J. Acoust. Soc. Jpn. (*E*) **19**, 2 (1998)

Scattering by a screen with fluid flow

S. Asghar and Tasawar Hayat

Mathematics Department, Quaid-i-Azam University,
Islamabad, Pakistan

(*Received 20 December 1996*)

Acoustic diffraction from a line source by a semi-infinite plane in the presence of a moving fluid is studied. A finite region in the vicinity of the edge has a soft (pressure release) boundary condition; the remaining part of the semi-infinite plane is rigid. The effects of convection are included at low Mach numbers. The problem which is solved is a mathematical model for a rigid barrier with a soft edge. The far field is calculated using integral transforms, the Wiener-Hopf technique and asymptotic approximations. The physical interpretation of the result is then discussed.

Keywords: Diffraction theory, Rigid screen, Moving fluid, Cylindrical wave, Asymptotic approximations

PACS number: 43. 40. Dx, 43. 40. Rj, 43. 20. Fn

1. INTRODUCTION

Noise from motorways, railways and airports can be shielded by a barrier which intercepts the line of hearing from the noise source to the receiver.[1] An ideal barrier should be such that it is a good attenuator of sound and economical at the same time. The latter requirement is not difficult to appreciate when one considers the miles of motorway in heavily built-up areas. One possible economic barrier construction is to have a rigid barrier (hence reducing transmitted noise) of cheap material which is robust and not necessarily a good attenuator of edge diffracted noise. The barriers having absorbing lining on the surfaces are good attenuator of sound. The provision of a barrier covered completely with an absorbing lining presents several difficulties, among them the cost of construction and maintenance. Since the acoustic field in the shadow region of a barrier (when transmission through the barrier is negligible) is due to diffraction at the edge alone. For this reason Butler[2] suggested that it would be more economic to cover the region only in the immediate vicinity of the edge with absorbent material to reduce the sound level in the shadow region. This technique has potential applications in engine noise shielding by aircraft wings. Recently, T. Okubo and K. Fujiwara[3] discussed the efficiency of noise with an acoustically soft cylindrical edge. In the case of noise radiated by aero engines and inside wind tunnels, it is necessary to discuss acoustic diffraction in the presence of a moving fluid.

The boundary condition on an acoustically absorbing lining is described by an impedance relation between the pressure (p) and the normal velocity fluctuation on the lining surface[4]. *i.e.*,

$$\partial p/\partial n = ik\beta p, \quad \mathcal{R}e(\beta) > 0\,, \tag{1}$$

where the sound wave has time harmonic factor $e^{-i\omega t}$. In Eq. (1), $k=\omega/c$ is the wave number with ω as the angular velocity, c the velocity of sound, n the normal pointing into the lining, $\beta(\rho_0 c/z)$ the complex specific admittance of the acoustic lining,[5] ρ_0 is the density of the ambient medium and z is the acoustic impedance of the surface. We remark that an acoustically hard (or perfectly reflecting) surface is given by $|\beta| \rightarrow 0$ and an acoustically soft surface (pressure fluctuation vanishing on surface) is given by $|\beta| \rightarrow \infty$. The case when the edge region of the barrier surface is soft is considered.

If the wavelength of the sound is much smaller than the length scale associated with the barrier, the diffraction process is governed to all intents and purposes by the solution to the canonical problem of diffraction by a semi-infinite rigid plane with soft edge. The aim here is to solve this mixed boundary value problem for a cylindrical wave in the presence of a moving fluid. The present analysis is also related to Wiener-Hopf solutions for structural elements composed of plates joined end to end,[6–15] although such configurations are quite distnict from the present one. The solution of the problem is obtained using Jones' method and the Wiener-Hopf technique.[16] It is found that results for the still air case can be deduced easily by taking the Mach number ($= U/C$, U is the fluid velocity) to be zero.

2. FORMULATION OF THE PROBLEM

We consider the scattering of a cylindrical wave by a half plane $y=0$, $x\leq 0$ as shown in the Fig. 1. The half plane is assumed to be infinitely thin, and over the interval $-l<x<0$ there is a soft substance satisfying

$$p=0 \qquad (*)$$

on both sides of the surface and the remainder $-\infty<x<-l$, of the half plane is rigid. In Eq. $(*)$ p is the pressure. The whole system is now assumed to be in a fluid moving with velocity U parallel to the x-axis. The perturbation velocity u of the irrotational sound wave can be expressed in terms of the total velocity potential η_t by $u=\text{grad}\ \eta_t$. The resulting pressure in the sound field is given by

$$p=-\rho_0(\partial/\partial t+U\partial/\partial x)\eta_t\,,$$

where ρ_0 is the density of the undisturbed stream. We consider a line source to be located at the position (x_0, y_0). The wave equation satisfied by $\eta_t(x, y)$ in the presence of a line source is

