Constitution of an Approximate Equation of Demagnetizing Factors for Cylinder Using Multiple Regression Analysis

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An approximate equation is constituted for a cylinder as a function of the aspect ratio of its length to diameter and its normalized axial position. In order to create reference points for the approximate equation, the demagnetizing factor distribution of the cylinder, which has several ratios in a uniform applied field, is computed using a finite element method. From the results of the computations, an approximate equation at an arbitrary Z value along the axial direction of the cylinder is constituted from a multiple regression analysis.

Key words: demagnetizing factor, finite element method, multiple regression analysis, approximate equation

1. Introduction

When a magnnetic material of finite length is magnetized in a uniform applied field, its ends carry magnetic poles. These magnetic poles induce a demagnetizing field, which is directed in the opposite direction of the applied field [1]. Therefore, a demagnetizing factor is applied to compensate for the effects of the demagnetizing field. An accurate method of estimating demagnetizing factors for a cylinder is described in [2]. The method estimates the demagnetizing factor as the function of the susceptibility and the aspect ratio of the length to the diameter, and it uses two types of models according to the aspect ratio. Therefore, it would be of great practical use if one approximate equation could estimate the demagnetizing factors. This paper studies a method for the approximation of the demagnetizing factor for a cylinder in a uniform applied field using a multiple regression analysis (MRA).

2. Derivation of the Approximate Equation for Demagnetizing Factors

The approximate equation is derived as a function of the aspect ratio of the length to the diameter of a cylinder and its normalized axial position. In order to create reference points of this approximate equation, the demagnetizing factor distribution of the cylinder, which has several ratios in a uniform applied field, is computed using a finite element method (FEM). The FEM estimates distribution of demagnetizing factors accurately. Based on the results of the FEM, approximate equations at arbitrary Z values along the axial direction of the cylinder are derived from the MRA.

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2.1 Computation of demagnetizing factors

The cylinder in a uniform applied field H_0 has a demagnetizing field H_d, as shown in Fig. 1. The demagnetizing factor N is defined as the fluxmetric demagnetizing factor, which is the ratio of an average demagnetizing field to the average magnetization at a plane perpendicular to the Z axis. The demagnetizing factor is given as Eq. (2) based on Eq. (1). The demagnetizing factor distribution in the cylinder is not uniform, and it varies according to the aspect ratio γ of the length L to the diameter D of the cylinder. In this paper, FEM is employed to obtain the demagnetizing factor distribution in the cylinder accurately.

$$\begin{cases} \boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M} \\ \boldsymbol{H}_d = N\boldsymbol{M} \\ \boldsymbol{H}_d = \boldsymbol{H}_0 - \boldsymbol{H} \end{cases} \tag{1}$$

$$N = \frac{\boldsymbol{H}_0 - \boldsymbol{H}}{\boldsymbol{H}(\mu_r - 1)} \tag{2}$$

We use an axisymmetric model of the upper half of a cylinder as the model for the FEM as shown in Fig. 1. The cylinder is placed in a square air region with the side length 5 times as long as either the cylinder length or its diameter. The diameter of the cylinder D is 10mm, and the aspect ratio varies

Table 1 Samples of coefficients of the multiple regression equation (4) at the specified z position.

z	C_0	C_1	C_2	C_3	C_4	C_5	C_6
0.000E+0	-1.285E-1	-1.349E+0	-4.279E-1	-6.359E-2	8.870E-4	-1.199E-5	3.556E-4
1.000E-1	-1.278E-1	-1.320E+0	-4.156E-1	-7.579E-2	8.037E-4	-9.655E-6	1.138E-3
2.000E-1	-1.242E-1	-1.243E+0	-3.844E-1	-1.082E-1	6.016E-4	-4.035E-6	3.231E-3
3.000E-1	-1.156E-1	-1.159E+0	-3.527E-1	-1.429E-1	4.155E-4	1.071E-6	5.492E-3
4.000E-1	-1.001E-1	-1.099E+0	-3.342E-1	-1.669E-1	3.494E-4	2.694E-6	7.115E-3
5.000E-1	-7.514E-2	-1.086E+0	-3.410E-1	-1.694E-1	4.988E-4	-1.912E-6	7.453E-3
6.000E-1	-4.245E-2	-1.087E+0	-3.583E-1	-1.632E-1	7.660E-4	-1.007E-5	7.254E-3
7.000E-1	-1.510E-3	-1.089E+0	-3.811E-1	-1.527E-1	1.134E-3	-2.144E-5	6.779E-3
8.000E-1	4.692E-2	-1.092E+0	-4.101E-1	-1.333E-1	1.639E-3	-3.740E-5	5.650E-3
9.000E-1	8.155E-2	-9.400E-1	-3.739E-1	-1.530E-1	1.805E-3	-4.497E-5	6.608E-3

