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231 VIBRATION DAMPING OF A FLEXIBLE STRUCTURE

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ABSTRACT

The University of Siegen is participating in several virtual laboratory projects in the field of automatic control engineering, mechatronics and telematics. The emphasis is on the remote control of real laboratory experiments via Internet. One of these online experiments is the "Swinging Rod" test-platform for vibration damping of a flexible structure. Robots, aerospace structures, active earthquake-damping devices of tall buildings and active sound suppression are examples, where the active vibration control and positioning control of the structures become evident. Therefore students should be more adverted to this topic during their studies at universities.

INTRODUCTION

In the sequel a detailed work about the Swinging Rod platform is given. It can be considered as a flexible robot arm, which moves in a vertical plane and gets excited with a DC motor. In order to achieve the goal of the vibration damping, several topics will be mentioned, which include the modeling and control of the Swinging Rod. Finally the E-Learning part will be explained, which consists of a remote control interface, tutorials and online-test for the students.

The vibration damping approach includes first the modeling of the Swinging Rod and the transformation of the derived model into modal coordinates. The modal representation is used intentionally due to its compactness, simplicity and explicit physical interpretation. Then properties of the structures, such as controllability and observability grammians, and norms of the modeled system, will also be derived and explained. These properties will be used for the sensor placement and model reduction procedure. The proper positions for a maximum of five sensors will be selected from the 40 candidates along the flexible beam. After model reduction and sensor placement, the LQG and optimal controllers will be designed for active damping and positioning control of the Swinging Rod.

The experimental setup is presented in figure 1, and shows that the flexible rod is fixed on the motor's shaft and points downward. The motor is used for disturbing the system (setting the rod into vibration) and also for controlling the system (as an actuator). Different sensors are used on the Swinging Rod for measuring the vibration. The motor itself has a four-quadrant encoder built-in for the rotational angle measurement, the tip position of the rod (bottom position) is measured with a position sensitive detector and finally the bending of the rod itself is analyzed with strain gauges. The physical properties of the flexible beam are specified in the annex at table 1.

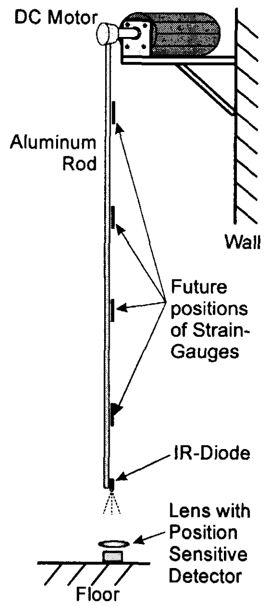

Figure 1: Schematic drawing of the Swinging Rod

SWINGING ROD MODELING

The mathematical model is derived using the Lagrange formalism. The beam was represented with Euler-Bernoulli beam, which helped to represent the model in desired number of modes for the following simplified model as shown in figure 2.

$$\underbrace{M_o}_{\substack{Inertia\\Matrix}} \ddot{q}(t) + \underbrace{D_o}_{\substack{Damping\\Matrix}} \dot{q}(t) + \underbrace{C_o}_{\substack{Spring\\Matrix}} q(t) + \underbrace{G_o}_{\substack{Gravity\\Matrix}} (q(t)) = B_o \tau(t) \qquad (1)$$

The parameters of the above equation for the beam are shown in figure 2, whereas the maximum rotation of the motor-shaft will be limited to ±45 degrees.

