+1} \neq \phi$, then let $i=i+1$ and go **Step 5**, else go to **Step 8**.

Step 8: Select minimal ES

make minimal subset $ES \subset \{I_1, I_2, ..., I_i\}$ which satisfies $\cup \{ RI_j \mid I_j \in ES\} = R$

In Steps from 1 to 3, we check the multi-attributes representability and addition of new evaluation attributes. In Step 4, we select minimal set of evaluation attributes A'' by repeating the following procedure. For each attribute a in A', check the

representability of R on $A'-\{a\}$. If R is representable on $A'-\{a\}$, then A' is replaced by $A'-\{a\}$. In Steps from 5 to 7, we select a family of minimal set of evaluation attributes $\{I_1, I_2, ..., I_i\}$. It is easy to show that if $R_{i+1} = \phi$ in Step 7, then $\cup \{ RI_j \mid j=1,2,...,i\} = R$. To construct a minimal set I_i in Step 6 is also trivial when A is finite. Indeed for each a in I_i, if $R(I_i - \{a\}) \subset R$, then I_i is replaced by $I_i - \{a\}$. Similarly to construct minimal ES in Step 8 is also easy. For each I_k in ES, if $\cup \{RI_j \mid I_j \in ES - I_k\} = R$, then ES is replaced by $ES - \{I_k\}$. In this procedure we get minimal evaluation structure ES, and for each I in ES, I is minimal. Moreover all selected evaluation attributes $\cup \{I_j \mid I_j \in ES\}$ is minimal, since A'' is minimal. If A and X are finite, the feedback loops of Steps from 1 to 3 and Steps from 5 to 7 terminate in finite iteration.

3. EXAMPLE OF THE ALGORITHM

We assume same $<X, R', f, A, \Pi\{X_a \mid a \in A\}, \{T_a \mid a \in A\}>$ as shown in Figure 1 and 2 except R' is shown in Figure 4 (1). Where $X=\{$car x, car y, car $z\}$, $A=\{$fuel efficiency, cabin space, engine power$\}$, $X_a=\{1,2,3\}$, and f is shown in Figure 1 (1). In this section, we show how we can construct evaluation structure applying algorithm for minimal evaluation structure proposed in Section 2.

Step 1: Selection of a preference pair

R' has preference pairs (x,y) and (x,z). Firstly we select (x,y) and secondly (x,z).

(1) R' (2) $RA(x,z)$ (3) XT_n (4) new $RA(x,z)$
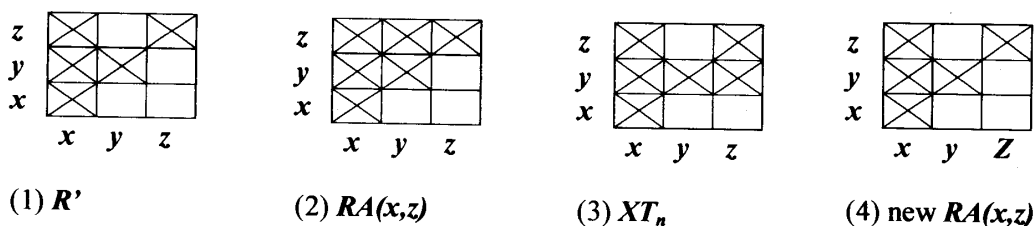
Figure 4 Example of R' and new attribute "noiselessness"

Step 2: Check of representability of a selected pair

As shown in Figure 2 (4), preference pair (x,y) is representable, since $RA(x,y)$ is subset of R'. But preference pair (x,z) is not representable since $RA(x,z)$ is not a subset of R' (See Figure 4 (2)). Note that the set of evaluation attributes compatible with preference pair (x,z) is {cabin space}, and $A(x,z)$={cabin spece}. We need addition of new evaluation attributes, and we must go to Step 3.

Step 3: Addition of new evaluation attributes

Since $(y,z) \in RA(x,z)-R'$, we must add new attribute b where $(y,z) \notin XT_b$ and $(x,z) \in XT_b$. Because we assume XT_b is pre-total order and satisfies comparability, $(y,z) \notin XT_b$ means $(z,y) \in XT_b$. We need new attribute on which $(z,y) \in XT_b$ and $(x,z) \in XT_b$ are satisfied. We find new attribute "noiselessness" and XT_n is shown in Figure 4 (3). Additional attributes set K={noiselessness}, and new attributes set $A'=A \cup K$={fuel efficiency, cabin space, engine power, noiselessness }. Now we go to Step 2.

