Ground state properties of unstable nuclei in the relativistic mean field framework

Kohsuke Sumiyoshi, Daisy Hirata,[*1] Satoru Sugimoto,[*2] Yuichi Sugahara,[*3]
Isao Tanihata,[*4] and Hiroshi Toki[*2]
Computational Science Laboratory

We make systematic studies of ground-state properties of unstable nuclei up to the proton and neutron drip lines in the nuclear chart in the relativistic mean field (RMF) framework taking account of the axial and triaxial deformations. We perform the RMF calculations of about 2000 even-even nuclei from $Z = 8$ to $Z = 120$ up to the drip lines with the axial symmetry and series of isotopes with the triaxial symmetry. We explore the global trend of nuclear properties such as masses, radii and deformations in the whole region of the nuclear chart. Appearance of axial and triaxial deformations in the nuclear chart is systematically examined.

Introduction

Study of unstable nuclei is fascinating since the discovery of exotic structures of nuclei far away from the stability line. The recent advance at the radioactive nuclear beam facilities has revealed novel phenomena such as neutron halos and neutron skins in light mass nuclei close to the neutron drip line and is bringing new findings in the frontier region toward the drip lines in the nuclear chart.[1)] Planned radioactive nuclear beam facilities are going to provide us with the information of more than a few thousand of nuclei in the nuclear chart. We are having the experimental data such as masses, radii and deformations of nuclei in a wider region of the nuclear chart than ever before. Having such experimental efforts, we are eager to exploit general feature of the nuclear properties in the whole region of the nuclear chart. It is extremely interesting to explore various questions on the nuclear chart; where new exotic phenomena, magic numbers and deformations appear and disappear.

From a theoretical point of view, it is desirable to explore the properties of all nuclei in the nuclear chart starting from nuclear many body framework. This seems a formidable task requiring huge computation, however, most recent supercomputers enable us to perform systematic study of many thousands of nuclei within many body framework to find out the global trend in the nuclear chart. To proceed this big task, we need to choose carefully a reliable framework based on the microscopic aspect of nuclear many body theories.

Recently, there has been a great progress in the relativistic many body framework.[2)] It has been shown[3)] that the relativistic Brückner Hartree Fock (RBHF) theory is capable of reproducing the saturation properties of nuclear matter starting from the nucleon-nucleon interaction determined by the scattering experiments. Strong density-dependent repulsion appears due to relativistic many body treatment and contributes automatically to the reproduction of nuclear saturation, which, in non-relativistic nuclear many body frameworks, is only attained by the introduction of extra three-body forces. Stable finite nuclei has been also satisfactorily reproduced using the RBHF theory in terms of local density approximation. Although it would be nice if we could work out all finite nuclei in the RBHF theory, this is still formidable task even with the current computing power. Therefore, we adopt the RMF framework, which is phenomenological version of the RBHF theory, to perform the global calculation of many nuclei in the nuclear chart.

The RMF framework has been successfully applied to the description of properties of nuclei as well as high energy proton-nucleus scattering.[2)] It has been shown that the RMF framework reproduces the properties of stable nuclei in a wide mass range of the periodic table.[4)] It is also remarkable that the RMF framework reproduces the properties of nuclei far away from stability as well as stable ones. The RMF framework has been also successfully applied to the study of deformed nuclei to describe the deformation as well as other properties both for stable and unstable nuclei.

We note that the RMF framework is also applicable to astrophysical problems such as the equation of state for neutron stars and supernovae. The relativistic treatment is essential to satisfy the causality condition of dense matter, which is the condition that the sound velocity should not exceed the light velocity and is often broken in non-relativistic nuclear many body frameworks. Global study of nuclei up to the drip lines in the RMF framework also provides us with the knowledge on neutron-rich nuclei for r-process nucleosynthesis.[5)] The consistent treatment of nuclei and equation of state within the same RMF framework enables us to probe the properties of dense matter in neutron stars and supernovae.