$$\left((1-M^2)\frac{\partial^2}{\partial x^2}+2ikM\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}+k^2\right)\eta_t =\delta(x-x_0)\delta(y-y_0)\,, \tag{1}$$

subject to the following boundary conditions:

$$\partial\eta_t(x, 0^{\pm})/\partial y=0\,, \qquad x<-l\,, \tag{2}$$

$$\eta_t(x, 0^{\pm})=0\,, \qquad -l<x<0\,, \tag{3}$$

$$\left.\begin{array}{l}\eta_t(x, 0^+)=\eta_t(x, 0^-)\,,\\ \partial\eta_t(x, 0^+)/\partial y=\partial\eta_t(x, 0^-)/\partial y\,,\end{array}\right\} \quad x>0\,, \tag{4}$$

where $M=U/c$ is the Mach number. For subsonic flow, $|M|<1$ and for analytic convenience we assume $k=k_r+ik_i$ (k_r, $k_i>0$). It is assumed that a solution can be written in the form

$$\eta_t(x, y)=\eta_0(x, y)+\eta(x, y)\,, \tag{5}$$

where η_0 is solution of inhomogeneous wave equation corresponding to the incident wave and η is the solution of homogeneous wave equation (1) which gives the diffracted field. *i.e.*,

$$\left((1-M^2)\frac{\partial^2}{\partial x^2}+2ikM\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}+k^2\right)\eta_0 =\delta(x-x_0)\delta(y-y_0)\,, \tag{6}$$

$$\left((1-M^2)\frac{\partial^2}{\partial x^2}+2ikM\frac{\partial}{\partial x}+\frac{\partial^2}{\partial y^2}+k^2\right)\eta=0\,. \tag{7}$$

In addition for a unique solution of the boundary value problem Eqs. (3)-(7),[17] we insist that η represents an outward travelling wave as $(x^2+y^2)^{1/2}\to\infty$ and also satisfies the 'edge condition'[18]

$$\left.\begin{array}{ll}\eta_t(x, 0)=O(1)\text{ and} & \\ \partial\eta_t(x, 0)/\partial y=O(x^{-1/2}) & \text{as } x\to 0^+\,,\\ \eta_t(x, 0)=O(1)\text{ and} & \\ \partial\eta_t(x, 0)/\partial y=O((x+l)^{-1/2}) & \text{as } x\to -l\,.\end{array}\right\} \tag{8}$$

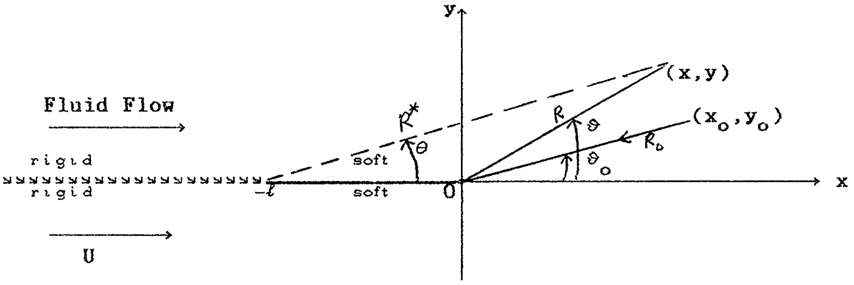

Fig. 1 Geometry of the diffraction problem.

3. SOLUTION OF THE PROBLEM

Since we are dealing with subsonic flow we can make the following real substitutions:

$$\left.\begin{aligned} &x=(1-M^2)^{1/2}X\,, \quad x_0=(1-M^2)^{1/2}X_0\,,\\ &y=Y\,, \quad y_0=Y_0\,,\\ &k=(1-M^2)^{1/2}K\,, \quad \beta=(1-M^2)^{1/2}B\,,\\ &l=(1-M^2)^{1/2}L\,, \end{aligned}\right\}$$

which, together with the substitution

$$\eta_t(x,\ y)=\phi_t(X,\ Y)e^{-iKMX}, \tag{9}$$

reduce the boundary value problem to

$$\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+K^2\right)\phi_0(X,\ Y) =a^*\delta(X-X_0)\delta(Y-Y_0)\,, \tag{10}$$

$$\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+K^2\right)\phi(X,\ Y)=0\,, \tag{11}$$

$$\partial\phi(X,\ 0^{\pm})/\partial Y=-\partial\phi_0/\partial Y\,, \qquad X<-L, \tag{12}$$

$$\phi_0(X,\ 0^{\pm})+\phi(X,\ 0^{\pm})=0 \qquad -L<X<0\,, \tag{13}$$

$$\left.\begin{aligned} &\phi_0(X,\ 0^+)+\phi(X,\ 0^+)\\ &\quad=\phi_0(X,\ 0^-)+\phi(X,\ 0^-)\,,\\ &\partial\phi_0(X,\ 0^+)/\partial Y+\partial\phi(X,\ 0^-)/\partial Y\\ &\quad=\partial\phi_0(X,\ 0^-)/\partial Y+\partial\phi(X,\ 0^-)/\partial Y\,, \end{aligned}\right\} \tag{14}$$

