

TECHNICAL NOTE

SHEARING DEFORMATION OF GRANULAR MATERIALS

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ABSTRACT

Based on the fundamental experimental properties, the author presents a new theory of plastic shear deformation of granular materials. The theory involves (a) yield function, (b) plastic potential, (c) stability condition and (d) deformation rule. The yield function is defined by the author's new state function S_s . The plastic potential ϕ is provided for the drained shear and the undrained shear. The stability condition can uniquely classify a experimental stress path into 'stable yielding' or 'unstable yielding' in terms of the functions of S_s and ϕ .

Key words: deformation, drained shear, granular material, plasticity, sand, stress path, stress-strain curve, yield

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INTRODUCTION

In order to build a rule of plastic shearing deformation, it is necessary to define (a) yield function, (b) plastic potential and (c) stability condition. Apart from previously proposed theory, the author constructs a theory of shearing deformation. In this paper, the elastic deformation is neglected. The discussions are limited to the following conditions

- (1) the 'triaxial' compression test,
- (2) isotropically consolidated samples,
- (3) monotonously and smoothly increasing stress paths, and
- (4) the virginal shear deformation.

YIELD CONDITION

The author considers that the plastic work increment due to shear is written as

$$dW_s^p = p \cdot dv_d^p + \frac{2}{3} \cdot q \cdot d\gamma^p \quad (1)$$

where p : mean principal stress (effective)

q : shear stress

v_d^p : plastic volumetric strain due to volume change by dilatancy effect of shear strain

γ^p : plastic shear strain

dv_d^p : increment of v_d^p

$d\gamma^p$: increment of γ^p

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Strains v_d^p and γ^p can be obtained as functions of stress ratio

$$\eta = q/p \quad (2)$$

This was confirmed by the authors experiments (Moroto et al., 1974). Hence we have

$$\gamma^p = G(\eta) \quad v_d^p = D(\eta) \quad (3)$$

and then

$$d\gamma^p = \frac{dG(\eta)}{d\eta} d\eta = G'(\eta) d\eta, \quad G'(\eta) = \frac{dG(\eta)}{d\eta} \quad (4)$$

$$dv_d^p = \frac{dD(\eta)}{d\eta} d\eta = D'(\eta) d\eta, \quad D'(\eta) = \frac{dD(\eta)}{d\eta} \quad (5)$$

Thus it follows,

$$dW_s^p = p \cdot X(\eta) d\eta \quad (6)$$

where

$$X(\eta) = G'(\eta) + D'(\eta) \quad (7)$$

$$G'(\eta) = \frac{dG(\eta)}{d\eta}$$

$$D'(\eta) = \frac{dD(\eta)}{d\eta}$$

The author has pointed out Eq. (6) is not a perfect differential so the plastic work done defined by Eq. (1) depends on stress path and he introduced a quantity S_s (Moroto, 1976) such that

$$dS_s = \frac{dW_s^p}{p} = X(\eta) d\eta, \quad S_s = \int \frac{dW_s^p}{p} \quad (8)$$

This quantity S_s is a stress path independent one. When several samples are compressed uniformly to the same state and then sheared along different stress paths until they reach equally the state of failure, the total plastic work done or total energy dissipated in each sample is different from each other depending on each stress path, but the quantity S_s for all samples which attain the final state by different stress path are equal. The plastic strains are expressed in Eq. (3), so that we can write

$$dS_s = \eta_w \cdot d\epsilon^p, \quad \eta_w = \frac{D'(\eta)}{\frac{2}{3}G'(\eta)} + \eta \quad (9)$$

where

$$d\epsilon^p = \frac{2}{3} d\gamma^p \quad (\text{scalar quantity}) \quad (10)$$

From the definition of S_s , we have

$$dS_s = \frac{dW_s^p}{p} = dv_d^p + \eta \cdot d\epsilon^p = \eta_w \cdot d\epsilon^p \quad (11)$$

As explained by Mogami (1977), Eq. (11) has the same form as Roscoe's flow rule. From Eq. (11), as easily recognized, the yielding condition can be given by

$$dS_s > 0 \quad (12)$$

PLASTIC POTENTIAL

From Eq. (11), the strain increment ratio can be written

$$\frac{dv_d^p}{d\epsilon^p} = \eta_w - \eta \quad (13)$$

This ratio depends only on the stress and not on the increments of stress. Hence we can introduce a function φ called the plastic potential, and write a deformation rule as

$$d\varepsilon^p = \frac{\partial \varphi}{\partial \sigma'} \cdot d\lambda \quad (14)$$

Where $d\lambda$ is a factor which determines the magnitude of the plastic strain increments.

