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Oxygen Transfer from Fine Bubbles Dispersed in Water

KOICHI FUJIE, TSUNEO SEKIZAWA,* and HIROSHI KUBOTA

Tokyo Institute of Technology, Midoriku, Yokohama 227 and
**Nagaoka Technical College, Nagaoka, Niigata 940*

Procedures to predict the utilization of oxygen in completely segregated fine gas bubbles, sparged into liquid, are presented and discussed. In such a system the change in mole fraction of oxygen and thus the change in volume of the bubbles with retention time cannot be ignored, so that the quantitative analysis becomes complicated.

Methods of determining the mass transfer coefficient from the experimental data in the fine bubbles system are also discussed. It is noted that besides the unsteady state method, commonly used in the previous works, the steady state method, in which the oxygen utilization of the exhaust gas is measured, can be recommended for application in practical operations.

In activated sludge waste water treatment and single-cell-protein production, the sparging of enriched oxygen gas into liquid is often practiced. In such cases, it is strongly required to increase the one-path yield of the oxygen utilization of the feed gas in the liquid. To accelerate oxygen transfer from gas bubbles, an effective way to increase the gas-liquid interfacial area is dispersing fine gas bubbles through the liquid.

In the previous investigations on the mass transfer from gas bubbles to liquid, however, bubbles of large diameter, usually more than 2 or 3 mm, have been employed. Little work has been reported on the fine-bubbles system, which involves a milky swarm of bubbles with submillimeter diameters.

In the fine-bubbles system, each bubble in the swarm is completely segregated and coalescence seldom occurs. The ratio of the surface area to the volume of the gas bubble is so large that a considerably high value of oxygen utilization can be expected even in a shallow tank. Therefore, the changes in the oxygen concentration and thus in the volume of the gas bubble in the process of oxygen transfer into the liquid must be taken into account.

In addition, the lack of information on the mass transfer coefficient on such fine gas bubble precludes the correct prediction of the oxygen utilization.

In the present paper, we propose procedures for quantitative treatments of the oxygen transfer from the swarm of fine bubbles to water. The procedures will allow prediction of oxygen utilization of the feed gas when the information on the mass transfer coefficient of the bubble is obtained. Methods of determining the mass transfer coefficient from the experimental data for the fine bubbles are also presented and discussed.

Oxygen Utilization in a Swarm of Sparged Bubbles

Fundamental relations for a single bubble As mentioned above, the gas bubbles in a swarm sparged into a liquid tank are completely segregated. Coalescence of bubbles will not occur unless something obstructs the free motion of the swarm. Therefore, the oxygen balance in a single bubble of the swarm leads the following equation.

$$(v_{p0}{}^* z_0/v_m)\frac{d\eta}{d\tau}=k_l a_p(C_0 z-C_l) \qquad (1)$$

The oxygen utilization η in the bubble will be given as a function of the residence time τ in the liquid, by solving Eq. (1) under the following initial conditions.

$$\tau=0;\quad \eta=0 \qquad (2)$$

The fine gas bubbles with diameters less than 2 mm were observed photographically to be spherical. The surface area a_p of the each bubble is thus given by

$$a_p=(6v_p)^{2/3}\pi^{1/3} \qquad (3)$$

The volume v_p as a function of the oxygen utilization η in the bubble, under the pressure P in the tank, is expressed by

$$v_p=v^*_{p0}(1-\delta z_0\eta)(P^*/P) \qquad (4)$$

Here v^*_{p0} is the initial bubble volume, converted under the atmospheric pressure P^*, and is given by

$$v^*_{P0}=(\pi d_{p0}{}^3/6)(P/P^*) \qquad (5)$$

The mole fraction of oxygen in the gas bubble z is related with the oxygen utilization η by[3)]

$$z=z_0(1-\eta)/(1-\delta z_0\eta) \qquad (6)$$

where $\delta=1-\beta$, and β is the number of moles of gaseous product yielded per mole of oxygen consumed in liquid. The pressure P at a vertical distance l from the gas sparger position, depth h, is expressed by

$$P=P^*+(h-l)(1-\tilde{\varphi}_l)(g/g_c)\rho_l \qquad (7)$$

The average gas holdup $\tilde{\varphi}_l$ is given by the following under the assumption of the uniform radial distribution of the gas holdup.

$$\tilde{\varphi}_l=\frac{1}{h-l}\int_l^h \varphi \mathrm{d}l \qquad (8)$$

The dissolved oxygen concentration saturated with pure oxygen in water C_0 in Eq. (1) is proportional to the pressure P as

$$C_0=C_0{}^*(P/P^*) \qquad (9)$$

where $C_0{}^*$ is the value of C_0 under atmospheric pressure P^*.

