

Regular Paper

Fundamental Study on a Vernier Motor

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This paper describes a fundamental study on the surface-permanent-magnet-type (SPM-type) vernier motor using the finite element method. The operating principle where the rotor is synchronized to the harmonics of the rotating magnetic field according to the reduction ratio is described. To satisfy $Z_2 = Z_1 \pm p$, the number of stator slots, pole pairs in the rotor, and winding pole pairs that were chosen were $Z_1=6$, $Z_2=5$, and $p=1$, respectively. With this setup, the rotor will rotate in the opposite direction of the fundamental rotating magnetic field, at $1/5^{\text{th}}$ of the speed. A comparison between using full and short pitch windings on the stator when it is used as a generator and a motor is discussed. As a motor, both winding types can generate the same output torque, but the former is more advantageous in terms of effectiveness in utilizing the magnetic flux.

Keywords: vernier motor, magnetic flux harmonics, full pitch winding, short pitch winding, power angle.

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1. Introduction

The vernier motor was developed by C. H. Lee in 1963 and is known as a direct drive motor which is suitable for low-speed and high-torque applications. These applications are absolutely free from mechanical friction because of its gear-less design, and as a result, maintenance-free operation becomes possible.

The vernier motor is structurally classified into three types. The first is the VR-type [1] which has no magnets and rotates only by the reluctance torque. The output torque is lower than the other types, but due to its simple structure, it is easy to assemble and the parts are rather inexpensive. The second is the PM-type [2, 3] which uses multi-pole permanent magnets. A high output torque can be generated. However since it requires many multi-pole magnets, it is expensive and difficult to assemble. The last is the HB-type [4], which is a hybrid structure of the VR and PM vernier motors. It uses only one single-pole permanent magnet sandwiched between the rotor cores. Compared to the PM-type, the output torque is lower and laminated steel cannot be used as the core material because the flux passes in the axial direction.

As shown above, each type of vernier motor has its own advantages and disadvantages, and has many possibilities for various applications. However the detailed operational principle based on the magnetic flux harmonics has not been clarified by numerical analysis. In particular, the relationship between the type of motor winding and characteristics as a generator and a motor has not been discussed. The PM-type vernier motor that has permanent magnets placed on the surface

of the rotor (SPM-type) is selected as it can produce a high output torque. This paper clarifies the operational principle based on the flux harmonics and investigates the performance comparison between the winding types using FEM analysis.

2. Operating Principle

Stator coils excited by a three-phase AC voltage generate a rotating magnetic field. The ν th harmonic component of the flux density in the air gap B_ν is given by the following equations [5]:

$$B_\nu = \frac{\pi}{4} \frac{B_m}{\nu} \left\{ \sin \omega t \sin \nu \theta + \sin \left(\omega t - \frac{2\pi}{3} \right) \sin \nu \left(\theta - \frac{2\pi}{3} \right) + \sin \left(\omega t - \frac{4\pi}{3} \right) \sin \nu \left(\theta - \frac{4\pi}{3} \right) \right\}, \quad (1)$$

$$B_m = \mu_0 \frac{w(\sqrt{2}I)}{2\alpha}, \quad (2)$$

where the rotor is regarded as a simple cylindrical core without permanent magnets, α is the air-gap length, w is the number of coil turns in each phase, and I is the coil current.

The fundamental harmonic component ($\nu = 1$) travels at the synchronous speed ω . The even-ordered harmonic components ($\nu = 2h$) and harmonic components that are multiples of three ($\nu = 6h - 3$) disappear, where h is a positive integer. The harmonic components $\nu = 6h - 1$ and $\nu = 6h + 1$ travel at the reduced speed ω/ν in the same and opposite directions of the fundamental component respectively. These flux harmonics are generated due to the permeance of the stator slots.

