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EVALUATION OF FUNCTION OF PASSIVE LOAD FOR TUNNEL LINING

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ABSTRACT

A method to analyze the concrete lining of tunnel taking passive loads into consideration is proposed. The basic assumptions used are given as follows: (1) The structural system of the tunnel lining is an arch under the plain strain condition supported elastically at the ends and (2) passive loads acting on the lining are induced in proportion to displacement of the lining toward the surrounding rock over void between the lining and the rock. Using a model, which is almost the same as the tunnel lining of the Shinkansen of Japanese National Railways, many numerical results are obtained for various loading patterns, heights of the rock mass, thickness of the lining and the voids. From the results, characteristics of passive loads associated with magnitude and shape of active loads, the thickness and the void are discussed. Stress reducing effects of the passive loads are significant. But the effects are reduced by the void. Finally, rough standards of the allowable height of the rock mass are presented.

Key words: concrete, lining, rock mass, subgrade reaction coefficient, tunnel
IGC: H 5

INTRODUCTION

The method to design the tunnel lining has been less well established. Main difficulties for this problem are given as follows: (1) Active and passive loads can not be estimated appropriately and (2) the structural system to analyze the lining is not clear.

The present author deals with a numerical analyzing method for the tunnel lining taking passive loads into consideration and discussions of characteristics of passive loads associated with magnitude and shape of active loads, thickness of the lining and void between the lining outside surface and the surrounding rock.

This problem has been investigated by researchers (Saito et al., 1973) using the finite element method. In this paper based on numerical results using the proposed method, more definite consequences are presented.

FORMULATION

The basic assumptions used can be summarized as follows: (1) The structural system of the tunnel lining is an arch under the plain strain condition supported elastically at the ends. (2) Passive loads acting on the lining are induced in proportion to displacement of the lining toward the surrounding rock over the void between the lining and the rock.

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(3) Skeleton of the lining is symmetrical with respect to vertical center line and the thickness is constant along the axial line, as usually the case.

The tunnel lining as an elastically supported arch can be analyzed by the method similar to the steel tunnel support proposed by the author (Chou, 1969). The present problem must be treated as the plain strain problem, while the arch support was treated as the plain stress problem.

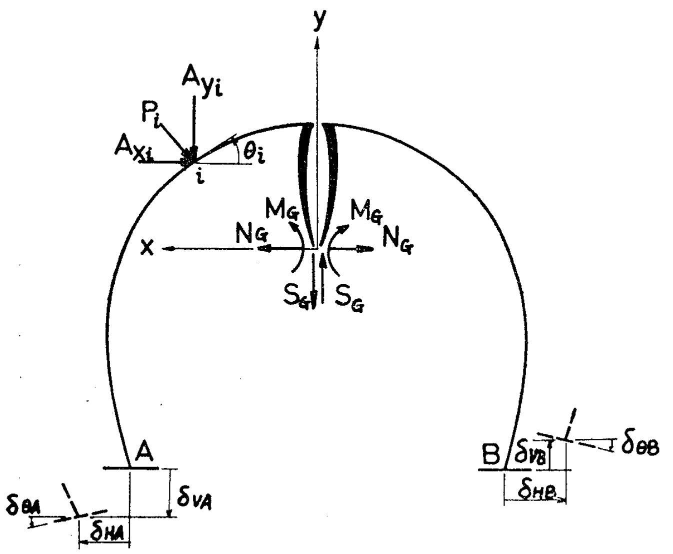

Fig. 1. Loads, redundants and displacements of ends

Referring to Fig. 1, we obtain the bending moment (M), the axial force (N) and the shearing force (S) at any section of the lining as follows.

$$M=M_0+M_G-N_Gy-S_Gx \qquad (1)$$

$$N=N_0+N_G\cos\theta+S_G\sin\theta \qquad (2)$$

$$S=S_0-N_G\sin\theta+S_G\cos\theta \qquad (3)$$

in which M_0, N_0 and S_0 are the bending moment, the axial force and the shearing force for the free body in Fig. 1 subjected to external loads and M_G, N_G and S_G are the redundants at the elastic center.