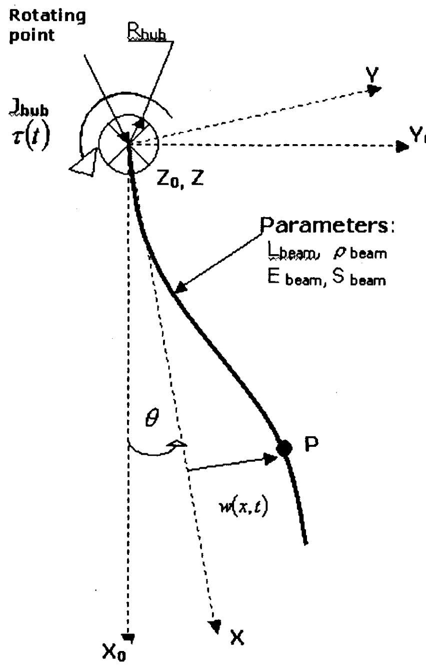

Figure 2: The simplified Swinging Rod model with the physical parameters

The transformed model into state space representation is more convenient for a later control approach and looks like:

$$\begin{aligned} \dot{x} &= A\,x + B\,u \\ y &= C\,x + D\,u \end{aligned} \tag{2}$$

Where the system and input matrix look like:

$$A = \begin{bmatrix} 0 & I \\ -M_o^{-1}C_o & -M_o^{-1}D_o \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M_o^{-1}B_o \end{bmatrix} \tag{3}$$

Two different C matrixes will be defined, which depend on the desired outputs (the location of the strain gauges along the beam). The strain gauges signals, which measure the bending, are transformed into displacement-values with the help of transformation blocks.

The Swinging Rod model is derived by using modal displacement, but it must be converted into modal state space representation, which allows a more convenient work [1]. The modal state space representation has the triple (A_m, B_m, C_m), characterized by the block-diagonal matrix A_m, and related input output matrices, which are defined as following:

$$A_m = diag(A_{mi}) \qquad B_m = \begin{bmatrix} B_{m1} \\ B_2 \\ \vdots \\ B_{mn} \end{bmatrix} \tag{4}$$

$$C_m = \begin{bmatrix} C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix}$$

$i=1,2,\ldots n$, where A_{mi}, B_{mi} and C_{mi} are 2x2, 2xs, and rx2 blocks respectively. The blocks A_{mi} according to [1] could occur in four different forms. The so called "modal form two" is used in this paper.

$$A_{mi} = \begin{bmatrix} -\varsigma_i \omega_i & \omega_i \\ -\omega_i & -\varsigma_i \omega_i \end{bmatrix} \tag{5}$$

The i'th state component of the modal form two is $x_i = \begin{Bmatrix} q_{mi} \\ q_{moi} \end{Bmatrix}$,

Where q_{mi} and $\dot{q}_{mi}$ are the modal displacement and velocity states, and $q_{moi} = \varsigma_i q_{mi} + \dot{q}_{mi} / \omega_i$.

PROBLEM STATMENT

The properties of the above system can now be used for a model reduction and sensor/actuator placement. The analytical solution for choosing the weighting matrices for later vibration damping is also possible for structures of such a type [1]. Therefore the main goal is to apply the theoretical approach to the swinging rod extended model (modal model) for model simplification sensor placement and vibration control.

The structure properties are concerned in the sequel with the controllability, observability and $H_{2,} H_\infty, H_{hankel}$ norms of the system. The system norms serve as a measure of the system "size" and in this capacity they are used in the model reduction and in actuator/ sensor placement procedures.

Controllability and observability are structural properties that carry useful information for system analysis and control. For example, a structure is controllable if the installed actuators excite all its structural modes. It is observable if the installed sensors detect the motions of all the modes. This information, although essential in many applications, is too limited. It answers the question of excitation or detection in terms of yes or no. The quantitative answer is supplied by controllability and observability grammians, which represent a degree of controllability and observability of each mode.

