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## SIMPLE OPTIMIZATION TECHNIQUES FOR EVALUATING DEFORMATION MODULI FROM FIELD OBSERVATIONS

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### ABSTRACT

A numerical procedure consisting of a combination of the finite element method and the mathematical programming, is proposed for estimating material constants of soil deposit based on field measurements. The procedure allows to correct the unknown material constants in a way that the differences between observed and the estimated values decrease sufficiently. Some examples of estimating elastic constants are also presented.

**Key words** : computer application, elasticity, excavation, field test, finite element method, measurement, natural ground (IGC : E 2/H 0)

### INTRODUCTION

The most difficult aspect in analysing the

elastic deformation of a soil deposit is the estimation of the material constants such as Young's modulus and Poisson's ratio. The

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laboratory investigation and the in-situ testing before construction have their limitations. To compensate for such deficiency observational methods have been adopted widely, in which material constants are estimated from data obtained during field measurements in the construction phase. When the finite element method (FEM) is employed in such a procedure, soil parameters must be calculated from observed nodal displacements and from element stresses. The conventional procedure is as follows:—

- (1) assume trial values of soil parameters;
- (2) calculate the displacements and stresses by FEM;
- (3) compare in-situ measurements with corresponding values obtained by FEM;
- (4) repeat (1) to (3) until the differences between experimental and numerical results decrease sufficiently.

It may be difficult to apply this procedure when the number of unknown parameters increase, as in the case where the soil deposit consists of many layers. A numerical technique is outlined here for estimating Young's modulus and Poisson's ratio from in-situ measurements of displacements, using a mathematical programming technique. A few reports of this type of research have been published to date in geotechnical engineering (see Domski and Wilk, 1979; and Gioda, 1979).

## PROBLEM FORMULATION AND NUMERICAL ANALYSIS

*Problem Formulations:* Consider a geotechnical problem in which Young's modulus  $E$  and Poisson's ratio  $\nu$  are unknown soil parameters to be estimated, based on limited numbers ( $N_d$  and  $N_s$ ) of measured displacements and stresses. Denote  $u^*$  and  $\sigma^*$  with the generic field measurements of displacements and stresses respectively. Assuming hypothetical set of elastic constants,  $E$  and  $\nu$ , one can obtain nodal displacements  $u$  and element stresses  $\sigma$  by means of FEM:

$$u = K^{-1}F \quad (1)$$

$$\sigma = DBu \quad (2)$$

where  $K$ =global stiffness matrix,  
 $F$ =nodal load force,  
 $D$ =matrix to specify stress-strain relation, and  
 $B$ =matrix to calculate strains from nodal displacements.

A set of soil parameters  $E$  and  $\nu$  may reasonably be determined so that it minimizes the sum of squares of differences between calculated and measured quantities if one accepts the assumptions that these measured quantities can be equally weighted regardless of their quality or reliability of measurement and the locations of measured points. Thus, the problem is replaced by an optimization problem with objective function:

$$\text{minimize } J = \sum_{i=1}^{N_d} (u_i^* - u_i)^2 + \sum_{i=1}^{N_s} (\sigma_i^* - \sigma_i)^2 \quad (3)$$

and

$$\text{constraints: } E > 0 \quad (4)$$

$$0 < \nu < 0.5 \quad (5)$$

Solving an optimization problem consisting of Eqs. (3), (4) and (5), a set of elastic constants would be obtained as the overall representative material parameters of the sub-soil being loaded.

*Numerical Analysis:* It is not easy to solve the formulated optimization problem analytically, because of its high nonlinearity in the objective function. There is a numerical procedure called a conjugate gradient technique proposed by Fletcher and Reeves (1964) which can effectively be used in mathematical programming of this optimization problem. The procedure is as follows:—

- (1) set the initial values of decision variables  $\mathbf{x} = (E, \nu)$ ;
- (2) calculate the gradients  $\mathbf{g}_m = (\partial J / \partial E, \partial J / \partial \nu)$  where  $m$  implies iteration number;
- (3)  $\mathbf{d}_m = \mathbf{g}_m + (\mathbf{g}_m^T \mathbf{g}_m) / (\mathbf{g}_{m-1}^T \mathbf{g}_{m-1}) \mathbf{d}_{m-1}$ ;
- (4)  $\mathbf{x}_{m+1} = \mathbf{x}_m + \alpha_m \mathbf{d}_m$ ; where  $\alpha_m$  has to be determined so that  $\mathbf{x}_{m+1}$  minimizes the objective function locally. When  $E$  or  $\nu$  violates the constraints,

take the boundary values;

- (5) repeat (1) to (4) until a chosen approximation becomes satisfactory.

