The nonlocal operator expansion for Drell-Yan process

Kazuhiro Tanaka
Department of Physics, Juntendo University

We develop an operator formulation for the Drell-Yan production of lepton pair in hadron-hadron collisions. We employ the generalized Wilson operator expansion where the main ingredients are the Keldysh formalism and the gauge-invariant nonlocal operators. Compared to conventional approaches based on the collinear expansion of the relevant diagrams, our approach has an advantage of preserving explicitly Lorentz and gauge invariance, and thus provides a convenient framework for systematic treatment of the Drell-Yan processes. Utilizing the background field method, we derive the coefficient functions at tree level in the generalized operator expansion, which are relevant to the asymmetry observables through order $1/Q$ (twist-3) for the case with polarized beams. We emphasize novel role played by the "color octet terms", which are generated by the Fierz rearrangement for the quark fields in the electromagnetic currents. We demonstrate that at higher twist the quark-antiquark color octet terms produce "color singlet terms" due to quark-antiquark-gluon correlation.

1. Introduction

Among hard processes, the deep inelastic lepton-hadron scattering (DIS) and the Drell-Yan process are the most familiar ones. Both processes are inclusive for hadrons in the final state; in the Drell-Yan process one observes a lepton pair with large invariant mass, which is via a high-mass virtual photon produced through hadron-hadron collisions.[1)] The Drell-Yan process yields complementary information to that revealed in the DIS, and plays a distinguished role in spin physics. By the use of polarized beams in *e.g.* RHIC, the Drell-Yan process allows us to open a new window to explore the various spin-dependent parton distributions in the nucleon.[2–6)]

The basis for the theoretical analysis of the hard processes is provided by the factorization theorems in QCD.[7)] Many processes are now known, to which the application of the factorization formalism is generally accepted. They include several inclusive reactions, and the above two processes constitute classic examples where the QCD factorization formulae have been successfully applied. In particular, for the DIS, the accurate theoretical analysis is available based on the operator product expansion (OPE), which provides a rigorous approach for the separation of the short-distance dynamics from the long-distance one.

For the Drell-Yan process, direct formulation based on the OPE is absent. The factorization theorem has been proved by indirect methods,[7–9)] where the theoretical analysis is considered to be less rigorous than that for the DIS. Furthermore, the absence of the operator methods makes it difficult to unravel higher-twist corrections that go as the inverse powers of the characteristic large momenta.

In this work we develop an operator formulation for the Drell-Yan process. We derive the generalized OPE including twist-3 effects for the case of polarized incoming hadrons. Our operator approach has an advantage of preserving Lorentz and gauge invariance, and is convenient for systematic treatment of the higher-twist corrections.

2. Factorization theorem and the space-time picture

In this section we recall the factorization theorem and the relevant space-time picture for the Drell-Yan process. We also discuss the reason why direct operator formalism has been absent from this process.

Let us consider the Drell-Yan process between two hadrons A and B with momenta P_A and P_B, producing a lepton pair with total momentum q_μ. The relevant kinematic invariants are $s \equiv (P_A + P_B)^2$ and $Q^2 \equiv q^2$. The Bjorken limit requires all these invariants to be very large and comparable, *i.e.*, we let Q^2 and s very large, while $\tau = Q^2/s$ remains fixed.* Then the factorization theorem tells us[7)] that the following picture ("parton model") holds for $Q^2 \to \infty$: we can view a high-energy beam of hadrons as if it were a beam of partons, and the hadronic collision is induced by the annihilation of two partons—say a quark and antiquark pair, one from each hadron—into the hard virtual photon. This leads to the well-known factorization formula of the Drell-Yan cross section, and is illustrated in Fig. 1. The corresponding cut Feynman diagram is dominated by the contribution factorized into short- and long-distance parts; other complicated contributions are suppressed by the powers of $1/Q$. The theorem has been proved based on the all-orders analysis of the cut diagrams in QCD perturbation theory.[7,8)] For the unpolarized Drell-Yan, the analysis has been extended to the first nonleading twist which gives $1/Q^2$ corrections.[9)]

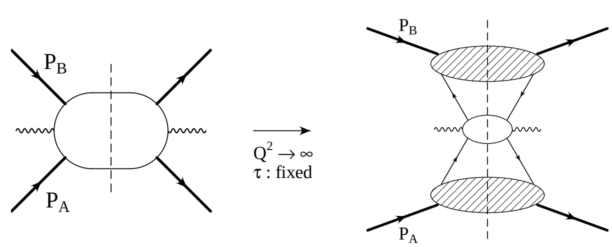

Fig. 1. The factorization theorem for the Drell-Yan process. Wavy lines denote virtual photons, which are reversed in direction for lack of space.