Given quantitative information on the residence time distribution of the gas bubbles and the liquid in the tank, and the mass transfer coefficient of the bubble, one can estimate the value of the oxygen utilization of the bubble swarm in the liquid.

In the following, for the sake of simplicity it is assumed that the oxygen utilization for a bubble of average diameter can represent that for the bubble swarm.

Plug flow of bubble swarm When plug flow of the bubbles in the swarm is assumed, the residence time of gas bubbles in water is the same for each bubble. The location l and the residence time τ of the bubbles in the tank are related by

$$\mathrm{d}l=(u_g/\varphi)\mathrm{d}\tau \qquad (10)$$

Here u_g is superficial gas velocity in the tank.

Substituting Eq. (10) into Eq. (1) gives the following equation:

$$\frac{\mathrm{d}\bar{\eta}}{\mathrm{d}l}=\left(\frac{v_m}{\bar{v}^*_{p0}z_0}\right)\frac{\varphi}{u_g}\bar{k}_l\bar{a}_p(C_0\bar{z}-C_l) \qquad (11)$$

where $\bar{a}_p$, $\bar{k}_l$, $\bar{v}^*_{p0}$, $\bar{z}$ and $\bar{\eta}$ represent the values of a_p, k_l, v^*_{p0}, z and η, respectively, for the bubble of average diameter. The boundary conditions are given by

$$l=0;\quad \bar{\eta}=0 \qquad (12)$$

The rising velocity of the bubble swarm, which is given by u_g/φ, is related with the apparent slip velocity of the bubbles swarm u_s, the gas holdup φ and the superficial liquid velocity u_l by

$$\frac{u_g}{\varphi}=\frac{u_l}{1-\varphi}+u_s \qquad (13)$$

The slip velocity of the bubble swarm, composed of completely segregated bubbles, is related with the interaction between bubbles, and thus with the gas holdup φ. This relationship has be en obtained empirically by Koide, Hirahara and Kubota,[2)] as follows:

$$u_s=u_{s0}[0.27+0.73(1-\varphi)^{2.80}] \qquad (14)$$

Here u_{s0} is the rising velocity of the single bubble, which has the average diameter of the bubbles in the swarm.

The superficial gas velocity in the tank u_g is also given as a function of the oxygen utilization and the pressure P as follows:

$$u_g=u^*_{g0}(1-\delta z_0\bar{\eta})(P^*/P) \qquad (15)$$

Eqs. (13), (14) and (15) show that the gas holdup φ can be obtained as a function of the oxygen utilization $\bar{\eta}$ and the distance l from the sparger under a given gas feed velocity u^*_{g0}. Since the average surface area of the bubbles $\bar{a}_p$ depends also on $\bar{\eta}$ and l, the oxygen utilization $\bar{\eta}$ as a function of l

can be obtained by solving Eq. (11), simultaneously with Eqs. (3)~(9) and (13)~(15), under the boundary conditions of Eq. (12). Numerical integration coupled with a trial and error procedure will be applied. It should be noted that the mass transfer coefficient k_l of the gas bubble also changes with the diameter and the slip velocity of the bubble. The experimental data for k_l will be reported in a forthcoming paper. The flow chart of the calculation procedure is shown in Fig. 1.

Completely mixed flow of bubble swarm When the flow of gas bubbles in the swarm is assumed to be completely mixed, the change in the oxygen utilization of the individual bubbles with their varied residence times must be noticed.