where

$$a^*=\frac{e^{iKMX_0}}{(1-M^2)^{1/2}} \tag{15}$$

The Fourier transform $\overline{\phi}\,(\alpha,\ Y)$ of $\phi\,(X,\ Y)$ can be written as

$$\begin{aligned}\overline{\phi}(\alpha,\ Y)&=\int_{-\infty}^{\infty}\phi(X,\ Y)e^{i\alpha X}dX\,,\\ &=\overline{\phi}_+(\alpha,\ Y)+e^{-i\alpha L}\overline{\phi}_-(\alpha,\ Y)+\overline{\phi}_1(\alpha,\ Y)\,,\end{aligned} \tag{15a}$$

and its inverse

$$\phi(X,\ Y)=\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau}\overline{\phi}(\alpha,\ Y)e^{-i\alpha X}d\alpha\,, \tag{15b}$$

where

$$\left.\begin{aligned} &\overline{\phi}_+(\alpha,\ Y)=\int_0^{\infty}\phi(X,\ Y)e^{i\alpha X}dX\,,\\ &\overline{\phi}_-(\alpha,\ Y)=\int_{-\infty}^{-L}\phi(X,\ Y)e^{i\alpha(X+L)}dX\,,\\ &\overline{\phi}_1(\alpha,\ Y)=\int_{-L}^{0}\phi(X,\ Y)e^{i\alpha X}dX\,, \end{aligned}\right\} \tag{15c}$$

and $\alpha=\sigma+i\tau$. A suitable representation for $\phi_0(X,\ Y)$ which satisfies Eq. (10) is

$$\begin{aligned}&\phi_0(X,\ Y)\\ &\quad=-\frac{a^*}{4i}H_0^{(1)}[K((X-X_0)^2+(Y-Y_0)^2)^{1/2}]\,,\\ &\quad=\frac{a^*}{4\pi i}\int_{-\infty+i\tau}^{\infty+i\tau}\frac{e^{-i\alpha(X-X_0)+i(K^2-\alpha^2)^{1/2}|Y-Y_0|}}{(K^2-\alpha^2)^{1/2}}d\alpha\,.\end{aligned} \tag{16}$$

Making change of variables

$$X_0=R_0\cos\vartheta_0\,, \quad Y_0=R_0\sin\vartheta_0\,, \quad (0<\vartheta_0<\pi)$$

in Eq. (16) and taking $R_0\to\infty$, we obtain using the asymptotic form for the Hankel function

$$\phi_0(X,\ Y)=be^{-iK(X\cos\vartheta_0+Y\sin\vartheta_0)}\,, \tag{17}$$

where

$$b=-\frac{a^*}{4i}\left(\frac{2}{\pi KR_0}\right)^{1/2}e^{i(KR_0-\pi/4)}\,. \tag{18}$$

In Eq. (15c), $\overline{\phi}_\pm(\alpha,\ Y)$ denote functions which are regular in the upper $\mathrm{Im}(\alpha)>-K_i$ and lower $\mathrm{Im}(\alpha)<K_i\cos\vartheta_0$, ($\vartheta_0$ is the angle of incidence) α plane and $\overline{\phi}_1(\alpha,\ Y)$ is an integral function and is analytic in the common region $-K_i<\mathrm{Im}(\alpha)<K_i\cos\vartheta_0$.

Applying the transform (15a) to Eq. (11) we have

$$\overline{\phi}(\alpha,\ Y)=A_1(\alpha)e^{i\kappa Y}\,, \quad Y>0, \tag{19a}$$

$$\qquad\quad=A_2(\alpha)e^{-i\kappa Y}\,, \quad Y<0\,, \tag{19b}$$

where $\kappa=(K^2-\alpha^2)^{1/2}$ is defined on the cut sheet for which $\mathrm{Im}(\kappa)>0$ when $|\mathrm{Im}(\alpha)|<K_i$.

After using Eq.(17), the Fourier transform of Eqs. (12)-(14) give

$$\overline{\phi}_-'(\alpha,\ 0^{\pm})=\frac{Kb\sin\vartheta_0 e^{iKL\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)}\,, \tag{20}$$

$$\overline{\phi}_1(\alpha,\ 0^{\pm})=\frac{ib}{(\alpha-K\cos\vartheta_0)}(1-e^{-i(\alpha-K\cos\vartheta_0)L})\,, \tag{21}$$

and

$$\left.\begin{aligned} &\overline{\phi}_+(\alpha,\ 0^+)=\overline{\phi}_+(\alpha,\ 0^-)=\overline{\phi}_+(\alpha,\ 0)\,,\\ &\overline{\phi}_+'(\alpha,\ 0^+)=\overline{\phi}_+'(\alpha,\ 0^-)=\overline{\phi}_+'(\alpha,\ 0)\,. \end{aligned}\right\} \tag{22}$$