When the following structural relation exists between the number of stator slots Z_1 , pole pairs of the permanent magnets Z_2 , and pole pairs of the rotating magnetic field p ,

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$$Z_2 = Z_1 \pm p, \quad (3)$$

the rotor operates at ω_m , which is a speed with a reduction ratio of p/Z_2 with respect to the synchronous speed.

$$\omega_m = \frac{\omega}{p} \times \frac{p}{Z_2} = \frac{\omega}{Z_2} \quad (4)$$

3. Verification of the Operating Principle by FEM

3.1 Analysis Model

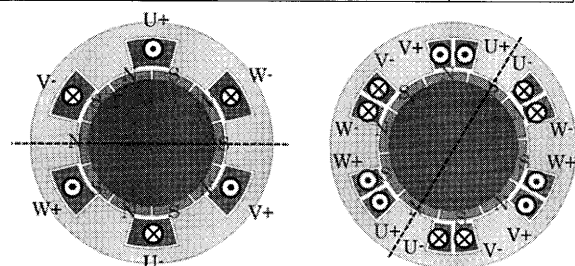
Figs. 1 and 2 show the cross section and the oblique drawing of the analysis model, respectively, and Table 1 gives the design parameters.

The rotor consists of a core made of laminated 50A400 steel sheets and NdFeB NEOMAX-39H sintered permanent magnets. The stator also consists of the same 50A400 laminated steel sheets and three-phase coil windings.

In this paper, full pitch and short pitch windings of the stator are also discussed. Full pitch winding is a type of winding where the two coil sides forming a complete coil of a winding, are 180 electrical space degrees apart. On the other hand, short pitch winding is when the coil span of the winding is less than 180 electrical space degrees i.e. the two coil sides forming a complete coil are less than 180 electrical space degrees apart. Generally under the same conditions, the EMF in the short pitch coil is less than that of the full pitch coil because the EMFs in the short pitch coil sides are not in phase. However, short pitch windings use less copper wiring, is easier to wind and assemble.

Table 1 Design parameters of the analysis model.

Number of stator slots Z_1	6
Number of pole pairs of permanent magnet Z_2	5
Number of pole pairs of magnetic field p	1
Outside diameter	50 mm
Thickness (exclude coil-end length)	15 mm
Air-gap length	0.5 mm
Radius of the air-gap center	15.25 mm
Number of coil turns in each phase per slot	
-Full pitch windings	600 turns
-Short pitch windings	300 turns
Coil diameter	0.28 mm
Space factor of coils	0.7



(a) Full pitch windings (b) Short pitch windings
Fig. 1. Cross section of the analysis model.

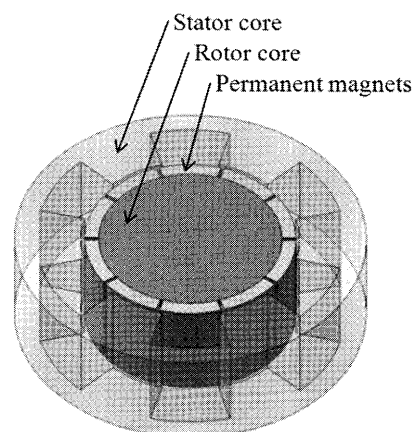
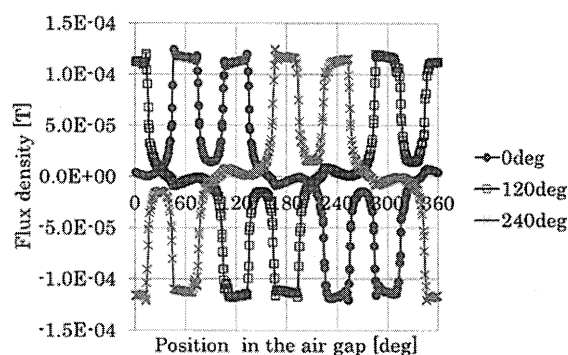
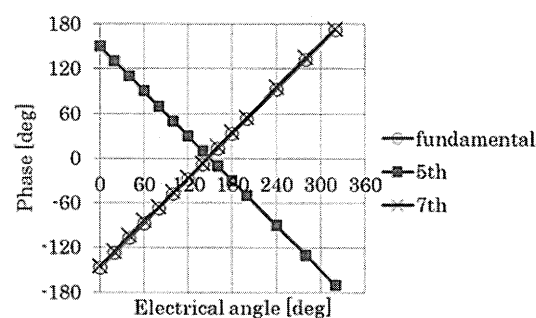