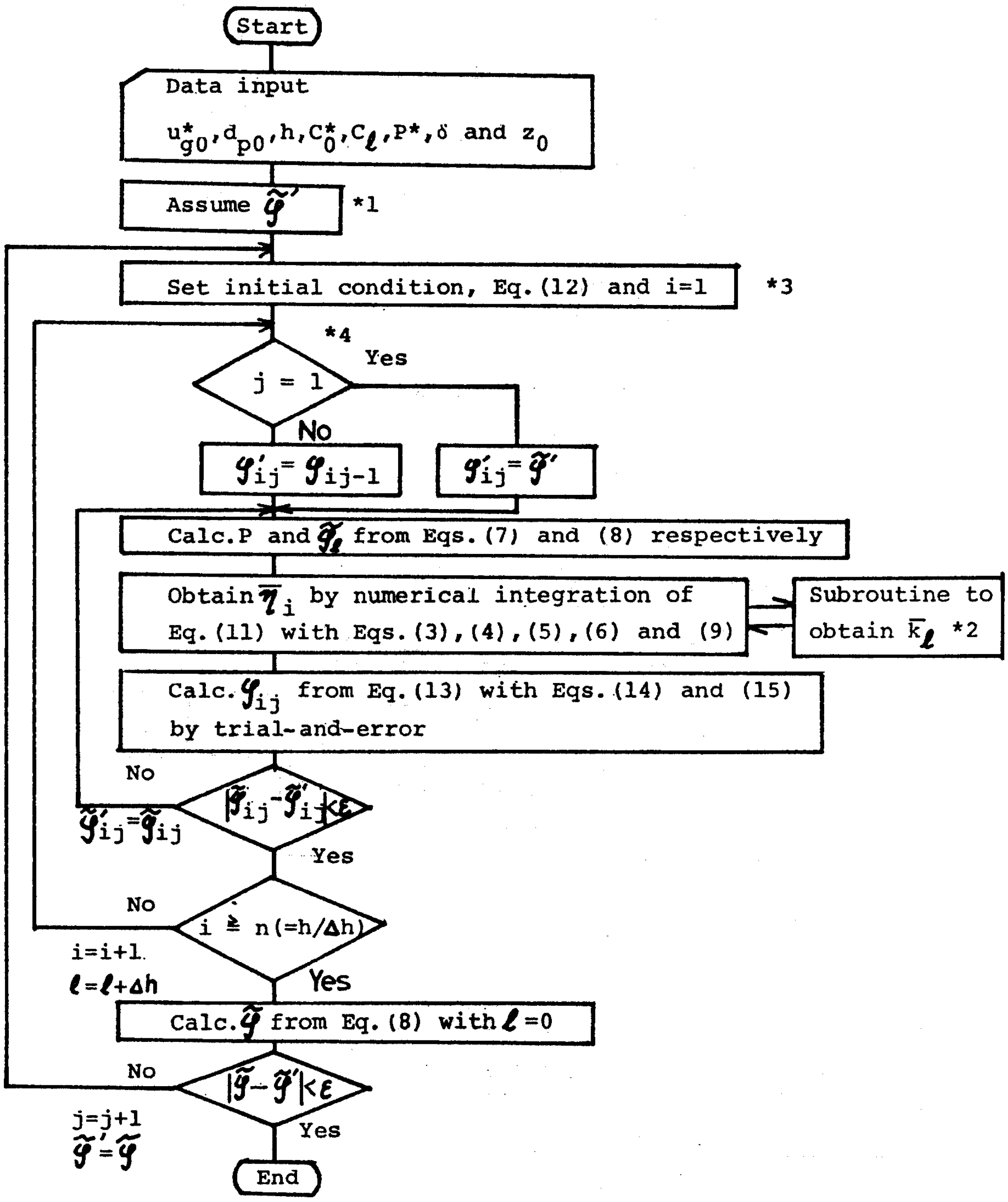

Fig. 1. Logic diagram of computer simulation to predict the oxygen utilization for plug flow of bubble swarm.
*1: an arbitraty value in the reasonable range is selected
*2: empirical correlation for k_l will be given in a authers' forthcoming paper
*3: i denotes the increment of Δh
*4: j denotes the number of repetition times

The oxygen balance equation (1) holds for a bubble of average diameter, and thus

$$\left(\frac{\bar{v}^*_{p0} z_0}{v_m}\right)\frac{d\bar{\eta}}{d\tau}=\bar{k}_l\bar{a}_p(\tilde{C}_0\bar{z}-C_l) \qquad (16)$$

where $\tilde{C}_0$ is the average value of C_0 in the tank. The initial conditions for Eq. (16) are

$$\tau=0;\quad \bar{\eta}=0 \qquad (17)$$

The arithmetic mean value of the pressure $\tilde{P}$ in the tank can reasonably be assumed to be given by

$$\tilde{P}=P^*+\frac{1}{2}(1-\tilde{\varphi})(g/g_c)\rho_l H \qquad (18)$$

where $\tilde{\varphi}$ is the average gas holdup through the tank. The oxygen utilization $\bar{\eta}$ of bubbles with residence time τ is obtained by solving Eq. (16) simultaneously with Eqs. (3)∼(6) and (18), under the initial conditions of Eq. (17).