Using Eqs. (15a) and (22) in Eqs. (19), we obtain

$$e^{-i\alpha L}\overline{\phi}_-(\alpha,\ 0^+)+\overline{\phi}_1(\alpha,\ 0^+)+\overline{\phi}_+(\alpha,\ 0)=A_1(\alpha)\,, \tag{23a}$$

$$e^{-i\alpha L}\overline{\phi}_-'(\alpha,\ 0)+\overline{\phi}_1'(\alpha,\ 0^+)+\overline{\phi}_+'(\alpha,\ 0)=i\kappa A_1(\alpha)\,, \tag{23b}$$

$$e^{-i\alpha L}\bar{\phi}_-(\alpha,0^-)+\bar{\phi}_1(\alpha,0^-)+\bar{\phi}_+(\alpha,0)=A_2(\alpha)\,, \tag{24a}$$

$$e^{-i\alpha L}\bar{\phi}_-'(\alpha,0)+\bar{\phi}_1'(\alpha,0^-)+\bar{\phi}_+'(\alpha,0)=-i\kappa A_2(\alpha)\,, \tag{24b}$$

where primes denote differentiation with respect to Y. From Eqs. (22), (23a, b) and (24a, b), we can write

$$\begin{aligned}&-e^{-i\alpha L}\bar{\phi}_-'(\alpha,0)+\bar{\phi}_1'(\alpha,0^\pm)+\bar{\phi}_+'(\alpha,0)\\&\quad=\pm i\kappa(e^{-i\alpha L}\bar{\phi}_-(\alpha,0^\pm)+\bar{\phi}_1(\alpha,0^\pm)+\bar{\phi}_+(\alpha,0))\,.\end{aligned} \tag{25a, b}$$

Using Eqs. (20) and (21) into Eqs. (25a, b) and then adding and subtracting the resulting equations, we get

$$\begin{aligned}&\Lambda_-(\alpha)+e^{i\alpha L}\Lambda_+(\alpha)\\&\quad=-\frac{Kb\sin\vartheta_0 e^{iKL\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K+\alpha}}\,,\end{aligned} \tag{26}$$

$$\begin{aligned}&e^{-i\alpha L}\gamma_-(\alpha)+N(\alpha)\gamma_1(\alpha)+\gamma_+(\alpha)\\&\quad=\frac{-ib}{(\alpha-K\cos\vartheta_0)}(1-e^{-i(\alpha-K\cos\vartheta_0)L})\,,\end{aligned} \tag{27}$$

where

$$\Lambda_-(\alpha)=\frac{-i\sqrt{K-\alpha}}{2}(\bar{\phi}_-(\alpha,0^+)-\bar{\phi}_-(\alpha,0^-))\,, \tag{28a}$$

$$\Lambda_+(\alpha)=\left\{\frac{[\bar{\phi}_1'(\alpha,0^+)+\bar{\phi}_1'(\alpha,0^-)]}{2\sqrt{K+\alpha}}+\frac{\bar{\phi}_+'(\alpha,0)}{\sqrt{K+\alpha}}\right\}, \tag{28b}$$

$$\gamma_-(\alpha)=\frac{1}{2}(\bar{\phi}_-(\alpha,0^+)+\bar{\phi}_-(\alpha,0^-))\,, \tag{29a}$$

$$\gamma_1(\alpha)=\frac{i}{2}(\bar{\phi}_1'(\alpha,0^+)+\bar{\phi}_1'(\alpha,0^-))\,, \tag{29b}$$

$$\left.\begin{aligned}&\gamma_+(\alpha)=\bar{\phi}_+(\alpha,0)\,,\\&N(\alpha)=\frac{1}{\kappa}=N_+(\alpha)N_-(\alpha)\,,\\&N_\pm(\alpha)=(K\pm\alpha)^{-1/2}\,.\end{aligned}\right\} \tag{29c}$$

The Eq. (26) can be written in the form

$$\begin{aligned}&\Lambda_-(\alpha)+\frac{Kb\sin\vartheta_0 e^{iKL\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K+K\cos\vartheta_0}}\\&\quad=-e^{i\alpha L}\Lambda_+(\alpha)-\frac{Kb\sin\vartheta_0 e^{iKL\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)}\\&\qquad\times\left\{\frac{1}{\sqrt{K+\alpha}}-\frac{1}{\sqrt{K+K\cos\vartheta_0}}\right\}.\end{aligned} \tag{30}$$

