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Comparison of loudness functions suitable for drawing equal-loudness-level contours

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Abstract: Five kinds of loudness functions were examined to determine a suitable function for drawing equal-loudness-level contours based on experimental data. First, equal-loudness relations were measured between 125-Hz and 1-kHz pure tones from 70 phons down to 5 phons and thresholds of hearing were measured at both frequencies. Then, five model equations expressing the equal-loudness relation were derived from the five loudness functions, and these equations were fitted to the data. The results showed that three of them could well explain the equal-loudness relation down to 5 phons. Among the three functions, the loudness function proposed by Zwislocki and Hellman [*J. Acoust. Soc. Am.*, **32**, 924 (1960)] and Lochner and Burger [*J. Acoust. Soc. Am.*, **33**, 1705–1707 (1961)] was regarded as the most appropriate function for drawing equal-loudness-level contours, because the number of parameters of the function was fewer than that in the other two and the threshold of hearing could be used for the fitting as a datum of the equal-loudness relation, resulting in a more stable estimation of the parameters of the model equation.

Keywords: Equal-loudness-level contour, Loudness function, Threshold of hearing, Pure tone, ISO 226

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1. INTRODUCTION

An equal-loudness-level contour is a curve that ties up sound pressure levels having equal loudness as a function of frequency. In other words, it expresses a frequency characteristic of loudness rating. One of the most famous sets of equal-loudness-level contours is that reported by Robinson and Dadson [1], which has been standardized as an international standard, ISO 226 [2]. Recent studies [3–11], however, have shown large discrepancies between the experimental data and the contours by Robinson and Dadson [1]. The remarkable differences between them are that the recent experimental data at frequencies below 1 kHz are significantly higher than the corresponding contours of Robinson and Dadson [1]. These differences are nearly as high as 15 dB. Therefore, it is extremely important that new set of equal-loudness-level contours is determined based on the experimental data of recent studies.

To draw equal-loudness-level contours from the experimental data, interpolation along the frequency axis is necessary, because all experimental data are discretely given for specific frequencies and for loudness levels. Moreover, as such data also shows some variances among subjects and studies, appropriate smoothing is required. When conducting such interpolation and smoothing, direct fitting of polynomial regression or spline functions to experimental data would, of course, be possible. In such simple methods, however, a contour for a loudness level is drawn independently of other contours, resulting in an ill-shaped representation as a set of equal-loudness-level contours. To avoid this, use of knowledge of a loudness

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function, i.e., a function representing the growth of loudness as a function of sound intensity level or sound pressure level, may be helpful. Loudness of a pure tone is a function of frequency and sound pressure level. Therefore, if the dependency of the loudness function on frequency is expressed by the changes of parametric variables in the loudness function depending on frequency, interpolation and smoothing of the parametric variables along the frequency axis make it possible to draw appropriate equal-loudness-level contours for any loudness level.

To date, several model functions have been proposed to describe loudness growth as a function of sound pressure [12–20]. All of these functions are essentially based on the power law [21] and show distinctive differences only at low sound pressure levels. Which function provides a correct description of loudness at low levels is an important unresolved question, because the recent experimental data show that the level difference between the hearing thresholds and the equal-loudness levels of 20 phons are generally larger than that between the equal-loudness levels of 20 phons and those of 40 phons in low frequency region.

Ross [22] measured equal-loudness relations for frequencies from 20 Hz to 5 kHz with the method of adjustment and examined the goodness of fit to the measured data for three loudness functions. Only three subjects, however, participated in the study. This seems to be insufficient to discuss a general law. Whittle *et al.* [23] studied loudness growth in a very low-frequency range. They examined three loudness functions by comparing the goodness of fit to the measured loudness growth data. However, they did this only at a frequency of 12.5 Hz, too low to examine pure loudness sensation. Humes and Jesteadt [24] studied loudness function in an attempt to account for growth of loudness near the threshold. They used, however, the equal-loudness-level contours proposed by Robinson and Dadson [1], which would contain large errors [9], for examining two loudness functions. Buus *et al.* [25] also studied loudness function at low levels. However, they fitted only Zwislocki's loudness function [19] to the experimental data.