The average value of the oxygen utilization $\bar{\eta}_e$ of the sparged gas at the outlet of the tank is obtained from

$$\bar{\eta}_e=\int_0^\infty \bar{\eta}(\tau)E(\tau)d\tau \qquad (19)$$

Here $\bar{\eta}(\tau)$ is the oxygen utilization of bubbles with residence time τ. $E(\tau)$ is the residence time distribution function of the bubbles and is given as follows for the completely mixed flow:

$$E(\tau)=\frac{1}{\bar{\tau}}\exp(-\tau/\bar{\tau}) \qquad (20)$$

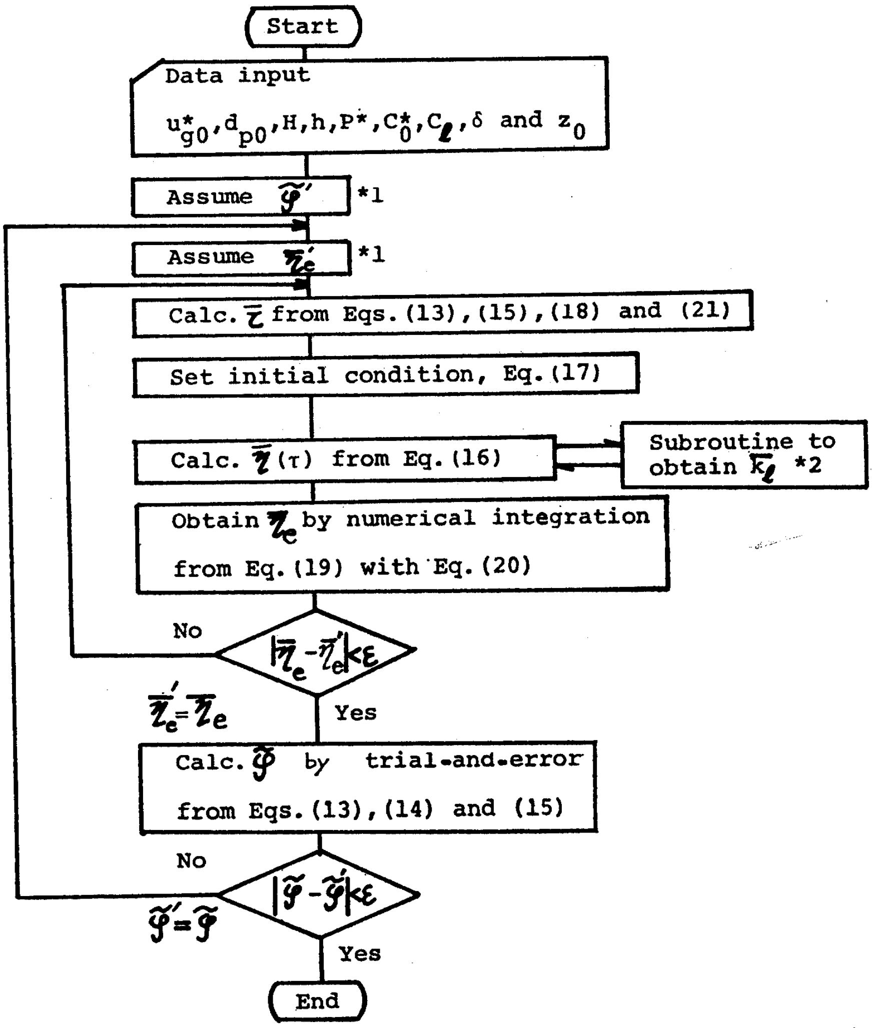

Fig. 2. Logic diagram of computer simulation to predict the oxygen utilization for completely mixed flow of bubble swarm.
*1 and *2 are the same as in Fig. 1.