Now before proceeding further, it is necessary to know that how the various quantities in Eqs. (26) and (27) grow as $|\alpha|\to\infty$. The edge condition (8) means that the transformed functions satisfy the following growth estimates as $|\alpha|\to\infty$:

$$\left.\begin{aligned}&\bar{\phi}_+(\alpha)\approx O(|\alpha|^{-1})\,,\ \bar{\phi}_+'(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&\qquad\qquad\text{in }\mathrm{Im}(\alpha)>-K_1\,;\\&\bar{\phi}_-(\alpha)\approx O(|\alpha|^{-1})\,,\ \bar{\phi}_-'(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&\qquad\qquad\text{in }\mathrm{Im}(\alpha)<K_1\cos\vartheta_0\,;\\&e^{i\alpha L}\bar{\phi}_1(\alpha)\approx O(|\alpha|^{-1})\,,\ e^{i\alpha L}\bar{\phi}_1'(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&\qquad\qquad\text{in }\mathrm{Im}(\alpha)>-K_1\,;\\&\bar{\phi}_1(\alpha)\approx O(|\alpha|^{-1})\,,\quad\bar{\phi}_1'(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&\qquad\qquad\text{in }\mathrm{Im}(\alpha)<K_1\cos\vartheta_0\,.\end{aligned}\right\} \tag{31}$$

Using (31) in (28) and (29), we have, as $|\alpha|\to\infty$,

$$\left.\begin{aligned}&\Lambda_-(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&\gamma_-(\alpha)\approx O(|\alpha|^{-1})\,,\\&\gamma_1(\alpha)\approx O(|\alpha|^{-1/2})\,,\\&N_-(\alpha)\approx O(|\alpha|^{-1})\,,\quad\text{in }\mathrm{Im}(\alpha)<K_1\cos\vartheta_0\,;\end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned}&\Lambda_+(\alpha)\approx O(|\alpha|^{-1})\,,\\&\gamma_+(\alpha)\approx O(|\alpha|^{-1})\,,\\&\gamma_1(\alpha)\approx O(|\alpha|^{-1}e^{-i\alpha L})\,,\\&N_+(\alpha)\approx O(|\alpha|^{-1})\,,\quad\text{in }\mathrm{Im}(\alpha)<-K_1\,.\end{aligned}\right\} \tag{33}$$

Using the asymptotic estimates (32) and (33), we see that left hand side of Eq. (30) is regular, analytic and asymptotic to $|\alpha|^{-1/2}$ as $|\alpha|\to\infty$ in $\mathrm{Im}(\alpha)<K_1\cos\vartheta_0$ and the right-hand side of Eq. (30) is regular, analytic, and asymptotic to $|\alpha|^{-1}$ as $|\alpha|\to\infty$ in $\mathrm{Im}(\alpha)>-K_1$. Hence the analytic function which is a continuation of the left-hand side and the right-hand side of Eq. (30) is zero in the whole plane. Thus from Eqs. (28) and (30) we arrive at

$$\begin{aligned}&\bar{\phi}_-(\alpha,0^+)-\bar{\phi}_-(\alpha,0^-)\\&\quad=-\frac{2ibe^{iKL\cos\vartheta_0}\sqrt{K-K\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K-\alpha}}\,.\end{aligned} \tag{34}$$

Now the second term on the left-hand side of Eq. (27) cannot be split by the standard Wiener-Hopf argument. Without going through the details, which in any case can be found, mutatis mutandis, in Noble's book [Ref. 16), pp. 196–199], we obtain

$$\begin{aligned}S_+^*(\alpha)&=\frac{ib\sqrt{K+K\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K+\alpha}}\\&\quad+\frac{[\sqrt{K+\alpha}\,]^{-1}}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}S_+^*(\varepsilon)}{(\varepsilon+\alpha)}d\varepsilon\,,\\&\qquad-\mathrm{Im}(\alpha)<a<K_1\cos\vartheta_0\,,\end{aligned} \tag{35}$$

$$\begin{aligned}D_+^*(\alpha)&=\frac{ib\sqrt{K+K\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K+\alpha}}\\&\quad-\frac{[\sqrt{K+\alpha}\,]^{-1}}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}D_+^*(\varepsilon)}{(\varepsilon+\alpha)}d\varepsilon\,.\end{aligned} \tag{36}$$

In Eqs. (35) and (36)

$$S_+^*(\alpha)=\gamma_+(\alpha)+\gamma_-(-\alpha)+\frac{ib}{(\alpha-K\cos\vartheta_0)}+\frac{ibe^{iKL\cos\vartheta_0}}{(\alpha+K\cos\vartheta_0)}, \tag{37}$$

$$D_+^*(\alpha)=\gamma_+(\alpha)-\gamma_-(-\alpha)+\frac{ib}{(\alpha-K\cos\vartheta_0)}-\frac{ibe^{iKL\cos\vartheta_0}}{(\alpha+K\cos\vartheta_0)}, \tag{38}$$

and asterisks denote that a pole is present at $\alpha=K\cos\vartheta_0$ in the region $\mathrm{Im}(\alpha)>-K_1\cos\vartheta_0$; otherwise the functions are regular and analytic in this domain.