Therefore, we have to look for the best loudness function appropriate for drawing equal-loudness-level contours. Requirements of the loudness function for this purpose are as follows: (1) It should be able to well describe the equal-loudness relationship, especially at low levels. (2) It should be able to estimate proper parametric values in the loudness function from the recent experimental data which include some variances and which are not enough in number at all frequencies.

Although data below 20 phons are crucial to examine the performance of the loudness functions, no data for equal-loudness relationships were contained in reports of the recent experiments [3–11]. Herein, therefore, we first measured equal-loudness relations between 125-Hz and 1-kHz pure tones from 70 phons down to 5 phons. The latter frequency was selected as the standard frequency for loudness levels, i.e., phons, and the former as the frequency where the form of the loudness function seems to be largely different from that at 1 kHz, in other words, where the equal-loudness relations on the plane of SPL v.s. phon are not on a straight line. Hellman *et al.* [26] reported that equal-loudness relations from 1 kHz to 10 kHz were almost on a straight line with a slope of 1.0. Therefore, we did not measure equal-loudness relations at frequencies higher than 1 kHz.

2. MEASUREMENT OF EQUAL-LOUDNESS RELATIONS

2.1. Experimental Setup

The experiments were performed in an anechoic room at Sendai National College of Technology. The room has an interior width of 5.4 m, a depth of 4.7 m, and a height of 2.6 m. Stimuli were presented through a loudspeaker with a diameter of 20 cm (Fostex, SLE24W). A subject sat on a chair facing the frontal axis of the loudspeaker. The distance between the subject and the loudspeaker was 2.0 m. The sound pressure level was calibrated at the midpoint between the subject's ears without a subject and a chair.

2.2. Subjects

Ten male subjects, 19 to 32 years of age, participated in the experiments. Prior to the experiments, monaural hearing threshold levels were measured with an audiometer. All of the subjects were judged as otologically normal according to the preferred test conditions specified for the measurement of the threshold of hearing [27].

2.3. Experimental Procedure

2.3.1. Measurement of thresholds of hearing

The thresholds of hearing for a pure tone at 125 Hz and those at 1 kHz were measured prior to the measurement of equal-loudness relations.

The bracketing method [28] was used for the measurement with a step size of 2 dB. A signal with a duration of 1.0 s including 50-ms rise and decay times was presented repeatedly. The pause between the signals was changed randomly from 600 ms to 1,500 ms to prevent the subject from using steady timing of the presentation as a cue. The subjects were asked to respond by pushing a button when the stimulus was audible.

In a session, four descending and three ascending sequences were alternately tested, starting with a descending sequence. In a descending sequence, when two consecutive "inaudible" responses were obtained, the signal level of the last "audible" response was employed as the judgment of threshold. On the other hand, the level of the first response in two consecutive "audible" responses was employed as the judgment of threshold in an ascending sequence. In the first descending sequence, the initial sound pressure level of the signal was set 20 dB higher than the normal free-field threshold of hearing specified in ISO 389-7 [29]. In other sequences, the initial level was set randomly 6 to 12 dB higher/lower than the judgment obtained in the previous ascending/descending sequence. The average of the last six judgments of threshold was considered to be the threshold of hearing for the subject.

2.3.2. Measurement of equal-loudness relations

The randomized maximum likelihood sequential procedure with 2AFC was used [10,30]. This method satisfies the requirements described in the preferred test conditions recommended by ISO/TC43/WG1 [31] and is one of the most robust methods against the range effect [10]. Two pure tones were presented sequentially to a subject; one of the two was a 1 kHz pure tone and the other was a 125-Hz pure tone. The order of presentation in a pair was randomly chosen. The duration of each pure tone was 1.0 s including 50-ms rise and decay times. The inter-stimuli interval was 0.7 s and the interval between two successive pairs of stimuli was 1.5 s. The subjects were asked to judge which stimulus in a pair was louder.