* The transverse momentum $q_\perp$ of the lepton pair should be integrated over or be of order Q.

The quark-antiquark annihilation into the virtual photon is a localized process with a size of order $1/Q$. Correspondingly, in principle, the long-distance part connected to the annihilation vertices could be given by matrix element of local operators. However, the participation of the momenta P_A, P_B of the initial hadrons makes the annihilation process sensitive to these momenta; as a consequence of the uncertainty principle, the annihilation vertices are no longer localized in the coordinates conjugate to P_A and P_B. For example, when P_A^+ and P_B^- are the dominant components of these respective momenta, the long-distance part is given by nonlocal operators along "−" and "+" directions. This logic is similar to that leading to the light-cone dominance in the DIS, but the dominance of the *two* light-like directions in the present case requires more sophisticated analysis. In particular, as in Fig. 1, the long-distance part has to be expressed by the *two* parton distribution functions, one for each hadron (light-like direction), in order to ensure the universality of the long-distance part as well as the factorization itself (see below).

The inclusiveness of the Drell-Yan process allows us to write the cross section in terms of leptonic ($L_{\mu\nu}$) and hadronic ($W_{\mu\nu}$) tensor, $d\sigma \sim L_{\mu\nu}W^{\mu\nu}$, and $W^{\mu\nu}$ is expressed with the product of the two electromagnetic currents $J^\mu(x)J^\nu(0)$ as[1,2)]

$$W^{\mu\nu} = \int d^4x e^{-iq\cdot x}\, {}_{\rm in}\langle P_A, P_B|J^\mu(x)J^\nu(0)|P_A, P_B\rangle_{\rm in}\ , \quad (1)$$

where $|P_A, P_B\rangle_{\rm in}$ denotes the in-state of the two hadrons. We recognize that up to this point the general treatment is formally similar to that in the DIS. Then, why the factorization formalism based on the OPE is absent from the Drell-Yan process?

In contrast to the DIS, the hadronic tensor (1) cannot be related to the T-product of currents, because the T-product has to be sandwiched between the in- and out-states. Thus it is not straightforward to construct convenient operator formulation, which allows Feynman diagram calculation of the short-distance contribution (coefficient function). Still, one might consider the possibility to apply the OPE directly to the current product of Eq. (1) as $J^\mu(x)J^\nu(0) = \sum_n C_n^{\mu\nu}(x)O_n(0)$, where $O_n(0)$ denotes a series of local operators and $C_n^{\mu\nu}(x)$ is the corresponding coefficient function; on the analogy of the DIS, the relevant OPE will be the light-cone expansion. In fact, such application of the OPE was attempted in Ref. 10 long time ago, but resulted in failure: In this case, one obtains ${}_{\rm in}\langle P_A, P_B|O_n(0)|P_A, P_B\rangle_{\rm in}$ as "long-distance part". However, this matrix element generally depends on $P_A \cdot P_B \cong s/2 \sim Q^2$, and thus the simple light-cone expansion fails to guarantee the factorization.

As mentioned above, Eq. (1) should be dominated by the contributions along the two light-like directions; the failure of the simple-minded OPE is eventually due to the fact that it is not suitable for extracting these leading contributions. A possible remedy for this point would be given by rewriting the operators O_n in the OPE in terms of pairs of color-singlet operators $O_{ni}^{(1)}$, $O_{ni}^{(2)}$ as $O_n = \sum_i c_{ni}O_{ni}^{(1)}O_{ni}^{(2)} + \cdots$, with c_{ni} some coefficients. In Eq. (1), this would give two single-hadron matrix elements of type $\langle P_A|O_{ni}^{(1)}|P_A\rangle\langle P_B|O_{ni}^{(2)}|P_B\rangle$, which depend on soft scale only. Then the final results would be expressed by the two parton distribution functions for the hadrons A and B, convoluted with the hard coefficient function, and would obey the factorization in conformity with Fig. 1. Unfortunately, however, infinitely many local operators O_n contribute to the OPE even for the leading twist; one has to perform the relevant procedure infinitely many times, which would be impossible.