The integrals appearing in Eqs. (35) and (36) can be solved asymptotically for $KL\gg1$. The physical meaning of this condition is that the soft tip covers a region greater than a wavelength. Now the path of integration in expression (35) is moved vertically in such a way that it crosses the pole $\alpha=K\cos\vartheta_0$ of $S_+^*(\alpha)$ but is not allowed to cross the branch point $\alpha=K$. Thus the third term of (37) will give a residue contribution so that

$$S_+^*=\frac{ib\sqrt{K-K\cos\vartheta_0}}{(\alpha-K\cos\vartheta_0)\sqrt{K+\alpha}}+\frac{ib\sqrt{K-\cos\vartheta_0}\,e^{iKL\cos\vartheta_0}}{(\alpha+K\cos\vartheta_0)\sqrt{K+\alpha}}+\frac{1}{2\pi i\sqrt{K+\alpha}}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}S_+^*(\varepsilon)}{(\varepsilon+\alpha)}d\varepsilon,$$
$$K_1\cos\vartheta_0<a<K_1. \tag{39}$$

Since $KL\gg1$, the dominant term of the asymptotic expansion for the integral appearing in Eq. (39) comes from the region near $\varepsilon=K$ so that

$$\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}S_+^*(\varepsilon)}{(\varepsilon+\alpha)}d\varepsilon\approx\frac{S_+^*(K)}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}}{(\varepsilon+\alpha)}d\varepsilon, \tag{40}$$

provided $S_+^*(K)$ is well behaved in this region, which it will be provided $\vartheta_0\neq0, \pi$. For calculating the integral (40), we wrap the contour of integration around the branch cut $\varepsilon=K$ and then

$$\frac{1}{2\pi i}\int_{-\infty+ia}^{\infty+ia}\frac{e^{i\varepsilon L}(K-\varepsilon)^{1/2}}{(\varepsilon+\alpha)}d\varepsilon=\frac{e^{iKL}}{\pi}\int_0^{\infty}\frac{e^{itL}t^{1/2}dt}{(\varepsilon+K+t)},\quad \mathrm{Im}(\alpha)>-a,$$
$$=W_0[\sqrt{z_1L}], \tag{41}$$

where

$$W_0[\sqrt{z_1L}]=\frac{e^{i(KL+\pi/4)}}{\sqrt{L\pi}}\{1+2i\sqrt{z_1L}\,F(\sqrt{z_1L})\},$$
$$L>0,\ |\arg(z_1)|<\pi,$$
$$z_1=(K+L),$$

and

$$F(|\tilde{m}|)=e^{-i\tilde{m}^2}\int_{|\tilde{m}|}^{\infty}e^{it^2}dt,$$

is the Fresnel integral. For large $\tilde{m}$ the Fresnel integral can be replaced by the dominant term of its asymptotic series: *i.e.*,

$$F(|\tilde{m}|)\approx\frac{i}{2|\tilde{m}|}+O(|\tilde{m}|^{-3}),$$
$$\text{as } |\tilde{m}|\to\infty,\ |\arg(\tilde{m})|<\pi.$$

Thus, from Eqs. (39) and (41) one obtains

$$S_+^*(\alpha)=\frac{ib(K+K\cos\vartheta_0)^{1/2}}{(K+\alpha)^{1/2}(\alpha-K\cos\vartheta_0)}+\frac{S_+^*(K)}{(K+\alpha)^{1/2}}W_0[\sqrt{(K+\alpha)L}]+\frac{ib(K-K\cos\vartheta_0)^{1/2}e^{iKL\cos\vartheta_0}}{(K+\alpha)^{1/2}(\alpha+K\cos\vartheta_0)}. \tag{42}$$

Similarly Eq. (36) gives

$$D_+^*(\alpha)=\frac{ib(K+K\cos\vartheta_0)^{1/2}}{(K+\alpha)^{1/2}(\alpha-K\cos\vartheta_0)}-\frac{D_+^*(K)}{(K+\alpha)^{1/2}}W_0[\sqrt{(K+\alpha)L}]-\frac{ib(K-K\cos\vartheta_0)^{1/2}e^{iKL\cos\vartheta_0}}{(K+\alpha)^{1/2}(\alpha+K\cos\vartheta_0)}, \tag{43}$$

where the terms $S_+^*(K)$ and $D_+^*(K)$ are obtained by putting $\alpha=K$ in expressions (42) and (43) and then solving the resulting expressions for $S_+^*(K)$ and $D_+^*(K)$ respectively.