In a single run, the sound pressure level of 1 kHz was kept constant at one of nine levels, i.e., 5, 10, 15, 20, 25, 30, 40, 50 and 70 dB, so that the equal-loudness levels of a 125-Hz pure tone at 5, 10, 15, 20, 25, 30, 40, 50 and 70 phons could be determined. The sound pressure level of the 125-Hz pure tone was selected adaptively from 21 levels in 2-dB steps ranging from −20 to +20 dB around a provisional PSE (Point of Subjective Equality) determined by preliminary experiments. As the randomized maximum likelihood sequential procedure was used, the sound pressure levels of the 125-Hz pure tone were given as follows: The levels for the first and the second presentations in a run were the highest and the lowest of the range, respectively. For the third presentation and later, the sound pressure level of the 125-Hz pure tone was randomly selected within ±3 levels, i.e., within ±6 dB, around the level showing maximum likelihood as PSE calculated from responses up to the current phase in the run. The logistic function with the spread parameter of 5 dB was assumed to be the psychometric function for calculating the likelihood during a run. Fifty pairs of stimuli were presented in a run. After the run, PSE for the loudness level for the subject was calculated from all responses in the run by using the method of maximum likelihood [32]. In this estimation, the cumulative normal distribution function was assumed to be a psychometric function, whose mean corresponds to PSE.

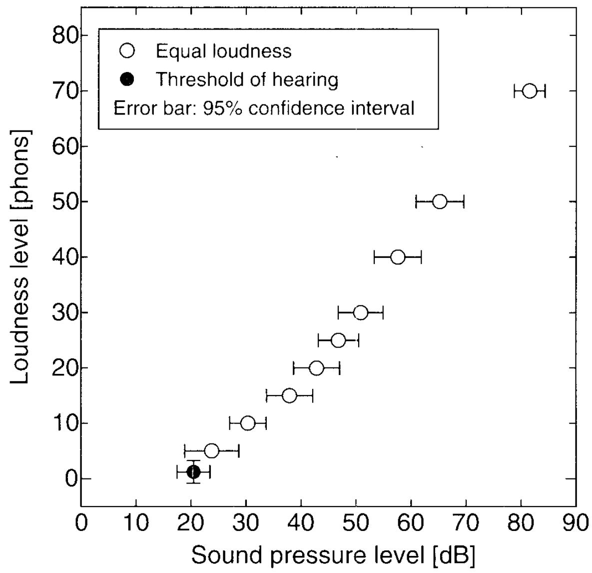

Fig. 1 Equal-loudness levels for a 125-Hz pure tone. Open circles represent average PSEs and the filled circle represents an average threshold of hearing. The horizontal bars and the vertical bar show 95% confidence intervals.

2.4. Experimental Results

Figure 1 shows the average results for the ten subjects. The abscissa shows the sound pressure level of the 125-Hz pure tone while the ordinate shows the loudness level of this tone in phons. The open circles show the averages of the equal-loudness levels and the filled circle shows the average of the thresholds of hearing. The horizontal bars and the vertical bar show 95% confidence intervals. The averages of the thresholds of hearing at the frequency of 125 Hz and 1 kHz were 20.5 dB SPL and 1.2 dB SPL, respectively.

The results show that the average equal-loudness levels seem to lie mainly on a straight line above 20 phons. The slope above 20 phons is 1.30 by linear fitting. This means that Stevens' power law is valid for this level of range and that the exponent of the power function at 125 Hz is 1.30 times larger than that for 1 kHz, resulting in narrower spacing of equal-loudness-level contours at 125 Hz above 20 phons. Below 20 phons, however, the equal-loudness levels deviate from the straight line and asymptotically approach the hearing threshold. This means that Stevens' power law requires some modification in this level range. Moreover, it also implies that the equal-loudness-level contours at low loudness levels exhibit wider spacing than those at higher loudness levels. For example, the difference of PSEs between the equal-loudness level for 5 and 20 phons is 19.0 dB, while that between 25 and 40 phons is 10.8 dB. This tendency is consistent with that observed in our previous study [4].