The unknown quantities of this problem are not only the redundants but also displacements of the ends, i.e., δ_{VA}, δ_{HA}, $\delta_{\theta A}$, δ_{VB}, δ_{HB} and $\delta_{\theta B}$ as shown in Fig. 1. Therefore, the number of the unknowns is nine, and nine simultaneous equations are necessary to solve the problem.

Applying the Castigliano's theorem, the compatibility conditions at the elastic center of the lining yield the following equations.

$$\frac{\partial W}{\partial M_G}+\delta_{\theta A}+\delta_{\theta B}=0 \qquad (4)$$

$$\frac{\partial W}{\partial N_G}+\delta_{HA}+\delta_{HB}-y_A\delta_{\theta A}-y_B\delta_{\theta B}=0 \qquad (5)$$

$$\frac{\partial W}{\partial S_G}+\delta_{VA}+\delta_{VB}-x_A\delta_{\theta A}-x_B\delta_{\theta B}=0 \qquad (6)$$

in which W indicates total strain energy of the arch and subscripts, i.e., A and B, denote the ends. The following relations between the end reactions and displacements can be expressed.

$$M_A=k_\theta J\delta_{\theta A} \qquad (7)$$

$$M_B=k_\theta J\delta_{\theta B} \qquad (8)$$

$$R_{HA}=k_HA_H\delta_{HA} \qquad (9)$$

$$R_{HB}=k_HA_H\delta_{HB} \qquad (10)$$

$$R_{VA}=k_VA_V\delta_{VA} \qquad (11)$$

$$R_{VB}=k_VA_V\delta_{VB} \qquad (12)$$

in which M_A, M_B, R_{HA}, R_{HB}, R_{VA} and R_{VB} are the bending moments, the horizontal and vertical reactions at the ends, k_θ, k_H and k_V are the subgrade reaction coefficients to rotation, horizontal and vertical displacements, J is moment of inertia of the end base, and A_H and A_V are horizontally and vertically projected areas of the end base, respectively.

The total strain energy of the arch as the plane strain problem ignoring the shearing effect is given as:

$$W=\frac{(1-\nu^2)}{2E}\int\left(\frac{M^2}{I}+\frac{N^2}{A}\right)ds \tag{13}$$

in which E=Young's modulus, ν=Poisson's ratio, I=moment of inertia of the lining, A=area of the lining and s=length of the lining along the axial line. Substituting Eqs. (1)～(3) and (13) into Eqs. (4)～(12), we obtain:

$$\frac{(1-\nu^2)}{EI}\left(\int ds\right)M_G+\delta_{\theta A}+\delta_{\theta B}=-\frac{(1-\nu^2)}{EI}\int M_0\,ds \tag{14}$$

$$\frac{(1-\nu^2)}{E}\left\{\int\left(\frac{y^2}{I}+\frac{\cos^2\theta}{A}\right)ds\right\}N_G-y_A\delta_{\theta A}-y_B\delta_{\theta B}+\delta_{HA}+\delta_{HB}=\frac{(1-\nu^2)}{E}\int\left(\frac{yM_0}{I}-\frac{\cos\theta\,N_0}{A}\right)ds \tag{15}$$

$$\frac{(1-\nu^2)}{E}\left\{\int\left(\frac{x^2}{I}+\frac{\sin^2\theta}{A}\right)ds\right\}S_G-x_A\delta_{\theta A}-x_B\delta_{\theta B}+\delta_{VA}+\delta_{VB}=\frac{(1-\nu^2)}{E}\int\left(\frac{xM_0}{I}-\frac{\sin\theta\,N_0}{A}\right)ds \tag{16}$$