Step 2: Check of representability of a selected pair

Preference pair (x,y) is representable as shown before, and (x,z) becomes representable on A' since new $RA'(x,z)$ is subset of R' (See Figure 4 (4)). Then go to Step 4.

Step 4: Selection of minimal attributes set by which the preference is representable

We can delete "fuel efficiency" and "engine power". And then we select minimal attributes set A'' ={cabin space, noiselessness} by which R' is representable.

Let $i=1$, and $R_i = R$.

Step 5: Selection of a preference pair

We firstly select preference pair (x,y) in R_i

Step 6: Construction of minimal attributes set which represents selected preference pair

$A''(x,y)$ ={cabin space, noiselessness} and

$RA''(x,y) \subset R'$ is satisfied. $I_i = A''(x,y)$ is minimal, since we can not delete any attribute from $A''(x,y)$.

Step 7: Check of representability of all preference pair of R

Let $R_{i+1}=R_i-RI_i$, then $R_{i+1}=\phi$, since $R_i=RI_i=R'$. Then go to Step 8

Step 8: Select minimal ES

We get minimal $ES =\{I_1\}$, since ES is singleton. And ES satisfies $\cup \{ RI_j| I_j \in ES\}= R'$.

According to the algorithm for minimal evaluation structure, we added new attribute "noiselessness" and construct evaluation structure ES={{cabin space, noiselessness}}.

4. GRID EVALUATION STRUCTURE APPROACH (GESA)

In this section, we introduce Grid Evaluation Structure Approach (GESA) that is practical approach to construct an evaluation structure from the result of pair-wise comparison by the use of algorithm for minimal evaluation structure. During the questionnaire survey of the pair-wise comparison, we apply evaluation grid method. So we call this approach GESA. In the algorithm of making minimal evaluation structure, we must delete some redundant evaluation attributes. This procedure is mathematically simple, but this means that minimal evaluation structure may not be unique. Depending on the way of deletion, different evaluation structure will be made. The reason why we apply grid evaluation method is to get meaningful evaluation structure in real questionnaire survey. We also construct a software system that implements GESA, and we call this system as GESA system. The main steps for GESA system is almost same as the algorithm for minimal evaluation structure as shown in Figure 3. Main difference from the

algorithm is the way of pair-wise comparison and the way of deletion of evaluation attributes. In our example the set of decision-making entities is a set of mobile telephones offered on the Japanese market. The example is applied for evaluation of 7 mobile phones with 27 attributes. Figure 5 shows the interface of GESA system. We prepare product profile table as similar table as Figure 1 (1). The product profile table shows the value of each evaluation attribute. For each evaluation attribute total order is naturally introduced on value set of attributes, e.g., evaluation attribute "weight" introduces total order that we will prefer the light weight phone. We prepare $<X, f, A, \Pi\{X_a|a\in A\}, \{T_a|a\in A\}>$. We will get preference relation R by pair-wise comparison. Based on the information $<X, R, f, A, \Pi\{X_a|a\in A\}, \{T_a|a\in A\}>$, GESA system construct evaluation structure ES according to the algorithm for minimal evaluation structure proposed in Section 2.