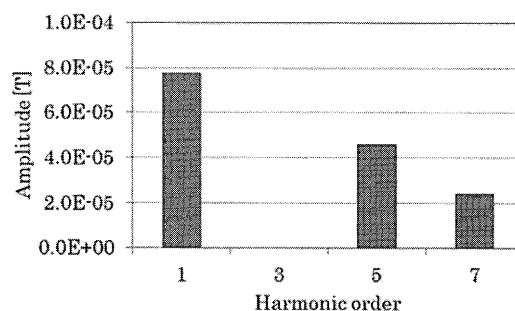
Fig. 2. Oblique drawing of the analysis model.
(Without coils)



(a) Flux density distribution

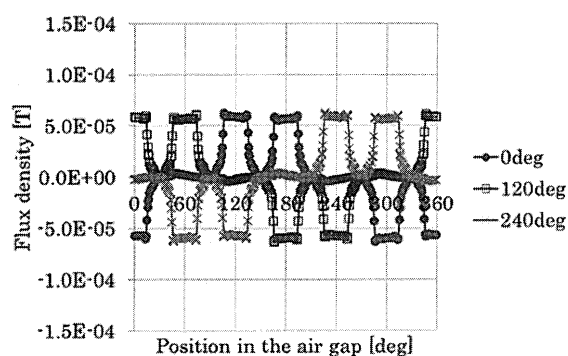


(b) Phase angle of the harmonics

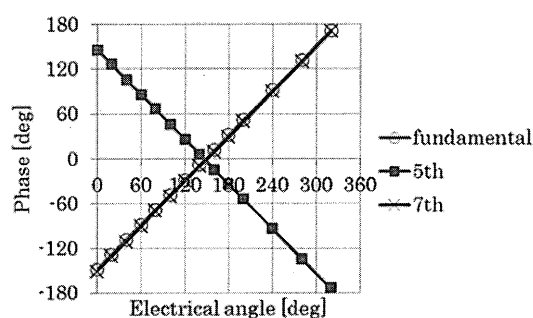


(c) Amplitude of the harmonics

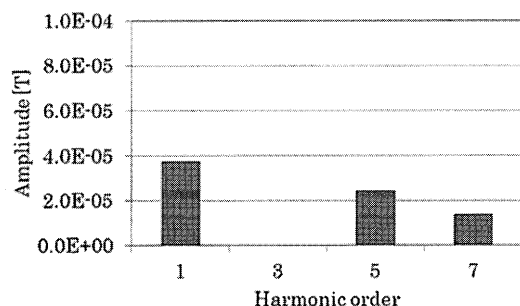
Fig. 3. Flux density distribution in the air gap.
(Full pitch windings)



(a) Flux density distribution



(b) Phase angle of the harmonics



(c) Amplitude of the harmonics

Fig. 4. Flux density distribution in the air gap.
(Short pitch windings)

3.2 Generation of a Rotating Magnetic Field

In the analysis models we regarded the rotor as a simple cylindrical magnetic core having a constant air-gap length of 0.5 mm. The stator coils were excited by a three-phase AC voltage.

The flux density in the air gap was computed using FEM, and the flux harmonics were calculated using fast-fourier-transform (FFT). The results of the full and short pitch windings are shown in Figs. 3 and 4, respectively.

The following results are observed regardless of the winding form.

- The third harmonic component disappears.
- The 5th harmonic component travels in the opposite direction of the fundamental component.
- The 7th harmonic component travels in the same direction as the fundamental component.

Rotating magnetic field

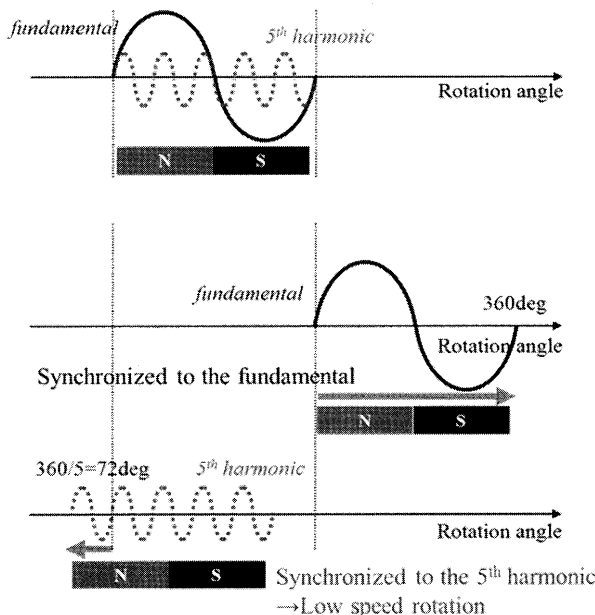


Fig. 5. Operating principle of a vernier motor.