The grammians are defined as

$$\begin{aligned} W_c(t) &= \int_0^t e^{A\tau} BB^T e^{A^T\tau}\, d\tau, \\ W_o(t) &= \int_0^t e^{A^T\tau} C^T C\, e^{A\tau}\, d\tau, \end{aligned} \tag{6}$$

and depend on the system coordinates, but the eigenvalues of the grammian product are invariant and defined as

$$\gamma_i = \sqrt{\lambda_i(W_c W_o)}, \quad \text{with} \quad i = 1, \ldots, N. \tag{7}$$

They are known as Hankel singular values of the system, which have strong connection to the system norms. It is also possible to balance the system so that its controllability and observability grammians are equal, diagonal and defined by Moor as:

$$\begin{aligned} &W_c = W_o = \Gamma, \\ &\Gamma = diag(\gamma_1, \ldots, \gamma_N), \\ &\gamma_i \geq 0, \quad i = 1, \ldots, N, \end{aligned} \tag{8}$$

where γ_i is the i'th Hankel singular value of the system. The complete transformation algorithm is represented in [1] and based on the singular value decomposition of the controllability and observability grammians. This approach will be used to balance the model in the model reduction procedure. The Hankel singular value is also called Hankel norm. The following approximation equality is valid for modal representation [1]:

$$\|G_i\|_\infty \cong 2\|G_i\|_h \cong \sqrt{\varsigma_i \omega_i}\,\|G_i\|_2 \tag{9}$$

Graphically it could be represented as follows:

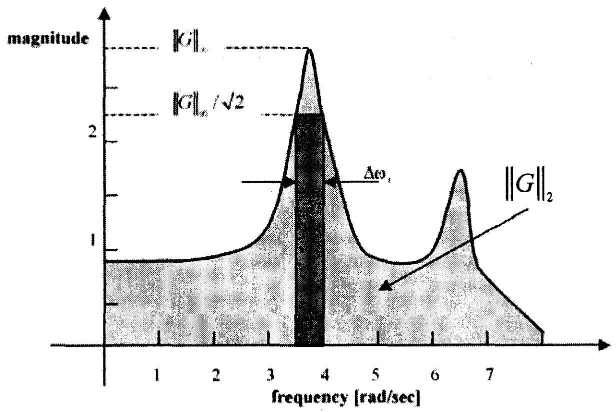

Figure 3: H_2 and H_{inf} norm of the system, shown in a bode plot

MODEL REDUCTION

The H_{hankel} system norm is approximately half of the H_∞ norm, see equation 9 and [1]. Hence the reduction using one of those norms is identical with the reduction using the other one.

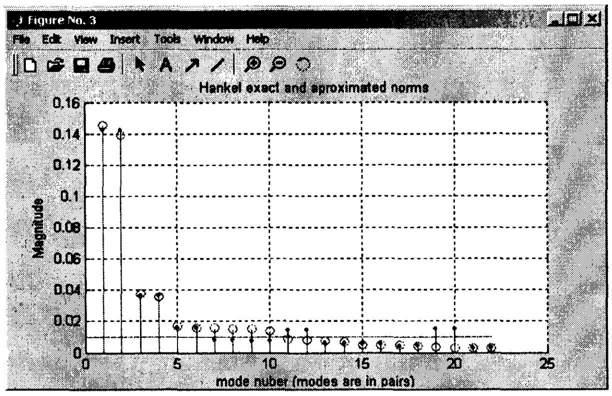

Figure 4: The H_{hankel} norms of the Swinging Rod with 10 flexible modes, approximated (small dots) and exact (circles) values

In the sequel we are only considering only the H_∞ norm. The H_∞ reduction error in modal coordinates is estimated as

$$e_\infty = \|G - G_r\|_\infty = \|G_t\|_\infty = \|G_{k+1}\|_\infty . \tag{10}$$

The model reduction procedure considers 10 flexible modes and is reduced with the help of the Hankel norms, as shown in figure 4.

The following figure shows the bode plot for original and the reduced system. Here are the neglected modes nicely shown, which only occur in higher frequencies above 100 Hz (w=6x10^2 1/s).