Adopting the RMF framework based on the RBHF theory, we make systematic study of even-even nuclei up to the drip lines taking account of deformation. In order to clarify the systematics of the appearance of the deformation of nuclei together with the behavior of other properties such as masses, charge radii and neutron skins, we perform RMF calculations with axial deformation for about 2000 even-even nuclei from oxygen isotopes to superheavy isotopes up to the model proton and neutron drip lines. We perform also RMF calculations with triaxial deformation for even-even nuclei in several re-

*1 Japan Synchrotron Radiation Research Institute (JASRI)
*2 RCNP, Osaka University
*3 Genome Science Laboratory, RIKEN
*4 Linear Accelerator Laboratory, RIKEN

gions, where the shape coexistence and the shape transition in axial symmetry occur, in order to clarify whether another type of deformation appears.

Relativistic mean field framework

In the RMF framework,[2] the system of nucleons is described by fields of mesons and nucleons under the mean field approximation. Strong interaction between nucleons is mediated by mesons, which are created in turn by the existence of nucleons. We start with the effective lagrangian, which is relativistically covariant, composed of meson and nucleon fields. We adopt the lagrangian with the non-linear σ and ω terms.[6] On top of the Walecka's σ - ω model with isovector-vector ρ meson, the non-linear terms of σ meson is introduced to reproduce the properties of nuclei quantitatively and a reasonable value for the incompressibility. The inclusion of the non-linear term of ω meson is motivated by the recent success of the relativistic Brueckner Hartree Fock theory.[3,6] Deriving the Euler-Lagrange equations from the lagrangian under the mean field approximation, we obtain a set of the Dirac equation for nucleons and the Klein-Gordon equations for mesons. The self-consistent Dirac equations and Klein-Gordon equations are solved by expanding the fields in terms of the harmonic oscillator wave functions.[4]

The RMF model contains the meson masses, the meson-nucleon coupling constants and the meson self-coupling constants as free parameters. We adopt the parameter set TMA, which was determined by fitting the experimental data of masses and charge radii of nuclei in a wide mass range.[7,8] A novel feature of this parameter set is its mass dependence so as to reproduce nuclear properties quantitatively from the light mass region to the superheavy region.

In order to take account of the deformation, the fields are expanded in terms of the eigenfunctions of a deformed harmonic oscillator potential.[4,9] Axially and triaxially deformed harmonic oscillator potentials are used for axial and triaxial RMF calculations, respectively. The solution of the Dirac equation, which is a main computing load, reduces to the eigenvalue problem of hermitian matrix. Due to the increase of the size of matrices according to the increase of the number of basis eigenfunctions, the computation becomes more expensive as the assumption of axial symmetry is extended to triaxial symmetry and the mass number of nuclei increases. We perform the constrained calculations on the quadrupole moments of the nucleon distribution to survey the coexistence of multiple shapes. We use the quadratic constraint to get full part of the energy surface due to the deformation.[9] This requires RMF calculations at many points in the parameter space of deformation and makes the whole computation even more expensive.

In the present calculation, we drop the pairing correlation, which causes serious problems for nuclei close to the neutron drip line. Although we have to stress that the pairing correlation plays an important role for the nuclear deformation, we regard that the present study aims to explore the general trend up to the drip lines as a first trial in the relativistic many body framework. We would need further investigations in order to incorporate the pairing correlation in the relativistic many body framework in a consistent way. It is challenging to examine the whole nuclides having such a framework in near future.

Numerical results

We perform the RMF calculation with axial deformation of 2174 even-even nuclei ranging from $Z = 8$ to $Z = 120$ up to the proton and neutron drip lines. We extract the nuclear properties of each nuclide for all possible equilibrium deformations from the energy curves obtained by the constrained calculations. We then perform non-constrained calculations starting from the deformation parameters of all selected minima as an initial guess and obtain the physical quantities as final values. We tabulate the quantities thus obtained for all minima of each nuclide. To discuss the general trend of the table of nuclear properties, we select a solution which has the deepest (absolute) minimum among the multiple solutions. More details of the table and its analysis will be published elsewhere.[8]