The mean residence time of the bubbles in the liquid tank $\bar{\tau}$ is given by

$$\bar{\tau}=h/\tilde{u}_s \qquad (21)$$

The average slip velocity of the gas bubbles $\tilde{u}_s$ is related with the average gas holdup $\tilde{\varphi}$ in the tank by Eq. (13). The average superficial gas velocity $\tilde{u}_g$ is also related with the average oxygen utilization $\bar{\eta}_e$ and the pressure $\tilde{P}$ in the tank by Eq. (15).

The oxygen utilization of the gas bubble swarm in this case is thus obtained by the procedure which is shown in Fig. 2.

Determination of Mass Transfer Coefficient

To predict the oxygen utilization of the fine gas bubble swarm in water, it is important to obtain correct information on the mass transfer coefficients for the bubbles.

As mentioned above, since sufficient information on fine gas bubbles has not been reported experimental studies are required. In the following we discuss methods of determining the mass transfer coefficients for the fine bubble swarm. It is assumed that a uniform radial distribution of bubbles is sparged at the bottom of the tank in these experiments.

Steady-state method The dissolved oxygen concentration C_l is maintained negligibly low by adding an oxygen-demand reagent to the liquid. The difference in the mole fraction of oxygen between the inlet and outlet gas is measured, from which oxygen utilization η can be obtained and the volumetric oxygen transfer coefficient k_la evaluated. This is called here the steady state method. In the fine bubble swarm system, the coalescence of bubbles can be negligible. If the bubble diameter and hence the surface area a, is obtained photographically, the mass transfer coefficient $\bar{k}_l$ can separately be determined.

a. *For plug flow of bubble swarm* When air is sparged, the change in diameter of the gas bubbles with the oxygen consumption is negligibly small, and bubble diameter can be taken as the average diameter of the bubbles in the tank space above the sparger. Also for the gas holdup and the pressure, constant average values in the tank can be used. Therefore,

$$\bar{d}_p=\tilde{d}_p, \quad \varphi=\tilde{\varphi}, \quad P=\tilde{P} \qquad (22)$$

The average bubble volume $\tilde{v}_p$ and the average superficial gas velocity $\tilde{u}_g$ in the tank are also expressed as a function of the average oxygen utilization $\tilde{\eta}$ and the pressure $\tilde{P}$, respectively, as

$$\tilde{v}_p=\bar{v}^*_{p0}(1-\delta z_0\tilde{\eta})(P^*/\tilde{P}) \qquad (23)$$

$$\tilde{u}_g=u^*_{g0}(1-\delta z_0\tilde{\eta})(P^*/\tilde{P}) \qquad (24)$$

The ratio of the average surface area $\tilde{a}_p$ to $\tilde{v}_p$ of the bubbles is also related with the averaged diameter $\tilde{d}_p$ by

$$\tilde{a}_p/\tilde{v}_p=6/\tilde{d}_p \qquad (25)$$

By substituting Eqs. (6), (9) and (22)~(25) into Eq. (11) with $C_l=0$,

$$\frac{d\bar{\eta}}{dl}=\bar{k}_lC^*{}_0A\left(\frac{\tilde{P}}{P^*}\right)\frac{1-\bar{\eta}}{1-\delta z_0\bar{\eta}} \qquad (26)$$

$$A=\frac{6\tilde{\varphi}v_m}{\tilde{d}_pu^*_{g0}} \qquad (27)$$

The boundary conditions of Eq. (26) are

$$l=0; \quad \bar{\eta}=0 \qquad (28)$$

Taking the average constant value $\bar{\bar{k}}_l$ for the mass transfer coefficient and integrating Eq. (26) with the boundary conditions of Eq. (28) gives

$$\bar{\bar{k}}_l=\frac{1}{C^*{}_0hA}\left(\frac{P^*}{\tilde{P}}\right)[\delta z_0\bar{\eta}_e-(1-\delta z_0)\ln(1-\bar{\eta}_e)] \qquad (29)$$

From the measured value of the oxygen utilization of the exit gas $\bar{\eta}_e$, the average mass transfer coefficient for the bubble of average diameter $\bar{\bar{k}}_l$ can easily be obtained from Eq. (29). The same relation between the volumetric mass transfer coefficient $K_{La}=k_la$ and the oxygen utilization as Eq. (29) has been derived by Urza and Jackson[4] for deep tank aeration.