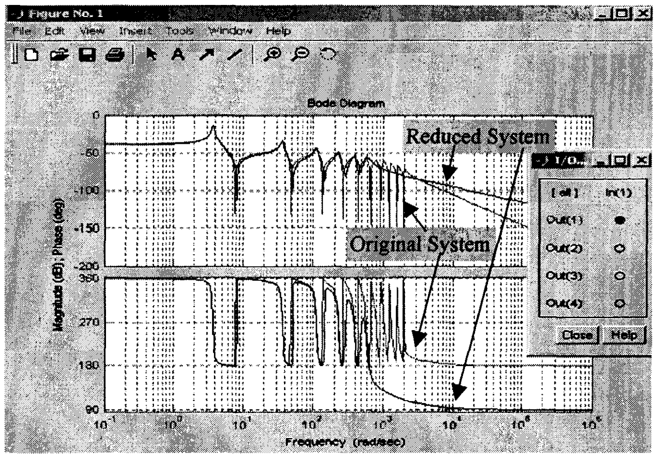

Figure 5: The Bode plot for one of the outputs, reduced (blue) and full (red) systems. The upper plot shows the magnitude and the lower plot the phase.

SENSOR PLACEMENT

The sensor-placement is done with the help of modal matrices and placement indices, according to [1]. Using this approach, the 5 strain gauge sensor locations were chosen from 40 possible candidate locations along the flexible beam, see figure 6.

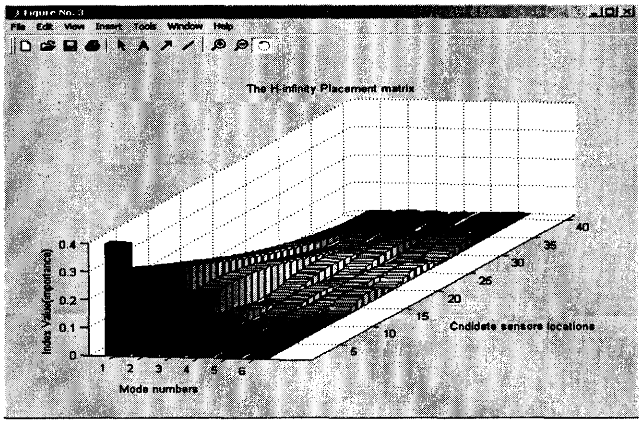

Figure 6: The H-infinity placement matrix, which shows the first 6 modes and their 40 possible sensor-locations

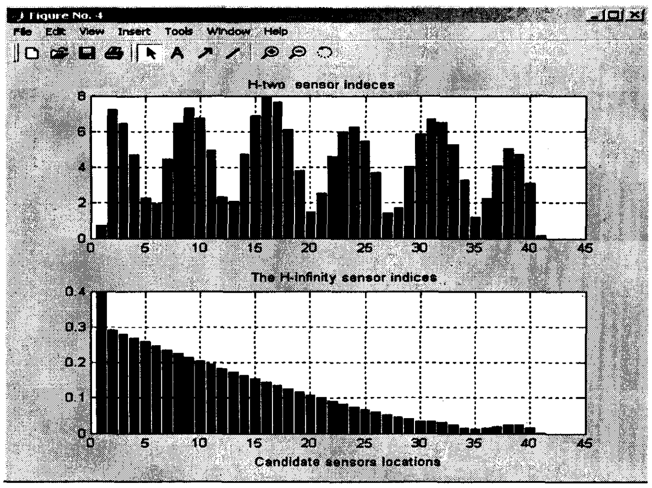

Figure 7: Possible sensor locations for the 40 different locations. The upper diagram shows the H_2 and the lower diagram the H_∞ sensor indices for all modes.

Figure 6 shows the distribution of the involvement of each sensor for measuring each mode (each sensor is represented by a certain position along the flexible beam). As higher the beam in the plot, as better it is to place a sensor at this location for certain mode sensing. In reality only 5 sensors are available, therefore only 5 positions can be chosen based on Figure 6 and 7.

The first sensor, which is shown on both figures, represents the motor angle θ. It can be seen that H_2 and H_∞ sensor indices contradict with each other. Sensor positions 2, 8, and 16 look quite good on the both norm indices (see figure 6 and 7). The last two sensor locations can now be taken according to the H_∞ placement matrix from figure 6 as 24 and 33.