Applying the normality condition

$$(d\varepsilon^p / dv_a^p) \cdot (dq/dp) = -1 \quad (15)$$

to Eq. (13), we obtain

$$d\varphi = \frac{d\eta}{\eta_w} + \frac{dp}{p} \quad (16)$$

We can easily understand that $d\varphi$ is the total differential. The plastic potential φ can be expressed by integrating Eq. (16) as

$$\varphi = \int \frac{d\eta}{\eta_w} + \ln p \quad (17)$$

In case of

$$\eta_w = \text{constant} = M \quad (18)$$

we get

$$\varphi = \frac{\eta}{M} + \ln p \quad (19)$$

This function coincides with the plastic potential proposed by Roscoe et al. (Schofield and Wroth, 1968).

One can consider that the plastic potential given by Eq. (19) is applicable to the drained shear because

$$\frac{\partial \varphi}{\partial p} = \frac{1}{p} \left(-\frac{\eta}{M} + 1 \right) \neq 0 \quad \text{for } 0 < \eta < M \quad (20)$$

On the other hand, in case of

$$\eta_w = \eta \quad (dv_a^p = 0) \quad (21)$$

we get

$$\varphi = \ln q \quad (22)$$

The plastic potential given by Eq. (22) can be used for the undrained shear or the constant volume test.

STABILITY CONDITION

Two kinds of stress paths can be considered on the p - q stress space.

$$\text{One is (i) :} \quad d^2 W_s^p \geq 0 \quad (23)$$

$$\text{the other is (j) :} \quad d^2 W_s^p < 0 \quad (24)$$

where

$$d^2 W_s^p = dv_a^p dp + d\varepsilon^p dq \quad (25)$$

The author calls state belonging to (i) as 'stable yielding' and state belonging to (j) as 'unstable yielding'.

Applying relation given by Eq. (13), we can write Eq. (25) as

$$d^2 W_s^p = \left(\frac{dv_a^p}{d\varepsilon^p} \cdot dp + dq \right) d\varepsilon^p = \{ (\eta_w - \eta) dp + dq \} d\varepsilon^p \quad (26)$$

Then, using the following relation

$$d\eta = (dq - \eta dp) \frac{1}{p} \tag{27}$$

we get

$$d^2 W_s^p = q_w \cdot d\varphi \cdot d\varepsilon^p = p \cdot d\varphi \cdot dS_s \tag{28}$$

where

$$q_w = p \cdot \eta_w \tag{29}$$

The quantity q_w and p are always positive. Therefore, the conditions (i) and (j) become equivalent to

$$d\varphi \cdot dS_s \geq 0 \tag{30}$$

and

$$d\varphi \cdot dS_s < 0 \tag{31}$$

respectively.

When the function φ depends upon stress path (there is no plastic potential), we can not specify uniquely the stability conditions.