3. MODELS OF EQUAL-LOUDNESS RELATION

3.1. Loudness Functions

If Stevens' power law stands, the loudness function of a pure tone is given by

$$S = kP^{2\alpha}, \tag{1}$$

where S is loudness, P is the sound pressure of the pure tone, k is a dimensional constant and α is the exponent of the power law. As described above, however, this simple power law does not explain the loudness growth at low levels. Therefore, several derivatives of Stevens' power law have been proposed [12–20].

The function proposed by Ekman [12], Luce [13], Stevens [14] and Sharf and Stevens [15] and that by Zwislocki and Hellman [16] and Lochner and Burger [17] are as follows, respectively:

$$S = k(P^2 - P_\mathrm{t}^2)^\alpha, \tag{2}$$

$$S = k(P^{2\alpha} - P_\mathrm{t}^{2\alpha}), \tag{3}$$

where P_t is the threshold of hearing denoted in sound pressure. In Eqs. (2) and (3), a constant corresponding to the threshold is subtracted from the value given by the original power law. The difference between these equations lies in the domain where the subtraction is executed: the constant is subtracted in the physical domain in Eq. (2), while it is subtracted in the loudness domain in Eq. (3).

Zwicker [18] considered that the power law stands between the sum of the excitation by a sound and the internal noise and the sum of the specific loudness of the sound and that of the internal noise. He derived another loudness function by applying the above concept. This function originally describes the relation between the specific loudness and the excitation. We consider that this function may be rewritten as shown in the following equation to describe the relation between loudness and intensity so long as a pure tone or a narrow-band noise is considered:

$$S = k\{(P^2 + CP_\mathrm{t}^2)^\alpha - (CP_\mathrm{t}^2)^\alpha\}, \tag{4}$$

where C is the noise-to-tone energy ratio required for a just detectable tone embedded in intrinsic or masking noise. Zwislocki [19] introduced internal noise into Eq. (3), resulting in the same function as Eq. (4). It should be noted that the loudness at the threshold of hearing given by Eq. (4) is not null and depends on the parameters. This idea is supported by experiments by Hellman and Zwislocki [33,34], the result of their study showing that the loudness of a pure tone at threshold is not null and slightly depends on its frequency. Buus *et al.* [25] also clarified that loudness of a pure tone at the hearing threshold was not null.

Zwicker [20] further modified Eq. (4) so that the loudness at the threshold is null, resulting in the following loudness function:

$$S = k\left[\{P^2 + (C-1)P_\mathrm{t}^2\}^\alpha - (CP_\mathrm{t}^2)^\alpha\right]. \tag{5}$$

3.2. Equal-loudness Relation Based on a Loudness Function

Our basic assumption is that the same loudness function is applicable to pure tones with different frequencies. Thus, if the loudness of an f-Hz pure tone is equal to that of an r-Hz pure tone (reference tone usually of 1 kHz) with the sound pressure of P_r, the sound pressure of P_f at the frequency of f Hz is given by the following functions which correspond to Eqs. (1) to (5), respectively:

$$P_f^2 = K_f^{1/\alpha_f} P_r^{2\alpha_r/\alpha_f}, \tag{6}$$

$$P_f^2 = K_f^{1/\alpha_f} (P_r^2 - P_{r\mathrm{t}}^2)^{\alpha_r/\alpha_f} + P_{f\mathrm{t}}^2, \tag{7}$$

$$P_f^2 = \{K_f(P_r^{2\alpha_r} - P_{r\mathrm{t}}^{2\alpha_r}) + P_{f\mathrm{t}}^{2\alpha_f}\}^{1/\alpha_f}, \tag{8}$$

$$P_f^2 = \left[K_f\{(P_r^2 + C_r P_{r\mathrm{t}}^2)^{\alpha_r} - (C_r P_{r\mathrm{t}}^2)^{\alpha_r}\} + (C_f P_{f\mathrm{t}}^2)^{\alpha_f}\right]^{1/\alpha_f} - C_f P_{f\mathrm{t}}^2, \tag{9}$$

$$P_f^2 = \left(K_f\left[\{P_r^2 + (C_r - 1)P_{r\mathrm{t}}^2\}^{\alpha_r} - (C_r P_{r\mathrm{t}}^2)^{\alpha_r}\right] + (C_f P_{f\mathrm{t}}^2)^{\alpha_f}\right)^{1/\alpha_f} - (C_f - 1)P_{f\mathrm{t}}^2, \tag{10}$$

where suffixes r and f indicate that the parameters are for the reference frequency (usually 1 kHz) and f Hz, respectively, and K_f represents k_r/k_f. Similar derivations have been carried out by Ross [22] and by Humes and Jesteadt [24].