VIBRATION CONTROL

A LQG controller has been used for vibration control. Apart of using trail and error approach for guessing the values of the weighting matrix Q, it can be found analytically [1]. The weight Q shifts the *i'th* pair of complex poles of the system, and leaves the remaining pairs of poles almost unchanged (for structures represented in modal coordinates), according to [1]. Only the real part of the pair of poles is changed (just moving the poles form the imaginary axis away to the left and stabilizing the system), whereas the imaginary part of the poles remain unchanged. The result of the LQG vibration damping controller is shown at the bode plot in figure 8.

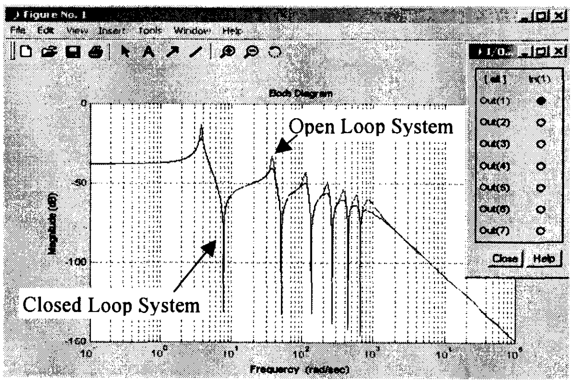

Figure 8: Bode plot for the first mode (theta) of the closed loop and open loop systems.

The controlled and uncontrolled system output is shown in the figures 9 and 10. The system was disturbed with sinusoidal torque input during the first 3 seconds, and after 3 seconds the closed control loop was directly activated with the designed controller.

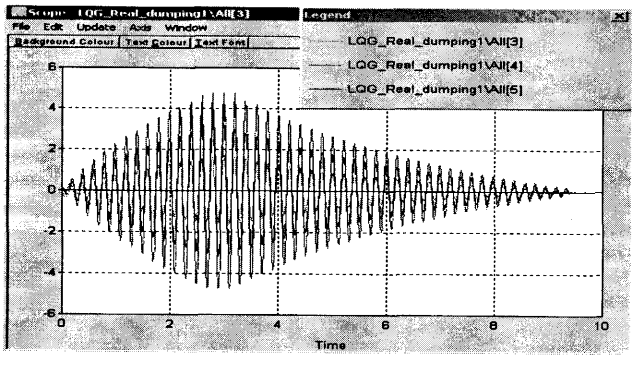

Figure 9: Output of the strain gauges no. 3, 4 and 5; response for uncontrolled vibration after 5Hz sinusoidal excitation for duration of 3 seconds, afterwards the excitation was switched off.

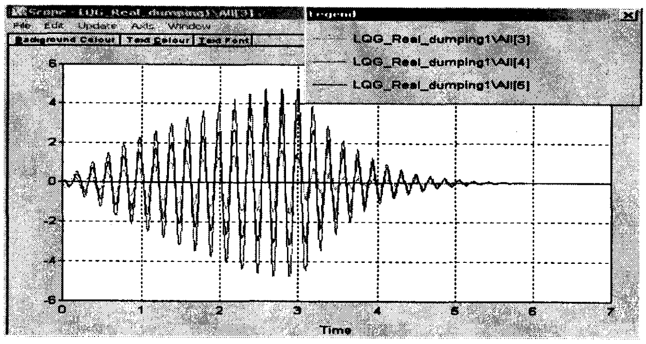

Figure 10: Output of the strain gauges no. 3, 4 and 5; response for controlled vibration damping after 5Hz sinusoidal excitation, where the controller was started at $t = 3$ *sec.*

THE EDUCATIONAL UNIT OF THE SWINGING ROD

The above presented Swinging Rod experiment is fully accessible over the internet for eLearning purposes. It consists of a complete system description and of theoretical teaching units with multiple choice questionnaires and extended text-questions. The remote control is implemented in Java and usable inside any Web-browser, from any locations over the internet. A video stream shows the movements of the rod during the experiment procedure, and especially how well the controller works. To see the stable system behavior with the controller, the rod gets first excited for duration of 5 seconds, afterwards the control mode is switched on and the vibration should be damped as fast as possible (right part in figure 11).

