Invitation to tiling art

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Two artistic patterns of a quasi-periodic lattice (QP) with decagonal symmetry are presented. They are built using a QP generated by the self-similar inflation method. Two patterns originate from the same inflated pattern but they are entirely different from each other. One is a quasi-periodic Escher-like pattern and the other is an illustration using a freeline of fractal growth. Since eight types of vertices exist in the decagonal QP, we can sketch an illustration like animals by tracing the edges of quasi-periodic tiles in the direction that we wish to draw an outline along the lattice line. This pattern is typical of local isomorphism.

Introduction

The properties of a quasi-periodic lattice (QP) were first discussed in detail for Penrose tiling by de Bruijn,[1] and those of QP have subsequently been studied in even greater detail. Further details of the properties of this pattern have also been studied by Watanabe and Zobetz.[2,3,4] In this review, it is shown that a quasi-periodic design can be presented using self-similarity and local isomorphism in decagonal tiling. Two applications of a QP to pictorial art have been surveyed.[5,6]

2D Matching rule for generation of QP

The base tiles which are shown as black tiles in Fig. 1 are derived from a regular decagonal zonogon. The first-generation tile (inflated tile) is presented as a self-similar assembly of base tiles. If the internal dividing point of an inflated tile along a shared edge takes an asymmetric position, for example, $1 : \tau$ ($\tau = (1+\sqrt{5})/2$) golden number, the edge is said to have a sense or polarity. The sense of the edge of the first generation shown by an arrow must be consistent with that of the base tile. Once the assembly satisfies this rule, a 2D QP is automatically generated by a self-similar operation to form a 2D QP of any generation. This is called the matching rule in a self-similar operation.

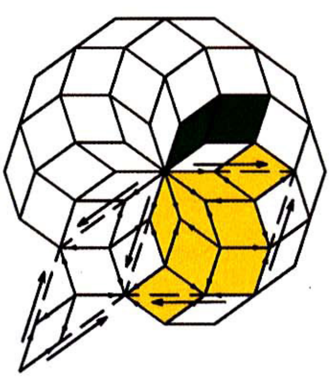

Fig. 1. Black base tiles and first-generation tiles drawn in a decagonal zonogon.

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Design using local isomorphism and matching rule

The properties of a QP are characterized in terms of local isomorphism and the matching rule. First, an application of local isomorphism to an artistic pattern is described. Local isomorphism refers to reappearance of the same pattern at short intervals. This can be demonstrated in an illustration titled "Which came first, the chicken or the egg?" drawn in the third generation of a thick rhombic tile (Fig. 2). An egg is drawn in the first-generation decagonal pattern of a thick rhombic tile. The egg hatched into two chickens which are drawn in the second-generation tile of a thick rhombus. The two chickens are similar to each other, but are not related by any symmetry operation in the second-generation pattern. The two chickens separately grow further into a hen and a cock. The cock and the hen are also similar to each other in size and shape. Both seem to occupy positions related by an approximate inversion center in the third-generation pattern, but no symmetry element can be found in the thick rhombic tile. In this illustration an egg can be found in the belly of a hen. The growth process from an egg to a cock and hen drawn with bold lines in every generation is presented by illustration in Fig. 2. Eight kinds of vertices exist (Fig. 3) which facilitate contour drawing in any direction in the thirdgeneration of the rhombic base tile. These properties of a QP are applicable to drawing of a curved line such as a sketching line.

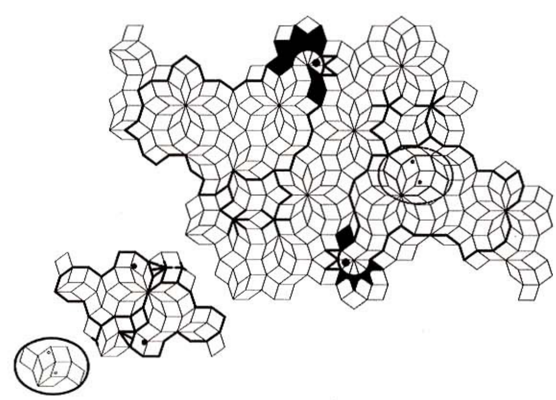

Fig. 2. Which came first, the chicken or the egg?

Second, an application of self-similarity is demonstrated. Each arrow (Fig. 1) on each tile edge is replaced by a sketch-

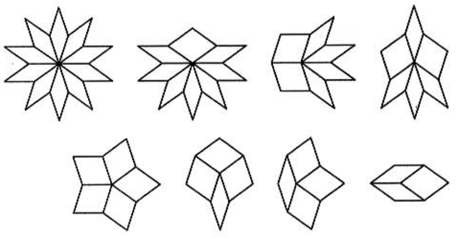

Fig. 3. Eight vertex types in decagonal QP.