b. *For completely mixed flow of bubble swarm* By substituting Eqs. (6), (9), (22), (23) and (25) into Eq. (16) with $C_l=0$, one obtains

$$\frac{d\bar{\eta}}{d\tau}=\bar{k}_l\frac{6v_mC^*_0}{\tilde{d}_p}(1-\bar{\eta}) \tag{30}$$

The initial conditions of Eq. (30) are

$$\tau=0;\quad \bar{\eta}=0 \tag{31}$$

Solving Eq. (30) with Eq. (31) by taking constant $\bar{\bar{k}}_l$ gives

$$\bar{\eta}=1-\exp\left(-\frac{6\bar{\bar{k}}_lC^*_0v_m}{\tilde{d}_p}\tau\right) \tag{32}$$

Substituting Eqs. (20) and (32) into Eq. (19), integrating and rearranging leads to

$$\bar{\bar{k}}_l=\frac{\tilde{d}_p}{6C^*_0v_m}\frac{1}{\bar{\tau}}\frac{\bar{\eta}_e}{1-\bar{\eta}_e} \tag{33}$$

When the liquid flow velocity in the tank is neglected, the mean residence time of the bubbles $\bar{\tau}$ is expressed by

$$\bar{\tau}=h/\tilde{u}_s=h\tilde{\varphi}/\tilde{u}_g \tag{34}$$

and hence

$$\bar{\bar{k}}_l=\frac{1}{C^*_0hA}\left(\frac{P^*}{\tilde{p}}\right)\frac{\bar{\eta}_e(1-\delta z_0\bar{\eta}_e)}{1-\bar{\eta}_e} \tag{35}$$

Since the flow of the gas bubbles is assumed to be completely mixed, the exit value η_e is taken as the average oxygen utilization in the tank and thus the mass transfer coefficient $\bar{\bar{k}}_l$ is easily obtained from the measured value of $\bar{\eta}_e$.

It must be noted that the steady-state method should be applied to obtain the volumetric mass transfer coefficient $K_La=k_la$ in tanks under practical operation, although a small change in oxygen concentration in the sparged gas must be detected.

Unsteady-state methods These methods are commonly used. First, by bubbling nitrogen or by adding a small amount of oxygen-demand reagent, such as sodium sulphite, dissolved oxygen concentration in water is made negligibly low. Next, after starting to feed air into the water, the change in the dissolved oxygen concentration in the water is measured.

In these methods, the following assumptions are made.

1) The time required for the liquid mixing is small compared with the time over which the oxygen concentration in water is measured. This allows the assumption of complete mixing, that is, of uniform distribution of the dissolved oxygen in water.

2) The residence time of the gas bubbles in water is also short compared with the experimental run.

a. *Conventional method* Under these assumptions, the mass balance of oxygen in the liquid phase leads to the following:

$$(1-\tilde{\varphi})\frac{dC_l}{d\theta}=\frac{1}{H}\int_0^h\bar{k}_l\bar{a}(C_0\bar{z}-C_l)dl \tag{36}$$

As mentioned above, in the experiments, the depth of the sparger h is only slightly less than the depth of water H. The average value of the gas-liquid interfacial area per unit liquid volume $\bar{a}$, as the averaged value through the tank, is given by

$$\bar{a}=\tilde{a}=6\tilde{\varphi}/\tilde{d}_p \tag{37}$$

In the previous investigations, carried out on larger gas bubbles, the change in the oxygen mole fraction in the bubbles $\bar{z}$ is usually small and the constant oxygen mole fraction in the sparged gas z_0, has been taken. And since the gas-liquid interfacial area is not measured, the volumetric oxygen transfer coefficient is defined as

$$K_La=\frac{1}{H}\int_0^h\bar{k}_l\bar{a}/(1-\varphi)dl \tag{38}$$

Eq. (36) is thus expressed by

$$\frac{dC_l}{d\theta}=K_La(C_0z_0-C_l) \tag{39}$$

By taking the average value $\tilde{C}_0$ in the tank as C_0 and integrating Eq. (39), one obtains

$$K_La=\frac{1}{\theta-\theta_0}\ln\frac{\tilde{C}_0z_0-C_l}{\tilde{C}_0z_0} \tag{40}$$

where θ_0 is the time when the dissolved oxygen concentration C_l acends from zero.