In order to synchronize the rotor to the flux harmonics, the number of permanent magnet pole pairs must be 5 or 7. This requirement is indicated in Eq. (3). We chose $Z_2 = 5$ to obtain a larger amplitude, which incidentally results in our model rotating in the opposite direction of the fundamental rotating magnetic field, at $1/5^{\text{th}}$ the speed, as can be seen in Fig. 5.

Figs. 3 and 4 show that full pitch windings generate twice as much flux density in the air gap as that of short pitch windings. Therefore full pitch windings are superior in terms of the magnetizing force by the permanent magnets.

4. Characteristic Analysis of a Vernier Machine

4.1 Generator Characteristics

The generator characteristics were computed when the rotor was rotated at 100 rpm, where the initial position is defined as the position shown in Fig. 1. Fig. 6 shows the no-load line voltage. The period of these waveforms corresponds to the magnetic pole pitch of 72 mechanical degrees regardless of the winding form. The amplitude of the full pitch winding is approximately twice as large as that of the short pitch winding because of the short pitch factor $k_\beta = 1/2$.

$$k_\beta = \sin\left(\frac{\beta\pi}{2}\right) \quad (5)$$

where β is $1/3$, which is equivalent to the coil pitch per full winding pitch.

This difference can be also described as follows. While the rotor rotates by one magnetic pitch, the full pitch windings (w) are crossed by two magnetic pole pairs ($2\phi_m$), where ϕ_m is the internal linkage flux per

pole pair of the permanent magnet. On the other hand, the short pitch windings that are connected in a series ($w/2$) are crossed by one magnetic pole pair (ϕ_m). As a result, the induced voltage of each phase is calculated using the following equations.

$$E_{of} = -w \frac{d(2\phi_m)}{dt} = -2w \frac{d\phi_m}{dt}, \quad (6)$$

$$E_{os} = -2 \left(\frac{w}{2} \frac{d\phi_m}{dt} \right) = -w \frac{d\phi_m}{dt}. \quad (7)$$

From the equations, we can see that the induced voltage of the full pitch windings is twice as large as that of the short pitch windings.

Fig.7 shows the cogging torque waveform of the motor. It should be noted that cogging torque is not influenced by the winding form. The least common multiple of the number of stator slots $Z_1 = 6$ and the number of permanent magnet poles $2Z_2 = 10$ is 30. Therefore the period of the cogging torque is 12 mechanical degrees, which corresponds to that of the computed result. The amplitude is approximately 0.06 Nm.

4.2 Motor Characteristics

Generally synchronous machines define the power angle (internal phase angle) δ as a phase difference between the supply voltage V and the induced voltage E_o , as is shown in Fig. 8. The circuit equation is expressed in Eq. (8), where r_a is the coil resistance, x_s is the coil reactance, and I is the coil current.

$$V = E_o + I(r_a + jx_s). \quad (8)$$

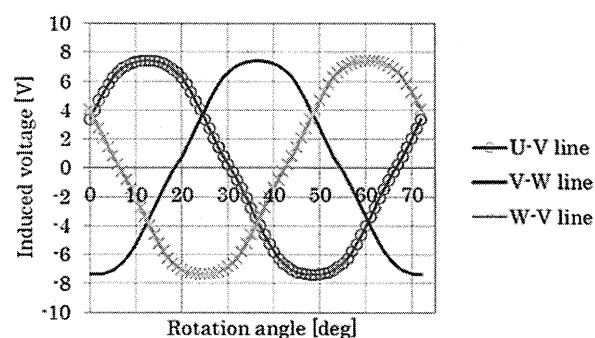
When the stator coil is excited by the open-loop control, the rotor rotates at a constant speed in accordance with δ , which increases in proportion to the load increase, until the load exceeds the limit torque, in which case the rotor will slip.