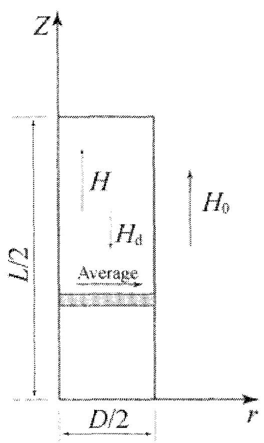

Fig. 1 The computation model for the finite element analysis.

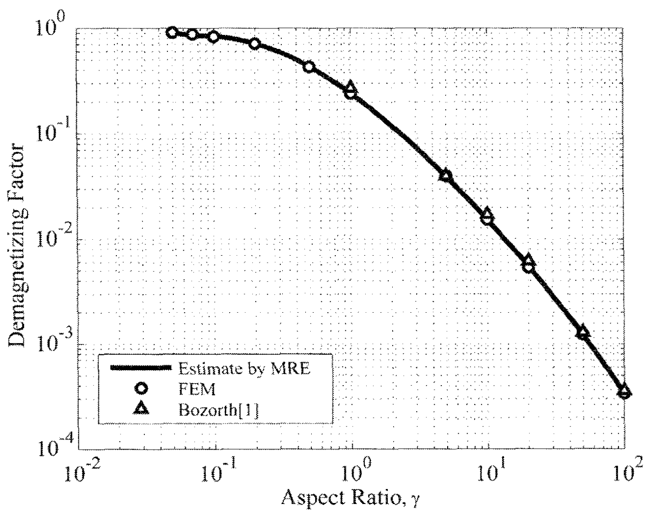

Fig. 2 Comparison of approximated fluxmetric demagnetizing factors at the midplane calculated using FEM, Eq. (3), and data from [1].

from 0.05 to 100. A uniform field of 1mT is applied as a boundary condition. The cylinder is composed of a ferromagnetic material, which has high susceptibility. The magnetic field and the magnetization are solved for each finite element, then the demagnetizing factor is obtained from their average values at the plane perpendicular to the axis.

2.2 The approximate equation at the midplane

The demagnetizing factor depends on the aspect ratio of the length to the diameter of a cylinder. Therefore, the magnetic field depending on the aspect ratio is solved using FEM, and based on the results of the FEM, the demagnetizing factor at the midplane of a cylinder, z=0, is calculated. The MRA is employed in order to create an approximate equation that gives the demagnetizing factor at an arbitrary aspect ratio from 0.05 to 100. Furthermore, the MRA refers to several points of the demagnetizing factor calculated by the FEM. The approximate equation of the demagnetizing factor at the midplane of the cylinder is obtained as the following multiple regression equation (MRE) [3].

$$\log_{10} N = -0.13398 - 1.36759\log_{10}\gamma - 0.42801/\sqrt{\gamma} - 0.05588\sqrt{\gamma} + 0.00060\gamma^{-2} \quad (3)$$

Equation (3) has a multiple correlation coefficient (MCC) of 0.999. Fig. 2 shows a comparison of the approximated demagnetizing factors at the midplane calculated by the FEM, with the factors in Eq. (3). It also shows experimental data from [1]. The demagnetizing factor in Eq. (4) has an error of plus or minus 2 % of the factor given by the FEM. The calculated demagnetizing factors agree well with experimental data of [1].