Actually, these obstacles to operator formalism can be overcome by employing the generalized OPE, which has been developed for studying the DIS and the inclusive particle production in $e^+ e^-$ annihilation:[11,12)] in this generalized operator formalism, it is possible to express Eq. (1) by the "T-product of currents" and to construct the corresponding OPE having a few terms, which are expressed by the gauge-invariant nonlocal operators. Before going into the detail, we mention previous treatment of higher-twist contributions to the Drell-Yan cross section in the next section.

3. Collinear expansion approach and the higher twist effects

The $O(1/Q)$ corrections to the Drell-Yan cross section with polarized beams have been computed in Ref. 3 at the tree level for the short-distance part. The results involve both twist-2 and twist-3 parton distribution functions, and indicate that these $O(1/Q)$ contributions are measurable as leading effects for certain asymmetries.

In that calculation, the relevant cut diagrams have been evaluated through $O(1/Q)$ corrections by "collinear expansion". The leading terms from a tree diagram of Fig. 2(a) correctly reproduce the known results.[2)] On the other hand, for the $O(1/Q)$ corrections from Fig. 2(a), the situation is not simple: $q_\mu \delta W_{2a}^{\mu\nu} \neq 0$ with $\delta W_{2a}^{\mu\nu}$ the corresponding contribution to the hadronic tensor (1). It has been argued[3)] that explicit gluon effects of the type shown in Figs. 2(b) and (c) have to be added in order to recover electromagnetic gauge invariance.

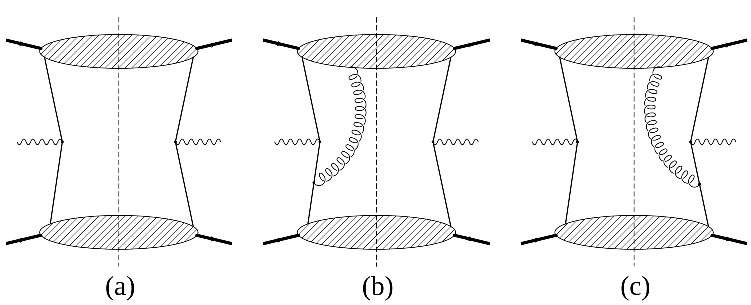

Fig. 2. Cut diagrams for polarized Drell-Yan process through $O(1/Q)$ corrections.

At a glance, Figs. 2(b) and (c) look like one-loop diagrams. However, they do not represent loop effects because the gluons are absorbed into a hadron as "long-distance fields". Thus they actually represent tree-level contributions due to quark-antiquark-gluon correlation in a hadron. In this connection, we note that the quark-gluon three-point vertices in these diagrams depend linearly on short-distance "quantum fields"; as is well known, such vertices can be eliminated in a "well-organized" perturbation expansion. In the following, we call the contributions involving such eliminable vertices, *e.g.* those from Figs. 2(b) and 2(c), the "spurious loop terms".

Also in the calculation of the $O(1/Q^2)$ corrections to the unpolarized Drell-Yan by a similar diagrammatic method, the spurious loop terms with one or two additional long-distance fields have been considered explicitly.[9] Although these diagrammatic approaches are known to give consistent results,[9] there exist some uncomfortable points: the spurious loop terms have to be explicitly taken into account as above; the collinear expansion does not preserve Lorentz as well as gauge invariance manifestly; the decomposition of the dynamical degrees of freedom into the short- and long-distance contributions is not treated in a systematic framework. As demonstrated in the next section, our operator approach makes it possible to get rid of these points.

4. Generalized operator product expansion

Now we explain the relevant techniques for our operator approach, and derive the generalized OPE including $O(1/Q)$ corrections for the polarized Drell-Yan process.