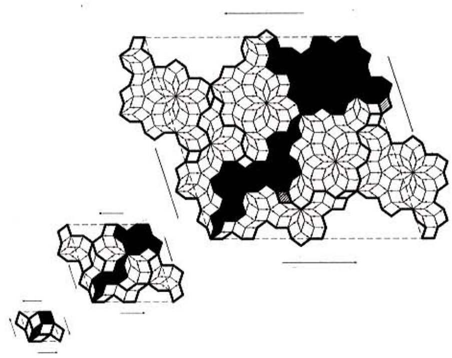

Fig. 4. Fractal growth of zigzag domain in thick rhombus tile.

ing line so as to generate a pattern like an animal; that is, the algorithm used to draw the sketching lines follows the matching rule determined by the arrow notation on the edges of the QP. This illustration using a QP is essentially different from Escher's pattern. While Escher's pattern is periodic, this one is aperiodic or quasi-periodic. We tried to draw a QP with the outline of several dogs and succeeded in inflation of the dogs by iteration of a self-similar operation. The procedure is as follows. First, consider the shaded area of the first-generation thick rhombic tile in Fig. 1. This yellow shaded area in Fig. 1 is found in a second-generation tiles $A^{(2)}$, shown as the black tile or tiles surrounded by bold line in Fig. 4. We can recognize fractal growth of the outline of the black tile in the third-generation in Fig. 4. At this stage we can imagine something like an animal pattern from the black zigzag pattern. For a thin rhombic tile, the situation of fractal growth is the same as that for a thick tile. The problem is how to generate an outline in the shape of a dog. In order to make up the dog image, reformation of the fractal domain is required. The reformation is carried out by adjusting the neighboring tiles. For example, the zigzag pattern of thin tiles which corresponds to the second-generation tile $B^{(2)}$ gives a shaded thin base tile $B^{(0)}$ to $A^{(2)}$ and takes a thick base tile $A^{(0)}$ from neighboring $A^{(2)}$.

As a result of the adjustment, a new matching rule is necessary; that is, the number of arrows increases from one to three (single, double and triple arrows). The three sketching lines (s_1, s_2, s_3) in Fig. 5 corresponding to the three arrows play an important role in the self-similar dog illustration. The adjustment has the effect of splitting an A tile into three tiles $A_1^{(3)}$, $A_2^{(3)}$ and $A_3^{(3)}$. In this stage, four third-generation tiles $A_1^{(3)}$, $A_2^{(3)}$, $A_3(3)$ and $B^{(3)}$ (Fig. 6), are redefined as new base tiles having the outline of a dog $A_1^{(0)}$, $A_2^{(0)}$, $A_3^{(0)}$ and $B^{(0)}$ (Fig. 7).

It is proved that a new matching rule is necessary due to

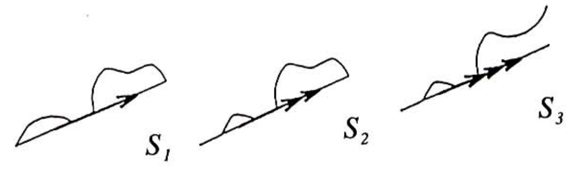

Fig. 5. Three sketching lines corresponding to three arrows.

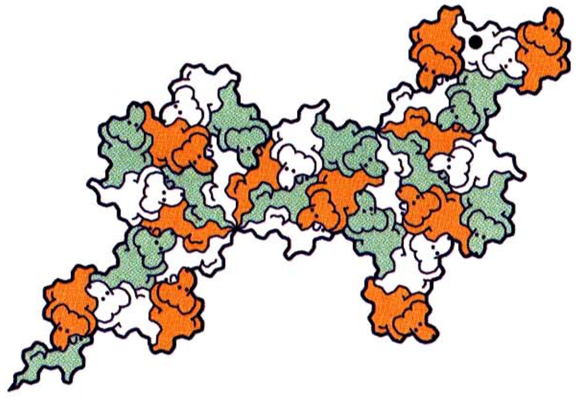

Fig. 6. Details of assembling base tiles: "Standing dog" $B^{(3)}$.

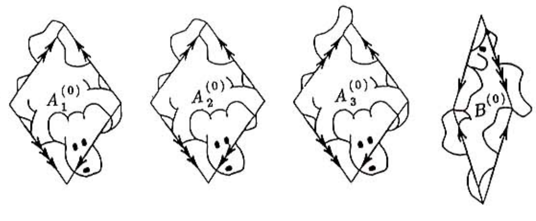

Fig. 7. Four new base tiles.

the adjustment effect. Each new base tile can be obtained by replacing the three arrows by sketching lines s_1, s_2 and s_3. Thus, inflation of four dog patterns is completed and any generation of a new dog illustration can be obtained using the new matching rule.

Concluding remarks

In this review we presented an easy method of drawing artistic illustrations using the matching rule of self-similarity in a decagonal QP. The coding of the pattern generation program was developed by Professor. T. Soma of South Pacific University. This work is partly supported by Special Coordination Funds for Promoting Science and Technology Agency of the Government of Japan.

References

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