

Two-particle wave function on lattice in scalar QED

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Main purpose of this study is to develop the method to identify the bound state on the lattice. The identification of the bound state is quite difficult in finite volume in the case of a very small binding energy. This is because the two-particle scattering state associated with attractive interactions in finite volume has negative energy shift due to the two-particle interaction. The ground state energy E of the attractive scattering state in finite volume is smaller than the two-particle threshold, $E < 2M$, as if it were the bound state, where M is the mass of the single particle. In this context, it is difficult to distinguish the bound state to the ground state in the finite volume system.

In the previous work we proposed a method [1] where the energy level of the second lowest state in the scattering system is determined with particular interest, and demonstrated the method works well in actual numerical simulations with the scalar QED. In this study we propose alternative method using the two-particle wave function in finite volume.

The two-particle wave function has useful information of the scattering state [2], *e.g.*, the scattering length and phase shift, the effective scattering potential. In finite volume the wave function is essentially described in terms of two specific variables [3], the relative coordinate of the two particles \vec{r} and the normalized relative momentum $q^2 = (pL/2\pi)^2$ where $p^2 = E^2/4 - M^2$, $\phi(\vec{r}) = C \sum_{\vec{n} \in Z^3} e^{i\vec{r}\cdot\vec{n}(2\pi/L)} / (\vec{n}^2 - q^2)$, where C is a normalization constant. Using the equation, it is easy to see that there is large difference in the wave functions for the bound and scattering states on large volume. The wave function of the scattering state behaves as a constant in the large r region, while the one of the bound state behaves as $e^{-\gamma r}$. Such difference should be useful for the identification of the bound state.

We calculate the wave function of the fermion-antifermion state in the scalar QED with the different charge of the fermion, $Q = 3, 4$, corresponding to the system without and with a bound state. We choose the simulation parameters in such a way that the binding energy is very small, $\Delta E/2M = 0.01$. In order to investigate the volume dependence of the wave function, the two volumes, $L = 16, 32$, are used. The simulations are carried out SX-8 at RCNP, Osaka University.

On the smaller volume, the wave functions in both the systems reveals similar r dependence in the large r region as shown in the left panel of Fig. 1, so that it is hard to identify the bound state by the wave function. On the larger volume, however, there is a clear difference in the wave functions for the bound and scattering states shown in the right panel of Fig. 1. The wave function of the bound state exhibits the localized behavior with an exponential fall-off in the large r region as expected. Thus, the proposed method in this study becomes definitely effective for the identification of a bound state on relatively large volume.

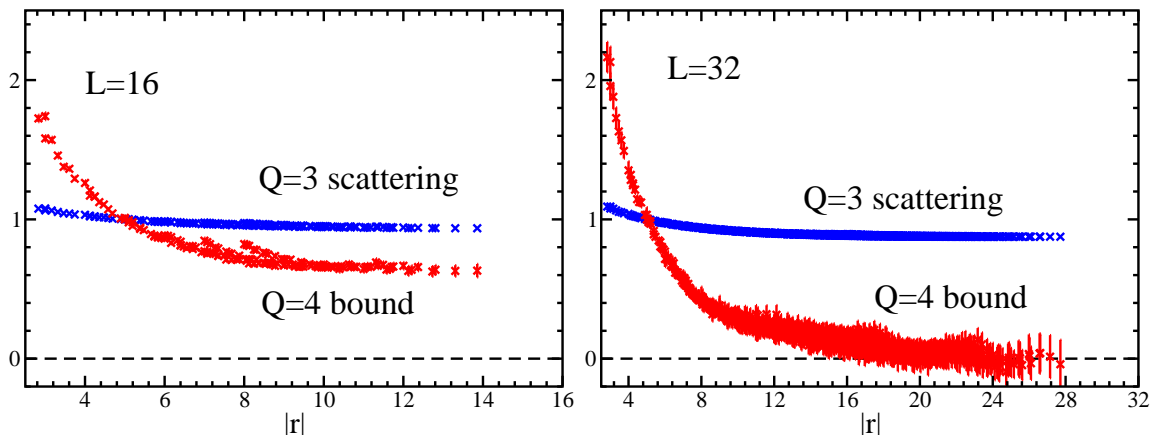


Figure 1: Left(right) panel is two-particle wave functions of the ground state in $Q = 3, 4$ cases on $L=16(32)$.

References

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