4.1 Keldysh formalism

Because the usual T-product is useless for Eq. (1), one has to treat quantities corresponding to the cut diagrams like Figs. 1 and 2 for the Drell-Yan process. A cut diagram is composed of two amplitudes—"direct" and "conjugate": a "direct" process evolves from $t=-\infty$ to $t=+\infty$ in the left of the cut, and, going over to the right of the cut, a "conjugate" process from $t=+\infty$ to $t=-\infty$. It is possible to describe the whole of these two processes as a "single process" with the time evolution over both sides of the cut by the trick of doubling the quark and gluon fields. We employ the quark fields ψ_+ (ψ_-), the gluon fields A_+^μ (A_-^μ), and the corresponding lagrangian $\mathcal{L}_+$ ($\mathcal{L}_-$) to the left (right) of the cut; $+$ and $-$ are the "Keldysh subscripts" to distinguish the fields to the left and to the right of the cut. The total system is now governed by $\mathcal{L}_{\rm tot}=\mathcal{L}_+-\mathcal{L}_-$ with the minus sign in front of $\mathcal{L}_-$ because of the complex-conjugation of $\exp(i\mathcal{L}_-)$ for conjugate amplitude. Here the "T-product" can be introduced as obvious generalization of the usual one: the "$+$ fields" are time-ordered, while all "$-$ fields" stand in front of any $+$ field and are ordered according to the inverse time. Then it is easy to see that the perturbation expansion can be constructed using this T-product, and the corresponding "Feynman rules" reproduce the Cutkosky rules for the cut diagrams.[12] Obviously, Eq. (1) can be expressed as the T-product of currents with different Keldysh subscripts:

$$W^{\mu\nu}=16\int d^4x e^{-2iq\cdot x}{}_{\rm in}\langle P_A,P_B|{\rm T}J_-^\mu(x)J_+^\nu(-x)|P_A,P_B\rangle_{\rm in}, \quad (2)$$

where the currents have been translated for later convenience. We note that the contents of the Keldysh formalism discussed in this section can be elegantly formulated by employing the "time-loop contour" in the complex time plane.[13]

4.2 The nonlocal operator expansion and the color octet term

We derive the generalized OPE for the T-product of currents in Eq. (2). The background field method in the coordinate space representation provides a systematic framework for this purpose.[11,12,14] This method is adequate because of its explicit Lorentz and gauge invariance at all steps of calculations. For simplicity, we neglect the flavor structure of quarks so that $J_\pm^\mu=\bar\psi_\pm\gamma^\mu\psi_\pm$.

First of all, we decompose the field operators into the short- and long-distance contributions. We divide all the fields into two parts—"quantum" and "classical", corresponding to the short- and long-distance contributions: $\psi_\pm=\psi_\pm^{(q)}+\psi_\pm^{(c)}$, $A_\pm^\mu=A_\pm^{(q)\mu}+A_\pm^{(c)\mu}$. The "quantum" operators $\psi_\pm^{(q)}$, $A_\pm^{(q)\mu}$ are constructed from the "modes" which are off-shell by the factorization scale Λ or more, while the "classical" background fields $\psi_\pm^{(c)}$, $A_\pm^{(c)\mu}$ have the virtualities up to Λ. We get

$$\begin{aligned}{\rm T}\,J_-^\mu(x)J_+^\nu(-x) &= {\rm T}\,\bar\psi_-^{(c)}(x)\gamma^\mu\psi_-^{(c)}(x)\bar\psi_+^{(c)}(-x)\gamma^\nu\psi_+^{(c)}(-x)\\ &\quad+(\text{quantum corr.})\,, \end{aligned}\quad (3)$$