An introduction for modeling the flexible systems is given, so that the student can get familiar with the theory. Multiple choice questions are used to verify the student's knowledge.

The mathematical model of the flexible rod must be derived by the Lagrangian approach, whereas the link is modeled as an Euler–Bernoulli beam. The approach above with the different mode shapes would be too difficult for regular students. The approach at the beginning of this paper only used for advanced students in intensive workshops.

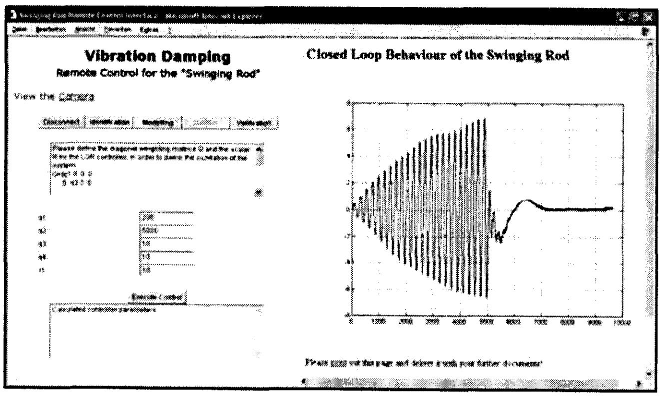

Figure 11: Remote control interface for the swinging rod experiment, running inside a web browser without additional software. Left part: control interface area; right part: system output after control is finished.

With the help of a remote control software is a test run performed, where a torque impulse is sent to the motor to get the impulse-response. These sampled sensor data will then be used for parameter-identification, which can be either used for only the determination stiffness of the rod (with the natural frequency), or for advanced users with the help of the System Identification

Toolbox inside MATLAB. The identified model has to be verified with the real one inside the remote control, and if necessary accordingly adapted. The controller design is the next step for the students in this virtual laboratory, where the aim is to set the complete rod in quiet mode as fast as possible (left part of figure 11). The rod gets therefore excited for 5 seconds (in open loop), and after these 5 seconds the control-loop gets activated and the vibration should get damped, as can be seen in the right part of figure 11.

CONCLUSION

The Swinging Rod test platform is used for vibration analysis and vibration damping. The simple setup of the experiment gives the students a very good overview about the vibration theory itself. Due to the remote control possibility, it can be demonstrated during lectures and students can perform the experiment alone from their home computer as part of the control lectures. The modal analysis explained at the beginning of the paper gives a good chance for advanced students, to enter this subject.

ACKNOWLEDGEMENT

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ANNEX

Table 1: Flexible beam properties

Material	AlMgSi0.5/6063(AA)/ A96063(UNS)
Density	2700 Kgm^{-3}
Elasticity module	65.5×109 … 68.9×109 Nm^{-2}
Rectangular section	20×4 mm^2
Length	163.5 mm

Table 2: Symbol Description

Symbol	Description
$\|G\|_2$	System norm H_2
$\|G\|_\infty$	System norm H_∞
$\|G\|_h$	System norm H_{hankel}
γ_i	i 'th Hankel singular value
$\gamma_{\max}$	The larges Hankel singular value of a system
G	System transfer matrix for (MIMO)
G_r	System transfer matrix with the retained modes
G_t	System transfer matrix with the truncated modes
ω_i	i'th natural frequency
ϕ_i	i'th natural mode
Ω	$\Omega = diag(\omega_1, \omega_2, ..., \omega_m)$ Matrix on natural frequencies
Φ	$\Phi = [\phi_1 \quad \phi_2 \quad \dots \quad \phi_m]$ Modal matrix
A_m, B_m, C_m	Modal state space triple
$\Delta\omega_i$	The half-power of i'th frequency