The equal-loudness relation between P_r^2 and P_f^2 can be drawn by these equations. Equation (6) is expressed as a straight line in the $\log P_r - \log P_f$ plane, while the other four equations are illustrated as curvilinear lines at low sound pressure levels. It should be noted that in Eq. (9) P_f is not equal to $P_{f\mathrm{t}}$ when P_r is at the hearing threshold, $P_{r\mathrm{t}}$, unlike in the other equations. This means that the loudness at the hearing threshold is not constant across frequencies [33,34].

It should be also noted that the plots in Fig. 1 are illustrated as a function of the sound pressure level of a 125-Hz pure tone. Equations (6) to (10) are, however, expressed as functions of the sound pressure of a 1-kHz pure tone corresponding to phon. This is a convenient way to represent the equal-loudness-level contours of a specific phon level. In Fig. 1, however, the opposite way of expressing function was chosen because loudness matching functions are usually drawn as in this figure [1,22,26].

3.3. Fitting of the Loudness Functions to the Experimental Data

To fit Eqs. (6) to (10) to the present experimental data, the exponent at 1 kHz (α_r) is required. Stevens [35] reported that the exponent at the 1-kHz pure tone was 0.3. From more detailed experiments, however, other investigators reported the exponent to be 0.23 [36], 0.27 [17,37–39] or 0.29 [40]. Therefore, in the present report, the exponents from 0.20 to 0.30 were adapted for the fitting.

The fitting was performed by the modified Mayquandt method, which is an algorithm of the nonlinear least squares method, provided by a computer program package, SALS (Statistical Analysis with Least Squares fitting) [41]. In the fitting, the reciprocal number of the variation for each equal-loudness (PSE) datum was used as weighting for the evaluation of square errors.

Equation (6) was fitted to the data by estimating K_f and α_f. For Eqs. (7) and (8), three parameters, namely, K_f, α_f and P_{ft}, were estimated while P_{rt} was fixed at the experimental value, 1.2 dB SPL. As for Eq. (9), K_f, α_f and C_f were estimated while P_{rt} and P_{ft} were fixed at the present experimental results, 1.2 and 20.5 dB SPL, respectively, and C_r was fixed at 2.5 [19]. Equation (10) was fitted to the data under the same conditions as those for Eq. (9), except that P_{ft} was an estimated value. Equations (6), (7), (8) and (10) were fitted to the present experimental data of equal-loudness relations as well as those of the threshold of hearing, while Eq. (9) was fitted only to the data of equal-loudness relations because, as stated in the previous subsection, loudness at the threshold of hearing is regarded as not being constant across frequencies for this equation. Due to the same reason, the threshold of hearing of the f-Hz pure tone, P_{ft}, was estimated in the cases of Eqs. (6), (7), (8) and (10), but not for Eq. (9).

Figure 2 shows the results of the fitting of these equations ($\alpha_r = 0.27$) to the experimental data shown in Fig. 1. The exponent of 0.27 was selected as a typical value after several studies [17,37–39]. It is clear that the simple power law, Eq. (6), fails to explain the equal-loudness relation at low levels. Equation (7) does not show good agreement with the data because the fitted line with this equation is almost straight above 10 phons. In contrast, the curves calculated by Eqs. (8), (9) and (10) well coincide with the experimental data.