These days, due to the development of switching power supply circuits and the widespread use of inverter control in motor drive, with feedback control, operation with a stable δ has become possible. Therefore varying δ is useful to gain knowledge on how to design the optimal inverter circuit condition.

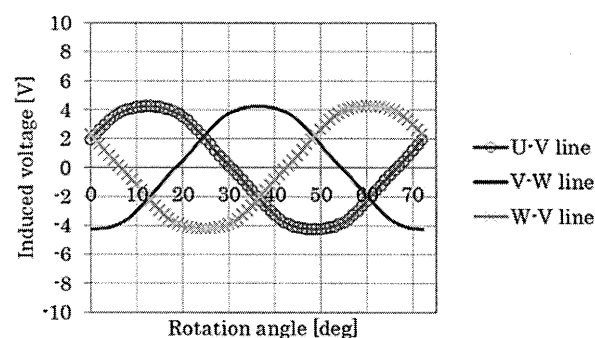
We computed the motor characteristics when the magnetic field was generated by a supply voltage of 6 V rotated at 500 rpm (CW) and the rotor synchronously rotated at 100 rpm (CCW).

Fig. 9 shows the output torque waveforms and Fig. 10 shows the variation of the output average torque according to the power angle. The output torque contains the same periodic ripples as the cogging torque, and both waveforms have approximately the same amplitude. The maximum torque of 0.28 Nm was achieved at $\delta = 5$ degrees (full pitch windings) and $\delta = 0$ degree (short pitch windings).

Fig. 11 compares the U-phase coil currents of the full pitch and short pitch windings at their respective optimal power angles, at 100 rpm.



(a) Full pitch windings



(b) Short pitch windings

Fig. 6. Line voltage waveforms as a generator.

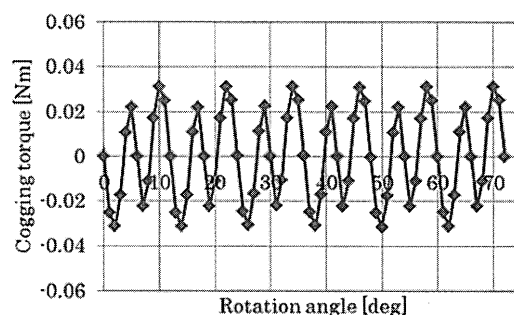


Fig. 7. Cogging torque waveforms.

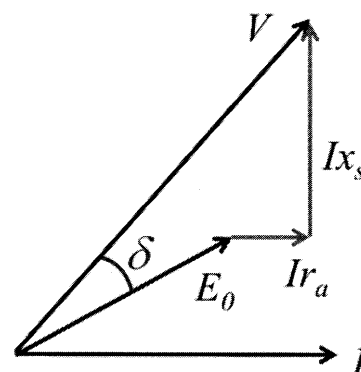
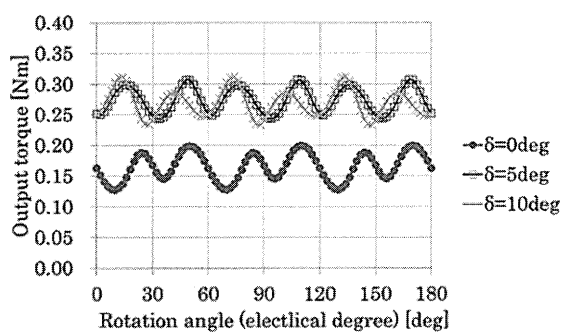
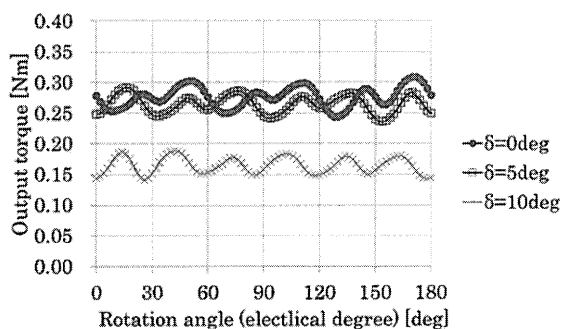


Fig. 8. Phaser diagram of the synchronous motor.