The rms deviation of the masses from experimental data turns out to be 2.22 MeV. The average of the deviation is 0.61 MeV. Besides the overall underbinding due to the lack of the pairing correlation, the agreement with experimental data is fairly good taking account of the fact that the masses are calculated by the self-consistent many body framework starting from the effective lagrangian. We note that the number of free parameters in the present framework is 14. The proton and neutron drip lines are found very close to the ones of the mass formula by Tachibana et al.[5] The neutron skin can be seen generally in the neutron rich region and very thick neutron skin up to 1 fm appears for nuclei along the neutron drip line. Proton skin can be seen for light mass nuclei along the proton drip line.

One of the most interesting quantities to discuss is on the deformation. Figure 1 shows the calculated deformation parameter, β_{2p}, in the nuclear chart. Patterns of prolate and oblate region can be seen generally. The characteristic of the neutron magic number is preserved having spherical shapes except for the neutron rich nuclei with $N = 20$, 28, 50 where the deformed shapes appear. The proton magic numbers $Z = 50$, 82 lose their characteristic in the regions away from the neutron magic numbers. Several lines along $N = 70$ close to the neutron drip lines, $N = 138$, 184 around $Z = 82$, and $Z = 58$, 92 around $N = 82$, 126, 184 can be seen suggesting magicness. It is interesting to see that there is a strong deformed region

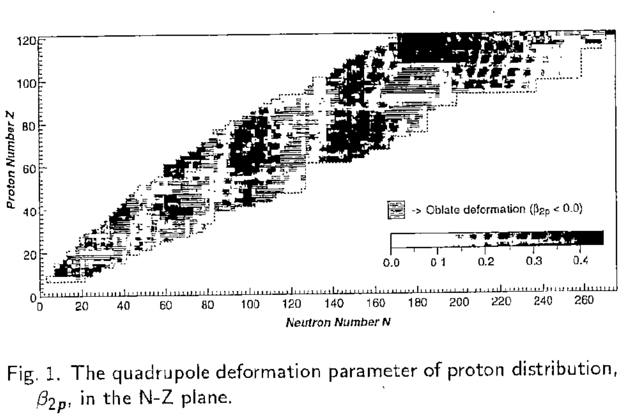

Fig. 1. The quadrupole deformation parameter of proton distribution, β_{2p}, in the N-Z plane.

around $Z = 38$, 40, 42 with $N = 38$, 60, which are suggested to be deformed magic numbers by the experiments.[1]

Through the global study under the axial symmetry, we have found that many nuclei have solutions of both prolate and oblate deformations with similar binding energies. In order to examine whether another type of deformation beyond the axial symmetry occurs, we perform the RMF calculation with triaxial symmetry by increasing the degree of freedom for deformation. We perform the constrained calculations to obtain the energy surface as a function of the parameters of β and γ deformations. Since the calculation is more expensive than that with the axial symmetry, we make systematic calculations of nuclei in selected regions suggested by the result of the axial RMF calculations.

One example of the systematic study of the triaxial deformation is on neutron-rich S isotopes.[9] Series of even mass nuclei, $^{42-56}$S, have been found to have both prolate and oblate deformations with similar binding energies in the RMF calculation with the axial symmetry. This result itself does not clarify which shape is realized as the ground state configuration or whether another type of deformation appears during the shape transition. The RMF calculation with the triaxial symmetry have revealed the shape change from prolate to oblate through triaxial deformation along the neutron-rich S isotopes.