4. 1 Pair-wise comparison

(Kelly 1963) proposed personal construct theory (PCT), and based on PCT evaluation grid method was proposed. Kelly proposed the evaluation grid technique as a way of getting people to exhibit their construct systems. The evaluation grid method uses a simplified interview technique as used in the PCT. The method is to ask not only pair-wise comparison but also the reason of the result of the comparison. The user interfaces based on evaluation grid method is realized by GESA as shown in Figure 6. By the use of this interface, the decision-maker can choose the preferences from the three combinations of possible checks; both phones are preferred, one phone is preferred, or two phones are not comparable. At the same time, the evaluator is asked to give the reason for the preference by the selection of pre-defined attribute or add a new attribute name in the list box. The attribute
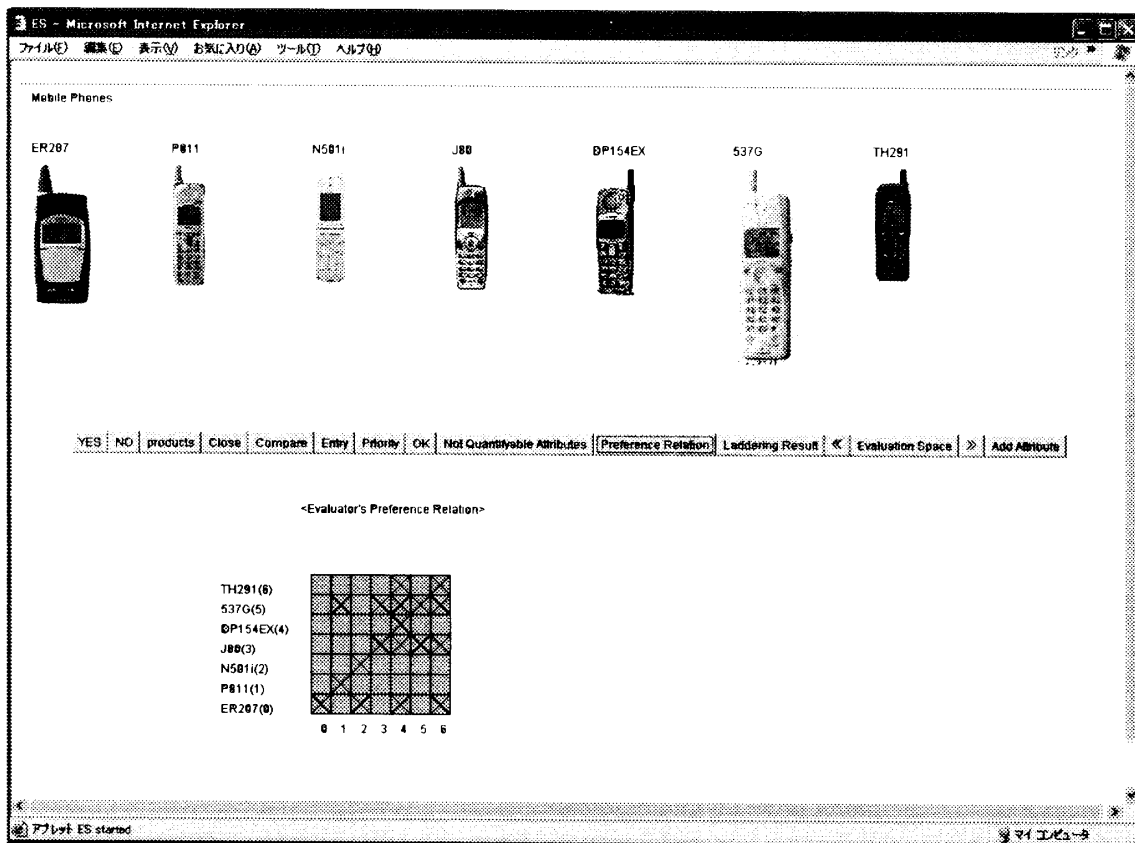

Figure 5 GESA system

"nice design" in Figure 6 is a new additional attribute name, which is entered here. In this situation, only attribute name is registered. A set of new additional attributes names is useful to construct new additional evaluation attributes, if the preference relation can not be malti-attributes representable by pre-defined 27 evaluation attributes. By the pair-wise comparison, we can get the preference relation, the selected pre-defined evaluation attributes, and the new additional attributes names. The selected pre-defined evaluation attributes can be regarded as important and appropriate evaluation attributes for evaluation structure.

4.2 Results of grid evaluation structure approach

After the pair-wise comparison, GESA system follows the algorithm for minimal evaluation structure. GESA system firstly checks the representability of a given preference relation. If the preference relation is not representable, then we must add new additional evaluation attributes (Step 3). The new additional attributes names will give us hints to construct new evaluation attributes K which satisfies the Condition 2 in Section 2. In order to construct new evaluation attribute, we must define attribute name a, its value set X_a, pre-totally order T_a, and function $f_a : X \to X_a$. GESA system also has interface which help us to input the above information. This procedure is very same as the example discussed in Section 3. If the preference relation is representable, then GESA system constructs a minimal attributes set. In the deletion of evaluation attributes, evaluation attributes which are not selected in evaluation grid method are deleted and checked representability firstly. As a result, minimal evaluation attributes consist of appropriate evaluation attributes. In Steps from 5 to 7, GESA constructs minimal attributes sets which represent selected preference pairs by the use of the selected attributes in evaluation grid method. And then GESA system constructs

Figure 6 pair-wise comparison in GESA system

a minimal evaluation structure. Figure 7 shows the inconsistent preference relation of decision-maker. Indeed in this preference relation, the preference 537G is preferred to P811, and J80 is preferred to 537G, but J80 is not preferred to P811. Figure 8 and 9 are the evaluation structure of the preference relation by the use of GESA system. This show that the inconsistent preference relation can be representable by the minimal evaluation structure ES={{Height, Thickness, Charge Time}, {Alert Melody, 10 yen per area, color}}.