$$M_G-y_AN_G-x_AS_G-k_\theta J\delta_{\theta A}=-M_{0A} \tag{17}$$

$$M_G-y_BN_G-x_BS_G-k_\theta J\delta_{\theta B}=-M_{0B} \tag{18}$$

$$N_G-k_HA_H\delta_{HA}=-R_{0HA} \tag{19}$$

$$N_G-k_HA_H\delta_{HB}=-R_{0HB} \tag{20}$$

$$S_G-k_VA_V\delta_{VA}=-R_{0VA} \tag{21}$$

$$S_G-k_VA_V\delta_{VB}=-R_{0VB} \tag{22}$$

in which R_0 is end reaction of the free body subjected to external loads. Substituting Eqs. (17)～(22) into Eqs. (14)～(16), the unknown quantities of the displacements of the ends are eliminated, and we can obtain three simultaneous equations as follows:

$$\begin{pmatrix} \frac{(1-\nu^2)}{EI}L+\frac{2}{k_\theta J} & -\frac{2y_A}{k_\theta J} & 0 \\ -\frac{2y_A}{k_\theta J} & \frac{(1-\nu^2)}{E}\left\{\int\left(\frac{y^2}{I}+\frac{\cos^2\theta}{A}\right)ds\right\}+\frac{2}{k_HA_H}+\frac{2y_A{}^2}{k_\theta J} & 0 \\ 0 & 0 & \frac{(1-\nu^2)}{E}\left\{\int\left(\frac{x^2}{I}+\frac{\sin^2\theta}{A}\right)ds\right\}+\frac{2}{k_VA_V}+\frac{2x_A{}^2}{k_\theta J} \end{pmatrix}\begin{pmatrix} M_G \\ N_G \\ S_G \end{pmatrix}$$

$$=\begin{pmatrix} -\frac{(1-\nu^2)}{EI}\int M_0\,ds-\frac{1}{k_\theta J}(M_{0A}+M_{0B}) \\ \frac{(1-\nu^2)}{E}\left\{\int\left(\frac{y}{I}M_0-\frac{\cos\theta\,N_0}{A}\right)ds\right\}-\frac{1}{k_HA_H}(R_{0HA}+R_{0HB})+\frac{1}{k_\theta J}(y_AM_{0A}+y_BM_{0B}) \\ \frac{(1-\nu^2)}{E}\left\{\int\left(\frac{x}{I}M_0-\frac{\sin\theta\,N_0}{A}\right)ds\right\}-\frac{1}{k_VA_V}(R_{0VA}+R_{0VB})+\frac{1}{k_\theta J}(x_AM_{0A}+x_BM_{0B}) \end{pmatrix} \tag{23}$$

By solving Eq. (23) we can obtain the redundants, i.e., M_G, N_G and S_G. M, N and S at any section of the elastically supported arch can be determined by Eqs. (1)～(3) using

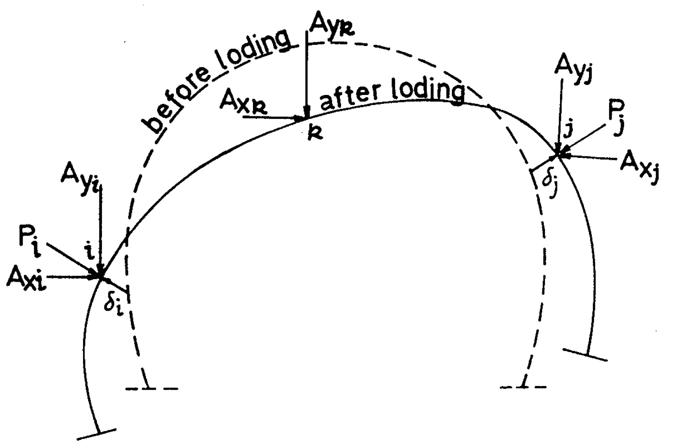

Fig. 2. Displacement of lining

M_G, N_G and S_G obtained.