Therefore, K_{La} can be determined from the observed change in C_l with time.

b. *Direct measurement of gas-phase oxygen concentration* In the completely segregated fine bubble swarm the gas-liquid interfacial area is so large that the oxygen utilization, that is the change in the oxygen mole fraction $\bar{z}$, in the sparged air is too large to be ignored. The integration of the right hand side of Eq. (36), therefore, must be carried out with varying values of $\bar{z}$ in the direction of the tank depth.

For simplicity's sake, the average value in the tank is taken as $(C_0\bar{z}-C_l)_{av}$, and Eq. (36) becomes:

$$(1-\tilde{\varphi})\frac{dC_l}{d\theta}=\bar{\bar{k}}_l\tilde{a}(C_0\bar{z}-C_l)_{av} \tag{41}$$

The constant average mass transfer coefficient k_l thus can be obtained from

$$\bar{\bar{k}}_l=\frac{(1-\tilde{\varphi})(dC_l/d\theta)}{\tilde{a}(C_0\bar{z}-C_l)_{av}} \tag{42}$$

When the oxygen mole fraction in the exhaust gas $\bar{z}_e$ is observed, the following logarithmic mean for $(C_0\bar{z}-C_l)_{av}$ can be applied for the assumed plug flow of the bubbles swarm.

$$(C_0\bar{z}-C_l)_{av}=\frac{C_{0b}z_0-C^*{}_0\bar{z}_e}{\ln[(C_{0b}z_0-C_l)/(C^*{}_0\bar{z}_e-C_l)]} \tag{43}$$

where C_{0b} is the value of C_0 at the tank bottom. For the completely mixed flow, on the other hand, $(C_0\bar{z}-C_l)_{av}$ can be directly obtained by substituting the observed value of the oxygen mole fraction in the exhaust gas $\bar{z}_e$ as $\bar{z}$.

However, the simultaneous observation of the oxygen mole fraction in the exhaust gas along with that of the dissolved oxygen concentration in water is not easy, because of the considerable delay involved in the measurement of the oxygen composition of the exhaust gas.

c. *Prediction of gas phase oxygen concentration* Without direct measurement of gas phase oxygen concentration, the value of $(C_0\bar{z}-C_l)_{av}$ can be predicted by the following methods. These are more straightforward approaches than the approximate methods presented by Jackson and Shen.[1)]

When a plug flow of the bubble swarm is assumed, the oxygen balance is given by Eq. (11) at any instant. By substituting Eqs. (22)~(25) into Eq. (11), and taking the average observed values for $\tilde{\varphi}$ and $\tilde{d}_p$ in the tank, one obtains

$$\frac{d\bar{\eta}}{dl}=\frac{A\bar{k}_l}{z_0}(C_0\bar{z}-C_l) \tag{44}$$

where A is given by Eq. (27).

Substituting Eq. (44) into Eq. (36) gives

$$(1-\tilde{\varphi})\frac{dC_l}{d\theta}=\frac{1}{H}\int_0^h\frac{\tilde{a}z_0}{A}\frac{d\bar{\eta}}{dl}dl=\frac{\tilde{a}z_0}{AH}\bar{\eta}_e \tag{45}$$

By comparing Eq. (45) with Eq. (41) and substituting Eq. (37), one obtains

$$(C_0\bar{z}-C_l)_{av}=\frac{z_0}{\bar{\bar{k}}_lAH}\bar{\eta}_e \tag{46}$$

It is apparent that the value of $(C_0\bar{z}-C_l)_{av}$ can be easily obtained if $\bar{\eta}_e$ is calculated for a given value of $\bar{\bar{k}}_l$. This can be simply done by solving Eq. (44) with Eq. (6) and with constant C_l, under the boundary condition of $\bar{\eta}=0$ at $l=0$. The solution is

$$\delta z_0\bar{\eta}_e-\left\{1-\frac{\delta(\tilde{C}_0z_0-C_l)}{\tilde{C}_0-C_l}\right\}\ln\left|1-\frac{z_0(\tilde{C}_0-C_l)}{\tilde{C}_0z_0-C_l}\bar{\eta}_e\right|=(\tilde{C}_0-C_l)\bar{\bar{k}}_lAh \tag{47}$$

Since $\bar{\eta}_e$ is not explicitly obtained as a function of h in Eq. (47), trial- and-error calculation must be used.

