Systematic study of triaxial deformation in the relativistic mean field theory

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We study the change of deformation of even-even nuclei in the proton-rich Xe region in the relativistic mean field theory (RMF). We investigate the appearance of triaxial deformation in 25 nuclei in the region covering $Z = 50$–58 and $N = 64$–72 by performing constrained, triaxially symmetric RMF calculations of their energy surfaces. We include the pairing correlation using the BCS formalism.

The recent progress of radioactive nuclear beam facilities has provided us with marvelous findings in nuclear physics. Exotic structures such as neutron halos and neutron skins have been found in experimental studies of light unstable nuclei in the neutron-rich region. Much new information on the shapes and structures of nuclei far from stability is being revealed by the systematic measurement of radii and moments of unstable nuclei. Planned facilities in the world will access a large number of unstable nuclei in the whole region of the nuclear chart and enable us to explore where and how exotic phenomena of nuclear structure appear in the region far from the stability line. One of the great interests is to know where the deformation of unstable nuclei appears and how the shape of these nuclei changes along the isotopic and isotonic chains. Thus, we have chosen to explore the proton-rich Xe region for the appearance of triaxial deformation. We have made a systematic study of 25 even-even nuclei covering $Z = 50$–58 and $N = 64$–72, using the RMF theory with triaxial deformation, in order to clarify how their shapes change as a function of N and Z in this region. We have calculated the energy surface of those nuclei as a function of the deformation parameters, β and γ, to explore the ground state deformation. This region has been discussed as a possible region for triaxial deformation in studies using conventional frameworks. There have also been experimental efforts to measure excitation energies in order to study the collective nature of the nuclei in this region.

We briefly describe the framework of the RMF theory and the procedure of the calculation.[1–3] In the RMF theory, the system of nucleons is described by fields of mesons and nucleons under the mean field approximation. We start with the effective lagrangian, which is relativistically covariant, composed of meson and nucleon fields. We adopt a lagrangian with non-linear σ and ω terms,

$$\begin{aligned}\mathcal{L}_{\rm RMF} = \ & \bar\psi\left[i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu\right.\\ & \left. -g_\rho\gamma_\mu\tau_a\rho^{a\mu} - e\gamma_\mu\frac{1-\tau_3}{2}A^\mu\right]\psi\\ & +\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4\\ & -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3\left(\omega_\mu\omega^\mu\right)^2\\ & -\frac{1}{4}R_{\mu\nu}^a R^{a\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu^a\rho^{a\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \end{aligned} \qquad (1)$$

where the notation follows the standard one. On top of the Walecka σ-ω model with photons and isovector-vector ρ mesons, non-linear σ meson terms are introduced to reproduce the properties of nuclei quantitatively and give a reasonable value for the incompressibility. The inclusion of the non-linear term of ω meson is motivated by the recent success of the relativistic Brueckner-Hartree-Fock theory. Deriving the Euler-Lagrange equations from the lagrangian under the mean field approximation, we obtain the Dirac equation for the nucleons and Klein-Gordon equations for the mesons. The self-consistent Dirac equation and Klein-Gordon equations are solved by expanding the fields in terms of harmonic-oscillator wave functions.[1,3]

The RMF model contains the meson masses, the meson-nucleon coupling constants and the meson self-coupling constants as free parameters. We adopt the parameter set TMA, which was determined by fitting the experimental data of masses and charge radii of nuclei in a wide mass range.[2] We remark that this parameter set has a mass dependence so as to reproduce nuclear properties quantitatively from the light mass region to the superheavy region. With the TMA parameter set, the symmetry energy is 30.68 MeV and the incompressibility is 318 MeV. Note that the bulk properties of nuclear matter at saturation with the parameter set TMA is calculated for uniform matter in the limit of infinite mass number.

The TMA parameter set has been used for the systematic study of all even-even nuclei up to the drip lines in the nuclear chart within the RMF framework under the assumption of axial symmetry.[4] It has been shown that the overall agreement of the calculated results using TMA with the experimental data of masses and charge radii is excellent and is found to be much better than the results of spherical RMF calculations with TMA.