2.3 The approximate equation at the arbitrary Z position

The demagnetizing factor varies according to the axial z position. Therefore, when measured, the thickness of a search coil and the position where it

Table 2 Coefficients of the multiple regression equation (5).

	a_0	a_1	a_2	a_3	a_4	a_5	a_6
C_0	-1.28420E-1	2.56512E-3	5.42176E-1	-2.48177E-1	-2.85268E-1	2.32600E-1	-2.86943E-1
C_1	-1.35298E+0	4.56150E+0	-9.82726E+0	5.78427E+0	9.17079E-1	-4.17154E-1	1.43591E-1
C_2	-4.29523E-1	1.94467E+0	-4.42562E+0	2.53542E+0	4.47874E-1	-2.05257E-1	1.48416E-1
C_3	-6.19736E-2	-1.90916E+0	4.08811E+0	-2.30327E+0	-7.55592E-2	-1.43162E-1	1.88767E-1
C_4	8.95938E-4	-1.31600E-2	3.13545E-2	-1.67914E-2	-2.46179E-3	5.10937E-4	-1.30909E-3
C_5	-1.22203E-5	3.66676E-4	-8.75018E-4	4.58895E-4	5.94293E-5	-3.30085E-6	3.64166E-5
C_6	2.63143E-4	1.20479E-1	-2.52127E-1	1.40251E-1	2.11389E-3	7.83708E-3	-1.03985E-2

winds around the cylinder should be considered. Accordingly, approximate equations should be created, not only at the midplane but also at arbitrary Z positions.

Approximate equations are obtained at normalized positions along the Z axis, as in the procedure mentioned above. We set up approximate equations for the demagnetizing factor in increments of 0.02 at 50 positions between 0 and 1, based on demagnetizing factors calculated from results of the FEM. The approximate equation is defined as the following multiple regression equation.

$$\log_{10}N = C_0(Z) + C_1(Z)\log_{10}\gamma + C_2(Z)/\sqrt{\gamma} + C_3(Z)\sqrt{\gamma} + C_4(Z)\gamma^{-2} + C_5(Z)\gamma^{-3} + C_6(Z)\gamma \tag{4}$$

where, $C_n(Z)$ is the coefficient of a parameter.

Table 1 shows coefficients of the multiple regression equation (4). All of the MCCs of each multiple regression equation are over 0.99999. As for coefficients C_4, C_5 and C_6, shown in Table 1, there are some cases of loss of significance in the T-test at some Z positions, but we include them in the regression equation (4) to standardize the expression of coefficients.

The approximate equation (4) gives the demagnetizing factor of the cylinder at a specified arbitrary Z position by constituting an approximate equation of the regression coefficient C_n as a function of the Z value. We assume that the multiple regression equation of the regression coefficient C_n is a linear polynomial expression of a power of Z, and take the terms into account up to the 50th power. The multiple regression equation of the regression coefficient C_n of Eq. (4) is shown as follows.

$$C_n(Z) = a_0 + a_1Z^2 + a_2Z^3 + a_3Z^4 + a_4Z^{21} + a_5Z^{30} + a_6Z^{43} \tag{5}$$

The value of the regression coefficient a_n of Eq. (5) is obtained as shown in Table 2. All the MCCs of each equation with coefficients in Table 2 are over 0.999. We calculate the demagnetizing factor from the distribution of a magnetic field solved by using FEM as shown in a previous section, and constitute the multiple regression equation which estimates the demagnetizing factor of a cylinder from the specified aspect ratio and the arbitrary Z axis position. Fig. 3 shows demagnetizing factors estimated by Eq. (4). The demagnetizing factors increase with the increase of the Z value from 0 to 1.

3. Result of Approximation from Using a Multiple Regression Equation

3.1 Approximation of the regression coefficient

Fig. 4 shows the results of approximating coefficient C_1 in Table 1 by the MRE of Eq. (5), which uses coefficient values in Table 2. The approximation of the distribution of the coefficient C_0 in Eq. (5) has good agreement with values in Table 1. With regards to all coefficients except C_0, they have discontinuous points in their distribution as shown in Fig. 4 (at Z from 0.4 to 0.7), so the approximation is not adequate in this part. There are

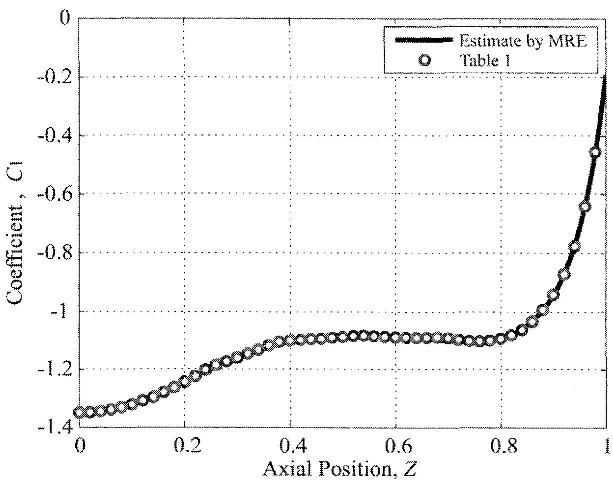

Fig. 4 The results of approximation of the coefficient C_1 in Table 1 from the multiple regression equation (5).