(a) Full pitch windings



(b) Short pitch windings

Fig. 9. Output torque waveforms at 100 rpm.

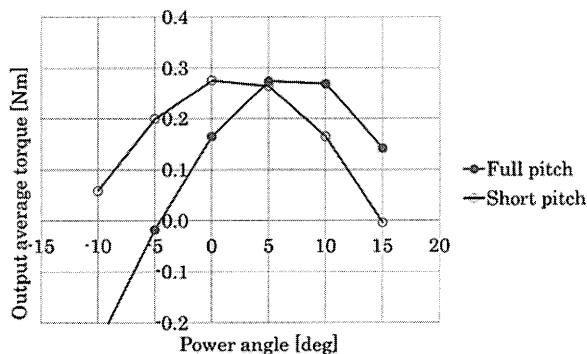


Fig. 10. Output average torque vs. power angle characteristics at 100 rpm.

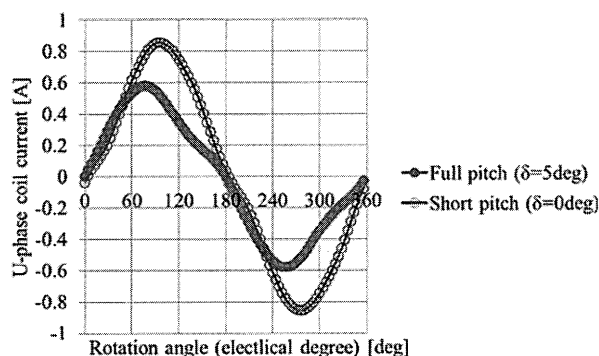


Fig. 11. Comparison of the coil current at 100 rpm.

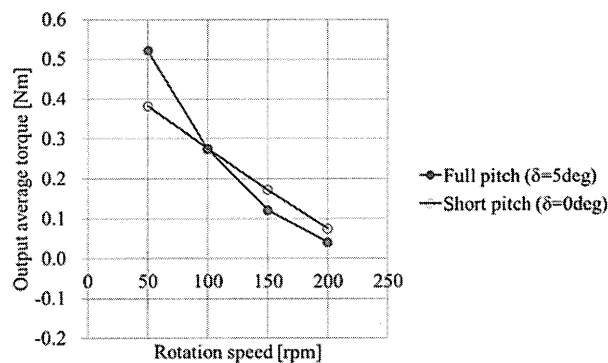


Fig. 12. N-T characteristics.

The short pitch windings needed a current 1.5 times larger than that of the full pitch windings.

Fig. 12 shows the rotor speed vs. the output average torque characteristics. When the rotor operates at 100 rpm, both winding types obtain the same output torque. Therefore we calculated the efficiency η by using the following equation.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{\omega T}{3 \sum V(t)I(t)}, \quad (9)$$

where T is the output average torque, ω is the angular velocity, $V(t)$ is the instantaneous phase voltage, and $I(t)$ is the instantaneous phase coil current.

The full pitch windings achieved a higher efficiency of $\eta=61.3\%$ compared to the $\eta=41.1\%$ of the short pitch windings. Since the coil current depends on the EMF, and the copper losses are proportional to the square of the coil current, it is clear that it is the copper losses that cause such a large difference in the efficiencies of the two motors.

On the other hand the iron loss of the rotor and stator cores was ignored because these cores consist of the laminated silicon steel sheets and the rotor rotates at low speed.

5. Conclusions

The operating principle and characteristics of the SPM-type vernier motor were clarified using FEM. We confirmed that the rotor is synchronized to the flux harmonics of the rotating magnetic field and operates according to the operating principle of a contactless gear.

Differences between full and short pitch windings in the stator were discussed as both a motor and a generator. Short pitch windings are inexpensive and easy to wind. However in terms of magnetic field intensity and the utilization of the magnetic flux, full pitch windings are superior. Also, when used as a motor, both windings produced the same output torque, but if we were to consider efficiency in terms of coil losses, full pitch windings are more advantageous.

In the future, 3-D FEM analysis will be conducted to take the influence of the coil ends and the estimation of the iron loss into account. In addition, a prototype will

be designed and manufactured to verify the analysis results.

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