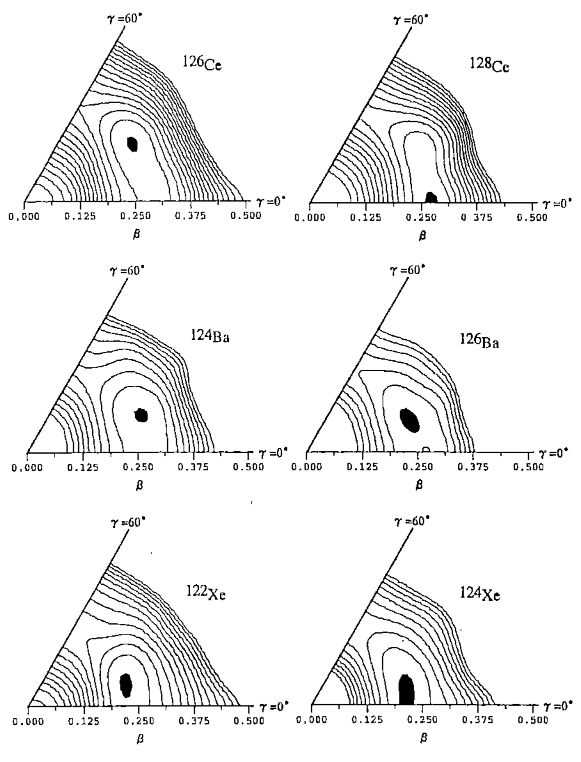

Fig. 2. The energy surfaces of nuclei in the Xe region in the β-γ plane.

Another example is the systematic study of even-even nuclei in neutron-deficient Xe region, where possibilities to have triaxial deformation have been suggested by experimental data, non-relativistic calculations and the axial RMF calculations.[10] We perform the triaxial RMF calculations of about 30 even-even nuclei around ^{122}Xe in the region of $Z = 50 - 58$ and $N = 64 - 72$. Figure 2 displays the energy surfaces of some nuclei in the Xe region as functions of the deformation parameters, β and γ. It is remarkable that the triaxial deformation appears in many nuclei in this region. Systematic calculations around this region suggest $N = 68$ and $N = 70$ are candidates for the magic numbers of triaxial deformation. Further studies with the RMF framework on the appearance of the triaxial deformation in the nuclear chart is currently being made.

Summary

We have systematically studied even-even nuclei in the nuclear chart by the RMF framework with deformations. We have performed the calculations of all even-even nuclei up to the drip lines from light mass to superheavy region under the axial symmetry assumption. The agreement of calculated masses with experiments is found successful and the global tendency of the appearance of neutron skins and deformations has been found. The result of the axial RMF calculation suggests possible appearance of triaxial deformation in many regions having the coexistence of prolate and oblate shapes. We have done the triaxial RMF calculations of even-even nuclei in the selected region; series of neutron-rich S isotopes and many nuclei around neutron-deficient Xe. The shape transition along S isotope from prolate to oblate via triaxial and the richness of appearance of triaxiality in the Xe region have been found through systematic calculations.

K. S. would like to express special thanks to the Computing Facility of RIKEN for a special allocation of computing time of VPP500/30.

References

1) J. H. Hamilton: *Treatise on Heavy Ion Science*, edited by D. A. Bromley (Plenum, New York, 1989), Vol. 8, p. 3.
2) B. D. Serot and J. D. Walecka: Adv. Nucl. Phys. **16**, 1 (1986).
3) R. Brockmann and R. Machleidt: Phys. Rev. C **42**, 1965 (1990).
4) Y. K. Gambhir, P. Ring, and A. Thimet: Ann. Phys. **198**, 132 (1990).
5) D. Hirata, K. Sumiyoshi, I. Tanihata, Y. Sugahara, T. Tachibana, and H. Toki: Nucl. Phys. A **616**, 438 (1997).
6) Y. Sugahara and H. Toki: Nucl. Phys. A **579**, 557 (1994).
7) H. Toki, D. Hirata, Y. Sugahara, K. Sumiyoshi, and I. Tanihata: Nucl. Phys. A **588**, 357 (1995).
8) D. Hirata, K. Sumiyoshi, I. Tanihata, Y. Sugahara, and H. Toki: submitted to At. Data Nucl. Data Tables.
9) D. Hirata, K. Sumiyoshi, B. V. Carlson, H. Toki, and I. Tanihata: Nucl. Phys. A **609**, 131 (1996).
10) D. Hirata, H. Toki, and I. Tanihata: Nucl. Phys. A **589**, 239 (1995).