The gradient of the objective function  $J$  on  $E$  is calculated as

$$\begin{aligned} \partial J / \partial E = & 2 \sum_{i=1}^{N_d} \{ (u_i^* - u_i) (-1) \{ (\partial K^{-1} / \partial E) F \}_i \} \\ & + 2 \sum_{j=1}^{N_e} \{ (\sigma_j^* - \sigma_j) (-1) \{ (\partial D / \partial E) B u \}_j \} \\ & + (DB \{ (\partial K^{-1} / \partial E) F \})_j \} \end{aligned} \quad (6)$$

$$\partial K^{-1} / \partial E = (-1) \{ K^{-1} (\partial K / \partial E) K^{-1} \}^T \quad (7)$$

where  $( )_i$  denotes the  $i$ -th element of vector.

As a global stiffness matrix  $K$  is formed by superposing the element stiffness matrix  $K^e$ ,  $\partial K / \partial E$  in Eq. (7) is also obtained by linear superposition of  $\partial K^e / \partial E$ . The gradient on  $\nu$  is given in the same way.

**EXAMPLES**

The following examples, only deal with the case of plane strain.

*Example 1:* Fig. 1 shows a model in which

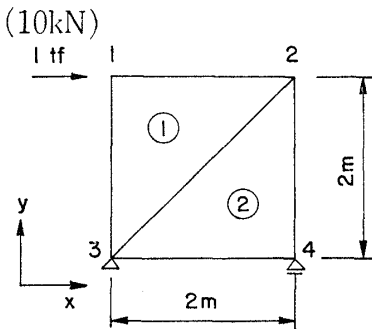


Fig. 1. Model in Examples 1 and 2

Table 1. Displacements computed by FEM (m)

node	horizontal (x-direc.)	vertical (y-direc.)
1	0.000048	0.000012
2	0.000036	-0.000012
3	0.0	0.0
4	0.000012	0.0

Table 2. Stresses computed by FEM (tf/m<sup>2</sup> : ×10kPa)

element	normal stress		shear stress $\tau_{xy}$
	$\sigma_x$	$\sigma_y$	
1	-0.500	0.500	0.500
2	0.500	-0.500	0.500

the strain is constant throughout each triangular element. The displacements and stresses calculated by FEM using the elastic constants,  $E=1 \times 10^5$  tf/m<sup>2</sup> ( $1 \times 10^6$  kPa); and  $\nu=0.2$ , are shown in Tables 1 and 2. Now, suppose these elastic constants  $E$  and  $\nu$  are unknown parameters to be estimated, and some of the displacements in Table 1, for instance, the horizontal and the vertical displacements of the nodal point 2 are measured in the field. Then,

$$\partial K^e / \partial E = \Delta B^T (\partial D / \partial E) B$$

$$\partial D / \partial E = \frac{1-\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$$

where  $\Delta$  is the area of the triangular element.  $\partial K^e / \partial \nu$  can be also calculated in a similar manner. Fig. 2 shows that the correct estimates of  $E$  and  $\nu$  are obtained after only a few iterations. It was confirmed that correct estimates of elastic constants can also be obtained under other conditions, i.e. when other displacements are measured, or when only one displacement is given. In addition, it was found that the appropriate scaling of  $E$  improved the efficiency of convergence, because  $E$  has much higher order of magnitude compared with  $\nu$ . For instance, put

$$E = 100,000 E'$$

and take  $E'$  as a new hypothetical decision variable which has equal order of magnitude as  $\nu$ , then the efficiency of convergence is