where the first term is the classical contribution, and the "quantum corr." denotes the quantum correction terms involving the operators $\bar\psi_\pm^{(q)}$, $\psi_\pm^{(q)}$. Substituting Eq. (3) into Eq. (2) and integrating over the quantum fields $\bar\psi_\pm^{(q)}$, $\psi_\pm^{(q)}$, $A_\pm^{(q)\mu}$, we obtain the perturbative diagrams in the external classical fields $\bar\psi_\pm^{(c)}$, $\psi_\pm^{(c)}$, $A_\pm^{(c)\mu}$. In particular, the "quantum corr." generates the perturbative series for the short-distance coefficient function. In this perturbation expansion, the vertices depending linearly on the quantum fields do not appear, because the background fields $\bar\psi_\pm^{(c)}$, $\psi_\pm^{(c)}$ and $A_\pm^{(c)\mu}$ satisfy the classical equations of motion. As a result, the contributions corresponding to Figs. 2(b) and (c) (spurious loop terms) cannot come from the quantum corr. of Eq. (3), but should be contained in the classical term. In the following, we analyze the classical term by the nonlocal OPE. (The investigation of the quantum corr. will be presented elsewhere.)

To start with, we consider the consequence of the Fourier transformation of Eq. (2) in the Bjorken limit $Q^2\to\infty$, where we can choose the factorization scale Λ as $Q\gg\Lambda\gg\Lambda_{\rm QCD}$ ($\Lambda_{\rm QCD}$ is the QCD scale parameter). The Fourier transformation forces the classical term of Eq. (3) to be off-shell by $\sim Q^2$. This is naively impossible and the classical contribution to Eq. (2) would vanish, because the classical fields are off-shell by Λ^2 or less. However, it is actually possible to satisfy the requirement: Divide the classical term of Eq. (3) into the two bilocal operators, and denote the momenta carried by them as k_1 and k_2, respectively. Then $(k_1+k_2)^2$ can be of order Q^2 although $k_1^2,k_2^2\le\Lambda^2$. For example, this can be realized when the two bilocal operators are dominated by contributions along opposite tangents to the light-cone. The kinematics of the Drell-Yan process, $P_A\cdot P_B\sim Q^2$, allows such configuration of the bilocal operators to produce the significant contribution to the matrix element in Eq. (2) when the relevant momenta are collinear, *e.g.* $k_1^\mu\propto P_1^\mu$, $k_2^\mu\propto P_2^\mu$. In principle, other complicated configurations of the classical quark fields could survive the Fourier transformation; but, it is easy to see that they are suppressed by taking the matrix element because of the kinematical mismatch between the classical fields and the initial hadron state.* The present arguments in the operator language are consistent with those in the parton language,[1)] and fit nicely with the space-time picture mentioned in section 2.

Following these arguments, we pair the fields with different Keldysh subscripts in the classical term of Eq. (3), and form the bilocal operators. To perform this in a gauge-invariant manner, we write $\bar{\psi}_-(x)\gamma^\mu\psi_-(x) = \bar{\psi}_-(x)[x,\infty]_-\gamma^\mu[\infty,x]_-\psi_-(x)$ (we drop the superscript "(c)" for the classical fields here and in the following) and similarly for $\bar{\psi}_+(-x)\gamma^\nu\psi_+(-x)$, with the straight-line ordered gauge phase factors:[12)] $[x,\infty]_\pm = P\exp\left(ig\int_{\infty x_0}^1 du\, x_\mu A_\pm^\mu(ux)\right)$. Neglecting the quantum corr. and using the Grassmann algebra for the classical quark fields, we obtain for Eq. (3)

$$
\begin{aligned}
\mathrm{T}\, J_-^\mu(x)J_+^\nu(-x)
&= -\bar{\psi}_{-,i'}(x)\left([x,\infty]_-\right)_{i'i}\left([\infty,-x]_+\right)_{ll'}\psi_{+,l'}(-x)\\
&\quad\times\bar{\psi}_{+,k'}(-x)\left([-x,\infty]_+\right)_{k'k}\left([\infty,x]_-\right)_{jj'}\\
&\quad\times\psi_{-,j'}(x)\left(\gamma^\mu\mathbf{1}\right)_{ij}\left(\gamma^\nu\mathbf{1}\right)_{kl}\ , \qquad (4)
\end{aligned}
$$

where $\mathbf{1}$ is the identity matrix in color space, and we have omitted the T-sign because it is actually irrelevant for the classical fields. It is convenient to rearrange the color and the Dirac indices by the Fierz transformation. The color Fierz is given by $\mathbf{1}_{ij}\mathbf{1}_{kl} = (1/N_c)\mathbf{1}_{il}\mathbf{1}_{kj} + 2t^a_{il}t^a_{kj}$, where t^a is the color matrix for N_c color. The Dirac Fierz is