To examine the goodness of fit quantitatively, residual sums of squares (RSSs) were calculated for each equation for the exponent values from 0.20 to 0.30. Figure 3 shows the results. To calculate the RSS, residuals were first normalized by the standard deviation of each datum, squared and then summed, where the residual was the difference along the horizontal axis between an experimental datum and an estimated value. The residual for the threshold of hearing was not included in the RSS for each equation. The results show that the goodness of fit for Eqs. (8), (9) and (10) is much better than that for Eqs. (6) and (7)

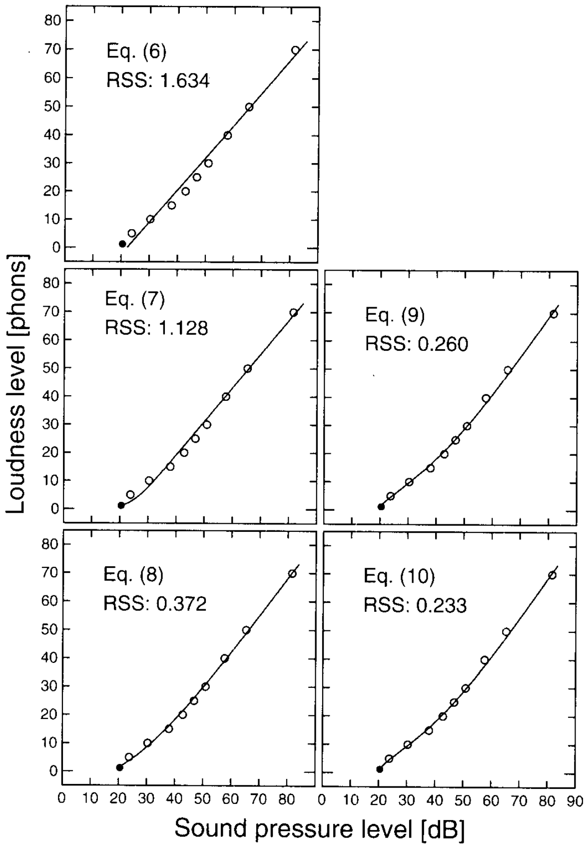

Fig. 2 Comparison between the experimental data and the model functions fitted to the experimental data shown in Fig. 1 by the method of nonlinear least squares. These curves were the results in the case of the exponent at 1 kHz (α_r) of 0.27. RSS means the residual sum of squares.

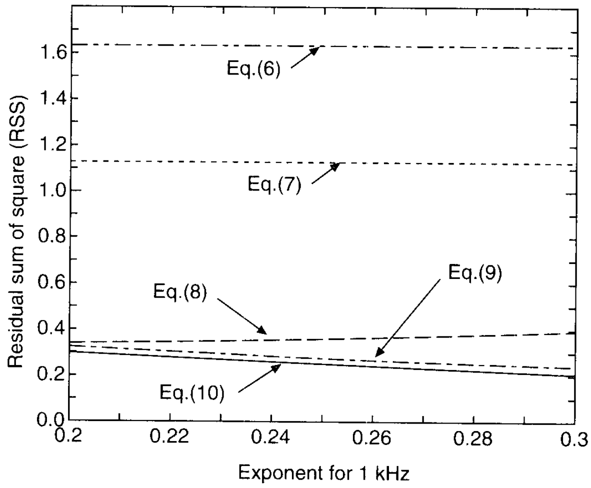

Fig. 3 Residual sum of squares (RSS) for each equation. The residual for the threshold of hearing was not included in the RSS for each equation.

independently of the exponent at 1 kHz.

Statistical tests were carried out to confirm the significance of the difference between the RSSs. Because RSS corresponds to variance, the hypothesis of equal variance was tested using the F distribution. The results showed that the differences between RSSs of Eqs. (6) and (8), (6) and (9), (6) and (10), (7) and (9), and (7) and (10) are significant beyond 0.05 (one-tailed test) within the exponents from 0.20 to 0.30.

4. DISCUSSION

Five model equations for equal-loudness relations derived from five kinds of loudness functions were fitted to the equal-loudness levels of the 125-Hz pure tone. The results show that Eqs. (8), (9) and (10) may be candidates for appropriate functions to describe the equal-loudness relation across frequencies. We thus examine which equation is the most suitable for the present purpose.