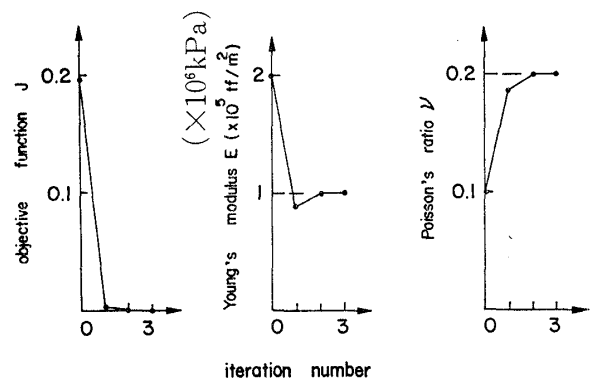


Fig. 2. Iteration behaviour in Example 1

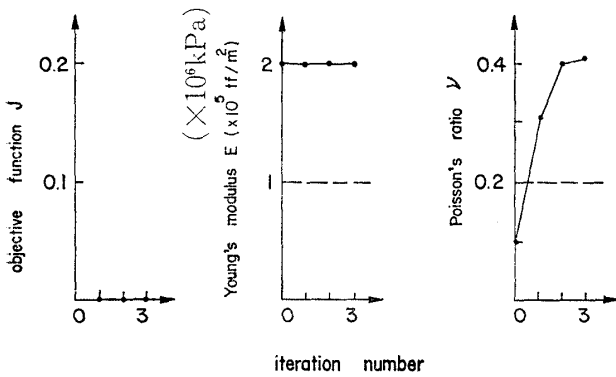


Fig. 3. Iteration behaviour in Example 2

much better than in the case of using  $E$  itself in the computation.

*Example 2:* In Fig.1, suppose that  $E$  and  $\nu$  are unknown parameters, and that some of the stresses in Table 2 are observed, for instance, horizontal and vertical normal stresses of element 1. Other conditions are identical with Example 1. As shown in Fig. 3, correct estimates were not obtained, since stresses are not primarily affected by elastic constants in the plain strain problems. This implies that it may be difficult to estimate elastic constants solely from the field measurements of stresses. From the practical point of view in monitoring techniques, it may be wiser to concentrate the monitoring efforts into the measurement of the deformation of the foundation rather than the measurement of stresses.

*Example 3:* A vertical cut of a soil deposit consisting of a single layer. Fig.4 illustrates the model together with Examples 5 and 6. Unknown parameters are elastic constants

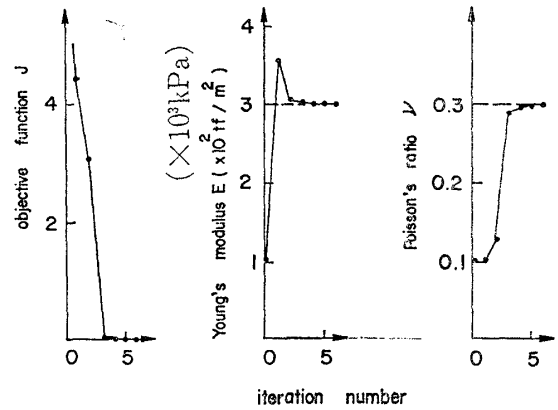


Fig. 5. Iteration behaviour in Example 3. (Case 1)

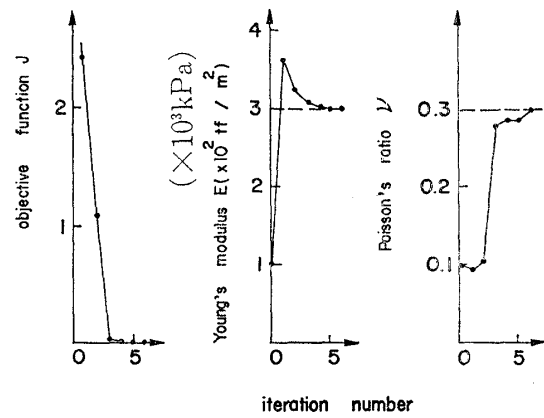


Fig. 6. Iteration behaviour in Example 3. (Case 2)

of the soil deposit.

*Case 1*

Field measurement data:  $u_4, u_5, u_6, u_9, v_1, v_4, v_5, v_6, v_9,$  and  $v_{14}$  where  $u_i$  and  $v_i$  denote horizontal and vertical displacements of node  $i$ . The results are shown in Fig.5.