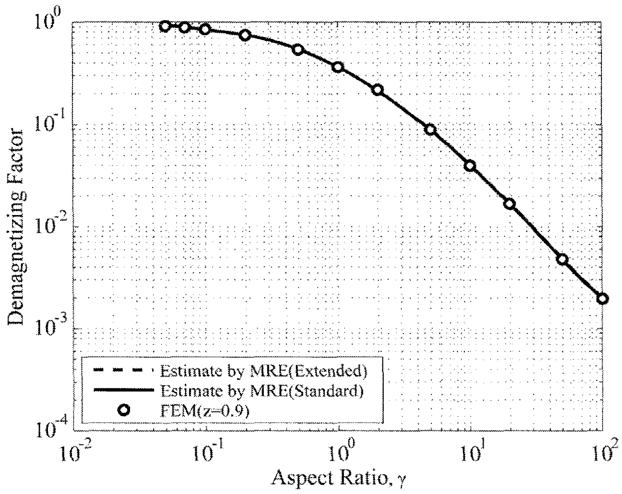

Fig. 5 Three types of demagnetizing factors at the Z position 0.9, which are estimated from Eq. (4) with coefficient equation (5) (extended), from Eq. (4) with coefficient values in Table 1 (standard), and from FEM.

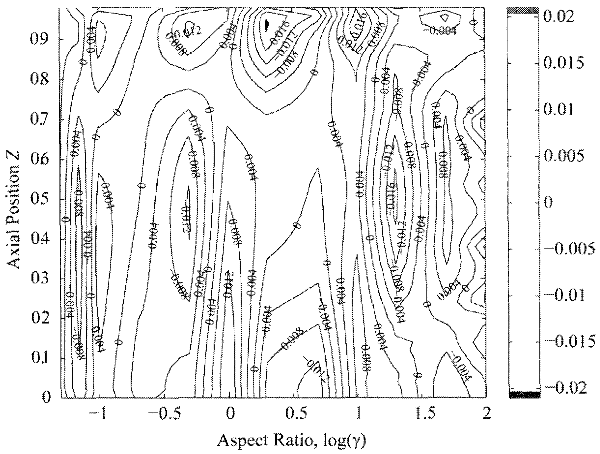

Fig. 6 Error ratio of the demagnetizing factor from Eq. (4) to that from FEM.

relatively large errors in coefficients C_3 and C_6. The fluctuation of the mesh qualities among models for the FEM should be reduced to lessen their discontinuous distribution. Therefore, a method to create homogeneous mesh models, such as an adaptive mesh procedure, is required.

3.2 Approximation of the demagnetizing factor

Fig. 5 shows the distribution of three types of demagnetizing factors at the Z position 0.9, each of which is estimated from Eq. (4) with the coefficient equation (5) (extended), from Eq. (4) with the coefficient values in Table 1 (standard), and from FEM. The estimated values produced by the above-mentioned equations agree well with the results from the FEM, but there is a small difference in the range of the large aspect ratio.

Fig. 6 shows the error distribution of the demagnetizing factor of the cylinder between the estimated values from Eq. (4) and those values using FEM. The error ratio is distributed in the range of -0.021 to 0.02, and relatively large error is distributed in the region at the end of the cylinder.

4. Conclusion

The demagnetizing factor distributions of a cylinder that has several ratios in a uniform applied field are computed using FEM in order to create reference data for the approximate equation. An approximate equation of demagnetizing factors at an arbitrary Z position of the cylinder is constituted by using a MRA based on the results of the FEM. Thus the approximate equation of the demagnetizing factor is obtained from the MRA, and it is defined as a function of the aspect ratio of the length to the diameter of a cylinder and its normalized axial position. A comparison of the estimated value from the approximate equation and the one from the FEM demonstrated that the equation has the ability to estimate the demagnetizing factor with an error of plus or minus 2 % of the performance of the FEM.

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Received: 27 November 2008/Revised: 18 March 2009