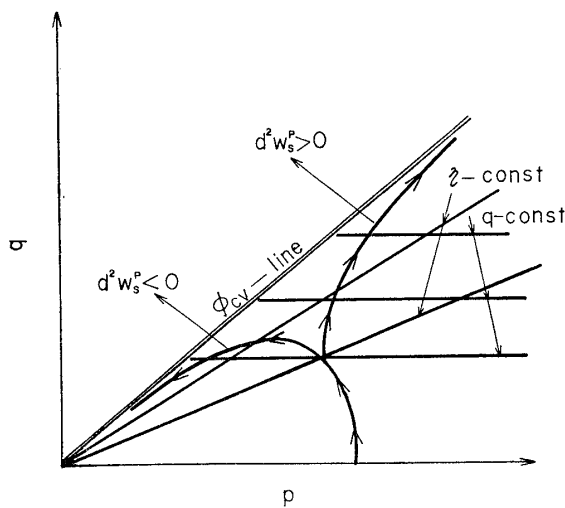


Fig. 1. Undrained stress path

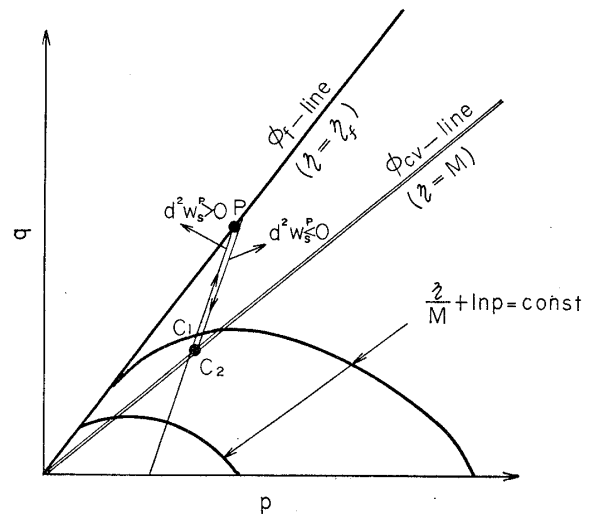


Fig. 2. Drained stress pass

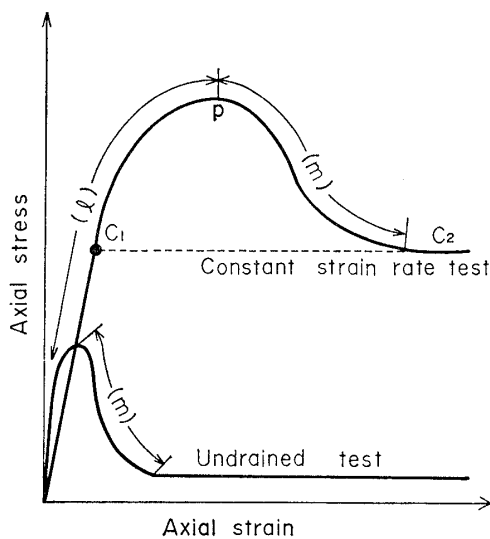


Fig. 3. Typical stress strain relationship

Here we assume that the condition of

$$dS_s = 0 \tag{32}$$

is specified in terms of

$$\eta = \text{constant} \tag{33}$$

and the condition of

$$d\varphi = 0 \tag{34}$$

is specified by

$$\eta + M \ln p = \text{constant (for drained shear)} \tag{35}$$

$$\ln q = \text{constant (for undrained shear or constant volume test)} \tag{36}$$

These assumptions are not unrealistic ones for not only glass beads but also sands and gravels. When we have these two state functions, the shearing deformation can be classified into two

kinds on p - q stress space as shown in Fig.1 and 2. The stable yielding corresponds to the curve (l) and the unstable yielding corresponds to the curve (m) in Fig.3

DEFORMATION RULE

As has been studied, the plastic shear deformation of granular materials can be represented by

$$d\varepsilon^p = p \cdot \frac{\partial \varphi}{\partial \sigma'} \cdot dS_s \quad (37)$$

By the way, Roscoe et al's equation can be expressed as

$$d\varepsilon = p \cdot h \cdot \frac{\partial e}{\partial \sigma'} de \quad (38)$$

$$h = \frac{1}{(M-\eta)(1+e)}$$

e : void ratio

NOTATION

$$\sigma' = \text{effective stress} \quad \sigma' = p \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{q}{3} \begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

$$p = \frac{1}{3}(\sigma_1' + 2\sigma_3') = \text{mean principal stress}$$

$$q = \sigma_1' - \sigma_3' = \text{shear stress}$$

$$\eta = q/p = \text{stress ratio}$$

$$\sigma_1', \sigma_3' = \text{principal stress}$$

$$\sigma_a = \sigma_1 = \text{axial stress}$$

$$\sigma_r = \sigma_3 = \text{radial stress}$$

$$\varepsilon = \text{strain}$$

$$v = \varepsilon_1 + 2\varepsilon_3 = \text{volumetric strain}$$

$$\gamma = \varepsilon_1 - \varepsilon_3 = \text{shear strain}$$

$$\varepsilon_1, \varepsilon_3 = \text{principal strain}$$

$$\varepsilon_a = \varepsilon_1 = \text{axial strain}$$

$$\varepsilon_r = \varepsilon_3 = \text{radial strain}$$

$$v_d = \text{volumetric strain due to dilatancy}$$

$$\varepsilon^p = \text{plastic strain}$$

$$d\varepsilon = \text{increment of strain} \quad d\varepsilon = \frac{dv}{3} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} + \frac{d\gamma}{3} \begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

$$dv = d\varepsilon_1 + 2d\varepsilon_3 = \text{increment of volumetric strain}$$

$$d\gamma = d\varepsilon_1 - d\varepsilon_3 = \text{increment of shear strain}$$

$$d\varepsilon^p = \text{increment of plastic strain}$$

Compression is taken positive for both of stress and strain.

$$dW_s^p = \text{plastic work increment due to shear force}$$

$$S_s = \text{entropy}$$

M = a material constant corresponding to M in the critical state defined by Roscoe et al.

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