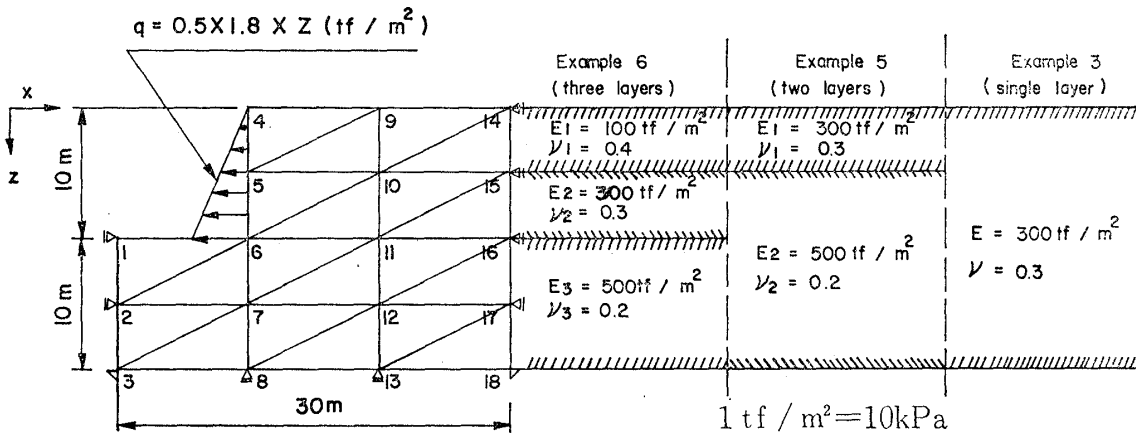


Fig. 4. Models in Examples 3, 5, and 6

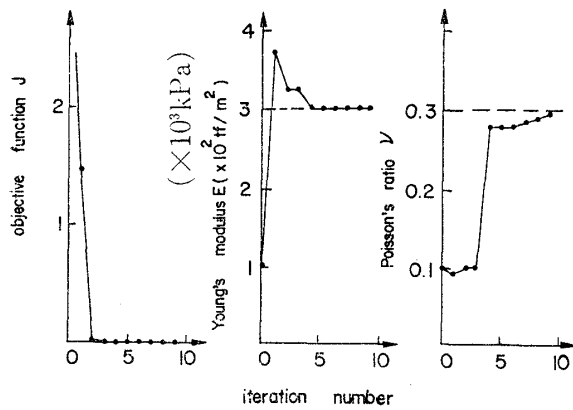


Fig. 7. Iteration behaviour in Example 3. (Case 3)

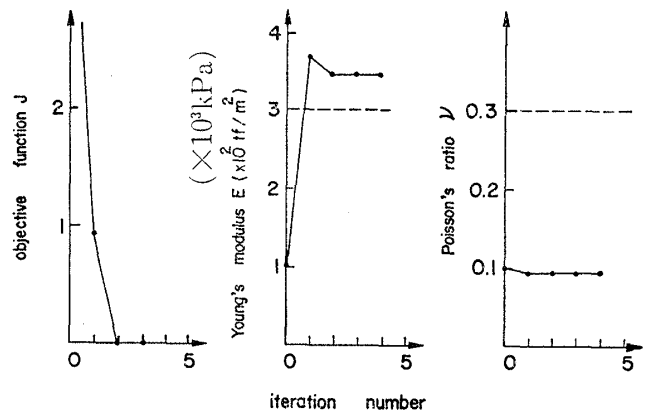


Fig. 10. Iteration behaviour in Example 4. (Case 2)