Substitution of Eqs. (42) and (43) into Eqs. (37) and (38), give expressions for $\gamma_\pm(\alpha)$, from which, in conjunction with equations (29) and (34), one can produce expressions for $\bar{\phi}_+(\alpha, 0)$ and $\bar{\phi}_-(\alpha, 0^{\pm})$. Substituting these latter expressions together, with expressions (21), into Eqs. (23a) and (24a) gives

$$A_1(\alpha)=\frac{ib}{(\alpha-K\cos\vartheta_0)}\left\{\frac{(K+K\cos\vartheta_0)^{1/2}}{\sqrt{K+\alpha}}-\frac{2(K-K\cos\vartheta_0)^{1/2}e^{-i(\alpha-K\cos\vartheta_0)L}}{\sqrt{K-\alpha}}\right\}+\frac{S_+^*(K)-D_+^*(K)}{2(K+\alpha)^{1/2}}W_0[\sqrt{K+\alpha)L}]+\frac{S_+^*(K)+D_+^*(K)}{2(K-\alpha)^{1/2}}W_0[\sqrt{(K-\alpha)L}]e^{-i\alpha L}, \tag{44}$$

$$
\begin{aligned}
A_2(\alpha)=&\frac{ib(K+K\cos\vartheta_0)^{1/2}}{(\alpha-K\cos\vartheta_0)\sqrt{K+\alpha}}\\
&+\frac{S_+^*(K)-D_+^*(K)}{2(K+\alpha)^{1/2}}W_0[\sqrt{(K+\alpha)L}\,]\\
&+\frac{S_+^*(K)+D_+^*(K)}{2(K-\alpha)^{1/2}}W_0[\sqrt{(K-\alpha)L}\,]e^{-i\alpha L}.
\end{aligned}
\tag{45}
$$

The solution to the boundary value problem is now known and is given by

$$
\phi_t=\phi_0(X,\ Y)+\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau}A_1(\alpha)e^{-i\alpha X+i\kappa Y}d\alpha,\quad Y>0,\tag{46}
$$

$$
=\phi_0(X,\ Y)+\frac{1}{2\pi}\int_{-\infty+i\tau}^{\infty+i\tau}A_2(\alpha)e^{-i\alpha X-i\kappa Y}d\alpha,\quad Y<0,\tag{47}
$$

where $-K_i<\tau<K_i\cos\vartheta_0$. The scattered far field in the expressions (46) and (47) are asymptotically evaluated for KR large by an application of the modified saddle point method because of the presence of the pole $\alpha=K\cos\vartheta_0$.[18] Thus, for $0<\vartheta_0<\pi$ and using Eq. (9) without going through the detailed analysis, one obtains

$$
\left.
\begin{aligned}
\eta_t(R,\ \vartheta)&=I+RF+D_1, &&\text{for } \pi-\vartheta_0<\theta<\pi,\\
&=I-RF+D_1, &&\text{for } \theta<\pi-\vartheta_0<\Theta,\\
&=I+D_1, &&\text{for } 0<\Theta<\pi-\vartheta_0,\\
&=I+D_2, &&\text{for } \vartheta_0-\pi<\Theta<0,\\
&=D_2, &&\text{for } -\pi<\Theta<\vartheta_0-\pi,
\end{aligned}
\right\}
\tag{48}
$$

where $X=R\cos\Theta,\ Y=R\sin\Theta,\ -\pi\le\Theta\le\pi,$

$$
\begin{aligned}
I&=\text{incident wave}\\
&=be^{-iKR\cos(\Theta-\vartheta_0)-iKMX},
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
RF&=\text{reflected wave}\\
&=be^{-iKR\cos(\Theta+\vartheta_0)-iKMX},
\end{aligned}
\tag{50}
$$

D_1=the diffracted edge wave in the illuminated region

$$
\begin{aligned}
&=\frac{e^{-iKM(X-X_0)+iK(R+R_0)}}{4\pi K\sqrt{RR_0}(1-M^2)^{1/2}}\\
&\quad\times\frac{2|Q|F(|Q|)(1+\cos\Theta)^{1/2}(1+\cos\vartheta_0)^{1/2}}{(\cos\Theta+\cos\vartheta_0)}\\
&\quad+\frac{e^{-iKM(X-X_0)+iK(R^*+R_0)}}{4\pi K(R_0R^*)^{1/2}(1-M^2)^{1/2}}\\
&\quad\times\frac{2|Q_1|F(|Q_1|)e^{iKL\cos\vartheta_0}(1-\cos\vartheta_0)^{1/2}}{(\cos\vartheta+\cos\vartheta_0)[2(1-\cos\vartheta)^{1/2}]^{-1}}\\
&\quad+\frac{e^{i(KR-\pi/4)-iKMX}}{(2\pi KR)^{1/2}}G_1(\Theta,\ \vartheta_0)\\
&\quad+\frac{e^{i(KR^*-\pi/4)-iKMX}}{(2\pi KR^*)^{1/2}}G_2(\vartheta,\ \vartheta_0),
\end{aligned}
\tag{51}
$$