$$
\gamma^\mu_{ij}\gamma^\nu_{kl} = \frac{1}{4}\sigma^{\mu\alpha\nu\beta}\left[(\gamma_\alpha)_{il}(\gamma_\beta)_{kj} + (\gamma_\alpha\gamma_5)_{il}(\gamma_\beta\gamma_5)_{kj} + \cdots\right], \qquad (5)
$$

with $\sigma^{\mu\alpha\nu\beta} = g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}$. Here we have shown explicitly the terms which involve the "chiral-even" Dirac matrices and are also symmetric for $\mu \leftrightarrow \nu$ (note that the relevant leptonic tensor is symmetric, $L_{\mu\nu} = L_{\nu\mu}$). Now we consider the contribution to Eq. (4) due to the axial vector term of Eq. (5) in detail. This reads

$$
\begin{aligned}
-\frac{\sigma^{\mu\alpha\nu\beta}}{4}\Bigg[&\frac{1}{N_c}\bar{\psi}(x)\gamma_\alpha\gamma_5\psi(-x)\bar{\psi}(-x)\gamma_\beta\gamma_5\psi(x)\\
&+2\bar{\psi}(x)\gamma_\alpha\gamma_5 t^a\psi(-x)\bar{\psi}(-x)\gamma_\beta\gamma_5 t^a\psi(x)\Bigg]\ , \qquad (6)
\end{aligned}
$$

where we have omitted the obvious Keldysh subscripts. The gauge phase factors have been also suppressed for simplicity. According to the color structure of Eq. (6), we call the first term the "color singlet" term, while the second term the "color octet" term.

In taking matrix element, the two bilocal operators in each term of Eq. (6) would act independently in different hadrons. At this stage, our exact formula is formally similar to that corresponding to Fig. 2(a) in the diagrammatic approach,[2,3)] except for the color octet term. As was assumed in the previous works, one would consider that the matrix element of the color octet term would vanish. We argue that this is actually correct only for the leading twist: at higher twist, the additional "color singlet" term is *hidden* in the color octet term.

To demonstrate this, we need the twist expansion of the color octet term in Eq. (6). The standard technique for this purpose would be the collinear expansion. We employ a more convenient approach by extending the nonlocal OPE for the color singlet bilocal operators[11,12)] to the color octet case. This is based on exact operator identities between the nonlocal operators, and allows us to perform the twist decomposition *off the light-cone*, *i.e.*, keeping manifest covariance. We obtain $\bar{\psi}(x)\gamma_\alpha\gamma_5 t^a\psi(-x) = [\bar{\psi}(x)\gamma_\alpha\gamma_5 t^a\psi(-x)]^{\mathrm{tw.2}} + [\bar{\psi}(x)\gamma_\alpha\gamma_5 t^a\psi(-x)]^{\mathrm{tw.3}} + O(\text{twist-4})$, where the first term of twist-2 can be expressed as an integral of the quark-antiquark color-octet bilocal operators. The twist-3 term can be expressed, using the equations of motion $\not{D}\psi = 0$, by the quark-antiquark-gluon correlation (we neglect the quark mass for simplicity)

$$
\begin{aligned}
&[\bar{\psi}(x)\gamma_\alpha\gamma_5 t^a\psi(-x)]^{\mathrm{tw.3}}\\
&\quad= -\frac{1}{2N_c}\int_0^1 du u^2\int_{-1}^1 dv\bar{\psi}(ux)\not{x}\left(i\gamma_5 v g G^a_{\alpha\lambda}(uvx)x^\lambda\right.\\
&\qquad\left.+\, g\tilde{G}^a_{\alpha\lambda}(uvx)x^\lambda\right)\psi(-ux) + \cdots\ , \qquad (7)
\end{aligned}
$$