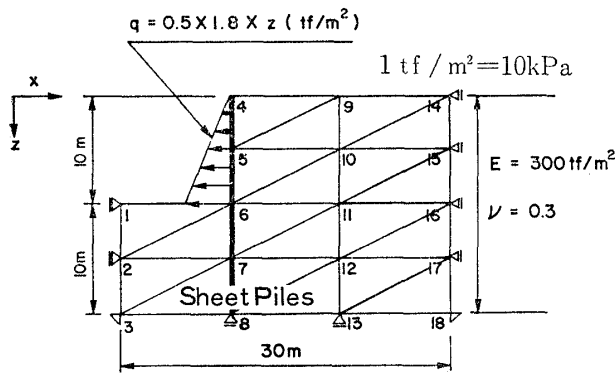


Fig. 8. Model in Example 4

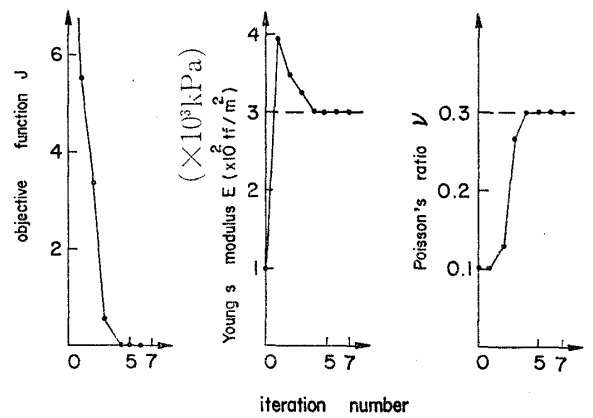


Fig. 11. Iteration behaviour in Example 4. (Case 3)

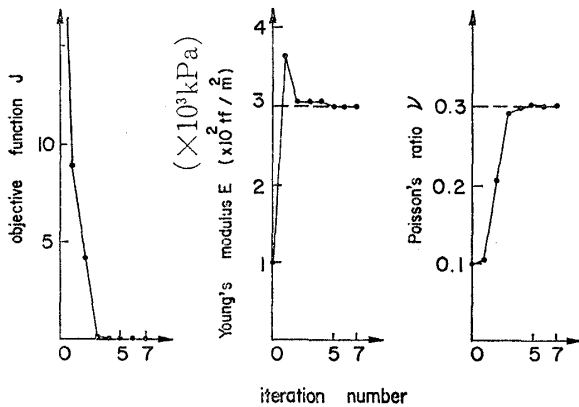


Fig. 9. Iteration behaviour in Example 4. (Case 1)

Case 2

Measured data:  $u_4, u_5, u_6, v_4, v_5,$  and  $v_6$ .  
Results: see Fig. 6.

Case 3

Measured data:  $u_4, u_5,$  and  $u_6$ .  
Results: see Fig. 7.

It is clearly seen from Figs. 5, 6 and 7 that the less the measured informations,

the more the iteration numbers are needed to obtain the correct estimates.

*Example 4:* The excavation of a soil deposit consisting of a layer supported by steel sheet piles. Fig. 8 shows the model in which steel sheet piles are replaced by beam elements in FEM. Unknown parameters are elastic constants of the soil deposit.

Case 1

Measured data: the same as those given in Example 3-Case 1. ( $u_4, u_5, u_6, u_9, v_1, v_4, v_6, v_9$  and  $v_{14}$ ).

Results: see Fig. 9.

Case 2

Measured data: the same as in Example 3-Case 2 ( $u_4, u_5, u_6, v_4, v_5,$  and  $v_6$ ).

Results: see Fig. 10.

*Case 3*

Measured data:  $u_4, u_5, v_1,$  and  $v_{14}$ .

Results: see Fig. 11.

In such a model which contains the beam like elements with a different order of stiffness from the soil deposit, we must carefully select the displacements to be measured in the field. As being obvious from Figs. 10 and 11, the given data of displacements (at the nodal points located at where they are restrained by much stiffer structure) may contribute little to the estimation of the elastic constants of the soil deposit.

*Example 5:* The excavation of a soil deposit consisting of two layers.

Model: see Fig. 4.

Unknown parameters:  $E_1, \nu_1, E_2,$  and  $\nu_2$  (elastic constants of each layer).

Measured data: identical with those in Example 3-Case 1.

Results: see Fig. 12.

*Example 6:* The excavation of a soil deposit consisting of three layers.

Model: see Fig. 4.

Unknown parameters:  $E_1, \nu_1, E_2, \nu_2, E_3,$  and  $\nu_3$ .

The measured data are the same in Example 3-Case 1.