In order to take into account triaxial deformation, the fields are expanded in terms of the eigenfunctions of a triaxially deformed harmonic oscillator potential.[1] We perform calculations that constrain the quadrupole moments of the nucleon distribution, in order to survey the coexistence of multiple shapes and to identify the ground state deformation. We use a quadratic constraint to calculate a complete map of the energy surface as a function of the deformation.[1,5]

We shall show the result. Figure 1 displays the energy surfaces

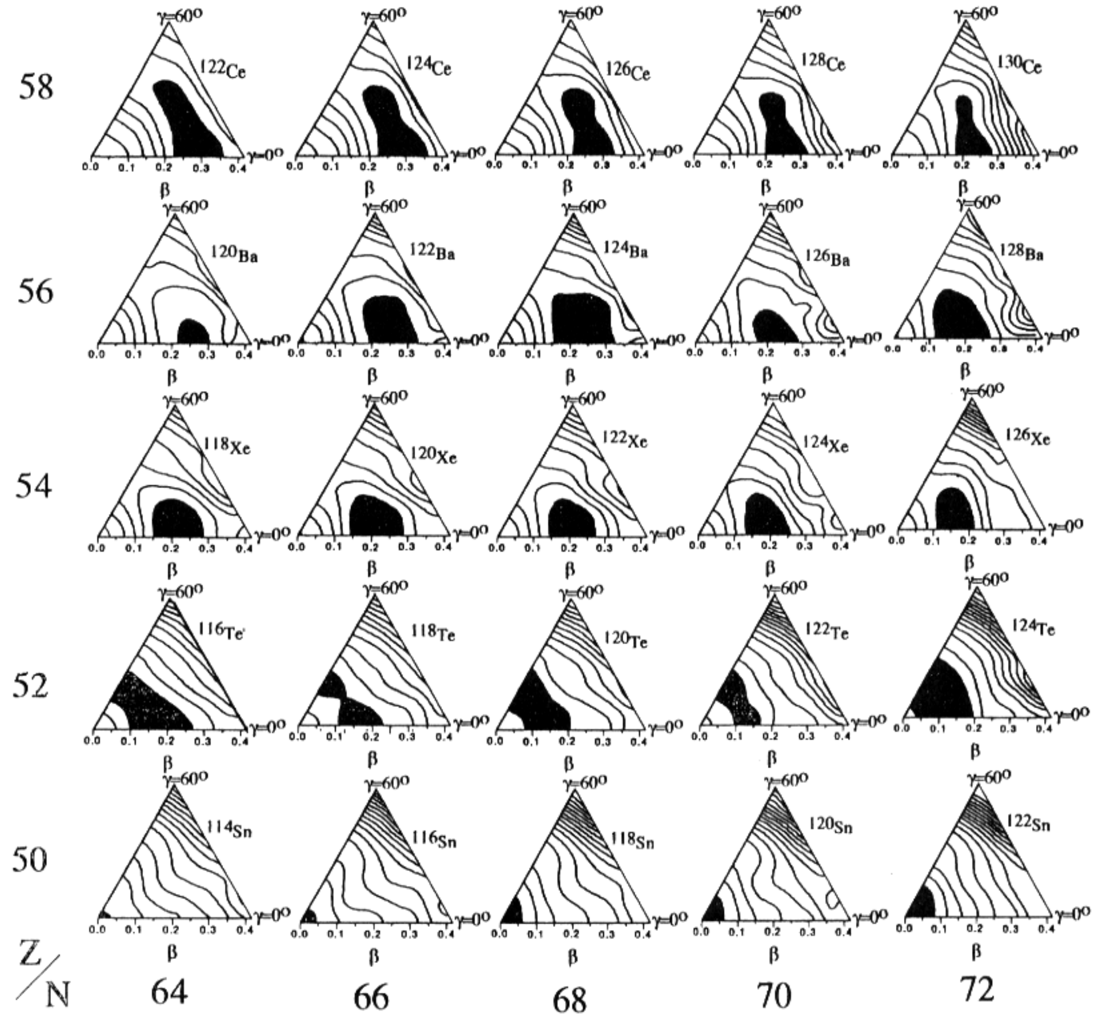

Fig. 1. The energy surface in the plane of deformation parameters, β and γ, calculated in the RMF theory with triaxial deformation for nuclei in the range of Z = 50–58 and N = 64–72, arranged in the form of the nuclear chart. The energy difference between the contours is 1 MeV in total energy. The energy minimum is marked by the black region, in which the energy difference is less than 1 MeV.

of the Sn, Te, Xe, Ba, Ce ($Z = 50$–58) isotopes with $N = 64$–72, arranged in the form of the nuclear chart. The spacing of contours is 1 MeV in total energy in all figures. The energy minimum is marked by the black region, in which the energy difference is less than 1 MeV from its absolute minimum energy. From this figure, we can see that the Sn isotopes are spherical and the Te isotopes are very gamma unstable with shallow minima around $\gamma = 60°$. Adding more protons, the Xe , Ba and Ce isotopes have prolate deformations with their size increasing with proton number.

References

1) D. Hirata et al.: Nucl. Phys. A **609**, 131 (1996).
2) Y. Sugahara: Doctor Thesis of Tokyo Metropolitan University (1995).
3) Y. K. Gambhir et al.: Ann. Phys. **198**, 132 (1990).
4) D. Hirata et al.: Nucl. Phys. A **616**, 438c (1997).
5) H. Flocard et al.: Nucl. Phys. A **203**, 433 (1973).