By the use of GESA system, we make another questionnaire survey in order to analyze the properties of evaluation structure depending on the number of the entities. In our survey GESA is applied to 3 – 8 computers as entities and 21 evaluation attributes are prepared. Figure 10 shows the number of entities and the degree of anti-transitivity, which indicates degree of inconsistency defined in Definition 4. Figure 10 shows the inconsistency is rapidly increase according to the number of entities. Figure 11 shows the relationships among the number of entities and the

Figure 7 Inconsistent preference relation

Figure 8 Evaluation structure (1)

Figure 9 Evaluation structure (2)

number of evaluation attributes sets in evaluation structure. It shows the number of evaluation attributes sets are linearly according to the number of entities.

5. SUMMARY

In real complex management decision-making such as project planning, product planning, we are frequently faced with the inconsistent preference relation of decision-makers. In this paper we focus on the concept of evaluation structure which gives the representing way of inconsistent preference relation in complex management decision-making. We usually have many evaluation attributes before the questionnaire survey of preference relation. We introduced framework for representing management decision-making situation mathematically. We introduced a concept for multi-attributes representability of prepared evaluation attributes, and a condition for new added evaluation attributes. Based on the mathematical background, we proposed an algorithm for minimal evaluation structure. In the algorithm, we can check multi-attributes representability of prepared evaluation attributes, and add new evaluation attributes, and select minimal evaluation structure. We showed the meaning of the algorithm by the use of simple example.
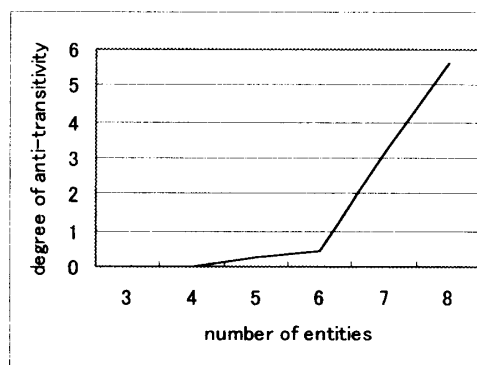
Minimal evaluation structure is not unique, and depending on the way of deletion different evaluation structure will be made. From a practical viewpoint, constructing appropriate minimal evaluation structure is important problem. In order to recognize appropriate evaluation attributes for decision-maker, we apply evaluation grid method. We proposed GESA system based on both algorithm of minimal evaluation structure and evaluation grid method. We showed the application of GESA system to the real questionnaire survey.

References

1. Atzeni, P., Antonellis V.D., "Relational Database Theory," Benjamin-Cummings Publishing Company,1993.
2. Cooper, W.W., "Data Envelopment Analysis - A Comprehensive Text with Models, Applications, References," Kluwer Academic Publishers, 1999.
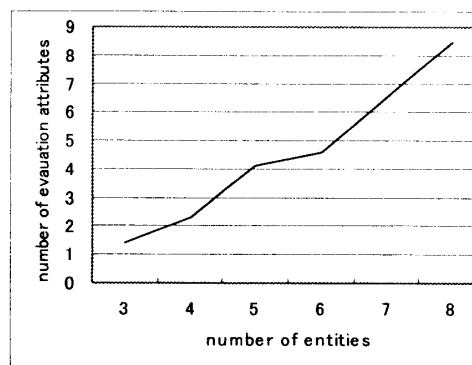
Figure 10 Relationships among number of entities and anti-transitivity

Figure 11 Relationships among number of entities and attributes of *ES*

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3. Fishburn, P.C., "Utility theory for decision making," John Wiley, 1970.

4. Gehrmann A., Ishizu S., "Representation of Evaluation Spaces over Attributes Structures - a practical example-," *Proc. of SCI 2001*, Vol.2 Information Systems, 2001, pp. 139-145.

5. Ikeshoji, H., Mesarovic M. D., and Furukawa, O., "Attributes and complexity in general systems," *Systems Sci.*, Vol. 8, No. 2/3, 1982, pp. 25-36.