Fig. 2 shows axial lines of the lining before being subjected to loads and after that. It is assumed that a passive load P_i at a point i is induced to the direction normal to the axis of the lining in proportion to a quantity, i.e., $\delta_i-\delta_{0i}$, toward the surrounding rock over the void, δ_{0i}, between the lining and the rock. Thus P_i is expressed as:

$$P_i=\begin{cases}K_i(\delta_i-\delta_{0i}) & \delta_i>\delta_{0i}\\ 0 & \delta_i\leq\delta_{0i}\end{cases} \tag{24}$$

in which K_i is a proportional constant. Using the subgrade reaction coefficient k_i can be expressed as:

$$K_i=k_iA_{gi} \tag{25}$$

in which A_{gi} is area of surrounding rock of the point i. The displacement of the point i toward the rock can be written in the following form.

$$\delta_i=\sum_j(A_{xj}\bar{\delta}_{ijx}+A_{yj}\bar{\delta}_{ijy}+P_j\bar{\delta}_{ij}) \tag{26}$$

in which A_{xj} and A_{yj} are the active loads at a point j to x and y directions, and $\bar{\delta}_{ijx}$, $\bar{\delta}_{ijy}$ and $\bar{\delta}_{ij}$ are influencing coefficients of displacement at the point i to normal direction, by being subjected to unit loads at a point j to x, y and normal directions. Using Eq. (24), Eq. (26) yields

$$-\frac{P_i}{K_i}+\sum_j^n P_j\bar{\delta}_{ij}=-\sum_j^n(A_{xj}\bar{\delta}_{ijx}+A_{yj}\bar{\delta}_{ijy})+\delta_{0i} \qquad (i=1,\ 2,\ \cdots n) \tag{27}$$

in which n is a number of elements divided. In Eq. (27), P_i and P_j are unknown quantities. Then the passive loads can be obtained by solving Eq. (27) simultaneously. But, since the passive loads are non-negative values defined by Eq. (24), they must be solved successively in an iterative process.

The influencing coefficients must be calculated taking account of the displacements of the ends. Applying the principle of virtual work they are obtained as follows.

$$\left.\begin{aligned}
\bar{\delta}_{ij}=&\alpha\int\frac{M_j\bar{M}_i}{EI}ds+\alpha\int\frac{N_j\bar{N}_i}{EA}ds+\bar{M}_{Ai}\delta_{\theta Aj}+\bar{M}_{Bi}\delta_{\theta Bj}\\
&+\bar{H}_{Ai}\delta_{HAj}+\bar{H}_{Bi}\delta_{HBj}+\bar{V}_{Ai}\delta_{VAj}+\bar{V}_{Bi}\delta_{VBj}\\
\bar{\delta}_{ijx}=&\alpha\int\frac{M_{jx}\bar{M}_i}{EI}ds+\alpha\int\frac{N_{jx}\bar{N}_i}{EA}ds+\bar{M}_{Ai}\delta_{\theta Ajx}+\bar{M}_{Bi}\delta_{\theta Ajx}\\
&+\bar{H}_{Ai}\delta_{HAjx}+\bar{H}_{Bi}\delta_{HBjx}+\bar{V}_{Ai}\delta_{VAjx}+\bar{V}_{Bi}\delta_{VBjx}\\
\bar{\delta}_{ijy}=&\alpha\int\frac{M_{jy}\bar{M}_i}{EI}ds+\alpha\int\frac{N_{jy}\bar{N}_i}{EA}ds+\bar{M}_{Ai}\delta_{\theta Ajy}+\bar{M}_{Bi}\delta_{\theta Bjy}\\
&+\bar{H}_{Ai}\delta_{HAjy}+\bar{H}_{Bi}\delta_{HBjy}+\bar{V}_{Ai}\delta_{VAjy}+\bar{V}_{Bi}\delta_{VBjy}
\end{aligned}\right\} \tag{28}$$

in which $\alpha=(1-\nu^2)$, i denotes that a unit load acts on the point i toward the surrounding rock and j, jx and jy denote that unit loads act on the point j toward inside direction normal to the axial line, x and y direction, respectively.