Available data of the equal-loudness levels subsequent to the research of Robinson and Dadson [1] have the following profiles: Most equal-loudness levels were measured at loudness levels up to 100 phons with a step-size of 10 phons at frequencies of the 1/3 octave series [42]. However, as not all of the studies examined all frequencies nor all loudness levels with 10-phon steps, only seven to nine loudness levels can be used in the fitting process of a loudness function for each frequency between 50 Hz and 12.5 kHz; for other frequencies, fewer levels can be used. Therefore, the smaller the number of parameters is the more advantageous to realize the stable estimation. In Eq. (8), the parameters to be estimated are K_f, α_f and P_{ft}, while in Eqs. (9) and (10), an additional parameter, C_f, must be estimated. Therefore, Eq. (8) would be better than Eqs. (9) and (10). Moreover, as stated in section 3, in Eq. (9), P_f is not equal to P_{ft} when P_r is at the hearing threshold, P_{rt}, unlike in the other equations. This means that loudness at the hearing threshold is not constant across frequencies [33,34], and thus the data of the hearing threshold cannot be used in the curve fitting process. In contrast, threshold data can be used in Eqs. (8) and (10) as one datum of the equal-loudness relation for the point of $S=0$. Because the dependency of the loudness at the hearing threshold on frequency is extremely small [33,34], we consider that the approximation that the threshold curve is one of the equal-loudness-level contours is rather beneficial. Consequently, Eq. (3), which is the origin of Eq. (8), would be the most appropriate loudness function for actually drawing new equal-loudness-level contours with presently available data.

New equal-loudness-level contours must be determined over the full range of audibility. Therefore, it is necessary to discuss whether Eq. (8) may be applied to frequencies other than 125 Hz. Whittle *et al.* [23] compared three loudness functions, equivalent to Eqs. (2), (3) and (4), with the data at 12.5 Hz and reported that Eq. (3), which is the origin of Eq. (8), could be fitted to the experimental data the best among the three. Moreover, they confirmed that this function well fitted to the experimental data from 3.15 Hz to 50 Hz. Hellman *et al.* [26] measured equal-loudness relations from 1 kHz to 16 kHz. The results show that equal-loudness relations from 1 kHz to 10 kHz were almost on a straight line with a slope of 1.0, meaning that they can be expressed with any of the equations from Eqs. (6) to (10). Consequently, we consider that Eq. (8) can be used for drawing equal-loudness-level contours over a wide frequency range from below 20 Hz to 10 kHz at least.

We should emphasize, however, that in principle, Eq. (9) must be the best model, at present, since it shows a smaller error than Eqs. (6) to (8). Moreover, it is known that the original equation, Eq. (4), is capable of expressing loudness of sound partially masked by noise [19] and that only this equation can explain the experimental results that the loudness at the hearing threshold is not null and is dependent on frequency [25,33,34].

5. CONCLUSION

In this study, we examined which of the five kinds of loudness functions reported in the literatures is most suitable for drawing the equal-loudness-level contours from available experimental data. First, we precisely measured the equal-loudness relations between 125-Hz and 1-kHz pure tones from 70 phons down to 5 phons. Five model equations expressing the equal-loudness relation were then derived from the five loudness functions, and these equations were fitted to the data. The results of the goodness of fit for these functions showed that three of the five equations could well explain the equal-loudness relation down to 5 phons. Among these, the authors concluded that the function proposed by Zwislocki and Hellman [16] as well as by Lochner and Burger [17] was the most appropriate for the present purpose because the number of parameters was fewer than that of the other functions and the threshold of hearing could be used for the fitting as a datum of the equal-loudness relation, resulting in a more stable estimation of the parameters of the loudness function with available data.

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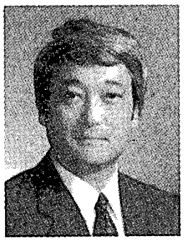

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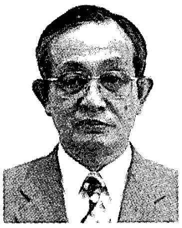

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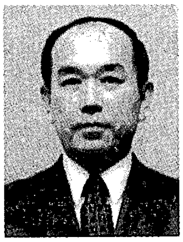

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