6. Ishizu, S., "Attributes structure in evaluation of complex decision making," *Trans. of the Society of Instrument and Control Engineers*, Vol. 28, No. 9, 1992, pp. 1118-1124. (in Japanese).

7. Ishizu, S., "An Attributes Structure on Evaluation Space," *Advances in Systems Study*, Vol. II, 1995, pp33-38.

8. Ishizu, S., H. Amakasu, Basic Concepts and Algorithms for Construction of Evaluation Structures, JASMIN, Vol.12, No.2, 2003, pp.21-36.

9. Kelly, G.A., "Theory of Personality: The Psychology of Personal Constructs," W. W. Norton & Company, 1963.

10. Keeney, R. L. and Raiffa, H., "Decision with multiple objectives, preferences and value tradeoffs," John Wiley & Sons, Inc, 1976.

11. Klir G.J., "Fuzzy Measure Theory," Kluwer Academic, Plenum Publishers, 1993.

12. Orlowska, E. (ed)., "Incomplete Information: Rough Set Analysis", Physica-Verlag, 1997.

13. Pawlak, Z., Slowinski, R., "Rough set approach to multi-attribute decision analysis," *European J. of OR.*,No.72, 1994, pp. 443-459.

14. Saaty, T.L., "Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World: 1999/2000 Edition Vol. 2," Rws Pubns, 1999.

15. Ullman, J.D., "Principles of database systems," Computer Science Press,1982.

Andreas Gehrmann

所　属：青山学院大学

連絡先：〒229-8558
　　　　神奈川県相模原市淵野辺 5-10-1
　　　　青山学院大学

電　話：045-948-4771

e-mail： gehrmann@yhc.att.ne.jp

石　津　昌　平

所　属：青山学院大学

連絡先：〒229-8558
　　　　神奈川県相模原市淵野辺 5-10-1
　　　　青山学院大学

電　話：042-759-6306

e-mail：ishizu@ise.aoyama.ac.jp

APPENDIXES

(Proof of Proposition 2)

Let ES be $ES = \{A(x,y) \mid (x,y) \in R\}$. If $(x,y) \in R$, then $A(x,y) \in ES$, and $(x,y) \in RA(x,y)$. This means that for any $a \in A(x,y)$, $(x,y) \in XT_a$ and then $((f(x))_a, (f(y))_a) \in T_a$. Conversely, if $(v,w) \in RA(x,y)$, then $(v,w) \in R$ since $RA(x,y) \subset R$. So we show the Condition 1 is satisfied and $<X,R>$ is multi-attributes representable.

(End of proof)

(Proof of Proposition 3)

Let $(x,y) \in R$. We can easily show that there exist the following T and T' very similar as the proof of Proposition 1, i.e., there exist total orders T and T' such that $(x,y) \in T$, $(x,y) \in T'$, and $T \cap T' = \Delta \cup \{(x,y)\}$. Since A is complete, then there exist a and $a' \in A$ such that $T = XT_a$ and $T' = XT_{a'}$. Next we show (x,y) is multi-attributes representable. Let $J = \{a, a'\}$. Since $(x,y) \in XT_a$ and $(x,y) \in XT_{a'}$, then $J \subset A(x,y)$, and $RA(x,y) \subset RJ$. Since $RJ = XT_a \cap XT_{a'} = \Delta \cup \{(x,y)\} \subset R$, so we get $RA(x,y) \subset RJ \subset R$, and this means a preference pair (x,y) is multi-attributes representable. For each $(x,y) \in R$, (x,y) is multi-attributes representable, then from Proposition 2 $<X,R>$ is multi-attributes representable.

(End of proof)

(Proof of Porposition 4)

Let assume pair (x,y) is not multi-attribute representable by $A \cup K$. This means $R(A \cup K)(x,y) \not\subset R$ and then there exists a pair $(v,w) \in R(A \cup K)(x,y)$ and $(v,w) \notin R$. Since $A \subset A \cup K$, then $R(A \cup K)(x,y) \subset RA(x,y)$ and $(v,w) \in RA(x,y)$. From the Condition 2, there exists $a \in K$ which satisfies that $(v,w) \notin XT_a$ and $(x,y) \in XT_a$. The facts $(v,w) \notin XT_a$ and $a \in K$ mean $(v,w) \notin R(A \cup K)(x,y)$. This contradicts from the assumption and we get $R(A \cup K)(x,y) \subset R$.

(End of proof)