MODEL FOR CALCULATION

Fig. 3 shows a skeleton used for the calculations, which is almost the same as the con-

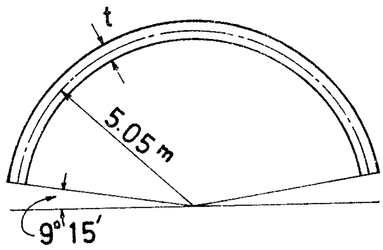

Fig. 3. Model for calculation

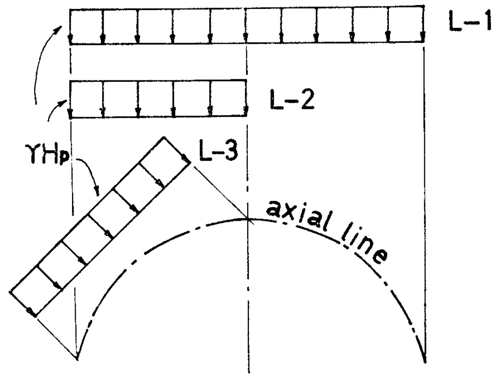

Fig. 4. Load patterns

crete arch of the tunnel lining of the Shinkansen of Japanese National Railways. The following three patterns of the active loads are used: (1) uniform vertical load which is produced by the mass of the rock having the constant height, H_p, along the horizontal line, (2) half side uniform vertical load which consists of half side portion of the uniform vertical load to the center line, and (3) oblique load which is the half side uniform load being inclined at 45° to the vertical line (See Fig. 4). These patterns will be hereinafter referred to as L-1, L-2 and L-3. The rock mass are taken from 1 meter to a few meters in height, H_p, at intervals of 1 meter. The unit weight of the rock used is 2.7 t/m³. The subgrade reaction coefficients of the ends, i.e., k_θ, k_V and k_H, are selected as 500 kg/cm³, and the subgrade reaction coefficient of the surrounding rock, k_t, is selected as 50 kg/cm³. These values were determined by taking account of the fact that some results of examinating calculations using larger values than them agree approximately with the results using them and orders of magnitudes of the coefficients can be considered to be so in fact. The Young's modulus of the lining concrete, E, used is 300000 kg/cm². The thicknesses of the lining used, t, are 30, 50 and 70 cm. Magnitudes of the void, δ_0, are increased up to a few milli-meters where the passive loads are not induced, at intervals of 1 milli-meter. The lining is divided into 19 pieces and the calculations of integration are conducted by the numerical integration. The accuracy of the calculation was ascertained by comparing with some results for 39 pieces.

NUMERICAL EXAMPLES AND DISCUSSIONS

Typical numerical results, which demonstrate the characteristics of the passive loads, are shown in Figs. 5~8. Fig. 5 shows the upper fiber stresses for a thickness of 50 cm relevant to the three loading patterns about two cases of the void where $\delta_0=0$ and $\delta_0=\infty$. This shows a good comparison of stress reducing effects of the passive loads for the different loading patterns. The effects for L-3 are much more than for L-1 and L-2 and magnitudes of stresses for the cases of $\delta_0=0$ in three loading patterns are approximately the same. This means that an inclined load induces much more passive loads in order to confine the displacement of the lining. Figs. 6 and 7 show fiber stresses, passive loads and displacements to normal direction by being subjected to L-3 for different thickness of the lining, where $H_p=2$ m and $\delta_0=1$ mm. The fiber stresses for a thin lining are not so much as predicted by a decrease of the section modulus as compared to those for a thick lining. This is caused by the passive loads induced much more for the thin lining (Fig. 7). Fig. 8 shows fiber stresses of the lining subjected to L-3 for various heights of the rock mass, where $t=50$ cm and $\delta_0=1$ mm. The fiber stresses do not increase proportionally to H_p. This is also caused by the passive loads induced much more for higher H_p. Therefore, it can be