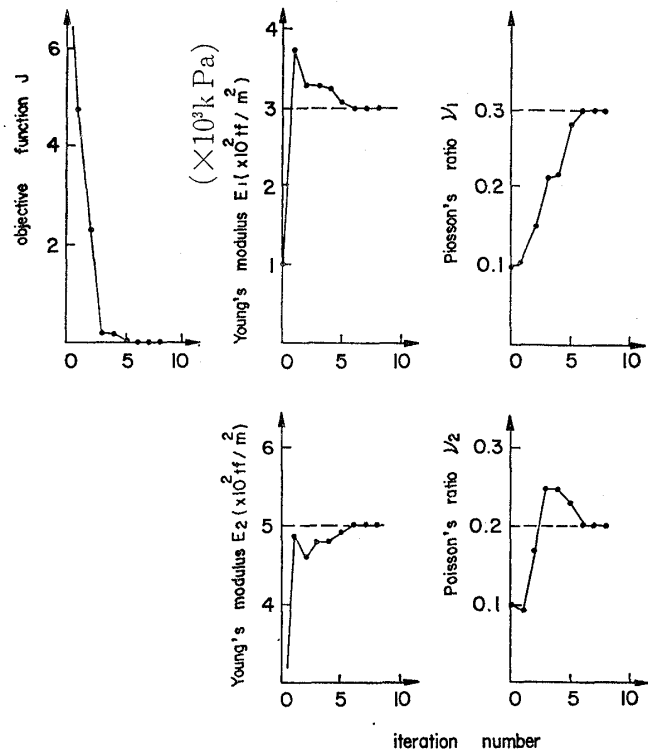
Results: see Fig. 13.

As shown in Fig. 13, the iterative solutions of  $E_3$  and  $\nu_3$ , which are elastic constants of the third layer in Fig. 4, cannot approach the correct values, because the observed displacements were given at the nodal points remote from the deep third layer.

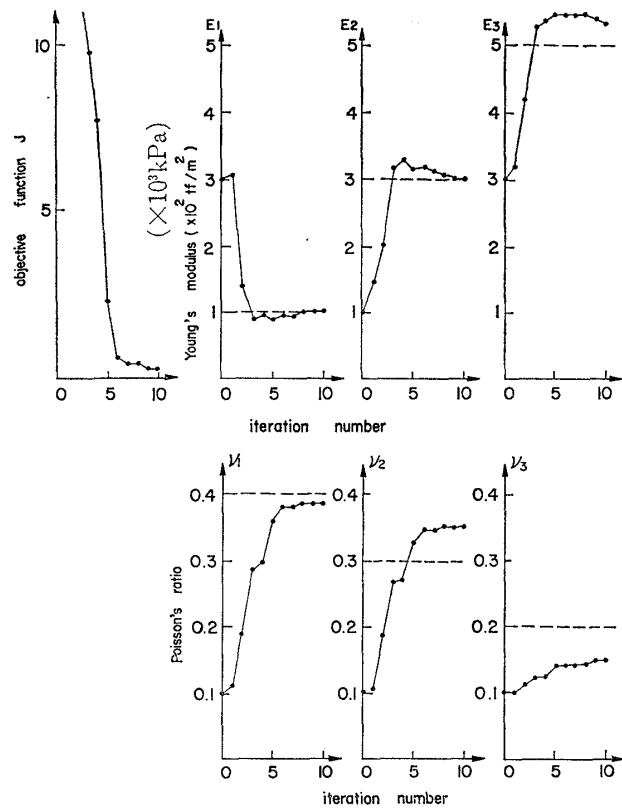
**CONCLUSIONS**

It is shown that a combination of the use of simple elastic finite element method and the conjugate gradient optimization technique can be successfully used in a trial to find the elastic parameters of the subsoil based on the information obtained through the measurements during construction. Concluding remarks are as follows: -

- (1) field measurements of displacements are useful for estimating elastic constants,
- (2) when soil-structure system includes



**Fig. 12. Iteration behaviour in Example 5.**



**Fig. 13. Iteration behaviour in Example 6.**

exceedingly stiff structures, attention should be paid to the location where displacements are to be measured,

(3) the proposed procedure is applicable to soil deposit with multiple layers. However, it will be difficult to obtain the correct elastic constants of deep layers considerably remote from the region where the measuring instruments are installed.

In the end of the conclusions, it must be emphasized that the technique proposed in this note should be proved to be useful through some of the trial applications in the real engineering practices.

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