Once $\bar{\eta}_e$ is obtained for an assumed $\bar{\bar{k}}_l$, $(C_0\bar{z}-C_l)_{av}$ is given by Eq. (46) and then the value of $\bar{\bar{k}}_l$ is determined from the experimentally measured time gradient of the dissolved oxygen in liquid $(dC_l/d\theta)$ from Eq. (42). The trial-and-error procedure is continued until the assumed value agrees with the obtained value for $\bar{\bar{k}}_l$.

When completely mixed flow of the bubble swarm is assumed, the relation between $(C_0\bar{z}-C_l)_{av}$ and $\bar{\eta}_e$ shown in Eq. (46) can be adopted. The value of $\bar{\eta}_e$ can be evaluated by substituting Eqs. (6), (9), (22), (23) and (25) into Eq. (16):

$$\frac{d\bar{\eta}}{d\tau}=\frac{6v_m\bar{k}_l}{\bar{d}_p}\left(\frac{P^*}{\bar{P}}\right)\left[\left(\tilde{C}_0-\frac{C_l}{z_0}\right)-(\tilde{C}_0-\delta C_l)\bar{\eta}\right] \qquad (48)$$

Then taking constant $\tilde{\bar{k}}_l$ and solving Eq. (48) with the initial condition of $\bar{\eta}=0$ at $\tau=0$, $\bar{\eta}$ is yielded as a function of residence time τ. Substituting $\bar{\eta}$ into Eq. (19) with Eq. (20) and integrating gives

$$\bar{\eta}_e=\frac{\tilde{\bar{k}}_l\boldsymbol{B}h(\tilde{C}_0-C_l/z_0)}{1+\tilde{\bar{k}}_l\boldsymbol{B}h(\tilde{C}_0-\delta C_l)} \qquad (49)$$

where

$$\boldsymbol{B}=\boldsymbol{A}/(1-\delta z_0\bar{\eta}_e) \qquad (50)$$

Therefore, the trial and error method must be applied to determine $\bar{\eta}_e$ from Eqs. (49) and (50).

Conclusion

1. Procedures for predicting the oxygen utilization in a completely segregated fine bubble swarm sparged into water are given for the both plug flow and completely mixed flow of swarm.
2. Methods to determine the oxygen transfer coefficient for the bubbles are discussed.
3. In the steady state method, the oxygen utilization of the exhaust gas is observed, while the dissolved oxygen in water is kept negligible.
4. The unsteady state method, which has commonly been used in previous works, is modified for the present system, in which the change in oxygen mole fraction in the gas bubbles cannot be ignored.

Nomenclature

$\boldsymbol{A}$	given in Eq. (27)	[cm·sec/mol]
a	gas-liquid interfacial area per unit liquid volume	[1/cm]
a_p	surface area of single bubble	[cm²]
$\boldsymbol{B}$	given in Eq. (50)	[cm·sec/mol]
C_0	saturated concentration with pure oxygen in water	[mol/cm³]
C_l	dissolved oxygen concentration	[mol/cm³]
d_p	diameter of bubble	[cm]
g	gravitational acceleration	[cm/sec²]
g_c	gravitational conversion factor	[g-cm/G·sec²]
h	depth of gas sparger	[cm]
H	liquid depth	[cm]
k_l	mass transfer coefficient	[cm/sec]
K_La	volumetric oxygen transfer coefficient	[1/sec]
l	distance from gas sparger	[cm]
P	pressure	[atm]
u_g	superficial gas velocity	[cm/sec]
u_l	superficial liquid velocity	[cm/sec]
u_s	gas-liquid slip velocity	[cm/sec]
u_{s0}	terminal velocity of single gas bubble in water	[cm/sec]
v_m	molar volume of gas	[cm³/mol]
v_p	bubble volume	[cm³]
z	mole fraction of oxygen in gas phase	[—]

Greek

β	number of moles of gas produced per mole of oxygen consumed	[—]
δ	$=1-\beta$	[—]
θ	time	[sec]
ρ_l	liquid density	[g/cm³]
φ	gas holdup	[—]
η	oxygen utilization	[—]
τ	residence time in liquid	[sec]

Superscipts and subscripts

—	averaged value for bubbles
~	averaged value throughout tank
*	under atmospheric pressure
0	under conditions of $\tau=0$ or $l=0$
e	the exit condition

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