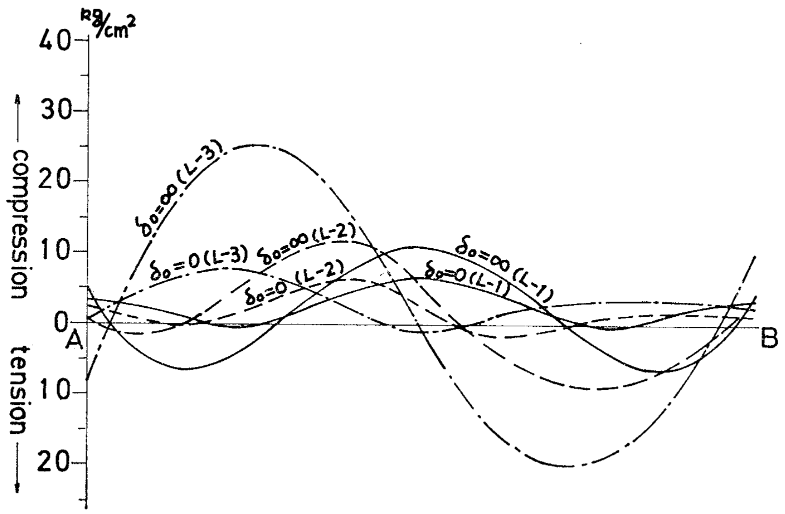

Fig. 5. Upper fiber stresses (H_p=1 m, t=50 cm)

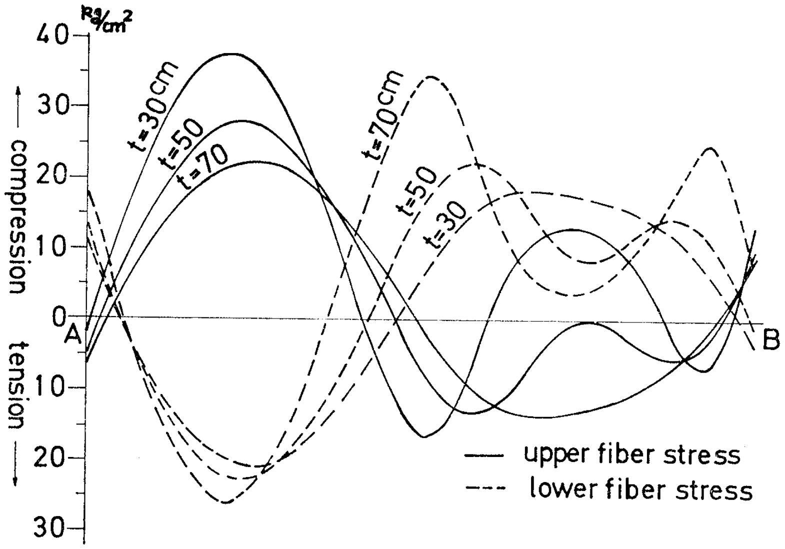

Fig. 6. Fiber stresses (L-3, H_p=2 m, δ_0=1 mm)

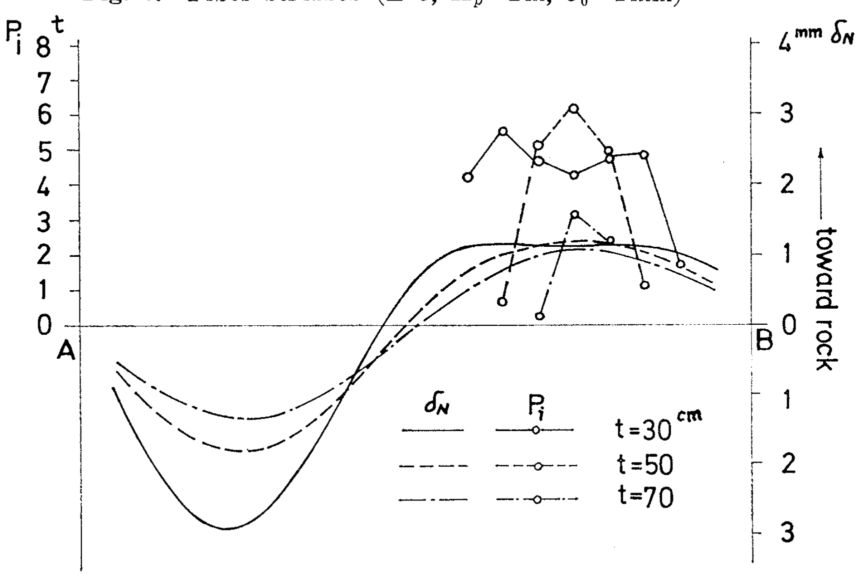

Fig. 7. Passive loads and displacements (L-3, H_p=2 m, δ_0=1 mm)