where $G^a_{\alpha\lambda}$ and $\tilde{G}^a_{\alpha\lambda}$ are the gluon field strength tensor and its dual, respectively. To be precise, they should be understood as $\int_{-1}^1 dvvG = \int_{-1}^{\infty x_0} dvvG_+ - \int_1^{\infty x_0} dvvG_-$, $\int_{-1}^1 dv\tilde{G} = \int_{-1}^{\infty x_0} dv\tilde{G}_+ - \int_{-1}^{\infty x_0} dv\tilde{G}_-$. The ellipses stand for the other types of quark-antiquark-gluon operators involving $d^{abc}G^b_{\alpha\lambda}t^c$, $f^{abc}G^b_{\alpha\lambda}t^c$, *etc.* and for the quark-antiquark operators involving total derivatives; both of them are irrelevant for the present purpose. By substituting the results into the color octet term of Eq. (6), we see that the color index "a" of $G^a_{\alpha\lambda}$ and $\tilde{G}^a_{\alpha\lambda}$ of Eq. (7) is contracted with that of $[\bar{\psi}(-x)\gamma_\beta\gamma_5 t^a\psi(x)]^{\mathrm{tw.2}}$, thereby giving a product of two color singlet nonlocal operators. It is straightforward to see that the similar results can be obtained for the contribution due to the vector term of Eq. (5). Combining these results, we obtain the "chiral-even terms" in the nonlocal OPE of Eq. (4) as

$$
\begin{aligned}
&\mathrm{T}\, J_-^\mu(x)J_+^\nu(-x)\\
&= -\frac{\sigma^{\mu\alpha\nu\beta}}{4N_c}\Bigg[\bar{\psi}(x)\gamma_\alpha\gamma_5\psi(-x)\bar{\psi}(-x)\gamma_\beta\gamma_5\psi(x)\\
&\quad - gx^\lambda\int_0^1 du u^2\int_{-1}^1 dv\Big\{\bar{\psi}(ux)\not{x}\gamma_5\psi(-ux)\bar{\psi}(-x)\gamma_\beta\\
&\quad\times\Big(ivG_{\alpha\lambda}(uvx)\gamma_5 + \tilde{G}_{\alpha\lambda}(uvx)\Big)\psi(x) - (x\to -x)\Big\}\Bigg]\ , \qquad (8)
\end{aligned}
$$

up to the corrections of order $x^2 \sim 1/Q^2$ and neglecting the terms which vanish when taking matrix element. Here we have shown explicitly the terms relevant for the spin asymmetries in the polarized nucleon-nucleon collisions. As shown above, the second term is generated by the "gluon transfer" from one nonlocal operator to another, and thus corresponds to the spurious loop terms of Figs. 2(b) and (c) in the diagrammatic approach; no need for adding them separately in our operator approach.

* Corresponding to the decomposition of the fields at Λ, the hadron state $|P_A,P_B\rangle_{\mathrm{in}}$ should be considered as composite of the classical long-distance fields. As usual, we assume $|P_A,P_B\rangle_{\mathrm{in}} = |P_A\rangle\otimes|P_B\rangle$ for $s \gg \Lambda^2$. For the construction of the OPE of Eq. (3), this assumption corresponds to taking into account only the operators composed of two color-singlet parts. The assumption would be justified if the short-distance coefficient functions for those operators are infrared safe. This is subject to the factorization proof, and a detailed study of this point goes beyond the scope of this paper. For discussions in conventional approaches, see Refs. 7–9.

By substituting Eq. (8) into Eq. (2) for polarized incoming nucleons, the contributions along opposite tangents to the light-cone are picked out of the nonlocal operators, as discussed above Eq. (4). The results are expressed by the chiral-even spin-dependent parton distribution functions of twist-2 and -3, and give the spin asymmetries through order $1/Q$ similarly to those of Ref. 3. It is straightforward to extend the results by including the "chiral-odd terms" as well as the order x^2 corrections to Eq. (8). The detail of these manipulations and the results will be reported elsewhere.

5. Summary

We have presented direct operator approach to the Drell-Yan process, and derived the nonlocal OPE including the next-to-leading twist terms. Our approach preserves Lorentz and gauge invariance at all steps of calculations, and therefore is convenient for systematic study of the Drell-Yan process. Our results of the OPE have revealed a novel role played by the "color octet terms" for the higher twist effects.

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