D_2=the diffracted edge wave in the shadow region

$$
\begin{aligned}
&=\frac{e^{-iKM(X-X_0)+iK(R+R_0)}}{4\pi K\sqrt{RR_0}(1-M^2)^{1/2}}\\
&\quad\times\frac{2|Q|F(|Q|)(1+\cos\Theta)^{1/2}(1+\cos\vartheta_0)^{1/2}}{(\cos\Theta+\cos\vartheta_0)}\\
&\quad+\frac{e^{i(KR-\pi/4)-iKMX}}{(2\pi KR)^{1/2}}G_1(\Theta,\ \vartheta_0)\\
&\quad+\frac{e^{i(KR^*-\pi/4)-iKMX}}{(2\pi KR^*)^{1/2}}G_2(\vartheta,\ \vartheta_0),
\end{aligned}
\tag{52}
$$

$$
Q=\left(\frac{KR}{2}\right)^{1/2}\left(\frac{\cos\Theta+\cos\vartheta_0}{\sin\Theta}\right),
$$

$$
Q_1=\left(\frac{KR^*}{2}\right)^{1/2}\left(\frac{\cos\vartheta+\cos\vartheta_0}{\sin\vartheta}\right),
$$

$$
R^{*2}=R^2+L^2+2RL\cos\Theta,
$$

$$
\vartheta=\operatorname{sgn}(\Theta)\cos^{-1}\left(\frac{L+R\cos\Theta}{R^*}\right),
$$

$$
\begin{aligned}
G_1(\Theta,\ \vartheta_0)=&\frac{W_0[(KL(1-\cos\Theta))^{1/2}]}{(2K)^{1/2}(1+\cos\Theta)^{-1/2}}\\
&\times\left\{\frac{K}{\sqrt{2}}(S_+^*(K)-D_+^*(K))\right\},
\end{aligned}
\tag{53}
$$

$$
\begin{aligned}
G_2(\vartheta,\ \vartheta_0)=&\frac{W_0[(KL(1+\cos\vartheta))^{1/2}]}{(2K)^{1/2}(1-\cos\vartheta)^{-1/2}}\\
&\times\left\{\frac{K}{\sqrt{2}}(S_+^*(K)+D_+^*(K))\right\}.
\end{aligned}
\tag{54}
$$

4. CONCLUDING REMARKS

The principal result obtained in this paper, Eq. (48) gives the total velocity potential of a cylindrical wave in a moving fluid. The expression for the total acoustic far field, gives a picture of the physical fields. I represents the incident cylindrical wave directly coming from a line source $(x_0,\ y_0)$ and RF is the reflected wave. The terms D_1 and D_2 represent the fields diffracted from the edges $x=0$ and $x=-l$. Further, the terms involving R and Θ represent the contribution to the diffracted field from the edge $x=0$. The terms involving R^* and ϑ give the field by the joint $x=-l$. From the first of expressions (48) it will be noticed that the sign of the reflected wave corresponds to reflection from the rigid part of the barrier. In the second of expressions (48) the reflected wave sign corresponds to reflection from the soft region of the barrier. It is also worth pointing out that the strength of the field dies down as $1/\sqrt{R_0}$. In addition, the magnitude of the far field on the source side of the semi infinite plane seems to depend more critically on kl, l representing the length of the soft region and k the wave number. This is due to the constructive/destructive interaction of the edge fields with the reflected wave from the hard region $-\infty<x<-l$, and the reflected wave from the region $-l<x<0$.

We very much like to present here the physical interpretation of Eqs. (51) and (52). As in case of moving fluid

$$R=(X^2+Y^2)^{1/2}, \quad R^*=[(L+X)^2+Y^2]^{1/2}, \tag{55a, b}$$

and for still fluid ($M=0$)

$$\hat{R}=(x^2+y^2)^{1/2}, \quad \hat{R}^*=[(l+x)^2+y^2]^{1/2}, \tag{56a, b}$$

Since $X=x/(1-M^2)^{1/2}$, $Y=y$, $l+x=(L+X)(1-M^2)^{1/2}$, therefore Eqs. (55a, b) can be rewritten as

$$R(1-M^2)^{1/2}=[x^2+(1-M^2)y^2]^{1/2}, \tag{57a}$$

$$R^*(1-M^2)^{1/2}=[(l+x)^2+(1-M)^2y^2]^{1/2}. \tag{57b}$$

From Eqs. (56a,b) and (57a,b), we can make the comparison of moving fluid situation to still fluid situation. We conclude that the motion of the fluid contracts R and R^* by the factor $(1-M^2)^{1/2}$ and that the field is enhanced as as a result of fluid motion. We further note that the fluid velocity U appears as $|U|^2$ in the factor $(1-M^2)$ and hence the enhancement of the field is independent of the direction of the flow. We also observe that the wave profile at $y=y_0$ moves along the direction of x-axis at the velocity $c+U$ due to the fact that the fluid is moving in the x direotion.

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