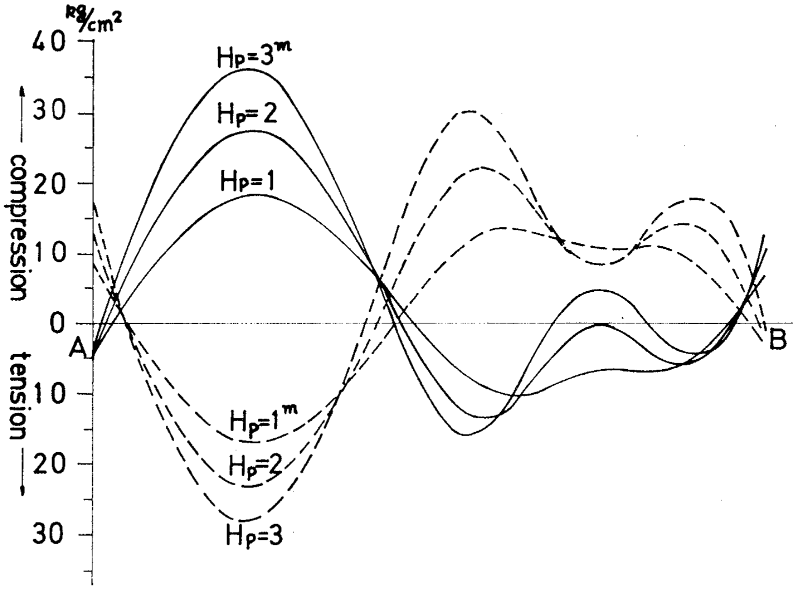

Fig. 8. Fiber stresses (L-3, t=50cm, δ_0=1mm)

said that in the cases where the active loads are inclined and high and thickness of the lining is thin, the passive loads contribute more effectively to the decreasing of the stresses. The contribution of the passive loads is reduced by the voids between the lining and the surrounding rock. Table 1 and Fig. 9 show allowable heights of the rock mass, H_p^a, against the various voids for the various loading patterns and thickness of the lining. The allowable height is determined by the assumption that the lining is cracked when the maximum tensile stress reaches 20 kg/cm². In Fig. 9 horizontal lines in H_p^a curves for the various thickness indicate that the passive load is not induced. The allowable heights are the least in the case of L-3, except a case of L-2 where t=30 and δ_0=0. This means that the inclined load gives unfavourable effects compared to the vertical loads. When the voids do not exist, the heights are very high against the cases not being confined by the surrounding rock, differences in H_p^a between various loading patterns are reduced compared to the cases where the voids exist, and the heights do not increase proportionally to the thickness, especially in the case of L-3. The last is caused by the fact that the thin lining produces much more passive loads. The ratios of H_p^a (δ_0=0) to H_p^a ($\delta_0=\infty$) amount to 2~15. Therefore, the strength of tunnel lining arch amounts to 2~15 times of that of the arch on the ground. But, when the voids of 1~2 mm exist, the strength gains vanish and in the case of L-3 the heights drop down to 0.3~1.6 m. This leads to emphasize

Table 1. Allowable heights of rock mass, H_p^a

t (cm)	δ_0 (mm)	L-1	L-2	L-3
30	0	5.26	3.28	4.55
	1	1.12	1.21	1.21
	2	0.94*	0.86	0.53
	3	0.94*	0.73*	0.29*
50	0	7.41	5.41	5.41
	1	2.56*	2.02*	1.56
	2	2.56*	2.02*	0.82*
70	0	9.09	7.69	6.25
	1	4.65*	3.85*	1.60*

* denote that passive loads are not induced.

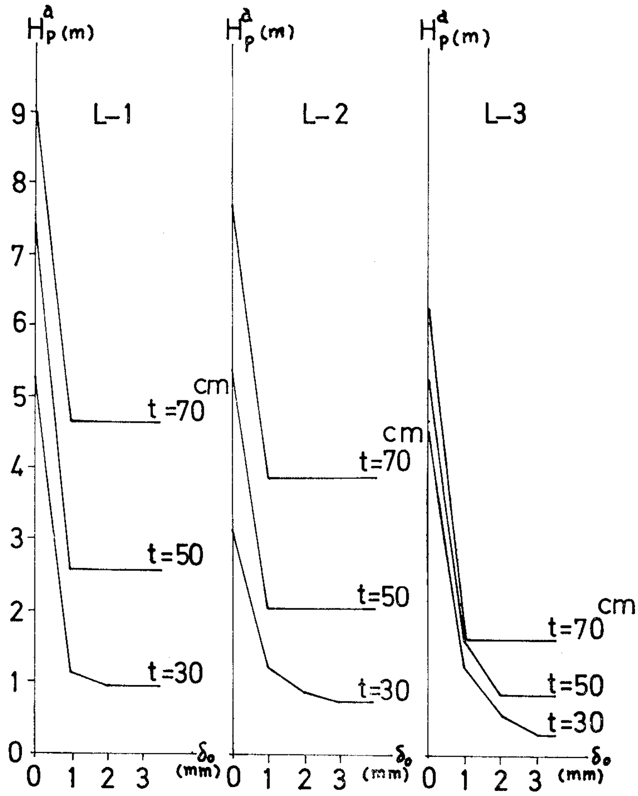

Fig. 9. Allowable heights of rock mass, H_p^a

that the back-filling must be set enough. Since shapes of the active load used involve critical shape such as L-3, the least values of H_p^a may be taken as rough standards on strength of the lining, which is evaluated by the height of the rock mass. When the back-filling is set enough, the standards are 3 m for $t=30$ cm, 5 m for $t=50$ cm and 6 m for $t=70$ cm.

CONCLUSIONS

A method to analyze the tunnel concrete lining is proposed and many examples are calculated. The following conclusions can be drawn:

1. The stress reducing effects of the passive load acting on the lining are significant, especially in the case of the inclined active load.
2. The contribution of the passive load on the decreasing of the stresses is reduced by the voids between the lining and the surrounding rock. When the voids of 1~2 mm exist, the contribution vanishes.
3. When the back-filling is set enough, rough standards of the allowable heights of rock mass may be taken on values of 3 m for $t=30$ cm, 5 m for $t=50$ cm and 6 m for $t=$ 70 cm.

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NOTATION

A_{gi} = area of surrounding rock of a point i

A_x, A_y = active loads to x and y direction
A_V, A_H = horizontally and vertically projected areas of the end base
E = Young's modulus of the lining concrete
H_p = height of rock mass
$H_p{}^a$ = allowable height of rock mass
J = moment of inertia of the end base
k_i = subgrade reaction coefficient of surrounding rock
$K_i = k_i A_{gi}$
k_θ, k_V, k_H = subgrade reaction coefficients of end base to rotation, horizontal and vertical displacements
M, N, S = moment, axial force and shearing force at any section of the lining
M_0, N_0, S_0 = M, N and S for the free body in Fig. 1
M_G, N_G, S_G = redundants at the elastic center
P_i = passive load at a point i
R_H, R_V = horizontal and vertical reaction at the end
R_0 = reaction for the free body
s = length measured along the axial line
t = thickness of lining
W = total strain energy
x, y = co-ordinates in two dimensions
$\alpha = 1 - \nu^2$
$\bar{\delta}$ = influencing coefficients of displacement
δ_i = displacement of a point i toward rock
δ_θ, δ_V, δ_H = end displacements defined in Fig. 1
δ_0 = void between the lining and rock
ν = Poisson's ratio of the lining concrete
θ = angle between the axial line and horizontal line

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